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Basic Notions of Algebra

With 45 Figures

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by M. Reid

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Preface

This book aims to present a general survey of algebra, of its basic notions and main branches. Now what language should we choose for this? In reply to the question 'What does mathematics study?', it is hardly acceptable to answer 'structures' or 'sets with specified relations'; for among the myriad conceivable structures or sets with specified relations, only a very small discrete subset is of real interest to mathematicians, and the whole point of the question is to understand the special value of this infinitesimal fraction dotted among the amorphous masses. In the same way, the meaning of a mathematical notion is by no means confined to its formal definition; in fact, it may be rather better expressed by a (generally fairly small) sample of the basic examples, which serve the mathematician as the motivation and the substantive definition, and at the same time as the real meaning of the notion.

Perhaps the same kind of difficulty arises if we attempt to characterise in terms of general properties any phenomenon which has any degree of individuality. For example, it doesn't make sense to give a definition of the Germans or the French; one can only describe their history or their way of life. In the same way, it's not possible to give a definition of an individual human being; one can only either give his 'passport data', or attempt to describe his appearance and character, and relate a number of typical events from his biography. This is the path we attempt to follow in this book, applied to algebra. Thus the book accommodates the axiomatic and logical development of the subject together with more descriptive material: a careful treatment of the key examples and of points of contact between algebra and other branches of mathematics and the natural sciences. The choice of material here is of course strongly influenced by the author's personal opinions and tastes.