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Baryons with Double Charm

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We use potential models and several versions of the bag model to calculate the mass spectrum of baryons with two charmed quarks surrounded by an ordinary or strange quark. Such a system is conveniently described in the Born-Oppenheimer approximation, where the light quark dynamics provides an effective potential in which the two heavy quarks move non-relativistically. We survey briefly the possibilities of producing such heavy baryons and discuss their decay properties.

§ 1. Introduction

The discovery of the J/Ψ in November 1974 has opened a new era in hadron spectroscopy.[1] The Charmonium and Upsilon families provide unique information on the interquark potential and Zweig-forbidden strong decays.[2] Particles with naked heavy flavour, like D, B or Λ_c,[3] offer an interesting situation where the light quarks acquire high velocities in the field of a static heavy quark,[4] whereas their weak decays are carefully scrutinized to test the standard model and the subtle interplay of strong interactions with electroweak processes.[5]

New type of hadrons containing heavy quarks are also impatiently awaited for. The (ccc) baryons will provide the first clean baryon spectrum with several narrow levels above the ground state.[6] The possibility of stable multiquarks, like the tetraquark $(QQ\bar{q}\bar{q})$[7] or the pentaquark $(\bar{Q}qqqq)$,[8] has also been studied.

In the mass range of 3–4 GeV, one should also find baryons with double charm, of quark content (ccq), where q denotes u, d or s. They combine the dynamics found in the D meson, with a fast moving light quark surrounding a static colour $\bar{3}$ core, and the dynamics of Charmonium, with two heavy quarks experiencing the short range QCD potential. It is thus rather interesting to confront in this sector various quark models which describe successfully the hadrons which are already known experimentally.

This paper is organized as follows. Section 2 is devoted to potential models. We recall some rigorous inequalities and also compare the exact solution of the three-body problem to the results obtained in the quark-diquark or in the Born-Oppenheimer approximation. The latter turns out to be extremely well suited for this problem. We end this section by the numerical predictions based on realistic potential models. In § 3, we show how to use the bag model for those particular baryons, and underline the sensitivity to the choice of parameters and to the strategy used to treat the centre-of-mass motion and the zero-point energy. The properties of the mass spectrum and, in particular, the stability of the levels are discussed at the end

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of § 3. We comment briefly in § 4 on the possibilities of producing these baryons in electron or hadron machines. We also examine the weak decays of double charm baryons in the light of the studies performed recently for mesons and baryons with single charm.

§ 2. Potential models

2.1. *Rigorous results*

Let us assume here that the (QQq) system is bound by a Hamiltonian

$$H=\frac{\boldsymbol{p}_1^2}{2M}+\frac{\boldsymbol{p}_2^2}{2M}+\frac{\boldsymbol{p}_3^2}{2m}+V(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)\,. \tag{1}$$

Under reasonable assumptions,[9] one may expect that the corresponding bound state energy $E(M, M, m)$ will satisfy the convexity relation

$$E(M, M, m)\leq 2E(M, m, m)-E(m, m, m)\,. \tag{2}$$

This gives, with appropriate spin averaging of experimental masses

$$\mathcal{M}(ccu)\leq 2(2.4\text{ GeV})-1.1\text{ GeV}=3.7\text{ GeV}\,. \tag{3}$$

If, furthermore, one accepts the so-called "1/2 rule"[10] relating the interquark potential in baryons to the quarkonium potential v via

$$V(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)=\frac{1}{2}\sum_{i<j}v(r_{ij})\,, \tag{4}$$

one gets:[11]

$$\mathcal{M}(ccu)\geq\frac{1}{2}\mathcal{M}(c\bar{c})+\mathcal{M}(c\bar{u})\approx 3.51\text{ GeV}\,. \tag{5}$$

More precisely, if one takes spin forces into account,[12]

$$\begin{aligned}&\mathcal{M}\left(ccu, \frac{2^+}{3}\right)\geq\frac{1}{2}\mathcal{M}(J/\Psi)+\mathcal{M}(D^*)\approx 3.56\text{ GeV}\,,\\ &\mathcal{M}\left(ccu, \frac{1^+}{2}\right)\geq\frac{1}{2}\mathcal{M}(J/\Psi)+\frac{1}{4}\mathcal{M}(D^*)+\frac{3}{4}\mathcal{M}(D)\approx 3.45\text{ GeV}\,.\end{aligned} \tag{6}$$

It is very remarkable that, from the above inequalities, one predicts almost unambiguously the mass of the ground state (ccu) as:

$$\mathcal{M}\approx 3.6+0.1\text{ GeV}\,. \tag{7}$$

Some generalizations involving excited states are possible. We refer to the papers quoted in Refs. 9)~12).

2.2. *Solving the three-body problem*

a) The reduced hamiltonian

The Hamiltonian (1) is greatly simplified by introducing Jacobi variables

$$\boldsymbol{R}=\frac{M(\boldsymbol{r}_1+\boldsymbol{r}_2)+m\boldsymbol{r}_3}{2M+m},$$

$$\boldsymbol{x}=\boldsymbol{r}_2-\boldsymbol{r}_1,$$

$$\boldsymbol{y}=(2\boldsymbol{r}_3-\boldsymbol{r}_1-\boldsymbol{r}_2)\sqrt{\frac{m}{2M+m}}. \tag{8}$$

Since the potential is invariant under translation, the centre-of-mass motion can be factorized in the wave-function, and one deals with the reduced hamiltonian

$$h=\frac{p_x^2}{M}+\frac{p_y^2}{M}+v(\boldsymbol{x},\boldsymbol{y}). \tag{9}$$

The details of the resolution of the three-body Schrödinger equation $h\varphi=E\varphi$ are given elsewhere.[13] Let us present briefly some possible methods.

b) Variational methods

One possibility consists of expanding the wave-function in terms of the eigenstates ψ of a symmetric harmonic oscillator (h.o.)

$$h_0=\frac{p_x^2}{M}+\frac{p_y^2}{M}+\frac{1}{2}K(\boldsymbol{x}^2+\boldsymbol{y}^2) \tag{10}$$

with K being adjusted to optimized the convergence of

$$\varphi(\boldsymbol{x},\boldsymbol{y})=\sum_{N,l_1,l_2,\cdots} C_{N,l_1,l_2,\cdots}\psi(N,l_1,l_2,\cdots;\boldsymbol{x},\boldsymbol{y}), \tag{11}$$

where the summation contains obvious restrictions due to parity and angular momentum conservation. The convergence as a function of the total number N of quanta in (11), is shown in Table I for a typical choice

$$V=\frac{1}{2}\sum_{i,j} r_{ij}^{0.1} \tag{12}$$

with $M=1.0$ and $m=0.2$, in arbitrary units.

The use of a symmetric h.o. allows one to make use of the powerful machinery of

Table I. Convergence of the harmonic oscillator expansion for the potential (12), for $m_1=m_2=1$ and $m_3=0.2$. The results concern the ground-state, whose energy and correlation coefficients $\delta_{ij}=\langle\delta(r_{ij})\rangle$ are displayed as a function of the maximum number of quanta N allowed in the expansion (11).

N	E	$\delta_{12}(\times 10^3)$	$\delta_{13}(\times 10^3)$
0	1.9502	0.9781	0.1882
2	1.9485	0.7495	0.2187
4	1.9459	0.8891	0.2495
6	1.9457	0.8739	0.2623
8	1.9454	0.9179	0.2736

Table II. Convergence of the gaussian expansion (14) for the ground-state of (qqq') bound by potential of Eq. (12). The notation is the following: $2S+1D$, for instance, means that 2 terms with $l=0$ and 1 term with $l=2$ have been introduced in the expansion of the wave-function.

	E	$\delta_{12}(\times 10^3)$	$\delta_{13}(\times 10^3)$
$1S$	1.9480	0.7100	0.2304
$2S$	1.9460	0.8078	0.2791
$3S$	1.9459	0.8284	0.2867
$1S+1D$	1.9479	0.7066	0.2384
$2S+1D$	1.9460	0.8079	0.2810

the Talmi-Brody-Moshinsky[14] coefficients to compute the matrix elements of $v(\boldsymbol{x}, \boldsymbol{y})$. However, the expansion (11) does not easily reproduce the propetry $\langle|\boldsymbol{x}|\rangle \ll \langle|\boldsymbol{y}|\rangle$ of the wave-function and, as seen in Table I, the convergence is rather slow.

A variant consists of gaussians with different range in the $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates. We restrict the discussion here to the ground state but the generalization to other levels is straightforward.[15] The first approximation consists of

$$\varphi^{(1)}(\boldsymbol{x}, \boldsymbol{y})=\sum_{\alpha,\beta} C_{\alpha,\beta}\exp\left(-\frac{1}{2}(\alpha \boldsymbol{x}^2+\beta \boldsymbol{y}^2)\right) \tag{13}$$

and then, the possibility of internal orbital excitations can be introduced

$$\varphi^{(2)}(\boldsymbol{x}, \boldsymbol{y})=\sum_{\alpha,\beta,l=0,2,\cdots} C_{\alpha,\beta,l}(xy)^l \exp -\frac{1}{2}(\alpha \boldsymbol{x}^2+\beta \boldsymbol{y}^2)[Y_l^m(\hat{x})Y_l^{m'}(\hat{y})]_0\,, \tag{14}$$

where the brackets denote the appropriate Clebsch-Gordan couplings. The results[15] are shown in Table II. Clearly this method which is empirical but flexible enough to adjust itself to the dissymmetry of the (ccq) baryon, turns out to be more efficient than the "brute force" (but eventually convergent) h.o. expansion (11).

The hyperspherical formalism[16] provides another variational method. The Jacobi variables $\boldsymbol{x}$ and $\boldsymbol{y}$ are re-expressed in terms of 6-dimensional spherical coordinates, consisting of an hyperradius $r=(\boldsymbol{x}^2+\boldsymbol{y}^2)^{1/2}$ and 5 angles $\Omega=\{\hat{x}, \hat{y}, tg^{-1}(x/y)\}$. The wave-function is thus expanded into generalized partial-waves

$$\psi(\boldsymbol{x}, \boldsymbol{y})=r^{-5/2}\sum_{[K]} u_{[K]}(r)P_{[K]}(\Omega)\,. \tag{15}$$

This results in an infinite set of coupled radial equations

$$\left[\frac{d^2}{dr^2}-\frac{(K+3/2)(K+5/2)}{r^2}+E-V_{[K][K]}(r)\right]u_{[K]}(r)=\sum_{[K']\neq[K]} V_{[K][K']}(r)u_{[K']}(r)\,, \tag{16}$$

where

$$V_{[K][K']}(r)=\int d\Omega P^*_{[K]}(\Omega)\,V(r,\Omega)P_{[K']}(\Omega)\,. \tag{17}$$

In the above equations, $[K]$ denotes the *grand* orbital momentum K as well as the associated magnetic numbers. In practice, a very good convergence is already achieved with only a few coupled partial-waves, as illustrated in Table III. Of course, the convergence is slower for the wave-function than for the binding energy.

Table III. Convergence of the hyperspherical expansion (15) for the potential of Eq. (12). The ground-state energy and correlation coefficients are shown as a function of the maximal grand orbital K. The case $K=8$ corresponds to 9 coupled radial equations.

K	E	$\delta_{12}(\times 10^3)$	$\delta_{13}(\times 10^3)$
0	1.9481	1.189	0.2289
2	1.9463	0.8866	0.2706
4	1.9453	0.9972	0.2932
6	1.9452	0.9853	0.2980
8	1.9452	0.9983	0.3018

One may mention that the h.o. expansion (11) is nothing but the hyperspherical expansion (14) with the $u(r)$'s taken as polynomials multiplied by a gaussian, the degree of the polynomial being consistently adjusted to the maximum number of quanta.

c) The quark-diquark approximation

There are many reasons for attempting a description of baryons as consisting of a quark and a diquark.[17] It has recently[18] been investigated to what extent the conventional three-quark picture induces a dynamical diquark clustering leading to a quasi two-body structure of baryons.

In the sector of interest here, it is undeniable that the ground state of (ccq) consists of a localized (cc) cluster surrounded by the light quark q, with the average distance $\langle r_{QQ}\rangle$ much smaller than $\langle r_{Qq}\rangle$. However, when radial or orbital excitation is involved in the baryon, it turns out to be much more economical to promote the relative motion between the heavy quarks rather than the motion of the light quark around them. As a consequence, the average separation between the heavy quarks increases, and the quark-diquark structure disappears. More importantly, a spectrum computed with the (QQ) diquark frozen would not approximate the true excitation spectrum.

d) The Born-Oppenheimer approximation

In fact, a dramatic simplification of the (QQq) dynamics is obtained in the Born-Oppenheimer or adiabatic approximation. The two heavy quarks have much lower velocity than the light quark. When they move, the light quark wave-function readjusts itself almost immediately to the state of minimal energy. The computation can thus be done in two steps: For any given $\boldsymbol{x}$, one computes the binding energy $\varepsilon(\boldsymbol{x})$, which is then used as the effective potential governing the relative motion of the heavy quarks.

To be more specific, one expands the wave-function on a complete basis with respect to the $\boldsymbol{y}$ variable

$$\psi=\sum_n \phi_n(\boldsymbol{x})f_n(\boldsymbol{x},\boldsymbol{y})\,, \tag{18}$$

where, for any given $\boldsymbol{x}$, the f_n are the eigenfunctions of

$$h_y f_n(\boldsymbol{x},\boldsymbol{y})=\left(\frac{\boldsymbol{p}_y{}^2}{2M}+v_{13}+v_{23}\right)f_n(\boldsymbol{x},\boldsymbol{y})=\varepsilon(\boldsymbol{x})f_n(\boldsymbol{x},\boldsymbol{y})\,. \tag{19}$$

Solving Eq. (19), which is a one-body problem in a non-central potential, can be done by partial-wave expansion or by using elliptic coordinates. Keeping only the first term in Eq. (18) results into the variational approximation (sometimes referred to as the *uncoupled adiabatic*[19]).

$$\left[-\frac{\Delta_x}{M}+v_{12}(\boldsymbol{x})+\varepsilon(\boldsymbol{x})-\left\langle f_0\left|\frac{\Delta_x}{M}\right|f_0\right\rangle\right]\phi_0(\boldsymbol{x})=E\phi_0(\boldsymbol{x})\,. \tag{20}$$

Now, if one neglects the last term in the effective potential, which is a part of the kinetic energy, one gets the simplest form of the approximation (sometimes referred to as the *extreme adiabatic*[19])

$$\left[-\frac{\Delta_x}{M}+v_{12}(\boldsymbol{x})+\varepsilon(\boldsymbol{x})\right]\phi_0(\boldsymbol{x})=E\phi_0(\boldsymbol{x}) \tag{21}$$

Table IV (a). Ground-state energy for (qqq') calculated from potential (12) with two versions of the Born-Oppenheimer approximation and compared to the exact result. The constituent masses are $m_1=m_2=m$ and $m_3=m'$.

m	m'	extreme adiabatic	uncoupled adiabatic	exact
1	0.2	1.9450	1.9453	1.9452
1	0.5	1.9037	1.9045	1.9042
1	1	1.8794	1.8810	1.8802

Table IV (b). Masses and properties of the double charm baryons calculated either exactly or in the Born-Oppenheimer approximation. We use the potential of Eqs. (22) and (23), and, for the ground-state, the hyperfine correction of Eq. (24).

		Mass ground-state	$\langle r_{12}^2\rangle^{1/2}$ ground-state	$\langle v_{12}^2\rangle^{1/2}$ ground-state	$M_{L=0 \atop n=1}$	$M_{L=1 \atop n=0}$
ccq	adia	3.6840	2.3303	0.1197	4.1092	3.9689
	exact	3.6848	2.3256	0.1337	4.1096	3.9712
ssq	adia	1.4272	3.7968	0.4478	1.9408	1.7669
	exact	1.4300	3.7808	0.5601	1.9333	1.7718
qqq	adia	1.0804	5.1084	0.9885	1.6489	1.4560
	exact	1.0856	5.0766	1.3430	1.6154	1.4636

which overestimates the binding, i.e., is antivariational (to obtain Eq. (21) from the initial hamiltonian, one gives up a positive definite piece of the kinetic energy operator).

For illustration, we use in Table IV the power law potential (12). In Table IV (a) a detailed comparison of the binding energies of the ground state in various approximations is presented, whereas (b) exhibits, in the extreme adiabatic approximation, some observables for this ground state as well as the masses of the first radial and orbital excitations. In (b), we use by anticipation the scale factors which will be presented in the next paragraph.

What is really astonishing is that the Born-Oppenheimer treatment works with such a high accuracy for (ccq) and also quite well for the Ξ baryons (ssq) and even for the ordinary baryons. This illustrates once more the well-known theorem[20] in few-body physics: *The Born-Oppenheimer approximation works always better than expected.*

This adiabatic treatment, which provides a nice simplification in non-relativistic posential models, will be very useful in bag models, allowing for a relativistic treatment of the light quark motion followed by a non-relativistic approximation for the heavy quarks. This will be done in § 3.

An interesting limit[21] consists of $m/M\to 0$, as already considered in Ref. 8) for different purposes. The contribution $\varepsilon(\boldsymbol{x})$ to the effective potential runs over a range in $\boldsymbol{x}$ which is small compared to the average separation $\langle \boldsymbol{x}\rangle$ between the flavoured core and the light quark q. The spectrum is essentially given by the direct interaction $v_{12}(\boldsymbol{x})$ between the heavy quarks, shifted by a constant $\varepsilon(0)$.

2.3. *Predictions of potential models*

A smooth potential $V(r)\propto r^{0.1}$ was introduced by Martin[22] to describe quarkonia like $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ or $c\bar{s}$ and also used in the baryon sector.[23] We adopt here the set of parameters used in Ref. 23) to fit the existing ground state baryons. They are:

$$V=\frac{1}{2}\sum_{i<j}A+Br_{ij}^{\beta} \tag{22}$$

with

$$\beta=0.1\,,\quad A=-8.337\,\mathrm{GeV}\,,\quad B=6.9923\,\mathrm{GeV}^{1+\beta}\,,\quad C=2.572\,\mathrm{GeV}^2\,,$$
$$m_q=0.300\,\mathrm{GeV}\,,\quad m_s=0.600\,\mathrm{GeV}\,,\quad m_c=1.895\,\mathrm{GeV}\,,\quad m_b=5.255\,\mathrm{GeV}\,. \tag{23}$$

The central potential, which is nothing but a rescaled version of the potential (12), is supplemented by standard hyperfine corrections, treated to first order:

$$V_{ss}=\frac{1}{2}\sum_{i<j}\frac{C}{m_i m_j}\delta(r_{ij})\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j \tag{24}$$

with $C=2.572$.

For comparison, we use also a potential model proposed by Badhuri et al.,[24] in an attempt (not fully successful) to reproduce all known hadrons, light and heavy. It reads

$$V=\frac{1}{2}\sum_{i<j}\left[\frac{-K}{r_{ij}}+\frac{r_{ij}}{a^2}-D+\frac{K_\sigma}{m_i m_j}\frac{e^{-r_{ij}/r_0}}{r_{ij}r_0^2}\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\right], \tag{25}$$

$$K=K_\sigma=-0.5203\,,\quad 1/a^2=0.1857\,\mathrm{GeV}^2\,,\quad D=-0.9135\,\mathrm{GeV}\,,\quad r_0^{-1}=0.4341\,\mathrm{GeV}\,,$$
$$m_q=0.337\,\mathrm{GeV}\,;\quad m_s=0.600\,\mathrm{GeV}\,;\ m_c=1.870\,\mathrm{GeV}\,;\ m_b=5.259\,\mathrm{GeV}\,. \tag{26}$$

The results for ground-states of (ccq) and (ccs) configurations are displayed in Table V. Some differences between the two models might be noted. The potential (25) with a smeared spin-spin interaction gives much smaller hyperfine splittings. Already, in the meson sector, the potential underestimates the J/Ψ-η_c splitting and thus fails in fitting accurately the heavy quark sector. On the other hand, both potentials give comparable radial and orbital excitation energies, which are shown in

Table V. Masses of ground-state baryons with double charm, as calculated from the power-law potential (22), supplemented by a contact term (24) for the hyperfine corrections, and from the potential (25) with a Coulomb-plus-linear central term and a spin-spin term with Yukawa shape. Masses are in GeV.

Potential		power-law		Coulomb
		centre-of-mass	+hyp. corr.	+Linear
ccq	Ξ_{cc}	3.685	3.613	3.635
	Ξ_{cc}^*		3.741	3.717
ccs	Ω_{cc}	3.776	3.703	3.740
	Ω_{cc}^*		3.835	3.802

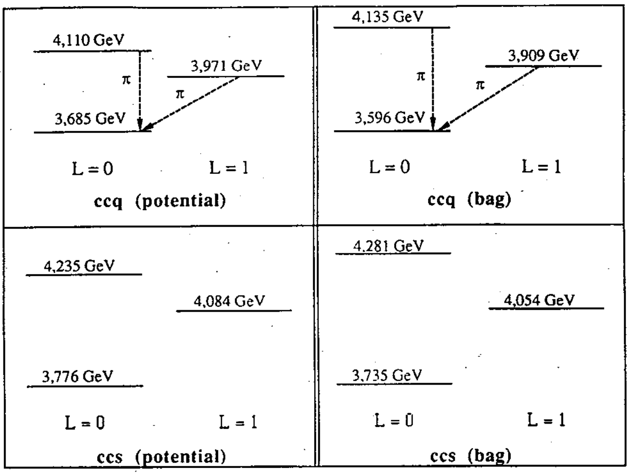

Fig. 1. Radial and orbital excitation spectrum of (ccq) and (ccs) baryons in a potential and in a bag model. Units are GeV and GeV^{-1}.

Fig. 1. The discussion on the spectrum is postponed to the end of the following section, where an alternative calculation is proposed, based on the bag model.

§ 3. Bag models

3.1. *Mini-review on bags*

The bag model[25] is a rather popular tool designed to describe the confinement of quarks and gluons inside hadrons. In the first phenomenological applications, devoted to the ground-state of light mesons and baryons,[26] the M.I.T. group used the approximation of a rigid cavity with spherical shape. Many variants have been proposed, to improve or to complicate the bag picture, with centre-of-mass corrections, gluonic corrections, partial restoration of the chiral symmetry, etc.[27]

Meanwhile, the bag model was adapted to heavy quarks systems by Kuti et al.[28] For each interquark separation, the bag parameters (size, shape) are adjusted to minimize the bag energy, which is used as a Born-Oppenheimer potential to compute the mass spectrum and wave-functions of the heavy hadrons such as $(Q\bar{Q})$ or (QQQ). The quarkonium potential exhibits the standard Coulomb behaviour at short distances and at large Q-Q separation, a linear rise. This latter effect is associated with an elongated bag. However, for most quarkonium calculations, the spherical approximation turns out to be quite accurate.[28],[29]

The so-called heavy-light or open-flavour sector was treated by Izatt et al.[30] In their approach, $(Q\bar{q})$ or (Qqq) systems are described by fixed spherical bags with the heavy quark Q fixed at the centre and the light quarks moving relativistically up to the border. More precisely, the mass M_0 is computed as the minimum with respect to the bag radius R of

$$M(R)=\sum_i \omega_i-\frac{Z_0}{R}+\frac{4\pi}{3}BR^3\,, \tag{27}$$

where $\omega_i=M$ for the heavy quark, and $\omega_i=\sqrt{\chi_i^2+(m_iR)^2}/R$ for the light quarks. As in the original MIT bag,[26] χ_i is determined by a boundary condition linking the upper and lower components of the free Dirac wave-function, leading for instance to $\chi_0 = 2.042$ for massless quarks.

Chromoelectric and chromomagnetic corrections are then applied to M_0. In Ref. 30), the coupling constant α_s was chosen to depend on the bag radius R, to imitate the effect of a "running" coupling constant.

3.2. *Bag model for double charm*

For (QQq) baryons, we have designed a model which combines the methods used for quarkonia and for naked flavours, and is reminiscent of the Born-Oppenheimer approximation to non-relativistic hamiltonians.

For any given QQ separation x, we compute the minimal bag energy. We restrict consideration to spherical bags, centred at the middle of the two heavy quarks, so that the expression of the bag energy is similar to Eq. (27). We learned from the charmonium case[28),29)] that the spherical approximation is adequate for the small heavy quark separations x involved in the lowest states of the spectrum.

As usual,[26),30)] the minimal bag energy is computed in the approximation where the light quark is free. Chromoelectric and chromomagnetic corrections to $E_0(x)$, due to the Qq interactions, are computed to first order.

Finally, the direct central potential between the heavy quarks is added, resulting in an effective $V_{\text{eff}}(x)$ which is used to compute the energy spectrum and the QQ relative wave-function. The ultimate correction accounts for the hyperfine repulsion between the heavy quarks, when they are in a relative s-wave.

For the numerical calculations, we selected three sets of parameters. The set (28) corresponds to the charmonium calculation of Hasenfratz et al.,[29)] and is well-suited for heavy quark systems. The light quark masses have been chosen empirically.

$$\begin{array}{lll} & \alpha_s=0.385\,, & m_c=1.35\,\text{GeV}\,, \\ (\text{Set }1) & B^{1/4}=0.235\,, & m_s=0.279\,\text{GeV}\,, \\ & Z_0=0\,, & m_q=0\,\text{GeV}\,. \end{array} \tag{28}$$

The second set corresponds to the original MIT bag, which reproduces the lightest mesons and baryons in the fixed cavity approximation. It is supplemented by an ad-hoc value of m_c, which gives a correct mass for the ground state of (cc), in the adiabatic approximation.

$$\begin{array}{lll} & \alpha_s=0.55\,, & m_c=1.55\,\text{GeV}\,, \\ (\text{Set }2) & B^{1/4}=0.145\,, & m_s=0.279\,\text{GeV}\,, \\ & Z_0=1.81\,, & m_0=0\,\text{GeV}\,. \end{array} \tag{29}$$

The last set of parameters is taken from the fit by Izatt et al. of ordinary and naked flavour hadrons[30)] (for ordinary hadrons, some centre-of-mass corrections are applied in Ref. 30))

Table VI. Comparison of ground-state baryons with double charm, using different set of parameters for the bag model; Set 1 given by Eq. (28), Set 2 given by Eq. (29) and Set 3 given by Eq. (30).

ccq		Set 1	Set 2	Set 3
$L=0$	$n=1$	3.596	3.249	2.895
	$n=2$	4.135	3.788	3.639
$L=1$	$n=1$	3.909	3.722	3.525
	$n=2$	4.422	4.034	3.943
$L=2$	$n=1$	4.183	3.939	3.829
	$n=2$	4.695	4.208	4.186

ccs		Set 1	Set 2	Set 3
$L=0$	$n=1$	3.735	3.377	2.978
	$n=2$	4.281	3.932	3.743
$L=1$	$n=1$	4.054	3.862	3.621
	$n=2$	4.571	4.184	4.056
$L=2$	$n=1$	4.332	4.087	3.937
	$n=2$	4.846	4.363	4.308

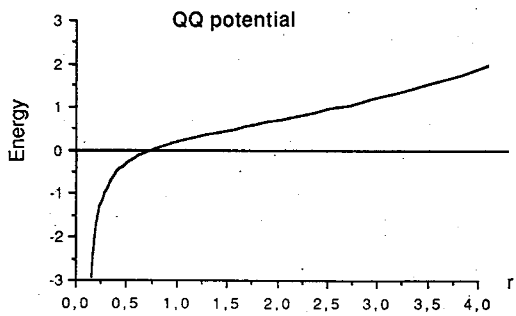

Fig. 2. Effective QQ potential for the (QQq) baryons in the bag model.

$$\text{(Set 3)}\quad \begin{aligned} &\alpha_s=\frac{2\pi}{9\ln(1+1/(\Lambda R))}, && m_c=2.004\ \text{GeV}, \\ &\Lambda=0.4199, && m_s=0.273\ \text{GeV}, \\ &B^{1/4}=0.1383, && \\ &Z_0=0.574, && m_q=0\ \text{GeV}. \end{aligned} \tag{30}$$

The comparison is shown in Table VI. Dramatic differences appear between the various calculations. First the ground state (ccq) lies much higher in models where m_c is fitted to the J/Ψ than in model (30) where m_c is adjusted from charmed hadrons D and Λ_c. The spacings obtained in model (30) are also systematically larger.

Unlike the naive potential model, which exhibits an unexpected extrapolating power, the bag model does not lead to very safe predictions. First, different approximations (sometimes rather drastic) are applied in the various sectors; rigid cavity for ordinary hadron, spherical shape, recoilless heavy quarks in the naked flavour sector, Born-Oppenheimer approximation for quarkonia, etc. This results in a renormalization of the parameters which differs from one sector to another. Also, with various improvements to the nucleon picture such as centre-of-mass corrections, zero point energy, gluonic corrections, running α_s, etc., the bag model involves now many parameters, whose phenomenological determination is ambiguous. As a consequence, the extrapolation towards other hadrons cannot be performed reliably.

For the rest of the calculation, we restrict ourselves to the set 1, which corresponds to the minimal number of parameters, and is fitted to the quarkonium spectrum.[29] The effective (QQ) potential is exhibited in Fig. 2. The spectrum, with or

Table VII. Bag model calculation (with parameter set P1) of s-wave (ccq) and (ccs) baryons. The results are obtained without chromomagnetic interaction (column 1), with chromomagnetic corrections to the light quark (column 2), and finally, with spin-spin repulsion between the heavy quark (column 3). Masses are in GeV.

	State L, n, S	M_0	M_0 +hyp. corr. on q	M_0 +hyp. corr. on q +hyp. corr. on QQ
ccq	$0, 0, \frac{3}{2}$	3.596	3.636	3.667
	$0, 0, \frac{1}{2}$		3.516	3.547
	$0, 1, \frac{3}{2}$	4.135	4.166	4.201
	$0, 1, \frac{1}{2}$		4.073	4.108
ccs	$0, 0, \frac{3}{2}$	3.735	3.774	3.805
	$0, 0, \frac{1}{2}$		3.657	3.689
	$0, 1, \frac{3}{2}$	4.281	4.311	4.347
	$0, 1, \frac{1}{2}$		4.221	4.257

Table VIII. Masses of ground-state baryons with double charm, calculated either with a potential model (Eq. (25)) or with the bag model, using the parameter given in Eq. (28).

		Potential	Bag
ccq	Ξ_{cc}	3.613	3.516
	Ξ_{cc}^*	3.741	3.636
ccs	Ω_{cc}	3.703	3.657
	Ω_{cc}^*	3.835	3.805

without hyperfine corrections, is given in Table VII.

3.3. *Discussion on double charm spectroscopy*

We restrict discussion to the two models we consider as the most representative: the potential model of Eqs. (22)~(24) and the bag model with the parameters (28). The ground state masses are summarized in Table VIII. The first spin average levels are displayed in Fig. 1. The following comments are in order:

i) The spin excitation Ξ_{cc}^* is stable against pionic decay $\Xi_{cc}^* \to \Xi_{cc} + \pi$. It should then decay radiatively to the ground state. The same is true for Ω_{cc}^*.

ii) The orbital and radial excitation of (ccq) are broad, since pionic transitions to the ground state are allowed.

iii) The radial excitation of $\Omega_{cc}(ccs)$ is also unstable, since the transition $\Omega_{cc}^* \to \Xi_{cc} + K$ is allowed. However, the orbital excitation should be quite narrow, since it can only decay by emitting a photon or through the isospin violating $\Omega_{cc}^{**} \to \Omega_{cc} + \pi$ reaction.

§ 4. Production and decay

4.1. *Remarks on production of double charm*

Charmed particles have been produced by various means:[3] e^+e^- collision, hadronic beams, neutrino production, decay of excited charmonia, weak decay of beauty, etc. It would be beyond the scope of the present paper to review the corresponding mechanisms and to compute the production rates. We shall restrict discussion to some common sense remarks concerning baryons with double charm.

i) Hyperon beam experiments[31),32)] have been successful in producing charmed-strange baryons.[3),23),33)] The next step in difficulty, beside beauty, will consist of two units of charm.

ii) (csq) configurations have also been identified using neutron beams,[34)] π^--beams[35)] and in e^+e^- collisions.[36)]

iii) In J/Ψ decay, baryonic modes $(B\overline{B})$ are not very rare. Also, hadronization does not distinguish between the different flavours, as seen in the approximate equality of rates[3)] (the phase-space corrections are minor here)

$$\Gamma_{J/\Psi}(p\bar{p}) \cong \Gamma_{J/\Psi}(n\bar{n}) \cong \Gamma_{J/\Psi}(\Lambda\bar{\Lambda}) \cong \Gamma_{J/\Psi}(\Xi\bar{\Xi})\,. \tag{31}$$

It is thus reasonable to expect a decent rate for Υ decay into $\Xi_{cc}\bar{\Xi}_{cc}$ or $\Omega_{cc}\bar{\Omega}_{cc}$.

iv) In fact, production rates of charm or double charm should be high enough with present day energies and intensities. The background should also be very small. The difficulties lies in the multiplicities associated with two cascades where charm decays into strangeness and strangeness, in turn, into ordinary hadrons. The reconstruction of double charm events seems a little delicate with present detectors and reconstruction programs. In view of the recent progress in the field, this should however become standard with the next generation of heavy flavour experiments.

v) The decay $\Lambda_b \to \Xi_{cc} + \bar{D}$ seems energetically possible since one expects[12)] $m(\Lambda_b) \cong 5.6\,\mathrm{GeV}$. Possible mechanisms are depicted in Fig. 3.

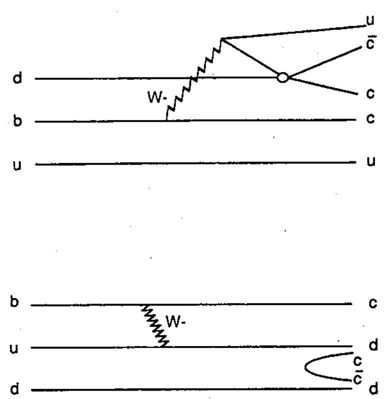

Fig. 3. Possible mechanisms for Λ_b decay into double charm baryon.

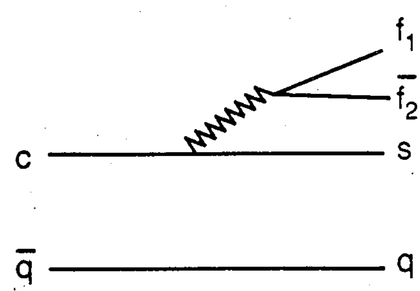

Fig. 4. Spectator diagram for charm decay.

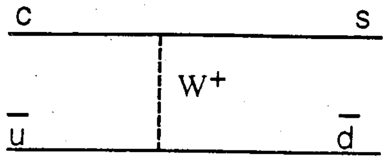

Fig. 5. W-exchange diagram.

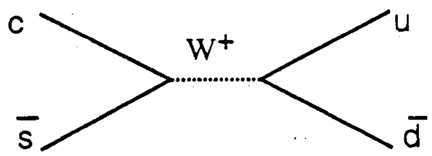

Fig. 6. Annihilation diagram.

4.2. *Decay of double charm baryons*

Charm decay is a rather hot topic. When it was announced that charged D and neutral D do not have the same lifetime,[3] many scenarios were proposed for explaining that the charmed quark does not decay independently of its environment. The starting point is the spectator diagram of Fig. 4, where $f_1\bar{f}_2$ denotes the standard fermion-antifermion pair in which the virtual W decays; $u\bar{d}$, $u\bar{s}$, $e^+\nu$, etc.

Strong interactions might now play a role. If, for instance, $\bar{f}_2=\bar{q}$ in Fig. 4 the Pauli principle applies so that interference effects are expected. Interferences alone act mostly on the hadronic decay of the D^+ (the mode $\bar{f}_2=\bar{s}$ is Cabbibo-suppressed, so the effect on the D_s is negligible), and lead to $\Gamma(D^0)\cong\Gamma(D_s)$, which is observed and also to a leptonic branching ratio $R_L=\Gamma(D\to e^+X)/\Gamma(D)$ normal for both D^0 and D_s and anomalous for D^+, in contradiction with experiment.[3] So, other effects have to contribute.

First, the D^0 can benefit from W-exchange, as depicted in Fig. 5. This effect, by itself, would produce

i) $\tau(D^0)<\tau(D^+)\cong\tau(D_s)$,

ii) $R_L(D^0)<R_L(D^+)\cong R_L(D_s)\cong 20\%$.

Annihilation diagrams (see Fig. 6) can also occur, mostly for the D_s, since they are Cabbibo-suppressed for the D^+.

Finally, there are Penguin diagrams, but they are not very important for charm and they affect equally all types of charmed particles.

The sector of single charm baryons allows further tests of the decay mechanisms. For instance, W-exchange diagrams contribute more for baryons, since the helicity suppression does not hold as in mesons. An analysis of charmed baryon decay has been carried out by Guberina et al.[37] Their conclusion is that

$$\tau(\Omega_c{}^0)\cong\tau(\Xi_c{}^0)<\tau(\Lambda_c{}^+)<\tau(\Xi_c{}^+)\,. \tag{32}$$

To obtain the above hierarchy, the effects accounted for are in order:

i) Interference effects of the s-quark obtained from c-decay with existing s-quark. This affects the $\Omega_c{}^0$ and, to a lesser extent, the Ξ_c's.

ii) W-exchange contributes to Λ_c and $\Xi_c{}^0$.

iii) There are some interference effects between existing quarks and quarks produced by the decay of the virtual W. This affects $\Lambda_c{}^+$ and $\Xi_c{}^+$.

The analysis of Ref. 37) is slightly model-dependent. For instance in the Λ_c lifetime, there is a cancellation between W-exchange and interference effects and the result depends on the wave-function and other hadronic parameters.

The extension to double charm is in fact much safer.

i) W-exchange gives a sizable contribution to Ξ_{cc}^{+} decay.

ii) Positive interference will occur between the s-quark resulting from c-decay and the exsting s-quark in Ω_{cc}^{+}.

iii) The Ξ_{cc}^{++} decays only via the spectator diagram.

Our prediction is thus:

$$\tau(\Xi_{cc}^{+})<\tau(\Omega_{cc}^{+})<\tau(\Xi_{cc}^{++}). \tag{33}$$

§ 5. Conclusion

Double charm baryons will certainly illuminate many aspects of quark physics. In spectroscopy, we have made quantitative predictions, based on the observation that in current models, the light quark dynamics decouples itself from the relative motion of the heavy quarks. A departure would indicate a more intimate connection between these interactions, for instance three-body forces not reducible to effective two-body terms.

The debate is even more open concerning the weak decays of those baryons. Our discussion on the comparison of the (ccu), (ccd) and (ccs) lifetimes and leptonic branching ratio remains rather qualitative. Detailed calculations of specific modes like $\Xi_{cc}^{++}\to\Lambda_c+K^*$ would help in testing our understanding of hadronization.

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