

# **Bayes and Empirical Bayes Methods for Data Analysis**

---

**BRADLEY P. CARLIN**

*Associate Professor,  
Division of Biostatistics,  
School of Public Health,  
University of Minnesota,  
Minneapolis, USA*

and

**THOMAS A. LOUIS**

*Professor and Head of Department,  
Division of Biostatistics,  
School of Public Health,  
University of Minnesota,  
Minneapolis, USA*



**CHAPMAN & HALL**

London · Weinheim · New York · Tokyo · Melbourne · Madras

---

# Contents

---

|  |             |
|--|-------------|
| <b>Preface</b>                                       | <b>xiii</b> |
| <b>1 Procedures and their properties</b>             | <b>1</b>    |
| 1.1 Introduction and motivation                      | 1           |
| 1.2 Structures for inference                         | 6           |
| 1.3 The general decision problem                     | 7           |
| 1.3.1 Risk and admissibility                         | 9           |
| 1.3.2 Unbiased rules                                 | 10          |
| 1.3.3 Bayes rules                                    | 12          |
| 1.3.4 Minimax rules                                  | 14          |
| 1.4 Procedure evaluation and other unifying concepts | 14          |
| 1.4.1 Mean squared error                             | 15          |
| 1.4.2 The variance-bias tradeoff                     | 15          |
| 1.5 Exercises  | 17          |
| <b>2 The Bayes approach</b>                          | <b>21</b>   |
| 2.1 Introduction                                     | 21          |
| 2.2 Prior distributions                              | 26          |
| 2.2.1 Elicited priors                                | 27          |
| 2.2.2 Conjugate priors                               | 30          |
| 2.2.3 Noninformative priors                          | 33          |
| 2.2.4 Other prior construction methods               | 37          |
| 2.3 Bayesian inference                               | 38          |
| 2.3.1 Point estimation                               | 39          |
| 2.3.2 Interval estimation                            | 42          |
| 2.3.3 Hypothesis testing and Bayes factors           | 45          |
| 2.3.4 Example: Consumer preference data              | 50          |
| 2.4 Model assessment                                 | 54          |
| 2.4.1 Diagnostic measures                            | 54          |
| 2.4.2 Model averaging                                | 57          |
| 2.5 Nonparametric methods                            | 59          |

|  |            |
|--|------------|
| 2.6 Exercises                                      | 62         |
| <b>3 The empirical Bayes approach</b>              | <b>67</b>  |
| 3.1 Introduction                                   | 67         |
| 3.2 Nonparametric EB point estimation              | 68         |
| 3.2.1 Compound sampling models                     | 68         |
| 3.2.2 Simple NPEB (Robbins' method)                | 69         |
| 3.2.3 Estimation of the prior (NPML method)        | 71         |
| 3.2.4 Example: Accident data                       | 72         |
| 3.3 Parametric EB point estimation                 | 74         |
| 3.3.1 Gaussian/Gaussian models                     | 75         |
| 3.3.2 Beta/binomial model                          | 80         |
| 3.3.3 EB performance of the PEB                    | 82         |
| 3.3.4 Stein estimation                             | 84         |
| 3.4 Computation via the EM algorithm               | 89         |
| 3.5 Interval estimation                            | 93         |
| 3.5.1 Morris' approach                             | 95         |
| 3.5.2 Marginal posterior approach                  | 95         |
| 3.5.3 Bias correction approach                     | 98         |
| 3.6 Generalization to regression structures        | 100        |
| 3.7 Exercises                                      | 102        |
| <b>4 Performance of Bayes procedures</b>           | <b>107</b> |
| 4.1 Bayesian processing                            | 108        |
| 4.1.1 Univariate stretching with a two-point prior | 108        |
| 4.1.2 Multivariate Gaussian model                  | 109        |
| 4.2 Frequentist performance: Point estimates       | 110        |
| 4.2.1 Gaussian/Gaussian model                      | 111        |
| 4.2.2 Beta/binomial model                          | 113        |
| 4.2.3 Generalization                               | 116        |
| 4.3 Frequentist performance: Confidence intervals  | 117        |
| 4.3.1 Beta/binomial model                          | 117        |
| 4.3.2 Fieller-Creasy problem                       | 124        |
| 4.4 Empirical Bayes performance                    | 128        |
| 4.4.1 Point estimation                             | 129        |
| 4.4.2 Interval estimation                          | 132        |
| 4.5 Design of experiments                          | 135        |
| 4.5.1 Bayesian design for Bayesian analysis        | 135        |
| 4.5.2 Bayesian design for frequentist analysis     | 137        |
| 4.6 Exercises                                      | 139        |

|   |            |
|---|------------|
| <b>5 Bayesian computation</b>                     | <b>141</b> |
| 5.1 Introduction                                  | 141        |
| 5.2 Asymptotic methods                            | 142        |
| 5.2.1 Normal approximation                        | 142        |
| 5.2.2 Laplace's method                            | 145        |
| 5.3 Noniterative Monte Carlo methods              | 150        |
| 5.3.1 Direct sampling                             | 150        |
| 5.3.2 Indirect methods                            | 152        |
| 5.4 Markov chain Monte Carlo methods              | 159        |
| 5.4.1 Substitution sampling and data augmentation | 159        |
| 5.4.2 Gibbs sampling                              | 163        |
| 5.4.3 Metropolis-Hastings algorithm               | 173        |
| 5.4.4 Hybrid forms and other algorithms           | 180        |
| 5.4.5 Convergence monitoring and acceleration     | 185        |
| 5.5 Exercises                                     | 197        |
| <b>6 Model criticism and selection</b>            | <b>205</b> |
| 6.1 Bayesian robustness                           | 206        |
| 6.1.1 Sensitivity analysis                        | 206        |
| 6.1.2 Prior partitioning                          | 212        |
| 6.2 Model assessment                              | 218        |
| 6.3 Bayes factors                                 | 220        |
| 6.3.1 Direct methods                              | 221        |
| 6.3.2 Marginal densities from the Gibbs sampler   | 223        |
| 6.3.3 Sampling over model and parameter space     | 225        |
| 6.3.4 Sampling over the model space alone         | 229        |
| 6.4 Predictive model selection                    | 230        |
| 6.5 Exercises                                     | 233        |
| <b>7 Special methods and models</b>               | <b>237</b> |
| 7.1 Ensemble estimates                            | 237        |
| 7.2 Order restricted inference                    | 246        |
| 7.3 Nonlinear models                              | 247        |
| 7.4 Longitudinal data models                      | 251        |
| 7.5 Continuous and categorical time series        | 253        |
| 7.6 Survival analysis and frailty models          | 255        |
| 7.6.1 Statistical models                          | 256        |
| 7.6.2 Treatment effect prior determination        | 257        |
| 7.6.3 Computation and advanced models             | 258        |
| 7.7 Spatial and spatio-temporal models            | 260        |

|  |            |
|--|------------|
| 7.8 Exercises                                      | 265        |
| <b>8 Case studies</b>                              | <b>273</b> |
| 8.1 Analysis of longitudinal AIDS data             | 274        |
| 8.1.1 Introduction and background                  | 274        |
| 8.1.2 Modeling of longitudinal CD4 counts          | 275        |
| 8.1.3 CD4 response to treatment at two months      | 285        |
| 8.1.4 Survival analysis                            | 287        |
| 8.1.5 Discussion                                   | 289        |
| 8.2 Robust analysis of clinical trials             | 289        |
| 8.2.1 Clinical background                          | 289        |
| 8.2.2 Interim monitoring                           | 290        |
| 8.2.3 Prior robustness and prior scoping           | 296        |
| 8.2.4 Discussion                                   | 302        |
| 8.3 Spatio-temporal mapping of lung cancer rates   | 303        |
| 8.3.1 Introduction                                 | 303        |
| 8.3.2 Data and model description                   | 305        |
| 8.3.3 Computational considerations                 | 307        |
| 8.3.4 Model fitting, validation and comparison     | 308        |
| 8.3.5 Discussion                                   | 315        |
| <b>Appendices</b>                                  | <b>317</b> |
| <b>A Distributional catalog</b>                    | <b>319</b> |
| A.1 Discrete                                       | 319        |
| A.1.1 Univariate                                   | 319        |
| A.1.2 Multivariate                                 | 321        |
| A.2 Continuous                                     | 321        |
| A.2.1 Univariate                                   | 321        |
| A.2.2 Multivariate                                 | 325        |
| <b>B Software guide</b>                            | <b>327</b> |
| B.1 Prior elicitation                              | 328        |
| B.2 Random effects models/Empirical Bayes analysis | 330        |
| B.3 Bayesian analysis                              | 334        |
| B.3.1 Early developments                           | 335        |
| B.3.2 Special purpose programs                     | 336        |
| B.3.3 Teaching programs                            | 341        |
| B.3.4 Markov chain Monte Carlo programs            | 342        |
| <b>C Answers to selected exercises</b>             | <b>349</b> |

|                      |            |
|----------------------|------------|
| <b>CONTENTS</b>      | <b>xi</b>  |
| <b>References</b>    | <b>363</b> |
| <b>Author index</b>  | <b>387</b> |
| <b>Subject index</b> | <b>393</b> |