Batch Arrival Queueing System

with Two Stages of Service

S. Maragathasundari

Sathyabama University, Chennai
&
Dept of maths, Velammal Institute of Technology, Chennai, India

S. Srinivasan

Dept of maths
B.S Abdur Rahman University, Chennai, India

A. Ranjitham

Dept of maths, Velammal Institute of Technology, Chennai, India

Abstract

This Paper studies batch arrival queue with two stages of service. Random breakdowns and Bernoulli schedule server vacations have been considered here .After a service completion, the server has the option to leave the system or to continue serving customers by staying in the system. .It is assumed that customers arrive to the system in batches of variable size, but served one by one. After completion of first stage of service, the server must provide the second stage of service to the customers. Vacation time follows general distribution, while we consider exponential distribution for repair time.We obtain steady state results in explicit and closed form in terms of the probability generating functions for the number of customers in the queue, average number of customers ,and the average waiting time in the queue.

1 Introduction

Queueing models with vacation plays a major role in manufacturing and production systems, computer and communication systems, service and distribution systems, etc. The studies on queues with batch arrival and vacations have been increased at present. Many real life situations of this model are mostly observed in supermarkets, factories and very large scale manufacturing industries.

Baba [1] has studied an $M^{[x]}$/G/1 queue with vacation time. Madan and Anabosi [8] have studied server vacations based on Bernoulli schedules and a single vacation policy. Madan and Choudhury [10] have studied a single server queue with two stage of heterogeneous service under Bernoulli schedule and a general vacation time. Thangaraj and Vanitha [17] have studied a single server M/G/1 feedback queue with two types of service having general distribution. Madan and Choudhury [9] proposed a queueing system with restricted admissibility of arriving batches. Igaki [4], Levi and Yechilai [6], Madan and Abu-Davyeh [13] have studied vacation queues with different vacation policies.

S.Maragathasundari and S.Srinivasan [14], they studied about analysis of M/G/1 feedback queue with three stage multiple server vacation. S.Maragathasundari and S.Srinivasan [15], they studied about analysis of triple stage of service having compulsory vacation and service interruptions. S.Maragathasundari and S.Srinivasan [16], they studied about three phase M/G/1 queue with Bernoulli feedback and multiple server vacation.

This paper is organized as follows. Model assumptions are given in section2.Notations are given in section 3.Steady state conditions have been obtained in section 4. Queue size distribution at random epoch is derived in section5.The average queue size and the average waiting time are computed in section 6.Conclusion is given in section7.

2. Model Assumptions

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a 'first come'-first served basis. Let $\lambda c_i (i = 1,2,3 ....)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t + dt)$.where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches.

b)There is a single server and the service time follows general(arbitrary)distribution with distribution function $G(s)$ and density function $g(s)$.Let $\mu(x)dx$ be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x,so that

$$\mu(x) = \frac{g(x)}{1 - G(x)} \tag{1}$$

and therefore

$$g(s) = \mu(s)e^{-\int_0^s \mu(x)dx} \tag{2}$$

c) As soon as a service is completed, then with probability p the server may take a vacation.

d) The server's vacation time follows a general(arbitrary) distribution with distribution function $B(v)$ and density function $b(v)$.Let $Y(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$,so that

$$Y(x) = \frac{b(x)}{1 - B(x)} \tag{3}$$

and, therefore

$$b(v) = Y(v)e^{-\int_0^v Y(x)dx} \tag{4}$$

e) The server may breakdown at random, and breakdowns are assumed to occur according to Poisson stream with mean breakdown rate $\alpha > 0$.Further we assume that once the system breaks down, the customer whose service is interrupted comes back to the head of the queue.

f) Once the system breaks down, it enters a repair process immediately. The repair times are exponentially distributed with mean repair rate $\beta > 0$.

g) Various stochastic process involved in the system are assumed to be independent of each other.

3. NOTATIONS

$P_n^{(1)}(x, t)$: Probability that at time t,the server is active providing service and there are $n(n \geq 0)$ customers in the queue excluding the one being served in the first stage and the elapsed service time for this customer is x.Consequently, $P_n^{(1)}(t) = \int_0^\infty P_n^{(1)}(x, t)\, dx$ denotes the probability that at time t there are n customers in the queue excluding the one customer in the first stage of service irrespective of the value of x.

$P_n^{(2)}(x, t)$: Probability that at time t,the server is active providing service and there are $n(n \geq 0)$ customers in the queue excluding being served in the second stage and the elapsed service time for this customer is x.Consequently, $P_n^{(2)}(t) = \int_0^\infty P_n^{(2)}(x, t)\, dx$ denotes the probability that at time t there are n customers in the queue excluding the customer in the second stage of service irrespective of the value of x.

$V_n(x,t)$:Probability that at time t,the server is on vacation with elapsed vacation time x and there are $n(n > 0)$ customers waiting in the queue for service. Consequently, $V_n(t) = \int_0^\infty V_n(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue and the server is on vacation irrespective of the value of x.

$R_n(t)$:Probability that at time t,the server is inactive due to system breadown and the system is under repair, while there are $n(n \geq 0)$ customers in the queue.

$Q(t)$:Probability that at time t,there are no customers in the system and the server is idle but available in the system.

4. STEADY STATE CONDITION

Let

$$\lim_{t\to\infty} P_n^i(x,t) = P_n^i(x), \lim_{t\to\infty} P_n(t) = \lim_{t\to\infty} \int_0^\infty P_n^i(x,t)dx = P_n^i$$

$$\lim_{t\to\infty} V_n(x,t) = V_n(x), \lim_{t\to\infty} V_n(t) = \lim_{t\to\infty} \int_0^\infty V_n(x,t)dx = V_n$$

$$\lim_{t\to\infty} R_n(x,t) = R_n$$

$$\lim_{t\to\infty} Q(t) = Q$$

denote the corresponding steady state probabilities.

Then, connecting states of the system at time $t + dt$ with those at time t and then takes limit as $t \to \infty$,we obtain the following set of steady state equations governing the system

$$\frac{d}{dx} P_n^{(1)}(x) + (\lambda + \mu_1(x) + \alpha)P_n^{(1)}(x) = \lambda \sum_{i=1}^{n-1} c_i\, P_{n-i}(x) \tag{5}$$

$$\frac{d}{dx} P_0^{(1)}(x) + (\lambda + \mu_1(x) + \alpha)P_0^{(1)}(x) = 0 \tag{6}$$

$$\frac{d}{dx} P_n^{(2)}(x) + (\lambda + \mu_2(x) + \alpha)P_n^{(2)}(x) = \lambda \sum_{i=1}^{n-1} c_i\, P_{n-i}(x) \tag{6a}$$

$$\frac{d}{dx} P_0^{(2)}(x) + (\lambda + \mu_2(x) + \alpha)P_0^{(2)}(x) = 0 \tag{6b}$$

$$(\lambda + \beta)R_n = \lambda \sum_{i=1}^{n-1} c_i\, R_{n-i} + \alpha \int_0^\infty P_{n-1}^{(1)}(x)\,dx + \alpha \int_0^\infty P_{n-1}^{(2)}(x)\,dx \tag{7}$$

$$(\lambda + \beta)R_0 = 0 \tag{8}$$

$$\frac{d}{dx} V_n(x) + (\lambda + \Upsilon(x))V_n(x) = \lambda \sum_{i=1}^{n-1} c_i\, V_{n-i}(x) \tag{8a}$$

$$\frac{d}{dx}V_0(x) + (\lambda + \Upsilon(x))V_0(x) = 0 \tag{8b}$$

$$\lambda Q = \beta R_0 + \int_0^\infty V_0(x)\Upsilon(x)dx + \int_0^\infty P_0^{(2)}(x)\,dx \tag{9}$$

$$P_n^{(1)}(0) = \int_0^\infty V_{n+1}(x)\Upsilon(x)dx + \int_0^\infty P_{n+1}^{(2)}(x)\mu_2(x)dx + \beta R_{n+1} + \lambda c_{n+1}Q \tag{10}$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx \tag{11}$$

$$V_n(0) = \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \tag{12}$$

5. Queue Size Distribution at a random Epoch

We define the following probability generating functions

$$\left.\begin{aligned}
P_q^{(i)}(x,z) &= \sum_{n=0}^\infty z^n P_n(x), \quad P_q^{(i)}(z) = \sum_{n=0}^\infty z^n P_n \quad i = 1,2 \\
V_q(x,z) &= \sum_{n=0}^\infty z^n V_n(x), \quad V_q(z) = \sum_{n=0}^\infty z^n P_n \\
R_q(z) &= \sum_{n=0}^\infty z^n R_n, \quad C(z) = \sum_{i=1}^\infty z^i c_i
\end{aligned}\right\} \tag{13}$$

Now, multiply equation (5) by z^n, and summing over n from $1\ to\ \infty$ we get, adding the result to equation (6), and performing the same in 6a and 6band using the generating functions,

$$\frac{d}{dx}P^{(1)}(x,z) + (\lambda - \lambda C(z) + \mu_1(x) + \alpha)P_q(x,z) = 0 \tag{14}$$

$$\frac{d}{dx}P^{(2)}(x,z) + (\lambda - \lambda C(z) + \mu_2(x) + \alpha)P_q(x,z) = 0 \tag{15}$$

Performing the similar operations to equations (7a), (7b), and (8a), (8b) we get

$$\frac{d}{dx}V(x,z) + (\lambda - \lambda C(z) + \Upsilon(x))V_q(x,z) = 0 \tag{16}$$

$$(\lambda - \lambda C(z) + \beta)R_q(z)$$
$$= \alpha z \int_0^\infty P_q(x,z)dx + \alpha z \int_0^\infty P_q^{(1)}(x,z)dx + \alpha z \int_0^\infty P_q^{(2)}(x,z)dx \tag{17}$$

For the boundary conditions, we multiply equations (10) & (11) by z^{n+1}, sum over n from $0\ to\ \infty$,and use the generating functions defined in (13), we get

$$zP^{(1)}(0,z) = \left(\int_0^\infty V(x,z)Y(x)dx + \beta R(z) + \lambda C(z)Q + \int_0^\infty P^{(2)}(x,z)\mu_2(x)dx \right)$$
$$- \left(\beta R_0 - \int_0^\infty V_0(x,z)Y(x)dx - \int_0^\infty P_0^{(2)}(x,z)\mu(x)dx \right)$$

By using equation (9) we get,

$$zP^{(1)}(0,z) = \int_0^\infty V(x,z)Y(x)dx + \beta R(z) + \int_0^\infty P^{(2)}(x,z)\mu_2(x)dx$$
$$+ (\lambda C(z) - 1)Q \qquad\qquad (18a)$$
$$P^{(2)}(0,z) = \int_0^\infty P^{(1)}(x,z)\mu_1(x)dx \qquad\qquad (18b)$$

Similarly, we multiply equation (12) by z^n and sum over n from $0\ to\ \infty$ and use the generating functions defined in (13)

$$V(0,z) = \int_0^\infty P^{(2)}(x,z)\mu_2(x)dx \qquad\qquad (19)$$

Integrating equations (14) and (15) from $0\ to\ x$ yields

$$P^{(1)}(x,z) = P^{(1)}(0,z)e^{-[\lambda-\lambda z+\alpha]x-\int_0^x \mu_1(t)dt} \qquad\qquad (20a)$$
$$P^{(2)}(x,z) = P^{(2)}(0,z)e^{-[\lambda-\lambda z+\alpha]x-\int_0^x \mu_2(t)dt} \qquad\qquad (20b)$$

where $P^{(1)}(0,z)\ and\ P^{(2)}(0,z)$ are given by (18a) and (18b).Again integrating equation (20a), (20b) by parts with respect to x yields

$$P^{(1)}(z) = P^{(1)}(0,z)\left[\frac{1 - \bar{G}_1(\lambda - \lambda c(z) + \alpha)}{\lambda - \lambda c(z) + \alpha} \right] \qquad\qquad (21a)$$

$$P^{(2)}(z) = P^{(2)}(0,z)\bar{G}_1(f_1(z))\left[\frac{1 - \bar{G}_2(f_1(z))}{f_1(z)} \right] \qquad\qquad (21b)$$

where $\bar{G}_1(\lambda - \lambda c(z) + \alpha) = \int_0^\infty e^{-[\lambda-\lambda C(z)+\alpha]x}\,dG_1(x)$ are the Laplace-Stieltjes transform of the service time $G_1(x)\ and\ G_2(x)$ .Now multiplying both sides of equation (20a) by $\mu_1(x)\ and\ \mu_2(x)$ and integrating over x we get

$$\int_0^\infty P^{(1)}(x,z)\mu_1(x)dx = P^{(1)}(0,z)\,\bar{G}_1(\lambda - \lambda c(z) + \alpha) \tag{22a}$$

$$\int_0^\infty P^{(2)}(x,z)\mu_2(x)dx = P^{(1)}(0,z)\,\bar{G}_1(\lambda - \lambda c(z) + \alpha)\bar{G}_2(\lambda - \lambda c(z) + \alpha) \tag{22b}$$

Using equation (22b), equation (19) becomes

$$V(0,z) = P^{(2)}(0,z)\bar{G}_2(\lambda - \lambda c(z) + \alpha) \tag{23}$$

Similarly, we integrate equation (16) from $0\ to\ x$, we get

$$V(x,z) = V(0,z)e^{-[\lambda - \lambda C(z) + \alpha]x - \int_0^x Y(t)dt} \tag{24}$$

Substituting the value of $V(0,z)$ from (23) in equation (24) we get

$$V(x,z) = P^{(2)}(0,z)\bar{G}_2(\lambda - \lambda c(z) + \alpha)e^{-[\lambda - \lambda C(z) + \alpha]x - \int_0^x Y(t)dt} \tag{25}$$

Again integrating equation (25) by parts with respect to x and using (4) we get

$$V(z) = \bar{G}_2(\lambda - \lambda c(z) + \alpha)\,P^{(1)}(0,z)\bar{G}_1(\lambda - \lambda c(z) + \alpha)\left[\frac{1 - \bar{B}[\lambda - \lambda c(z)]}{\lambda - \lambda c(z)}\right] \tag{26}$$

Where $\bar{B}[\lambda - \lambda c(z)] = \int_0^\infty e^{-[\lambda - \lambda C(z)]x}dB(x)$ is the Lapalce-Stieltjes transform of the vacation time $B(x)$.Now, multiplying bothsides of equation (25) by $Y(x)$ and integrating over x we get

$$\int_0^\infty V(x,z)Y(x)dx = P^{(2)}(0,z)\bar{G}_2(\lambda - \lambda c(z) + \alpha)\bar{B}[\lambda - \lambda c(z)] \tag{27}$$

Using equation (22), equation (17) becomes

$$R(z) = \alpha z \frac{1 - \bar{G}_1(\lambda - \lambda c(z) + \alpha)\bar{G}_2(\lambda - \lambda c(z) + \beta)}{(\lambda - \lambda c(z) + \beta)(\lambda - \lambda c(z) + \alpha)}\,P^{(1)}(0,z) \tag{28}$$

Now, using equations (22b),(27) and (28) in equation (18a) and solving for $P^{(1)}(0,z)$ we get

$$P^{(1)}(0,z) = \frac{\lambda Q[c(z) - 1]f_1(z)f_2(z)}{f_1(z)f_2(z)\left\{\begin{array}{c}z - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)]\\ \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\end{array}\right\} - \alpha\beta z[1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]} \tag{29a}$$

$$P^{(2)}(0,z) = \frac{\lambda Q[c(z)-1]f_1(z)f_2(z)(\lambda - \lambda c(z) + \alpha)}{f_1(z)f_2(z)\left\{\begin{array}{c} z - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)] \\ \bar{G}_1(f_1(z))\bar{G}_2(f_1(z)) \end{array}\right\}}$$

$$-\alpha\beta z[1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]$$

$$\times \left[\frac{1-\bar{G}_2(\lambda - \lambda c(z)+\alpha)}{(\lambda - \lambda c(z)+\alpha)}\right]\bar{G}_1(\lambda - \lambda c(z) + \alpha) \qquad (29b)$$

where $f_1(z) = \lambda - \lambda C(z) + \alpha$ and $f_2(z) = \lambda - \lambda C(z) + \beta$.From (21a),(21b),(26) and (28) we get

$$P^{(1)}(z)$$

$$= \left[\frac{\lambda Q[c(z)-1]f_1(z)f_2(z)}{f_1(z)f_2(z)\left\{\begin{array}{c} z - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)] \\ -\bar{G}_1(f_1(z))\bar{G}_2(f_1(z)) \end{array}\right\}} \right] \left[\frac{1 - \bar{G}_1(f_1(z))}{f_1(z)}\right] \qquad (30a)$$

$$\qquad\qquad -\alpha\beta z[1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]$$

$$P^{(2)}(z) = \left[\frac{\lambda Q[c(z)-1]f_1(z)f_2(z)}{f_1(z)f_2(z)\left\{\begin{array}{c} z-\bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)] \\ -\bar{G}_1(f_1(z))\bar{G}_2(f_1(z)) \end{array}\right\}}{-\alpha\beta z[1-\bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]}\right] \bar{G}_1(f_1(z)) \left[\frac{1-\bar{G}_2(f_1(z))}{f_1(z)}\right] \quad (30b)$$

$$V(z) = \frac{\lambda Q[c(z)-1]f_1(z)f_2(z)}{f_1(z)f_2(z)\left\{z - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)] - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\right\}}$$

$$-\alpha\beta z[1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]$$

$$\times \bar{G}_1(\lambda - \lambda c(z) + \alpha)\left[\bar{G}_2(\lambda - \lambda c(z) + \alpha)\left[\frac{1-\bar{B}[\lambda - \lambda c(z)]}{[\lambda - \lambda c(z)]}\right]\right] \qquad (31)$$

$$R_q(z)$$

$$= \alpha z \frac{\lambda Q[c(z)-1]}{f_1(z)f_2(z)\left\{z - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\bar{B}[\lambda - \lambda c(z)] - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))\right\}}$$

$$-\alpha\beta z[1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))]$$

$$\times \frac{1 - \bar{G}_1(f_1(z))\bar{G}_2(f_1(z))}{f_2(z)f_1(z)} \qquad (32)$$

Let $W_q(z)$ denote the probability generating function of the queue size irrespective of the state of the system.Then adding equations (30a), (30b), (31), and (32) we obtain

$$W_q(z) = P^{(1)}(z) + P^{(2)}(z) + V_q(z) + R_q(z)$$

$$W_q(z) = \frac{[-Q]\left\{\begin{array}{c} f_2(z)f_3(z)-f_2(z)f_3(z)\bar{G}_1f_1(z)\bar{G}_2f_1(z)+\bar{G}_1f_1(z)\bar{G}_2f_1(z)f_1(z)f_2(z) \\ -\bar{G}_1f_1(z)\bar{G}_2\,f_1(z)\bar{B}\,f_1(z)f_2(z)\alpha z f_3(z) \\ -\alpha z\bar{G}_1f_1(z)\bar{G}_2f_1(z)f_3(z) \end{array}\right\}}{f_1(z)f_2(z)\{z-\bar{G}_1f_1(z)\bar{G}_2f_1(z)\bar{B}-\bar{G}_1f_1(z)\bar{G}_2f_1(z)\}-\alpha\beta z[1-\bar{G}_1f_1(z)\bar{G}_2f_1(z)]} \tag{33}$$

In order to find Q,we use the normalization condition

$$W_q(1) + Q = 1$$

We see that for $z = 1, W_q(z)$ is indeterminate of $0/0$ form.Therefore, we apply $L'Hopitals$ Rule on equation (33), we get

$$W_q(1)$$
$$= \frac{\lambda Q\left\{\begin{array}{c} E(I)(\alpha+\beta)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] \\ +\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)E(v) \end{array}\right\}\bar{G}_1(\alpha)\bar{G}_2(\alpha)}{\begin{array}{c} -\alpha\lambda E(I)[1-2\bar{G}_1\bar{G}_2]-\beta E(I)\lambda[1-2\bar{G}_1\bar{G}_2]+\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha) \\ \left[1+\lambda E(I)\left[\bar{G}'_1(\alpha)\bar{G}_2(\alpha)+\bar{G}_1(\alpha)\bar{G}'_2(\alpha)+E(v)\right]\right] \end{array}} \tag{34}$$

where $C(1) = 1, C'(1) = E(I)$ is the mean batch size of the arriving customers. $\bar{B}[0] = 1, and\ \bar{B}[0] = -E(v)$, the mean vacation time.Therefore, adding Q to equation (34), equating to 1 and simplifying we get

$$Q$$
$$= 1 - \frac{\lambda Q\left\{E(I)(\alpha+\beta)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)]+\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)E(v)\right\}\bar{B}[\lambda-\lambda C(z)]}{\begin{array}{c} -\alpha\lambda E(I)[1-2\bar{G}_1\bar{G}_2]-\beta E(I)\lambda[1-2\bar{G}_1\bar{G}_2]+\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha) \\ \left[1+\lambda E(I)[\bar{G}'_1(\alpha)\bar{G}_2(\alpha)\bar{G}_1(\alpha)\bar{G}'_2(\alpha)+E(v)]\right] \end{array}} \tag{35}$$

gives the probability of server is idle. Substitute Q in (33), hence $W_q(z)$ is explicitly determined.

And hence, the utilization factor,ρ of the system is given by

$$\rho =$$
$$\frac{\lambda Q\{E(I)(\alpha+\beta)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)]+\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)E(v)\}\bar{B}[\lambda-\lambda C(z)]}{-\alpha\lambda E(I)[1-2\bar{G}_1\bar{G}_2]-\beta E(I)\lambda[1-2\bar{G}_1\bar{G}_2]+\alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)\left[1+\lambda E(I)\left[\bar{G}'_1(\alpha)\bar{G}_2(\alpha)\bar{G}_1(\alpha)\bar{G}'_2(\alpha)+E(v)\right]\right]} \tag{36}$$

where $\rho < 1$ is the stability condition under which the steady state exists.

6. The Average Queue Size and the Average Waiting Time

Let L_q denote the mean number of customers in the queue under the steady state.Then

$$L_q = \frac{d}{dz} W_q(z)\big|_{z=1}$$

Since the formula gives $0/0$ form, then we write $W_q(z)$ given in (33) as $W_q(z) = N(z)/D(z)$ where $N(z)$ and $D(z)$ are the numerator and denominator of the right hand side of (33) respectively.Then we use

$$L_q = \frac{D'(1)N''(1) - N'(1)D''(1)}{2(D'(1))^2} \tag{37}$$

where primes and double primes in (37) denote first and second derivatives at $z = 1$, respectively. Carrying out the derivatives at $z = 1$ we have

$$N'(1) = \lambda Q \left\{ E(I)(\alpha + \beta)[1 - \bar{G}_1(\alpha)\bar{G}_2(\alpha)] + \alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)E(v) \right\} \bar{B}[\lambda - \lambda C(z)] \tag{38}$$

$$N''(1) = \left\{ [\lambda E(I)]^2 2 + \bar{G}_1(\alpha)\bar{G}_2(\alpha)[2 - \alpha\beta E(v^2)] - (2\alpha - 2\beta)\left[\begin{matrix} \overline{G'}_1(\alpha)\bar{G}_2(\alpha) \\ +\bar{G}_1(\alpha)\overline{G'}_2(\alpha) \end{matrix} \right] \right\}$$

$$- \beta\lambda E(I(I-1)) \left\{ \begin{matrix} \beta + \bar{G}_1(\alpha)\bar{G}_2(\alpha)\beta - \alpha\bar{G}_1(\alpha)\bar{G}_2(\alpha)E(v)\beta \\ +\alpha - \alpha\bar{G}_1(\alpha)\bar{G}_2(\alpha) \end{matrix} \right\} \tag{39}$$

$$D'(1) = -\alpha\lambda E(I)[1 - 2\bar{G}_1\bar{G}_2] - \beta E(I)\lambda[1 - 2\bar{G}_1\bar{G}_2] + \alpha\beta\bar{G}_1(\alpha)\bar{G}_2(\alpha)$$
$$[1 + \lambda E(I) \ [\overline{G'}_1(\alpha)\bar{G}_2(\alpha)\bar{G}_1(\alpha)\overline{G'}_2(\alpha) + E(v)]] \tag{40}$$

$$D''(1) = \lambda E(I-1) \left\{ \begin{matrix} -\beta - \alpha + 2(\alpha + \beta)\bar{G}_1(\alpha)\bar{G}_2(\alpha) \\ \alpha\beta(\bar{G}_1(\alpha)\overline{G'}_2(\alpha) + \overline{G'}_1(\alpha)\bar{G}_2(\alpha)) \end{matrix} \right\}$$
$$+ \alpha\beta(\lambda E(I))^2 [\overline{G''}_1(\alpha)\bar{G}_2(\alpha) + 2\overline{G'}_1(\alpha)\bar{G'}_2(\alpha) + \overline{G''}_2(\alpha)\bar{G}_1(\alpha)]$$
$$- \alpha\beta\lambda \left\{ \begin{matrix} E(I)(\overline{G'}_1(\alpha)\bar{G}_2(\alpha) + \bar{G}_1(\alpha)\overline{G'}_2(\alpha))(2 + (\lambda E(I))) + (\bar{G}_1(\alpha)\bar{G}_2(\alpha)) \\ (E(I(I-1)E(v) - E(I)E(v^2)) \end{matrix} \right\} \tag{41}$$

where $E(v^2)$ is the second moment of the vacation time, $E(I(I-1)$ is the second factorial moment of the batch size of arriving customers, and Q has been found in (35).Then if we substitute the values of $N'(1), N''(1), D'(1) \ and \ D''(1)$ from (38),(39),(40),and (41)into (37)we obtain L_q in closed form.Futher,the mean waiting time of a customer could be found using $W_q = L_q/\lambda$.

7. Conclusion

In this paper we have studied a batch arrival, two stage heterogeneous service with random breakdown and Bernoulli schedule server vacation. This paper clearly analyses the steady state results and some performance measures of the queueing system. The result of this paper is useful for computer communication network, and large scale industrial production lines.

References

[1] Y.Baba, On the $M^{[x]}$/G/1 queue with vacation time, *operations Research Letters* **5** (1986), 93-98.

[2]G.Chadhury,An $M^{[x]}$/G/1 queueing system with a set up period and a vacation period,*Questa,***36,**(2000), 23-28.

[3] M.Cramer, Stationary distributions in a queueing system with a vacation times and limited service, *Queueing Systems*, **4**, (1989), 57-68.

[4] N. Igaki, Exponential two server queue with *N*- policy and general vacation, *Queueing Systems,* **10** (1992), 279-294.

[5] B.Krishnakumar, A.Vijayakumar and D.Arividainambi, An M/G/1 retrial queueing system with two phase service and preemptive resume, *Annals of Operations Research* **113**,(2002),61-79.

[6] Y. Levi and U.Yechilai, An *M/M/s* queue with server vacations, *Infor,* **14** (1976), 153-163.

[7] K.C. Madan On a single server with two stage heterogeneous service and binomial schedule server vacations, *Egyptian Statistical Journal,* **40**(1), (2000) 39-55.

[8] K.C.Madan and R.F. Anabosi, A single server queue with two types of service, Bernulli schedule server vacations and a single vacation policy, *Pakistan Journal of Statistics* **19**,(2003), 331-442.

[9] K.C Madan and G.Chodhury, An $M^{[x]}$/G/1 queue with Bernoulli vacation Schedule under restricted admissibility policy, *Sankhaya,* **66**, (2004), 172-193.

[10] K.C Madan and G.Chodhury, A two stage arrival queueing system with a modified Bernoulli schedule vacation under N-policy, *Mathematical and Computer Modeling* **42**, (2005), 71-85.

[11] K.C. Madan, F.A. Maraghi and K.D. Dowman Batch arrival queueing system with random breakdowns and Bernoulli schedule server vacations having general vacation time distribution, *Information and Management Sciences* **20** (2009), 55-70.

[12] K.C Madan, An M/G/1 queue with second optional service, *Queueing Systems*, **34**, (2000), 37-46.

[13] K.C. Madan and A.Z. Abu-Dayyeh, On a single server queue with optional phase type server vacations based on exhaustive deterministic service and a single vacation policy, *Applied Mathematics and Computation,* **149** (2004), 723-734.

[14] S.Maragathasundari and S.Srinivasan (2012), Analysis of M/G/1 feedback queue with three stage multiple sever vacation, *Applied mathematical sciences, vol.6, and pp.6221-6240.*

[15] S.Maragathasundari and S.Srinivasan(2012),Analysis of M/G/1 queue with triple stage of service having compulsory vacation and service interruptions, *Far East Journal of Mathematical Sciences,*Vol.**69**,pp.61-80.

[16] S.Maragathasundari and S.Srinivasan(2013),Three phase M/G/1 queue with Bernoulli feedback and multiple server vacation and service interruptions, *InternationalJournal of Applied mathematics and Statistics,*Vol.**33**,Issue no.3.

[17]   V.Thangaraj and S.Vanitha, Singe server Feedback queue with two types of service having general distribution, *International Mathematical Forum* **5**, (2010), 15-33.