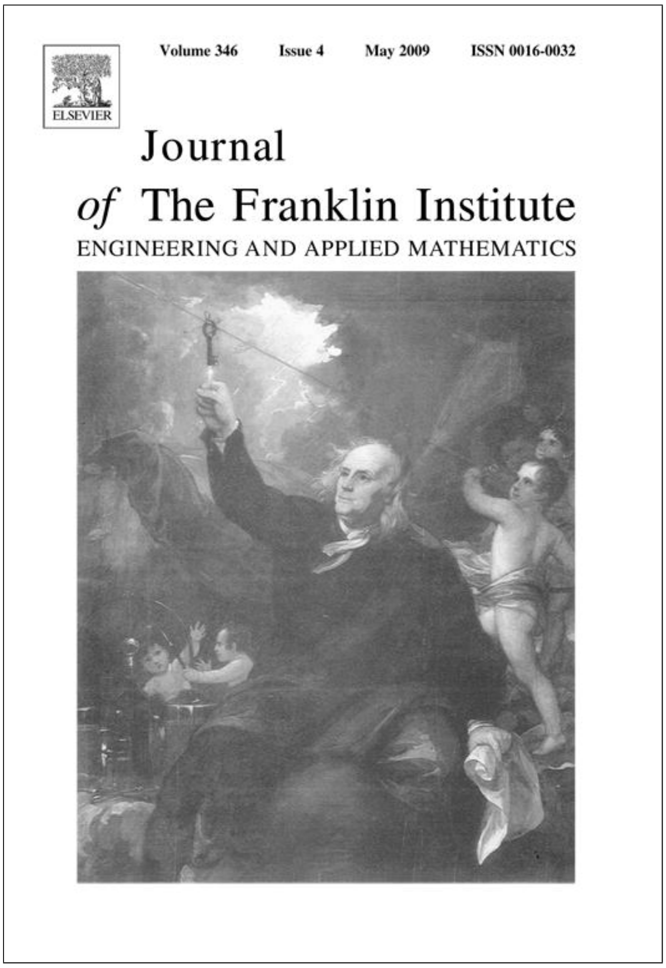
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Bayesian and frequentist estimation of the performance of free space optical channels under weak turbulence conditions

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Abstract

Approximate mathematical form expressions were derived for the estimation of the average (ergodic) capacity and the average bit error rate of a log-normal free space optical channel in the cases of weak to moderate atmospheric turbulence conditions. We investigate the average capacity, the average bit error rate and the outage probability of free space optical communication channels using the frequentist and the Bayesian approach. Emphasis is given on the cases of weak to moderate atmospheric turbulence leading to channels modeled by log-normal distributed intensity fading. Furthermore, accurate approximate closed-form expressions and estimation procedures for their achievable capacity as well as their bit error rate and the important parameters of interest are derived. The derived approximate analytical expressions are verified by various numerical examples and simulations. Moreover, each methodology is reviewed in terms of their analytic convenience and their accuracy is also discussed.

Keywords: Average capacity; Bit error rate; Bayesian statistics; Free space optical channels; Log-normal distribution

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1. Introduction

Free space optical communications (FSO) is a potentially high capacity and cost-effective technique that receives growing attention and commercial interest [1,2]. For an outdoor line-of-sight optical channel link, atmospheric turbulence represents an important capacity mitigation factor, in the sense that the received optical signal intensity fluctuates due to variation of the refractive index along the propagation path [3–9]. Hence, it is reasonable to expect that in a real FSO environment, optical channels will appear to have randomly time-varying characteristics. Thus, channel capacity becomes random variable. Moreover, the magnitudes that one should consider in order to design an FSO system are the average capacity, which indicates the average best rate for error-free transmission, [10–16], the average bit error rate (BER) [7,8], and the outage probability [16].

In this work, we examine the reliability and the performance of FSO channels, by investigating their outage probability and average (ergodic) capacity and average BER, respectively. We derive closed-form expressions for the estimation of the outage probability, the average capacity and the average BER of such optical links over atmospheric turbulence-induced fading channels as modeled by the log-normal distribution using the frequentist statistical approach. Furthermore, the Bayesian statistical viewpoint is employed as an alternative mode of obtaining the outage probability and the average capacity as well as the average BER, since in applications the parameters of interest may not be regarded as unknown constants but rather as time-varying quantities. Therefore, Bayesian statistical inference welcomes the use and application of the well-known prior statistical distributions on these parameters to reflect their variability.

The remainder of the paper is organized as follows: In Section 2, the FSO channel model is introduced while in Section 3 we present the mathematical expressions for the estimation of the outage probability, the average channel capacity and the average BER of the log-normal modeled FSO channels, using the frequentist statistical approach. In Section 4, employing the Bayesian viewpoint, we set up the appropriate prior statistical distributions and the resulting quantities that need to be estimated. Furthermore, in Section 5 numerical results under the frequentist and the Bayesian approach are displayed and the capabilities of the FSO systems are examined. Finally, concluding remarks are given in Section 6.

2. The channel model

We consider a binary input and continuous output FSO communication system using intensity modulation/direct detection (IM/DD) with on-off keying (OOK). The laser beams propagate along a horizontal path through a turbulent channel with additive white Gaussian noise (AWGN). The channel is assumed to be memoryless, stationary and ergodic, with independent and identically distributed (i.i.d.) intensity fast fading statistics. We also consider that the channel state information (CSI) is available at both transmitter and receiver. In this case, the statistical channel model is given by [7,9]

$$y = sx + n = \eta I x + n \tag{1}$$

where y is the signal at the receiver, $s = \eta I$ is the instantaneous intensity gain, η is the effective photo-current conversion ratio of the receiver, I the normalized irradiance, x the modulated signal which takes values "0" or "1", and n the AWGN with zero mean and variance $N_0/2$. For weak to moderate atmospheric turbulence conditions, the turbulence-induced fading can be assumed as a random process that follows the log-normal distribution model which has the following probability density function (pdf) with respect to I [3,17],

$$f_I(I) = \frac{1}{I\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(I)+\sigma^2/2)^2}{2\sigma^2}\right) \tag{2}$$

with σ^2 being the log-irradiance variance [17], which depends on the channel's characteristics and is given by the following expression:

$$\sigma^2 = \exp\left[\frac{0.49\delta^2}{(1+0.18d^2+0.56\delta^{12/5})^{7/6}} + \frac{0.51\delta^2}{(1+0.9d^2+0.62d^2\delta^{12/5})^{5/6}}\right] - 1 \tag{3}$$

where $d = \sqrt{kD^2/4L}$, $k = 2\pi/\lambda$ the optical wave number, L the length of the optical link, λ the operational wavelength and D the aperture diameter of the receiver. The parameter δ^2 is the Rytov variance for a plane wave in weak scintillation theory [6] and is given by

$$\delta^2 = 0.5C_n^2 k^{7/6} L^{11/6} \tag{4}$$

with ${C_n}^2$ being the atmospheric turbulence strength, which is altitude dependent and varies from 10^{-17} to $10^{-13}\,\mathrm{m}^{-2/3}$ for weak to strong turbulence conditions, respectively [2]. By integrating (2) we obtain the cumulative distribution function (cdf) with respect to I, which has the form:

$$F_I(I) = \frac{1}{2} erfc\left(-\frac{\ln(I)+\sigma^2/2}{\sqrt{2}\sigma}\right) \tag{5}$$

where $erfc(\cdot)$ is the complementary error function. The magnitude that can be easily measured is the instantaneous electrical signal-to-noise ratio (SNR) at the receiver and can be defined as $\mu = (\eta I)^2/N_0 = s^2/N_0$ [6]. Moreover, the average electrical SNR is given by $\bar{\mu} = (\eta E[I])^2/N_0$ [6], where $E[\cdot]$ is the expected value. Taking into account that $E[I] = 1$, since I is normalized to unity, with a power transformation of the random variable I, from (2) we obtain the following pdf for the instantaneous electrical SNR, μ [18]:

$$f_\mu(\mu) = \frac{1}{2\mu\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln(\mu/\bar{\mu})+\sigma^2)^2}{8\sigma^2}\right) \tag{6}$$

By integrating (6), we obtain the following form for the cdf with respect to μ:

$$F_\mu(\mu) = \frac{1}{2} erfc\left(\frac{\ln(\mu/\bar{\mu})-\sigma^2}{2\sqrt{2\sigma^2}}\right) \tag{7}$$

3. Frequentist statistical approach

The average (ergodic) capacity, $\langle C\rangle$, represents the practical capacity of an FSO channel with AWGN and it is a very significant metric for the evaluation of the performance of the link. The average capacity, with perfect CSI, at both the transmitter and the receiver of an FSO communication system is given by [16]

$$\langle C\rangle = \int_0^\infty B\log_2\left(1+\frac{(\eta I)^2}{N_0}\right) f_I(I)\,dI \tag{8}$$

where B is the signal transmission bandwidth. From (6) to (8) we obtain the following integral for the evaluation of the average capacity:

$$\langle C\rangle = \frac{B}{2\sigma\sqrt{2\pi}\,\ln(2)}\int_0^{+\infty}\frac{\ln(1+\mu)}{\mu}\exp\left[-\frac{(\ln(\mu/\bar{\mu})+\sigma^2)^2}{8\sigma^2}\right]d\mu \tag{9}$$

Using the equality $\ln(1+x)=\sum_{k=1}^{+\infty}(-1)^{k+1}(x^k/k)$, for $x\in[0,1]$ and the scaled complementary error function $erfcx(x)=\exp(x^2)erfc(x)$ [14,15], the above integral can be transformed to the following summation:

$$\begin{aligned}\langle C\rangle = BC_0\Bigg\{&\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left[erfcx\left(\sqrt{2}\sigma k+\frac{A}{2\sqrt{2}\sigma}\right)+erfcx\left(\sqrt{2}\sigma k-\frac{A}{2\sqrt{2}\sigma}\right)\right]\\ &+\frac{4\sigma}{\sqrt{2\pi}}+A\,\exp\left(\frac{A^2}{8\sigma^2}\right)\times erfc\left(-\frac{A}{2\sqrt{2}\sigma}\right)\Bigg\}\end{aligned} \tag{10}$$

where $A=\ln(\bar{\mu})-\sigma^2$ and $C_0=\exp(-A^2/8\sigma^2)/2\ln(2)$. In order to evaluate the result of (11) we should calculate the infinitely sum over k. However, taking into account the first twenty addends of the summation we obtain a very accurate approximation for the evaluation of the average capacity [19], which has the form

$$\begin{aligned}\langle\tilde{C}\rangle = BC_0\Bigg\{&\sum_{k=1}^{20}\frac{(-1)^{k+1}}{k}\left[erfcx\left(\sqrt{2}\sigma k+\frac{A}{2\sqrt{2}\sigma}\right)+erfcx\left(\sqrt{2}\sigma k-\frac{A}{2\sqrt{2}\sigma}\right)\right]\\ &+\frac{4\sigma}{\sqrt{2\pi}}+A\,\exp\left(\frac{A^2}{8\sigma^2}\right)\times erfc\left(-\frac{A}{2\sqrt{2}\sigma}\right)\Bigg\}\end{aligned} \tag{11}$$

Numerical evaluation of the estimation error $|\langle\tilde{C}\rangle-\langle C\rangle|/\langle C\rangle$ is found to be of order of 10^{-9} for all the cases of weak turbulence conditions, that we present below. Thus, we conclude that, for all practical purposes, (11) is a very accurate approximation [19], for the evaluation of the average capacity of a log-normal modeled FSO channel. Consequently, in practice, we consider $\langle C\rangle \cong \langle\tilde{C}\rangle$.

A very significant metric that describes the performance of an FSO communication system is the average bit error rate (BER) [7,8,20]. The BER of IM/DD systems with OOK in the presence of AWGN and perfect CSI at the receiver side is given by Sandalidis et al. [7], Navidpour et al. [20] and Tsiftsis et al. [21], $P=p(0)P(e|0)+p(1)P(e|1)$, where $p(0)$ and $p(1)$ are the probabilities of sending the bit "0" and "1", respectively, while $P(e|0)$ and $P(e|1)$ are the conditional bit error probabilities when the transmitted bit is "0" or "1". Taking into account that $p(0)=p(1)=0.5$, and $P(e|0)=P(e|1)$ [7], we obtain the following probabilities conditioned of the fading coefficient I [20]:

$$P=P(e|0,I)=P(e|1,I)=P\left(n>\frac{\eta I}{2}\right)=P\left(n<-\frac{\eta I}{2}\right)=Q\left(\frac{\eta I}{\sqrt{2N_0}}\right) \tag{12}$$

where $Q(\cdot)$ is the Gaussian Q-function. For the evaluation of the average BER by averaging, (12) over I, yields the integral

$$P_{av}=\int_0^{+\infty}f_I(I)Q\left(\frac{\eta I}{\sqrt{2N_0}}\right)dI \tag{13}$$

The integral (13) is very difficult, if not impossible, to be evaluated in closed mathematical form expression. Thus, we approximate the Q-function as in [22],

$$Q(x) \approx \frac{1}{12}\exp\left(\frac{-x^2}{2}\right) + \frac{1}{4}\exp\left(-\left(\frac{2x^2}{3}\right)\right)$$

and the above integral is taking the form

$$P_{av} \approx \tilde{P}_{av} = \frac{1}{12}\int_0^{+\infty} f_I(I)\exp\left(-\frac{\eta^2 I^2}{4N_0}\right)dI + \frac{1}{4}\int_0^{+\infty} f_I(I)\exp\left(-\frac{\eta^2 I^2}{3N_0}\right)dI \tag{14}$$

Eq. (14), can be rewritten also in terms of the instantaneous electrical SNR, μ, as

$$P_{av} \approx \tilde{P}_{av} = \frac{1}{12}\int_0^{+\infty} f_\mu(\mu)\exp\left(-\frac{\mu}{4}\right)d\mu + \frac{1}{4}\int_0^{+\infty} f_\mu(\mu)\times\exp\left(-\frac{\mu}{3}\right)d\mu \tag{15}$$

and from (15), using Eq. (6), we obtain

$$\begin{aligned}\tilde{P}_{av} = \frac{1}{8\sigma\sqrt{2\pi}}\left[\frac{1}{3}\int_0^{+\infty}\frac{1}{\mu}\exp\left(-\frac{\mu}{4}-\frac{(\ln(\mu)-A)^2}{8\sigma^2}\right)d\mu\right. \\ \left. + \int_0^{+\infty}\frac{1}{\mu}\exp\left(-\frac{\mu}{3}-\frac{(\ln(\mu)-A)^2}{8\sigma^2}\right)d\mu\right]\end{aligned} \tag{16}$$

where the parameters A and σ^2 are given above. For the evaluation of (16) we first transform the integrals to the following form:

$$\begin{aligned}\tilde{P}_{av} = \frac{1}{8\sigma\sqrt{2\pi}}\left[\frac{1}{3}\int_{-\infty}^{0}\exp\left(-\frac{\exp(y)}{4}-\frac{(y-A)^2}{8\sigma^2}\right)dy + \frac{1}{3}\int_0^{+\infty}\exp\left(-\frac{\exp(y)}{4}-\frac{(y-A)^2}{8\sigma^2}\right)dy\right. \\ \left. + \int_{-\infty}^{0}\exp\left(-\frac{\exp(y)}{3}-\frac{(y-A)^2}{8\sigma^2}\right)dy + \int_0^{+\infty}\exp\left(-\frac{\exp(y)}{3}-\frac{(y-A)^2}{8\sigma^2}\right)dy\right]\end{aligned} \tag{17}$$

Next, we approximate the first term of the exponential of each integral with another exponential function and thus Eq. (17) is taking the following form [8]:

$$\begin{aligned}\tilde{P}_{av} = \frac{1}{8\sigma\sqrt{2\pi}}\left[\frac{1}{3}\int_{-\infty}^{0}\exp\left(-\frac{(y-A)^2}{8\sigma^2}\right)dy - \frac{A_1}{3}\int_{-\infty}^{0}\exp\left(k_1 y-\frac{(y-A)^2}{8\sigma^2}\right)dy\right. \\ + \frac{A_2}{3}\int_0^{+\infty}\exp\left(-(k_2 y)^2-\frac{(y-A)^2}{8\sigma^2}\right)dy + \int_{-\infty}^{0}\exp\left(-\frac{(y-A)^2}{8\sigma^2}\right)dy \\ \left. -A_3\int_{-\infty}^{0}\exp\left(k_3 y-\frac{(y-A)^2}{8\sigma^2}\right)dy + A_4\int_0^{+\infty}\exp\left(-(k_4 y)^2-\frac{(y-A)^2}{8\sigma^2}\right)dy\right]\end{aligned} \tag{18}$$

where the parameters of (18) have been evaluated numerically and have the following values:

$$\begin{aligned}&A_1 = 0.22436, \quad A_2 = 0.74698, \quad A_3 = 0.28892, \quad A_4 = 0.67361 \\ &k_1 = 0.94751, \quad k_2 = 0.62532, \quad k_3 = 0.93127, \quad k_4 = 0.71433\end{aligned} \tag{19}$$

Now the integrals in (18) can be easily evaluated and, finally, will have the following form:

$$\tilde{P}_{av} = \frac{1}{8\sigma}\left[\frac{4\sigma}{3} erfc\left(\frac{A\sqrt{2}}{4\sigma}\right) + \frac{B_2}{3} erfc\left(\frac{-A}{2\sigma^2\sqrt{16k_2^2 + 2/\sigma^2}}\right) - \frac{A_1\sigma}{3} C_1\, erfc\left(\sqrt{2}\sigma\left(\frac{A}{4\sigma^2} + k_1\right)\right) + B_4\, erfc\left(\frac{-A}{2\sigma^2\sqrt{16k_4^2 + 2/\sigma^2}}\right) - A_3\sigma C_3\, erfc\left(\sqrt{2}\sigma\left(\frac{A}{4\sigma^2} + k_3\right)\right)\right] \quad (20)$$

where

$$B_j = \frac{A_j}{\sqrt{8k_j^2 + 1/\sigma^2}} \exp\left(\frac{A^2}{64\sigma^4\left(k_j^2 + \frac{1}{8\sigma^2}\right)} - \frac{A^2}{8\sigma^2}\right), \quad j = 2, 4$$
$$C_i = A_i\sigma\, \exp\left(-\frac{A^2}{8\sigma^2} + 2\sigma^2\left(\frac{A}{4\sigma^2} + k_i\right)^2\right), \quad i = 1, 3 \quad (21)$$

Eq. (20) is an approximate closed mathematical form expression for the evaluation of the average BER of an FSO channel with log-normal distribution. Numerical evaluation of the estimation error $|\tilde{P}_{av} - P_{av}|/P_{av}$ is found to be of the order of 10^{-6} for all the cases that we present below. Thus, (20) is an accurate approximate expression for the estimation of the average BER and we consider $P_{av} \cong \tilde{P}_{av}$.

Next, we evaluate the outage probability of an FSO channel, which is a critical metric for the design of a free space optical communication system. It represents the probability that the instantaneous electrical SNR falls below a critical threshold, μ_{th}, which, in practice, is the sensitivity limit of the receiver and is given by the following form, using Eq. (7):

$$P_{out} = Pr(\mu \leqslant \mu_{th}) = \int_0^{\mu_{th}} f_\mu(\mu) = F_\mu(\mu_{th}) = \frac{1}{2} erfc\left(\frac{\ln(\bar{\mu}/\mu_{th}) - \sigma^2}{2\sqrt{2\sigma^2}}\right) \quad (22)$$

4. Bayesian approach

In the analysis above, we considered all the parameters of the link to be invariable. Nevertheless, in a real FSO communication system most of them are changing their values according the atmospheric circumstances. Thus, in order to include the influence of the fluctuations of these parameters in the evaluation of the significant metrics of the performance of an FSO communication system studied above, we presume that the average electrical SNR at the receiver, $\bar{\mu}$, as well as the turbulence strength of the atmosphere, $C_n{}^2$—and resulting the Rytov variance, δ^2—vary over time. More specifically, these parameters can be viewed as fast varying with respect to time, allowing us to consider them as random variables. Hence, we use the Bayesian approach to evaluate their outage probability and average capacity. Taking into account the fluctuations of the values of $C_n{}^2$ [23] and $\bar{\mu}$, the conditional prior distributions of the average electrical SNR and the Rytov variance, are the normal and the inverse Gamma, respectively [24,25]. The former has the form

$$g(\bar{\mu}|\delta^2) = \frac{1}{\sqrt{2\pi\tau\delta^2}} \exp\left[-\frac{(\bar{\mu}-m)^2}{2\tau\delta^2}\right] \tag{23}$$

with parameters m and τ, while the latter distribution is given by

$$h(\delta^2) = \frac{g_2^{g_1}}{\Gamma(g_1)} (\delta^2)^{-(g_1+1)} \exp\left(-\frac{g_2}{\delta^2}\right) \tag{24}$$

with parameters g_1 and g_2. Thus, using (8), the average channel capacity with the Bayesian approach is given by the following triple integral:

$$\langle C \rangle = \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} B \log_2(1+\mu) f_{\mu}(\mu|\bar{\mu},\delta^2) g(\bar{\mu}|\delta^2) h(\delta^2)\, d\mu\, d\bar{\mu}\, d\delta^2 \tag{25}$$

where it is obvious that the pdf of Eq. (6), depends on the value of the Rytov variance because σ^2 is a function of δ^2 [i.e. $\sigma^2 = \sigma^2(\delta^2)$]. Using (6), (23) and (24), the above integral is taking the following form:

$$\langle C \rangle = \frac{B g_2^{g_1}}{4\pi\Gamma(g_1)\sqrt{\tau}} \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} \frac{\log_2(1+\mu)}{\mu\sigma^2(\delta^2)^{g_1+3/2}} \times \exp\left[-\left(\frac{(\ln(\mu/\bar{\mu})+\sigma^2)^2}{8\sigma^2} + \frac{(\bar{\mu}-m)^2}{2\tau\delta^2} + \frac{g_2}{\delta^2}\right)\right] d\mu\, d\bar{\mu}\, d\delta^2 \tag{26}$$

Solving numerically Eq. (26), using a Monte Carlo computational scheme [26,27], we obtain the average capacity of the free space optical link using the Bayesian approach.

The BER of the FSO link, using (13), is obtained from the following integral:

$$P_{av} = \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} Q\left(\sqrt{\frac{\mu}{2}}\right) f_{\mu}(\mu|\bar{\mu},\delta^2) g(\bar{\mu}|\delta^2) h(\delta^2)\, d\mu\, d\bar{\mu}\, d\delta^2 \tag{27}$$

which using (6), (23), (24) and the expression $erfc(x) = 2Q(\sqrt{2}x)$, is taking the following form:

$$P_{av} = \frac{g_2^{g_1}}{8\pi\Gamma(g_1)\sqrt{\tau}} \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} \frac{1}{\mu\sigma^2(\delta^2)^{g_1+3/2}} \times \exp\left[-\left(\frac{(\ln(\mu/\bar{\mu})+\sigma^2)^2}{8\sigma^2} + \frac{4(\bar{\mu}-m)^2}{2\tau\delta^2} + \frac{g_2}{\delta^2}\right)\right] d\mu\, d\bar{\mu}\, d\delta^2 \tag{28}$$

Moreover, the outage probability of the FSO link, using (22), can be evaluated from the following mathematical expression:

$$P_{out} = Pr(\mu \leqslant \mu_{th}) = \int_0^{+\infty} \int_0^{+\infty} \int_0^{1} f_{\mu^*}(\mu^*|\bar{\mu}^*,\delta^2) g(\bar{\mu}^*|\delta^2) h(\delta^2)\, d\mu^*\, d\bar{\mu}^*\, d\delta^2 \tag{29}$$

where $\mu^* = \mu/\mu_{th}$ and $\bar{\mu}^* = \bar{\mu}/\mu_{th}$ are the normalized instantaneous and average electrical *SNR*, respectively. The probability of (29), using (6), (23) and (24), is given by

$$P_{out} = \frac{g_2^{g_1}}{4\pi\Gamma(g_1)\sqrt{\tau}} \int_0^{+\infty} \int_0^{+\infty} \int_0^1 \frac{1}{\mu^* \sigma^2 (\delta^2)^{g_1+3/2}} \times \exp\left[-\left(\frac{(\ln(\mu^*/\bar{\mu}^*)+\sigma^2)^2}{8\sigma^2} + \frac{(\bar{\mu}^* - m^*)^2}{2\tau^*\delta^2} + \frac{g_2}{\delta^2}\right)\right] d\mu^*\, d\bar{\mu}^*\, d\delta^2 \tag{30}$$

where m^* and τ^* are the new parameters for the normal distribution for the normalized average electrical *SNR*.

5. Numerical results

As mentioned above, the performance of an FSO link depends strongly on the atmospheric conditions in the space between the transmitter and the receiver. We investigate the performance of such a system and we present our results using the above-mentioned statistical methods: the frequentist and the Bayesian approach. The results that we present below have been taken by considering a link with length $L = 5000\,\mathrm{m}$, operation wavelength $\lambda = 1.55\,\mu\mathrm{m}$, and aperture diameter of the receiver $D = 0.01\,\mathrm{m}$. It is clear that similar results can be obtained for any other value of the above-mentioned parameters.

In the frequentist approximation, we suppose that the atmospheric turbulence strength and the *SNR* at the receiver have steady values for long time. Thus, in order to evaluate the average capacity, the average BER and the outage probability we use four values for ${C_n}^2$, for weak to moderate turbulence conditions, for different values of the average electrical *SNR* at the receiver. However, the atmospheric conditions in the space between the transmitter and the receiver are constantly changing. As a result, the atmospheric turbulence strength varies over time, as the average electrical *SNR* at the receiver. In order to be more accurate in our estimations, we take into account these fluctuations around a mean value; at the estimation of the average capacity, the average BER and the outage probability we should use the Bayesian perspective.

In order to obtain the results that we present below we used experimental data [16], to model the fluctuations of the atmospheric turbulence strength. We assume that the distribution of the value of δ^2, which depends directly on ${C_n}^2$, follows an inverse gamma distribution, Eq. (24). In order to estimate the parameters of the distribution, we consider that the values of ${C_n}^2$, fluctuate 15% around the mean value which is in accordance with the experimental results of Ref. [23]. It is clear that the value of this rate depends on the atmospheric turbulence conditions in the space between the transmitter and the receiver of the FSO link and it can be smaller or larger. The results that we present in this work concern two values of ${C_n}^2$ for weak turbulence conditions [i.e. ${C_n}^2 = 4.0\times10^{-15}\,\mathrm{m}^{-2/3}$, ${C_n}^2 = 8.0\times10^{-15}\,\mathrm{m}^{-2/3}$] and two for moderate [i.e. ${C_n}^2 = 1.4\times10^{-14}\,\mathrm{m}^{-2/3}$, ${C_n}^2 = 1.8\times10^{-14}\,\mathrm{m}^{-2/3}$]. Thus, for ${C_n}^2 = 4.0\times10^{-15}\,\mathrm{m}^{-2/3}$, the corresponding value of δ^2 is 0.619 and the parameters of the distribution are $g_1 = 6.13$ and $g_2 = 3.17$, for ${C_n}^2 = 8.0\times10^{-15}\,\mathrm{m}^{-2/3}$, $\delta^2 = 1.238$, $g_1 = 10.25$ and $g_2 = 11.45$, for ${C_n}^2 = 1.4\times10^{-14}\,\mathrm{m}^{-2/3}$, $\delta^2 = 2.166$, $g_1 = 16.44$ and $g_2 = 33.45$, and for ${C_n}^2 = 1.8\times10^{-14}\,\mathrm{m}^{-2/3}$, $\delta^2 = 2.785$, $g_1 = 20.57$ and $g_2 = 54.51$. For the variation of the average electrical *SNR*, $\bar{\mu}$, and the normalized average electrical *SNR*, $\bar{\mu}^*$, at the receiver we use a normal distribution

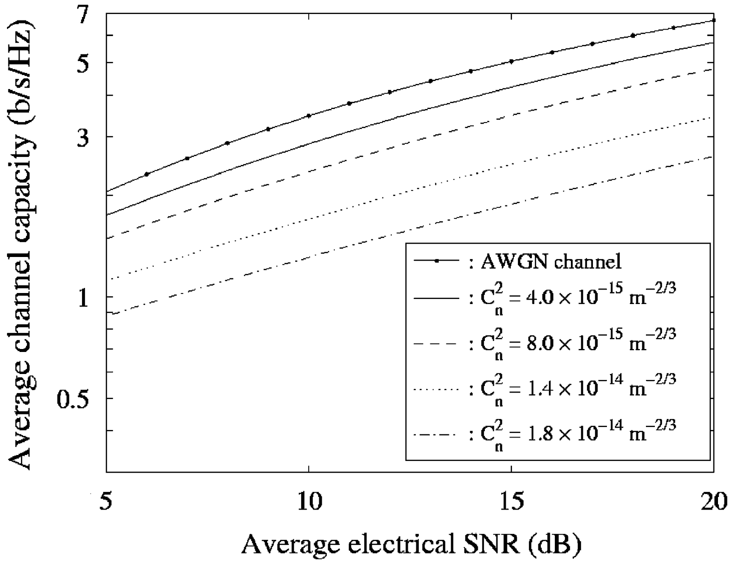

Fig. 1. Average channel capacity, $\langle C \rangle / B$, for weak and moderate atmospheric turbulence conditions, versus the average electrical SNR, $\bar{\mu}$. These results have been obtained with the frequentist approximation for $\lambda = 1.55\,\mu$m, $L = 5000$ m, and $D = 0.01$ m. The capacity of the AWGN channel is plotted too.

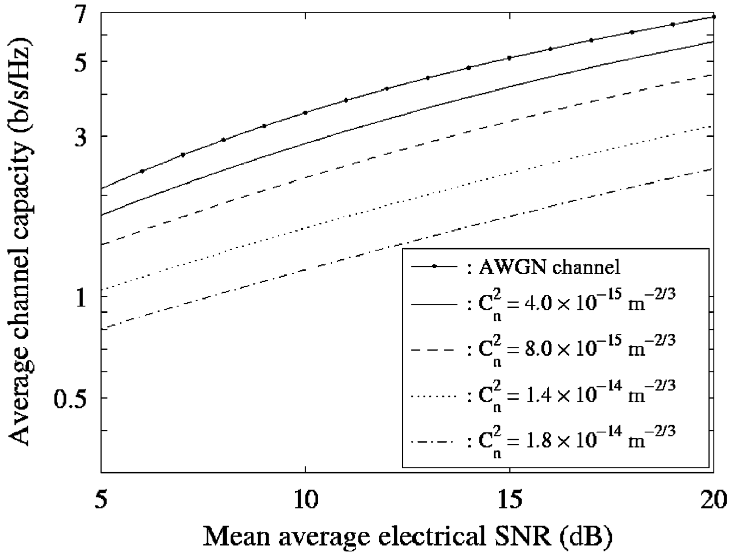

Fig. 2. Average channel capacity, $\langle C \rangle / B$, for weak and moderate atmospheric turbulence conditions, versus the mean average electrical SNR, m. These results have been obtained with the Bayesian approximation for $\lambda = 1.55\,\mu$m, $L = 5000$ m, and $D = 0.01$ m. The capacity of the AWGN channel is plotted too.

and for the evaluation of the distribution's parameter, τ, we consider that the SNR at the receiver fluctuates up to 1 dB, around the mean values m and m^*, respectively. Thus, we use a value of τ so that the product $\tau\delta^2$ always is 1.245.

Using the above-mentioned parameters we evaluate the average capacity, the average BER and the outage probability of an FSO channel, which are three significant metrics for performance and reliability and we study their dependence on the atmospheric turbulence conditions.

In Figs. 1 and 2, we present the normalized, average channel capacity, $\langle C \rangle / B$, of the FSO link, as a function of the average electrical SNR at the receiver, $\bar{\mu}$, for the frequentist

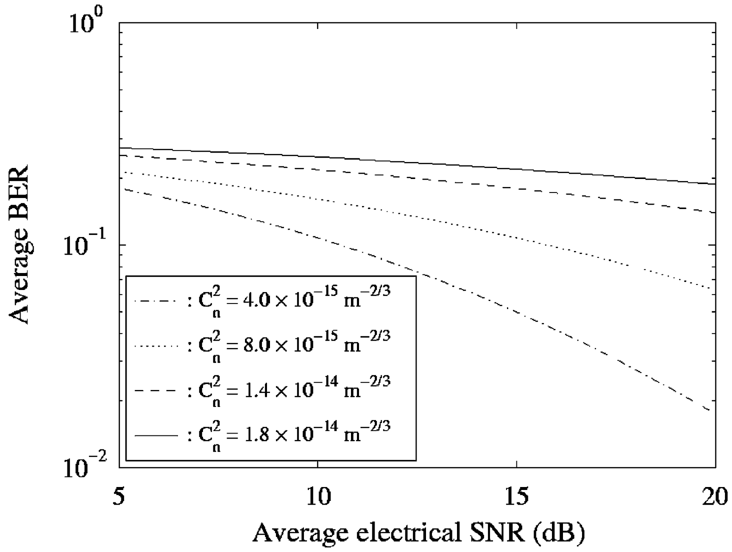

Fig. 3. Average BER, P_{av}, for weak and moderate atmospheric turbulence conditions, versus the average electrical *SNR*, $\bar{\mu}$. These results have been obtained with the frequentist approximation for $\lambda = 1.55\,\mu$m, $L = 5000$ m, and $D = 0.01$ m.

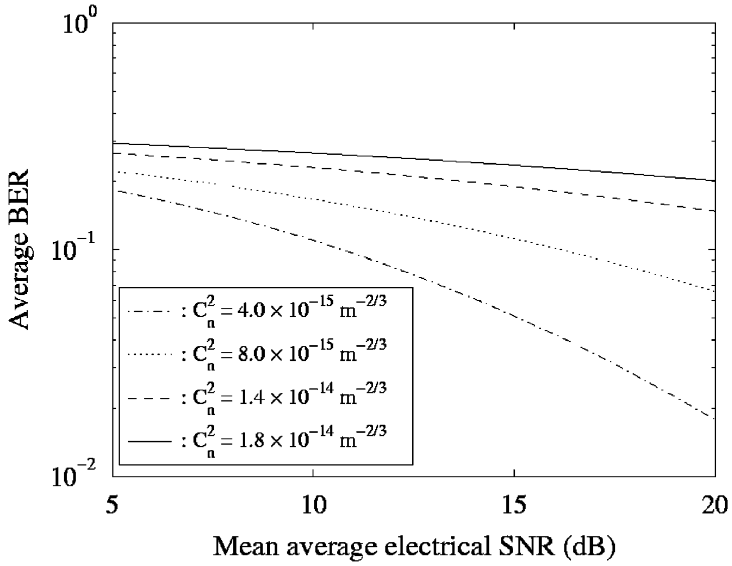

Fig. 4. Average BER, P_{av}, for weak and moderate atmospheric turbulence conditions, versus the mean average electrical *SNR*, m. These results have been obtained with the Bayesian approximation for $\lambda = 1.55\,\mu$m, $L = 5000$ m, and $D = 0.01$ m.

approximation case, and the mean average electrical *SNR*, m, for the Bayesian. For example, for the case of weak turbulence (i.e. ${C_n}^2 = 4.0 \times 10^{-15}\,\mathrm{m}^{-2/3}$), for (mean) average electrical *SNR* 20 dB, the Bayesian estimation of the average capacity gives 2.1% smaller result that this of the frequentist approximation, while for the case of moderate turbulence (i.e. ${C_n}^2 = 1.8 \times 10^{-14}\,\mathrm{m}^{-2/3}$), the respective rate is 8.2%. Moreover, in these figures, we plotted the capacity of an AWGN channel, $\langle C \rangle / B = \log_2 + (1 + \bar{\mu})$, which represents the capacity of the optical channel, without the influence of the atmospheric turbulence conditions.

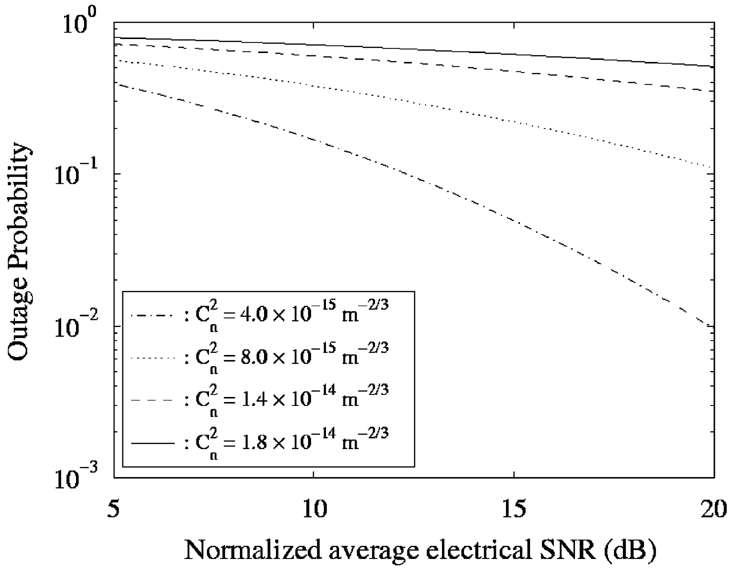

Fig. 5. Outage probability, P_{out}, for weak and moderate atmospheric turbulence conditions, versus the normalized average electrical SNR, $\bar{\mu}^*$. These results have been obtained with the frequentist approximation for $\lambda = 1.55\,\mu\text{m}$, $L = 5000\,\text{m}$, and $D = 0.01\,\text{m}$.

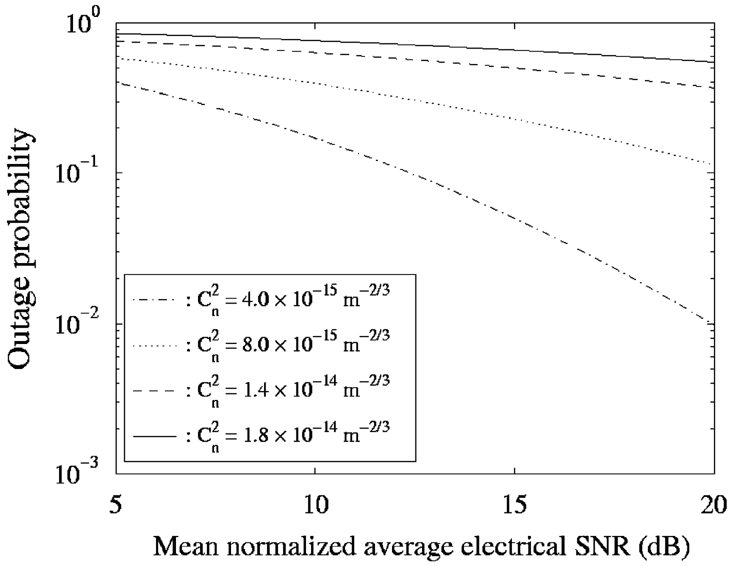

Fig. 6. Outage probability, P_{out}, for weak and moderate atmospheric turbulence conditions, versus the mean normalized average electrical SNR, m^*. These results have been obtained with the Bayesian approximation for $\lambda = 1.55\,\mu\text{m}$, $L = 5000\,\text{m}$, and $D = 0.01\,\text{m}$.

In Figs. 3 and 4, the average BER of the FSO link is plotted as a function of the average electrical SNR, $\bar{\mu}$, and the mean average electrical SNR, m, for the cases of the frequentist and Bayesian approximation, respectively. The results that obtained for invariable values of turbulence strength and average electrical SNR have smaller values than those, which have been evaluated with the Bayesian approximation. Thus, for weak turbulence conditions (i.e. $C_n^2 = 4.0 \times 10^{-15}\,\text{m}^{-2/3}$) and $\bar{\mu} = m = 20\,\text{dB}$, the Bayesian estimation of the average BER gives 1.8% larger result than that of the frequentist approximation. On the other hand, for the case of moderate turbulence (i.e. $C_n^2 = 1.8 \times 10^{-14}\,\text{m}^{-2/3}$), the respective rate is 6.9%.

In Figs. 5 and 6, the outage probability is plotted as a function of the normalized electrical SNR, $\bar{\mu}^*$ for the case of the frequentist estimation, and the mean normalized electrical SNR, m^*, for the Bayesian. This metric is less sensitive on the variation of the atmospheric turbulence conditions and SNR fluctuations. As a result, for weak turbulence conditions (i.e. ${C_n}^2 = 4.0 \times 10^{-15}\,\mathrm{m}^{-2/3}$) and $\bar{\mu}^* = m^* = 20\,\mathrm{dB}$, the frequentist estimation gives only 1.3% smaller result than that of the Bayesian approximation. This happens in the case of moderate turbulence conditions (i.e. ${C_n}^2 = 1.8 \times 10^{-14}\,\mathrm{m}^{-2/3}$) too, where the respective rate is 4.7%.

In all the cases presented above it is clear that with the frequentist approximation, the performance and the reliability of the FSO systems is found to be higher than in the Bayesian case. This result is due to the fact that with the Bayesian point of view, the parameters of the system tend to replicate natural circumstances more accurately. Thus, the system's computed average capacity is lower in the Bayesian case, while the average BER and the outage probability are higher.

6. Conclusions

In this work we derived accurate approximate closed-form expressions for the estimation of the average capacity and the average BER of a log-normal FSO channel in the cases of weak to moderate atmospheric turbulence conditions. We evaluate the reliability and the performance of FSO communications systems by means of the evaluation of the outage probability and the average channel capacity and average BER under real circumstances. Taking into account that the turbulence strength and the average electrical SNR at the receiver of a real optical channel cannot remain invariable for long time, we model these parameters following the Bayesian point of view. Thus, evaluating the outage probability, the average capacity and the average BER, using both statistical approaches (frequentist and Bayesian) and quoting the results provides a broader understanding of the evaluation of the performance and the reliability of the free space optical link. Finally, it is worth mentioning that the above methodology can be extended to many research areas besides optical communications such as social sciences and economical modeling where the researcher wishes to estimate average quantities given non-fixed parameters.

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