Basic Concepts of Formal Ontology

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Abstract:
The term 'formal ontology' was first used by the philosopher Edmund Husserl in his *Logical Investigations* to signify the study of those formal structures and relations – above all relations of part and whole – which are exemplified in the subject-matters of the different material sciences. We follow Husserl in presenting the basic concepts of formal ontology as falling into three groups: the theory of part and whole, the theory of dependence, and the theory of boundary, continuity and contact. These basic concepts are presented in relation to the problem of providing an account of the formal ontology of the mesoscopic realm of everyday experience, and specifically of providing an account of the concept of individual substance.

1. Introduction: The Formal Ontology of Edmund Husserl

We owe the idea of a formal ontology to the philosopher Edmund Husserl, whose *Logical Investigations* (1900/01) draws a distinction between *formal logic*, on the one hand, and *formal ontology,* on the other. Formal logic deals with the interconnections of truths (or of propositional meanings in general) – with inference relations, with consistency and validity. Formal ontology deals with the interconnections of things, with objects and properties, parts and wholes, relations and collectives. As formal logic deals with inference relations which are formal in the sense that they apply to inferences in virtue of their form alone, so formal ontology deals with structures and relations which are formal in the sense that they are exemplified, in principle, by all matters, or in other words by objects in all material spheres or domains of reality.

Husserl's formal ontology is based on mereology, on the theory of dependence, and on topology. The title of his third Logical Investigation is "On the Theory of Wholes and Parts" and it divides into two chapters: "The Difference between Independent and Dependent Objects" and "Thoughts Towards a Theory of the Pure Forms of Wholes and Parts". Unlike more familiar 'extensional' theories of wholes and parts, such as those propounded by Lesniewski, and by Leonard and Goodman (see Simons 1987), Husserl's theory does not concern itself merely with what we might think of as the vertical relations between parts and the wholes which comprehend them on successive levels of comprehensiveness. Rather, Husserl's theory is concerned also with the horizontal relations between co-existing parts, relations which serve to give unity or integrity to the wholes in question. To put the matter simply: some parts of a whole exist merely side by side, they can be destroyed or removed from the whole without detriment to the residue. A whole all of whose parts manifest exclusively such side-by-sideness relations with each other is called a heap or aggregate or, more technically, a purely summative whole. In many wholes, however, and one might say in *all* wholes manifesting any kind of unity, certain parts stand to each other in formal relations of what Husserl called *necessary dependence* (which is sometimes, but not always, necessary *interdependence*). Such parts, for example the individual instances of hue, saturation and brightness involved in a given instance of colour, cannot, as a matter of necessity, exist, except in association with their complementary parts in a whole of the given type. There is a huge variety of such lateral dependence relations, giving rise to correspondingly huge variety of different types of whole which more standard approaches of extensional mereology are unable to distinguish.

The topological background of Husserl's work makes itself felt already in his theory of dependence. (Fine 1995) It comes most clearly to the fore however in his treatment of the notion of *phenomenal fusion*: the relation which holds between two adjacent parts of an extended totality when there is no qualitative discontinuity between the two. (Casati 1991, Petitot 1994) Adjacent squares on a chess-board array are not fused together in this sense; but if we imagine a band of colour that is subject to a gradual transition from red through orange to yellow, then each region of this band is fused with its immediately adjacent regions. After distinguishing dependent and independent contents (for example, in the visual field, between a colour- or brightness-content on the one hand, and a content corresponding to the image of a moving projectile, on the other), Husserl goes on to note that there is in the field of intuitive data an additional distinction,

> between *intuitively* separated contents, contents *set in relief from* or *separated off from* adjoining contents, on the one hand, and contents which are *fused* with adjoining contents, or which *flow over* into them without separation, on the other (Investigation III, §8, 449).

He points out that independent contents

> which are what they are no matter what goes on in their neighbourhood, need not have this quite different independence of separateness. The parts of an intuitive surface of a uniform or continuously shaded white are independent, but not separated. *(Loc. cit.)*

Such content Husserl calls 'fused'; they form an 'undifferentiated whole' in the sense that the moments of the one pass 'continuously' ['*stetig*'] into corresponding moments of the other. (§9, 450)

That Husserl was at least implicitly aware of the topological aspect of his ideas, even if not under this name, is unsurprising given that he was a student of the mathematician Weierstrass in Berlin, and that it was Cantor, Husserl's friend and colleague in Halle during the period when the *Logical*

Investigations were being written, who first defined the fundamental topological notions of open, closed, dense, perfect set, boundary of a set, accumulation point, and so on. Husserl consciously employed Cantor's topological ideas, not least in his writings on the general theory of (extensive and intensive) magnitudes which make up one preliminary stage on the road to the third Investigation. (See Husserl 1983, pp. 83f, 95, 413; 1900/01, "Prolegomena", §§ 22 and 70.)

2 Mereology vs Set Theory

When modern-day philosophers turn their attentions to ontology, they standardly begin not with mereology or topology but with set-theoretic tools of the sort that are employed in standard model-theoretic semantics.

The rationale for insisting on a mereological rather than a set-theoretic foundation for the purposes of formal ontology can be stated as follows. Imagine that we are seeking a theory of the boundary-continuum structure as this makes itself manifest in the realm of everyday human experience. The standard set-theoretic account of the continuum, initiated by Cantor and Dedekind and contained in all standard textbooks of the theory of sets, will be inadequate to this purpose for at least the following reasons:

1. The application of set theory to a subject-matter presupposes the isolation of some basic level of urelements in such a way as to make possible a simulation of all structures appearing on higher levels by means of sets of successively higher types. If, however, as holds in the case of investigations of the ontology of the experienced world, we are dealing with mesoscopic entities and with their mesoscopic constituents (the latter the products of more or less arbitrary real or imagined divisions along a variety of distinct axes), then there are no urelements fit to serve as our starting-point. (Bochman 1990) Approaches based on mereology, in contrast, allow one to start not with atoms but with the mesoscopic objects by which we are surrounded in our normal day-to-day activities. The mereologist sees reality as being made up not of atoms, on the one

hand, and abstract (1- and n-place) ‘properties’ or ‘attributes’ or ‘worlds’, on the other, but rather of you and me, of your headaches and my sneezes, of your battles and my wars, or in other words of individuals of different sorts linked together by means of various different sorts of relations, including mereological and topological relations and relations of ontological dependence.

2. The experienced continuum is in every case a concrete, changing phenomenon, a phenomenon existing in time, a whole which can gain and lose parts. Sets, in contrast, are abstract entities, entities defined entirely via the specification of their members. Certainly, in order to do justice to the changing relations between parts and wholes in the realm of mesoscopic objects, we can conceive of a theory analogous to the theory of sets which would be constructed on the basis of a tensed or time-indexed membership relation. To this end, however, we would need to sacrifice extensionality and other features which had made the theory of sets attractive as a tool of ontology in the first place.

3. In the absence of points or elements, the experienced continuum does not sustain the sorts of cardinal number constructions imposed by the Dedekindian approach. The experienced continuum is not isomorphic to any real-number structure; indeed standard mathematical oppositions, such as that between a dense and a continuous series, here find no application.

4. Even if points or elements were capable of being isolated in the experienced continuum, the set-theoretical construction would still be predicated on the highly questionable thesis that out of unextended building blocks an extended whole can somehow be constructed. (Brentano 1988, Asenjo 1993, Smith 1987.) The experienced continuum, in contrast, is organized not in such a way that it would be built up out of particles or atoms, but rather in such a way that the wholes, including the medium of space, come before the parts which these wholes might contain and which might be distinguished on various levels within them.

Of course, set theory is a mathematical theory of tremendous power, and none of the above precludes the possibility of reconstructing topological and other theories adequate for ontological purposes also on a set-theoretic basis. Standard representation theorems indeed imply that for any precisely formulated topological theory formulated in non-set-theoretic terms we can find an isomorphic set-theoretic counterpart. Even so, however, the reservations stated above imply that the resultant set-theoretic framework could yield at best a *model* of the experienced continuum and similar structures, not a theory of these structures themselves (for the latter are after all *not sets,* in light of the categorial distinction mentioned under 2. above).

Our suggestion, then, is that mereotopology will yield more interesting research hypotheses, and in a more direct and straightforward fashion, than would be the case if formal ontologists were constrained to work with set-theoretic instruments.

3 The Ontology of Substance and Accident

3.1 Substances and Accidents

Mesoscopic reality is divided, at its natural joints, into what, following Aristotle, we shall call ‘substances’. Examples of substances are: animals (including human beings), logs of wood, rocks, potatoes, forks. Substances have various properties (qualities, features, attributes) and they undergo various sorts of changes (processes, events), for all of which we shall employ, again following Aristotle, the term ‘accident’. Examples of accidents are: whistles, blushes, speakings, runnings, my knowledge of French, the whiteness of this cheese, the warmth of this stone.

Accidents, like substances, are concrete, individual denizens of reality. My headache, like my lump of cheese, exists here and now, and both will cease to exist at some time in the future. Substances and accidents are nonetheless radically different in their ontological makeup. Substances are that which can exist on their own, where accidents require a support from substances in order to exist. Substances are the *bearers* or *carriers* of accidents, and accidents are said to 'inhere' in their substances; these relations between substance and accident will be defined more precisely in what follows in terms of the concept of *specific dependence*.

Substances are such that, while remaining numerically one and the same, they can admit contrary accidents at different times: I am sometimes hungry, sometimes not; sometimes suntanned, sometimes not. Substances thus endure through time. Accidents, in contrast, occur in time, and they have temporal parts, for example the first 5 minutes of my headache, the first three games of the match (entities which have no counterpart in the realm of substances: the first 5 minute phase of my existence is not a part of me but of that complex accident which is my life). Substances and accidents form two distinct orders of being. The former *endure* self-identically through time; the latter *occur:* they unfold themselves through time, and are never present in full at any given instant during which they exist.

3.2 Substances, Collectives, Relations

Substances are unities. They enjoy a certain natural completeness or rounded-offness, being neither too small nor too large – in contrast to the undetached parts of substances (my arms, my legs) and to heaps or aggregates and to complexes or collectives of substances such as armies or football teams. A substance has a complete, determinate boundary (the latter is a special sort of part of a substance; something like a maximally thin extremal or peripheral slice). A substance takes up space. It is an 'extended spatial magnitude' which occupies a place and is such as to have spatial parts. It is not merely spatially extended, but also (unlike other spatially extended objects such as places and spatial regions) such as to have divisible bulk, which means that it can in principle be divided into separate spatially extended substances. The material bulkiness of substances implies also that, unlike shadows and holes, substances compete for space, so that no two substances can occupy the same spatial region at the same time.

Substances are often joined together into more or less complex collectives, ranging from families and tribes to nations and empires. Collectives are real constituents of the furniture of the world, but they are not additional constituents, over and above the substances which are their parts. Collectives inherit some, but not all, of the ontological marks of substances. They can admit contrary accidents at different times. They have a certain unity. They take up space, and they can in principle be divided into separate spatially extended sub-collectives, as an orchestra, for example, may be divided into constituent chamber groups. Collectives may gain and lose members, and they may undergo other sorts of changes through time.

Accidents, too, may form collectives (a musical chord, for example, is a collective of individual tones). In the realm of accidents, however, we can draw another sort of distinction that offers a parallel to the distinction between collectives and individual substances. This is the distinction between relational accidents on the one hand and non-relational (or one-place) accidents on the other. Non-relational accidents are attached, as it were, to a single carrier, as a thought is attached to a thinker. Accidents are relational if they depend upon a plurality of carriers and thereby join the latter together into complex wholes of greater or lesser duration. Examples of relational accidents include a kiss, a hit, a dance, a conversation, a contract, a battle, a war. Note, again, that relational accidents, like accidents in general, are not abstract entities. All of the examples mentioned are denizens of reality which are no less individual than the substances which serve as their relata.

4 The Theory of Dependence

4.1 Mereology

The term 'object' in what follows will be used with absolute generality to embrace all substances, accidents, and all the wholes and parts thereof, including boundaries. Our basic ontological categories will be defined in terms of the primitive notion: *is part of* (for the theory of mereology) and *is necessarily such that* (for the theory of dependence). '*x is part of y*', which we shall symbolize by means of '$x \leq y$', is to be understood as including the limit case where x and y are identical. We shall use '$x \cup y$' to signify the mereological sum of two objects x and y. If we define overlap as the sharing of common parts:

DM1 $\mathrm{O}(x, y) := \exists z(z \leq x \wedge z \leq y)$ *overlap*

then the axioms for standard (non-tensed) mereology can be formulated as follows (Simons 1987):

AM1 $x \leq x$
AM2 $x \leq y \wedge y \leq x \rightarrow x = y$
AM3 $x \leq y \wedge y \leq z \rightarrow x \leq z$
AM4 $\forall z(z \leq x \rightarrow \mathrm{O}(z, y)) \rightarrow x \leq y$
AM5 $\exists x(\phi x) \rightarrow \exists y \forall z\, (\mathrm{O}(y, z) \leftrightarrow \exists x\, (\phi x \wedge \mathrm{O}(x, z)))$.

(Here and in the sequel initial universal quantifiers are to be taken as understood.)

Parthood is a reflexive, antisymmetric, and transitive relation, a partial ordering. In addition, AM4 ensures that parthood is extensional, whereas the schema AM5 guarantees that for every satisfied property or condition f (i.e., every condition f that yields the value true for at least one argument) there exists an entity, the sum or fusion, consisting precisely of all the fers. This entity will be denoted by '$\sigma x(\phi x)$' and is defined as follows:

DM2 $\sigma x(\phi x) := \iota y \forall z\, (\mathrm{O}(y, z) \leftrightarrow \exists x\, (\phi x \wedge \mathrm{O}(x, z)))$.

With the help of this operator, other useful notions are easily defined:

DM3 $x \cup y := \sigma z\, (z \leq x \vee z \leq y)$ *sum*
DM4 $x \cap y := \sigma z\, (z \leq x\ \&\ z \leq y)$ *product*

DM5 $x - y := \sigma z\,(z \leq x \;\&\; \neg O(z, y))$ *difference*
DM6 $-x := \sigma z\,(O(z, x))$ *complement*

4.2 Specific Dependence

As we have seen, substances and accidents may be compounded together mereologically to form larger wholes of different sorts. But substances and accidents are not themselves related mereologically: a substance is not a whole made up of accidents as parts. Rather, the two are linked together via the formal tie of specific dependence, which is defined as follows:

DD1 $SD(x, y) := \neg O(x, y) \wedge \Box_x(E!x \rightarrow E!y)$ *specific dependence*

where '$\Box_x$' signifies the *de re* necessity operator '*x* is necessarily such that' and 'E!' is the predicate of existence defined in the usual way in terms of the existential quantifier: $E!x := \exists y(x = y)$.

We can now define mutual and one-sided specific dependence, in the obvious way, as follows:

DD2 $MSD(x, y) := SD(x, y) \wedge SD(y, x)$
DD3 $OSD(x, y) := SD(x, y) \wedge \neg MSD(x, y)$

My headache, for example, is one-sidedly specifically dependent on me. Accidents in general stand to the substances which are their carriers in the formal tie of one-sided specific dependence only. Cases where objects are bound together via ties of *mutual* specific dependence would include, for example the relation between the north and south poles of a magnet or the relation between hue, saturation and brightness mentioned above. Equally, there are cases where an object stands in a relation of specific dependence to more than one object. These are cases of what we referred to above as relational accidents – kisses and hits – which are dependent simultaneously on a plurality of substances.

4.3 Separability

A further formal tie, in some respects the converse of that of specific dependence, is the relation of separability. We define:

DD4 $\mathrm{MS}(x, y, z) := x \leq z \wedge y \leq z \wedge \neg \mathrm{O}(x, y) \wedge \neg \Box_x(\exists w(w \leq y) \wedge \neg \Box_y(\exists w(w \leq x)$

mutual separability

x and y are non-overlapping parts of z and x is not necessarily such that any part of y exists and y is not necessarily such that any part of x exists.

z is, for example, a pair of stones, and x and y the stones themselves.

Separability, too, may be one-sided:

DD5 $\mathrm{OS}(x, y) := x \leq y \wedge x \neq y \wedge \exists w(w \leq y \wedge \neg \mathrm{O}(w,x) \wedge \mathrm{SD}(w,x)) \wedge \neg \exists w(w \leq y \wedge \neg \mathrm{O}(w,x) \wedge \mathrm{SD}(x,w))$

one-sidedly separability

(1) x is a proper part of y, and (2) some part of y discrete from x is specifically dependent on x, and (3) x is not specifically dependent on any part of y discrete from x.

x is for example a human being and y is the sum of x together with some one of x's thoughts.

5 Topology

5.1 The Concept of Transformation

Standard introductions to the basic concepts of topology take as their starting point the notion of *transformation.* We can transform a spatial body such as a sheet of rubber in various ways which do not involve cutting or tearing. We can invert it, stretch or compress it, move it, bend it, twist it, or otherwise knead it out of shape. Certain properties of the body will in general be invariant under such transformations – which is to say under transformations which are neutral as to shape, size, motion and orientation. The transformations in question can be defined also as being those which do not affect the possibility of our connecting two points on the surface or in the interior of the body by means of a continuous line. Let us use the term 'topological spatial properties' to refer to those spatial properties of bodies which are invariant under such transformations (broadly: transformations which do not affect the *integrity* of the body – or other sort of spatial structure – with which we begin). Topological spatial properties will then in general fail to be invariant under more radical transformations, not only those which involve cutting or tearing, but also those which involve the gluing together of parts, or the drilling of holes through a body.

The property of being a (single, connected) body is a topological spatial property, as also are certain properties relating to the possession of holes (more specifically: properties relating to the possession of tunnels and internal cavities). The property of being a *collection* of bodies and that of being an *undetached part* of a body, too, are topological spatial properties. It is a topological spatial property of a pack of playing cards that it consists of this or that number of *separate* cards, and it is a topological spatial property of my arm that it is *connected* to my body.

This concept of topological property can of course be generalized beyond the spatial case. The class of phenomena structured by topological spatial properties is indeed wider than the class of phenomena to which, for example, Euclidean geometry, with its determinate Euclidean metric, can be applied. Thus topological spatial properties are possessed also by mental images of spatially extended bodies. Topological properties are discernible also in the temporal realm: they are those properties of temporal structures which are invariant under transformations of (for example) stretching (slowing down, speeding up) and temporal translocation. Intervals of time, melodies, simple and complex events, actions and processes can be seen to possess topological properties in this temporal sense. The motion of a bouncing ball can be said to be topologically isomorphic to another, slower or faster, motion of, for example, a trout in a lake or a child on a pogo-stick.

5.2 The Concept of Boundary

5.3 The Concept of Closure

The two approaches briefly sketched above may be unified into a single system by means of the notion of *closure*, which we can think of as an operation of such a sort that, when applied to an entity *x* it results in a whole which comprehends both *x* and its boundaries. We employ as basis of our definition of closure the notions of mereology. Some of the reasons why we shun the set-theoretical instruments employed in standard presentations of the foundations of topology will be set out below.

An operation of *closure* (c) is defined in such a way as to satisfy the following axioms:

AC1 $x \leq \mathrm{c}(x)$ *expansiveness*
(each object is a part of its closure)

AC2 $\mathrm{c}(\mathrm{c}(x)) \leq \mathrm{c}(x)$ *idempotence*
(the closure of the closure adds nothing to the closure of an object)

AC3 $\mathrm{c}(x \cup y) = \mathrm{c}(x) \cup \mathrm{c}(y)$ *additivity*
(the closure of the sum of two objects is equal to the sum of their closures)

These axioms define a well-known kind of structure, that of a *closure algebra*, which is the algebraic equivalent of the simplest kind of topological space. (See Kuratowski 1922.)

5.4 The Concept of Connectedness

On the basis of the notion of closure we can now define the standard topological notion of (symmetrical) *boundary*, b(x), as follows:

DB2 $b(x) := c(x) \cap c(-x)$ *boundary*

Note that it is a trivial consequence of the definition of boundary here supplied that the boundary of an entity is in every case also the boundary of the complement of that entity.

It is indeed possible to define in standard topological terms an asymmetrical notion of 'border', as the intersection of an object with the closure of its complement:

DB3 $b^*(x) = x \cap c(-x)$ *border*

In fact, where Kuratowski's axioms were formulated in terms of the single topological primitive of closure, Zarycki showed (1927) that a set of axioms equivalent to those of Kuratowski can be formulated also in terms of the single primitive notion of border, and the same applies, too, in regard to the notions of interior and boundary.

The notion of interior is defined as follows:

DB4 $i(x) := x - b(x)$ (interior)

We may define a *closed object* as an object which *is identical with its closure.* An *open object,* similarly, is an object which is identical with its interior. The complement of a closed object is thus open, that of an open object closed. Some objects will be partly open and partly closed. (Consider for example the semi-open interval (0,1], which consists of all real numbers *x* which are greater than 0 and less than or equal to 1.) These notions can be used to relate the two approaches to topology distinguished above: topological *transformations* are those transformations which map open objects onto open objects.

A closed object is, intuitively, an independent constituent – it is an object which exists on its own, without the need for any other object which would serve as its host. But a closed object need not be *connected* in the sense that we can proceed from any one point in the object to any other and remain within the confines of the object itself.

The notion of *connectedness*, too, is a topological notion, which we can define as follows:

DCn1 $\mathrm{Cn}(x) := \forall yz(x = y \cup z \rightarrow \exists w(w \leq (\mathrm{c}(y) \cap \mathrm{c}(z))))$ *connectedness*

(a connected object is such that all ways of splitting the object into two parts yield parts whose closures overlap)

The following yields an alternative concept of connectedness which is useful for certain purposes:

DCn2 $\mathrm{Cn}^*(x) := \forall yz(x = y \cup \mathrm{z} \rightarrow (\exists w(w \leq x \wedge \mathrm{w} \leq \mathrm{c}(y)) \vee \exists w(w \leq \mathrm{c}(x) \wedge w \leq y)))$ *connectedness**

(a connected object is such that, given any way of splitting the object into two parts *x* and *y,* either *x* overlaps with the closure of *y* or *y* overlaps with the closure of *x*)

Neither of these notions is quite satisfactory however. Thus examination reveals that a whole made up of two adjacent spheres which are momentarily in contact with each other – if such is possible – will satisfy either condition of connectedness as thus defined. For certain purposes, therefore, it may be useful to operate in terms of a notion of *strong connectedness* which rules out cases such as this. This latter notion may be defined as follows:

DCn3 $\mathrm{Scn}(x) := \mathrm{Cn}^*(\mathrm{i}(x))$ *strong connectedness*

(an object is strongly connected if its interior is connected*).

On this basis we can now define a substance as a maximally strongly connected non-dependent entity:

DCn4 $S(x) := \mathrm{Scn}(x) \wedge \forall y(x \leq y \wedge \mathrm{Scn}(y) \rightarrow x = y) \wedge \neg \exists z \mathrm{SD}(x, z)$

This definition is still woefully incomplete. Thus in an ontology which admits spatial regions the constraint of maximal strong connectedness may be insufficient to do the job of separating substances from the spatial regions in which they are to be found. Moreover, there may be substances, for example a fetus inside a womb or a Siamese twin, which are not maximally connected but which yet would seem to rank as substances in virtue of an intrinsic causal integrity. Substances, too, are marked essentially by the fact that they can preserve their numerical identity even in spite of changes over time. For a full treatment, therefore, the present framework needs to be supplemented by an account of spatial location, of causal integrity, and of temporal identity.

5.5 Boundary-Dependence

One type of supplementation A full ontology of substance and of other spatial objects will require further the notion of boundary, a notion which we first of all introduce in terms of a new sort of dependence (first discussed by Brentano and Chisholm: see Smith 1997):

DB1 $\mathrm{BD}(x, y) := x \leq y \wedge \Box_x(\mathrm{E!}y \wedge \exists w(w \leq y \wedge x \leq w \wedge x \neq w) \wedge \forall z(z \leq x \rightarrow \Box_z(\mathrm{E!}y \wedge \exists w(w \leq y \wedge z \leq w \wedge z \neq w)$ *boundary-dependence*

(1) *x* is a proper part of *y*, and (2) *x* is necessarily such that either *y* exists or there exists some part of *y* properly including *x*, and (3) each part of *x* satisfies (2).

x is for example the surface of an apple and *y* the apple itself. Clause (2) is designed to capture the topological notion of neighborhood. Roughly, a boundary of given dimension can never exist alone but exists always only as part of some extended neighborhood of higher dimension. There are no points, lines or surfaces in the universe which are not the boundaries of three-dimensional material things. Boundaries are then just those objects which are boundary-dependent on some other object, (and ultimately on some substance).

The relation of boundary-dependence holds both between a boundary and the substance which it bounds and also among boundaries themselves. Thus zero-dimensional spatial boundaries (points) are boundary-dependent both on one- and two-dimensional boundaries (lines and surfaces) and also on the three-dimensional substances which are their ultimate hosts. Note that the relation of boundary-dependence does not hold between an accident and its substantial carrier. Certainly my current thought satisfies the condition that it cannot exist unless I or some suitably large part of me exists. And certainly each part of my current thought satisfies this condition also. But my current thought is also specifically dependent upon me, and thus, by the definition of specific dependence it is not a part of me.

Imagine a solid and homogeneous metal sphere. We can distinguish, with some intellectual effort, two parts of the sphere which do not overlap (they have no parts in common): on the one hand is its *boundary,* its *exterior surface*; on the other hand is its *interior*, the *difference* between the sphere and this exterior surface (that which would result if, *per impossibile,* the latter could be subtracted from the former). Similarly in the temporal realm we can imagine an interval as being composed of its initial and its final points together with the *interior* which results when these points are removed from the interval as a whole.

Define, now, the *complement* of an entity *x* as that entity which results when we imagine *x* as having been deleted from the universe as a whole. The boundary of an entity *x* is from the point of view of classical mathematical topology also the boundary of the complement of *x.* We can imagine, however, a variant topology which would recognize *asymmetric* boundaries, such as we find, for example, in the figure-ground structure as this is manifested in visual perception where the boundary of a figure is experienced as a part of the figure, and not simultaneously as boundary of the ground, which is experienced as running on behind the figure. Something similar applies also in the temporal sphere: the beginning and ending of a race, for example, are not in the same sense boundaries of any complement-entities (of all time prior to the race, and of all time subsequent to the race) as they are boundaries of the race itself.

References

Asenjo, F. G. 1993 "Continua without Sets*", Logic and Logical Philosophy*, 1, 95-128.

Bochman, Alexander 1990 "Mereology as a Theory of Part-Whole", *Logique et Analyse*, 129-130, 75-101.

Brentano, Franz 1988 *Philosophical Investigations on Space, Time and the Continuum*, translated by Barry Smith, London/New York/Sydney: Croom Helm.

Brentano, Franz 1988 *Philosophical Investigations on Space, Time and the Continuum*, ed. by R. M. Chisholm and S. Körner, Hamburg: Meiner, Eng. trans. by B. Smith, London: Croom Helm, 1988.

Chisholm, R. M. 1984 "Boundaries as Dependent Particulars", *Grazer Philosophische Studien*, 10, 87-95.

Casati, Roberto 1991 "Fusion", in H. Burkhardt and B. Smith (eds.), *Handbook of Metaphysics and Ontology*, Munich/Hamden/Vienna: Philosophia, 287-289.

Casati, Roberto and Varzi, Achille 1994 *Holes and Other Superficialities*, Cambridge, Mass. and London: MIT Press.

Fine, K. 1995 “Part-Whole”, in B. Smith and D.W. Smith (eds.), *The Cambridge Companion to Husserl*, Cambridge: Cambridge University Press, 463-485.

Guarino, N. 1995 “Formal Ontology, Conceptual Analysis and Knowledge Representation. *International Journal of Human and Computer Studies*, 43(5/6), 625-640.

Husserl, Edmund 1983 *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass, 1886-1901,* The Hague: Nijhoff (*Husserliana* XXI).

Husserl, Edmund 1900/01 *Logische Untersuchungen*, 1st ed., Halle: Niemeyer, 2nd ed., 1913/21 (both now available in a comparative edition as *Husserliana* XVIII-XIX, The Hague: Nijhoff, 1975, 1984). Page references unless otherwise indicated are to Findlay’s English translation of the 2nd ed. (*Logical Investigations*, London: Routledge, 1970), translation amended where necessary.

Kuratowski, Kazimierz 1922 “Sur l’opération A^- d’analysis situs”, *Fundamenta Mathematica*, 3, 182-99.

Petitot, Jean 1994 “Phenomenology of Perception, Qualitative Physics and Sheaf Mereology”, in R. Casati, B. Smith and G. White (eds.), *Philosophy and the Cognitive Sciences*, Vienna: Hölder-Pichler-Tempsky, 387-408.

Simons, Peter M. 1987 *Parts. A Study in Ontology*, Oxford: Clarendon Press.

Smith, Barry 1995 “On Drawing Lines on a Map”, in Andrew U. Frank und Werner Kuhn (Hrsg.), *Spatial Information Theory. A Theoretical Basis for GIS* (Lecture Notes in Computer Science 988), Berlin/Heidelberg/New York: Springer, 475-484.

Smith, Barry 1996 “Mereotopology: A Theory of Parts and Boundaries*”, Data and Knowledge Engineering*, 20, 287–303.

Smith, Barry 1997 “Boundaries: An Essay in Mereotopology”, in L. H. Hahn (ed.), *The Philosophy of Roderick Chisholm* (Library of Living Philosophers), Chicago and LaSalle: Open Court, 1997, 534–561.

Smith, Barry 1997 “On Substances, Accidents and Universals: In Defence of a Constituent Ontology”, *Philosophical Papers*, 26, 105–127.

Smith, Barry (ed.) 1988 *Foundations of Gestalt Theory*, Munich and Vienna: Philosophia.

Smith, Barry (ed.) 1982 *Parts and Moments. Studies in Logic and Formal Ontology*, Munich: Philosophia.

Smith, Barry and Varzi, Achille C. 1997 “Fiat and Bona Fide Boundaries: Towards on Ontology of Spatially Extended Objects”, *COSIT ’97: Conference on Spatial Information Theory* (Springer Lecture Notes), Berlin/Heidelberg/New York: Springer Verlag.

Varzi, Achille C. 1994 “On the Boundary between Mereology and Topology”, in R. Casati, B. Smith and G. White (eds.), *Philosophy and the Cognitive Sciences*, Vienna: Hölder-Pichler-Tempsky, 423–442.

Zarycki, Miron 1927 “Quelques notions fondamentales de l’Analysis Situs aux point du vue de l’Algèbre de la Logique”, *Fundamenta Mathematica*, 9, 3-15.