Bayesian Naturalness of the CMSSM and CNMSSM

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Based on PRD90 (2014) 055008 [arXiv:1312.4150]
in collaboration with P. Athron, C. Balázs, B. Farmer and E. Hutchison

Contents

- Address the naturalness problem
- Definitions of Fine-Tuning
- Compare Numerical results for CMSSM and CNMSSM
- Discussion and Summary

Naturalness Problem

- Fine-tuning problem of Higgs mass :
  Defined by a tension between gravity and weak interaction.

- In supersymmetric models, the big hierarchy problem is translated into the little hierarchy problem between the EWSB & SUSY breaking scale.

Naturalness Problem

- Fine-tuning problem of Higgs mass :
  Defined by a tension between gravity and weak interaction.

- In supersymmetric models, the big hierarchy problem is translated into the little hierarchy problem between the EWSB & SUSY breaking scale.

- How do we define the Fine-tuning problem?

Fine-tuned Higgs Mass & SUSY

- In the MSSM, it is hard to find a natural solution to the Higgs mass

$$m^2_{h\,\mathrm{Tree}} \le M_Z^2$$

- μ Problem

$$: \quad \frac{M_Z^2}{2} = \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

- Next-to-MSSM ameliorates the situation introducing additional scalar S
  : Lifts up m_h^2 more at tree level
  : All mass scales are introduced at the SUSY breaking scale

NMSSM better than MSSM

- It is generally believed that the additional F-term helps to relax the tension between Mz and Mh.

- For CMSSM, extensive studies have suggested the problem to get a realistic Higgs mass with low fine-tuning.

- If the singlet vev helps to increase the Mh, then it will reduce the fine-tuning. Then will the CNSSM also be better than the CMSSM?

Definitions of Fine-tuning

- $\frac{\delta m_h}{m_h}$,or $\frac{\delta m_Z}{m_Z}$: Compare the size of quantum fluctuation of $m_{h/Z}$, relative to its tree mass.

  H. Baer, et al., PRL 109, 161802 (2012) [arXiv:1207.3343]

- $\Delta_{BG} \equiv \max \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i^2} \right|$: Sensitivity of EW observable to model parameters.

  R. Barbieri and G. F. Giudice, NPB 306, 63 (1988)
  J. Ellis, et al., Mod. Phys. Lett. A 1, 57 (1986)
  G. F. Giudice, [arXiv:1307.7879]

- $\Delta_J = \left| \frac{\partial \ln \mathcal{O}_j^2}{\partial \ln p_i^2} \right|$: Counts the correlations bet. the observables.

  B. Allanach, et al., JHEP 0708, 023 (2007) [arXiv:0705.0487]
  M. Cabrera, et al., JHEP 0903, 075 (2009) [arXiv:0812.0536]

Definitions of Fine-tuning

- Δ_{EW} : Hierarch Based
  Focus on Radiative Stability & Cancellation

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

Each terms : C_i

e.g. $C_{H_u} = -m_{H_u}^2/(\tan^2\beta - 1)/(M_Z^2/2)$

$$\Delta_{EW} \equiv \max(C_i)$$

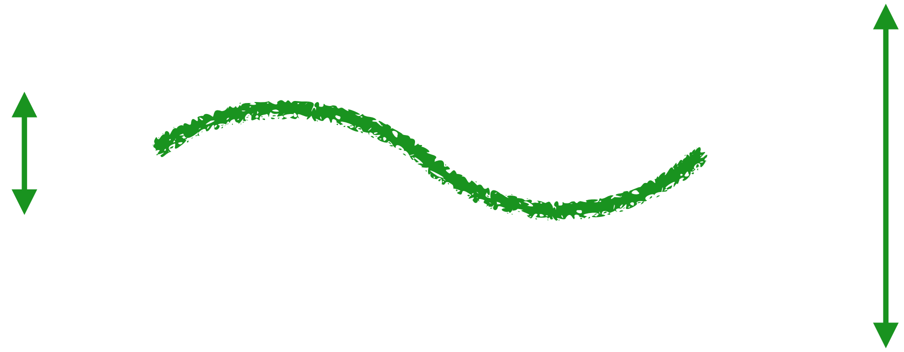
H. Baer et al, PRL 109, 161802 (2012) [arXiv:1207.3343]

Definitions of Fine-tuning

- Δ_{BG} : Focus on the Stable adjustment of parameters to fit data
  Usually Mz, Single variable

$$\Delta_{BG} \equiv \max \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i^2} \right|$$

M_Z



$p_i = \mu,\ B,\ m_0,\ m_{1/2}, A_0$

G. F. Giudice, [arXiv:1307.7879]

Definitions of Fine-tuning [arXiv:1312.4150]

- In Bayesian Analysis

$$p(\mathcal{M}|\mathcal{D}) = \frac{p(\mathcal{D}|\mathcal{M})}{p(\mathcal{D})}p(\mathcal{M}) = \frac{1}{p(\mathcal{D})}\int p(\mathcal{D}|p_i)p(p_i)dp_i$$

- For CMSSM

$$\int \mathcal{L}p(\mu, B, y)d\mu dBdy = \int \mathcal{L}\,|J_{\mathcal{T}_1}|\,p(M_Z, y, m_t)dM_Z dm_t dt$$

$$\mathcal{T}_1 : \{\mu, B, y\} \to \{M_z, t, m_t\}$$

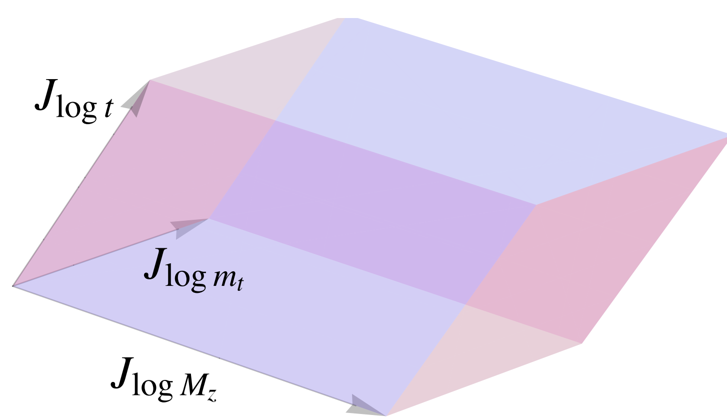
M. E. Cabrera, J. A. Casas and R. Ruiz de Austri, JHEP 1005, 043 (2010) [arXiv:0911.4686]

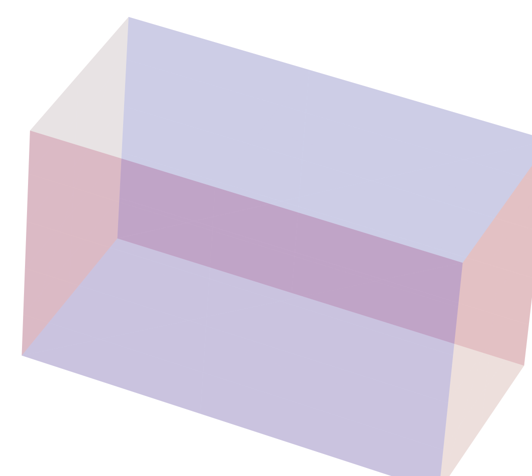
Definitions of Fine-tuning [arXiv:1312.4150]

- Δ_J : Same as Δ_{BG} but deals with a SET of low energy variables (all vevs)

  & the correlation among them

$$\Delta_J = \left| \frac{\partial \ln \mathcal{O}_j^2}{\partial \ln p_i^2} \right| \longrightarrow \frac{\delta V_{\mathcal{O}}}{\delta V_p}$$





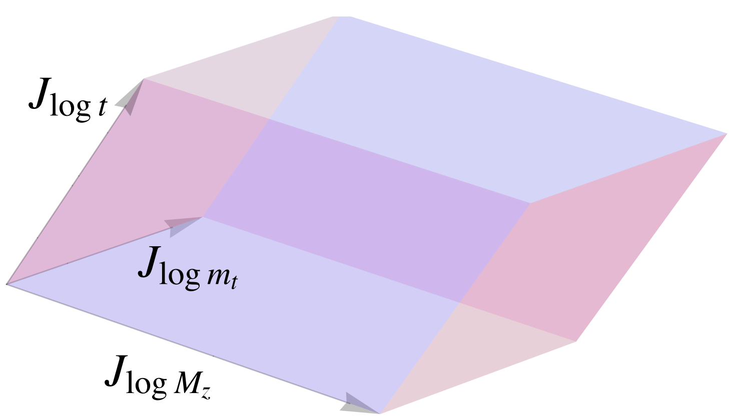
Definitions of Fine-tuning [arXiv:1312.4150]

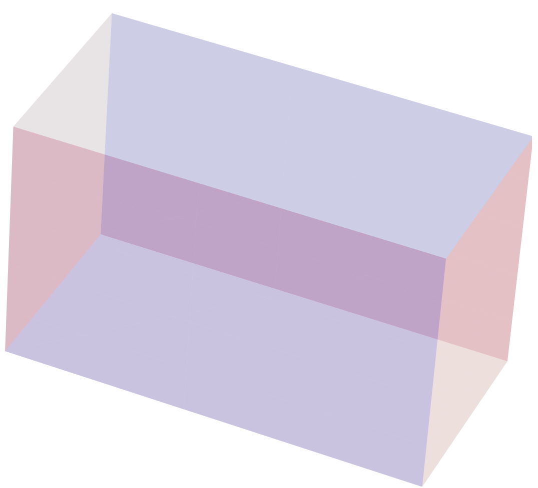
- For CMSSM

$$\Delta_J = \left| \frac{\partial \ln(M_Z^2, \tan^2\beta, m_t^2)}{\partial \ln(\mu^2, B^2, y_t^2)} \right|$$

- For CNMSSM

$$\Delta_J = \left| \frac{\partial \ln(M_Z^2, \tan^2\beta, s^2, m_t^2)}{\partial \ln(\lambda^2, \kappa^2, m_S^2, y_t^2)} \right|$$





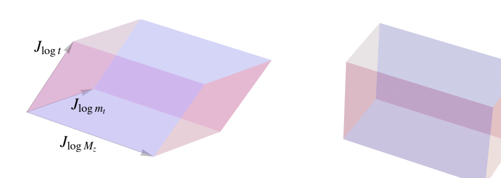
Definitions of Fine-tuning [arXiv:1312.4150]

- For CMSSM

$$\Delta_J^{-1}\Big|_{\mathrm{CMSSM}} = \frac{M_Z^2}{2\mu^2}\frac{B}{B_0}\frac{t^2-1}{t^2+1}\frac{\partial \ln y^2}{\partial \ln y_0^2}$$

- For CNMSSM

$$\Delta_J^{-1}\Big|_{\mathrm{CNMSSM}} = \begin{vmatrix} b_1 & e_1 & f_1 \\ b_2 & e_2 & f_2 \\ b_3 & e_3 & f_3 \end{vmatrix}$$



- Convergence of NMSSM to MSSM

$$\begin{array}{l} \lambda, \kappa \to 0 \\ m_S^2/A_\kappa^2 \to 0 \end{array} \longrightarrow \quad \Delta_J\big|_{\mathrm{CNMSSM}} \to \Delta_J\big|_{\mathrm{CMSSM}}$$

$$
\begin{aligned}
d\lambda = & -\frac{\lambda}{s}ds + \frac{1}{2\lambda s^2}\frac{2t}{(t^2-1)^2}(m_{H_u}^2 - m_{H_d}^2)dt - \frac{1}{4\lambda s^2}dM_z^2 + \frac{1}{2\lambda s^2}\frac{dm_{H_d}^2 - t^2 dm_{H_u}^2}{t^2-1} \\
& + \frac{1}{2\lambda s^2}\frac{\partial \mu^2}{\partial y_t^2}dy_t^2 \\
\equiv & \; b_1 ds + b_2 dt + b_3 dM_z^2 + b_5 dy_t^2 + b_7 dm_{H_u}^2 + b_8 dm_{H_d}^2,
\end{aligned}
$$

$$
\begin{aligned}
0 = & \left[\frac{A_\lambda + \kappa s}{\lambda s} - 2\sin 2\beta\left(1 + \frac{M_z^2}{\bar{g}^2 s^2}\right)\right] d\lambda - \frac{2\lambda s \sin 2\beta - (A_\lambda + 2\kappa s)}{s^2}ds \\
& - \frac{1-t^2}{(1+t^2)^2}\frac{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2\left(1 + \frac{M_z^2}{\bar{g}^2 s^2}\right)}{\lambda s^2}dt - \frac{\lambda \sin 2\beta}{\bar{g}^2 s^2}dM_z^2 + d\kappa \\
& + \frac{1}{\lambda s^2}\frac{t}{1+t^2}\frac{2\lambda^2 M_z^2}{\bar{g}^4}d\bar{g}^2 - \frac{1}{\lambda s^2}\frac{t}{1+t^2}(dm_{H_u}^2 + dm_{H_d}^2) + \frac{1}{s}dA_\lambda - \frac{1}{\lambda s^2}\frac{\partial B\mu}{\partial y_t^2}dy_t^2 \\
\equiv & \; e_0 d\lambda + e_1 ds + e_2 dt + e_3 dM_z^2 + e_4 d\kappa + e_5 dy_t^2 + e_6 d\bar{g}^2 + e_{7,8} dm_{H_{u,d}}^2 + e_9 dA_\lambda,
\end{aligned}
$$

$$
\begin{aligned}
dm_S^2 = & -\frac{4M_z^2}{\bar{g}^2 s}\left[\lambda s - \frac{t}{1+t^2}(A_\lambda/2 + \kappa s)\right] d\lambda - \left(4\kappa^2 s + \kappa A_\kappa + \frac{2M_z^2}{\bar{g}^2 s^2}\frac{t}{1+t^2}\lambda A_\lambda\right) ds \\
& + \frac{4M_z^2}{\bar{g}^2 s^2}\frac{1-t^2}{(1+t^2)^2}\lambda s(A_\lambda/2 + \kappa s)dt - \left[2\lambda^2/\bar{g}^2 - \frac{4\lambda}{\bar{g}^2 s}\frac{t}{1+t^2}(A_\lambda/2 + \kappa s)\right] dM_z^2 \\
& - \left[4\kappa s^2 + sA_\kappa - \frac{4M_z^2}{\bar{g}^2 s}\lambda s\frac{t}{1+t^2}\right] d\kappa + \frac{M_z^2}{\bar{g}^2}\left[2\lambda^2/\bar{g}^2 - \frac{4\lambda}{\bar{g}^2 s}\frac{t}{1+t^2}(A_\lambda/2 + \kappa s)\right] d\bar{g}^2 \\
& + \frac{2M_z^2}{\bar{g}^2 s}\frac{t}{1+t^2}\lambda dA_\lambda - \kappa s dA_\kappa + \frac{\partial m_S^2}{\partial y_t^2}dy_t^2 \qquad (2.35) \\
\equiv & \; f_0 d\lambda + f_1 ds + f_2 dt + f_3 dM_z^2 + f_4 d\kappa + f_5 dy_t^2 + f_6 d\bar{g}^2 + f_9 dA_\lambda + f_{10} dA_\kappa. \qquad (2.36)
\end{aligned}
$$

MSSM/NMSSM Scalar Potential

$$
\begin{aligned}
V_{\text{Higgs}} &= \left(|\mu|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\
&\quad + \left[B\mu\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{c.c.}\right] \\
&\quad + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right) \\
&\quad + \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 , \\
m_{H_u}^2 &= -|\mu|^2 + B\mu \cot\beta + (M_z^2/2)\cos 2\beta, \\
m_{H_d}^2 &= -|\mu|^2 + B\mu \tan\beta - (M_z^2/2)\cos 2\beta.
\end{aligned}
$$

$$W \supset \mu H_u H_d$$

$$V \supset B\mu h_u h_d$$

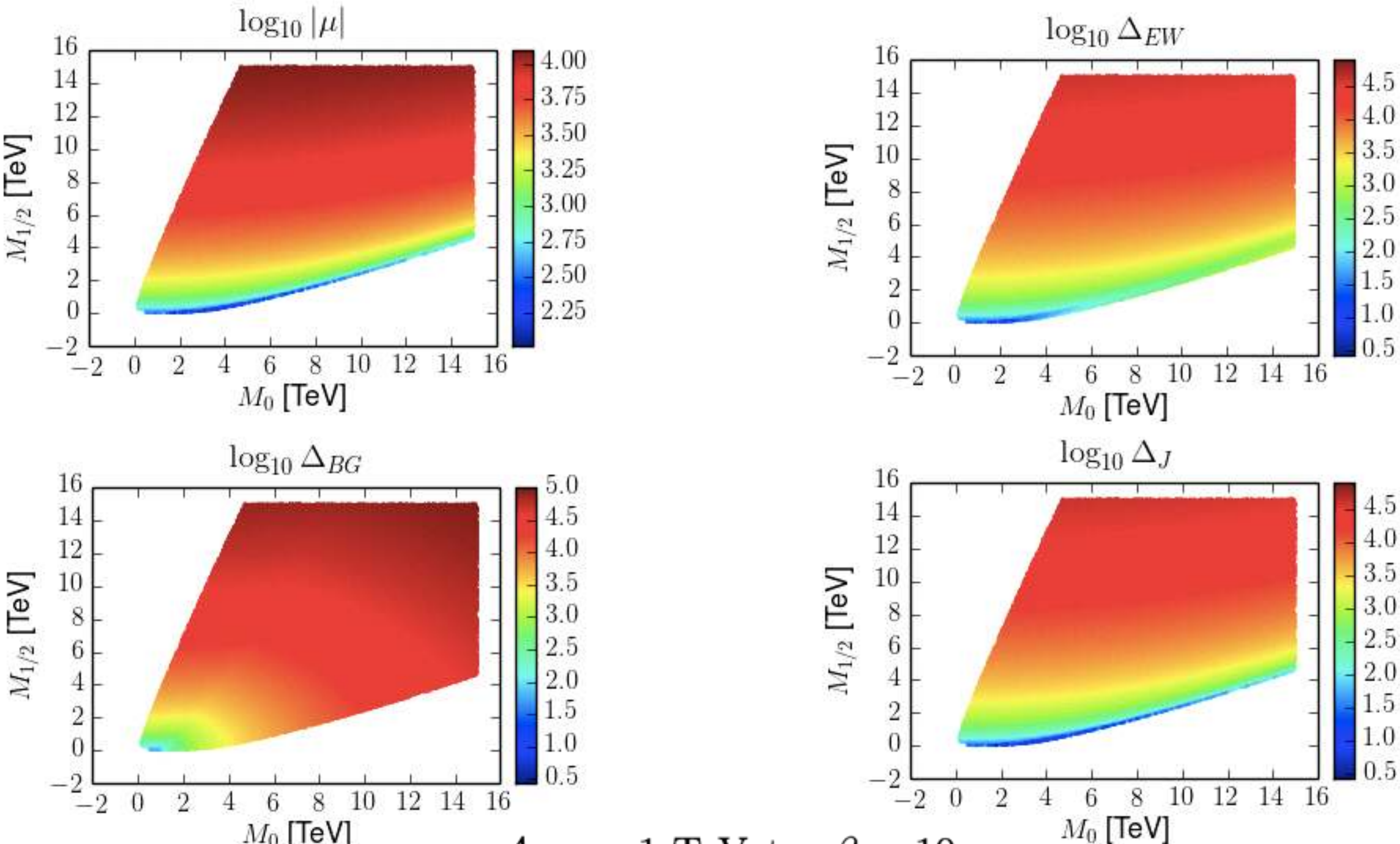
$$
\begin{aligned}
V_{\text{Scalar}}^{Z_3\ \text{NMSSM}} &= \left|\lambda\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \kappa S^2\right|^2 \\
&\quad + \left(|\lambda S|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\lambda S|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\
&\quad + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right) \\
&\quad + \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \\
&\quad + m_S^2 |S|^2 + \left(\lambda A_\lambda \left(H_u^+ H_d^- - H_u^0 H_d^0\right) S + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right), \qquad (2.49) \\
m_{H_u}^2 &= -\lambda^2 s^2 + \lambda s (A_\lambda + \kappa s)\cot\beta + (M_z^2/2)\cos 2\beta - 2\lambda^2 \frac{M_z^2}{\bar{g}^2}\cos^2\beta, \qquad (2.50) \\
m_{H_d}^2 &= -\lambda^2 s^2 + \lambda s (A_\lambda + \kappa s)\tan\beta - (M_z^2/2)\cos 2\beta - 2\lambda^2 \frac{M_z^2}{\bar{g}^2}\sin^2\beta, \qquad (2.51) \\
m_S^2 &= -2\kappa^2 s^2 + 2\lambda(A_\lambda/2 + \kappa s)\frac{M_z^2}{\bar{g}^2 s}\sin 2\beta - \kappa A_\kappa s - 2\lambda^2 \frac{M_z^2}{\bar{g}^2}. \qquad (2.52)
\end{aligned}
$$

$$W \supset \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$V \supset \lambda A_\lambda h_u h_d$$

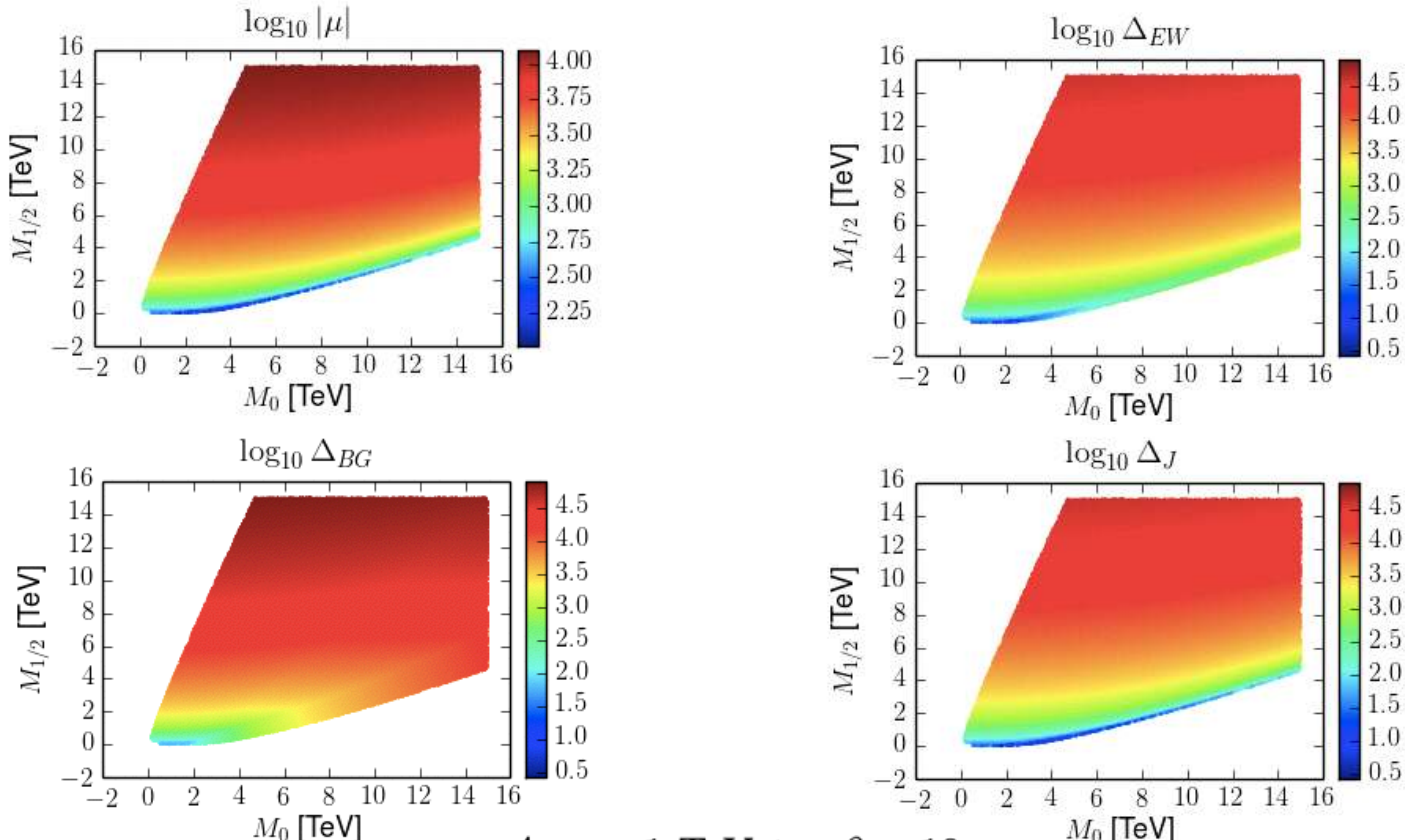
Numerical Results

CMSSM w/o Yukawa



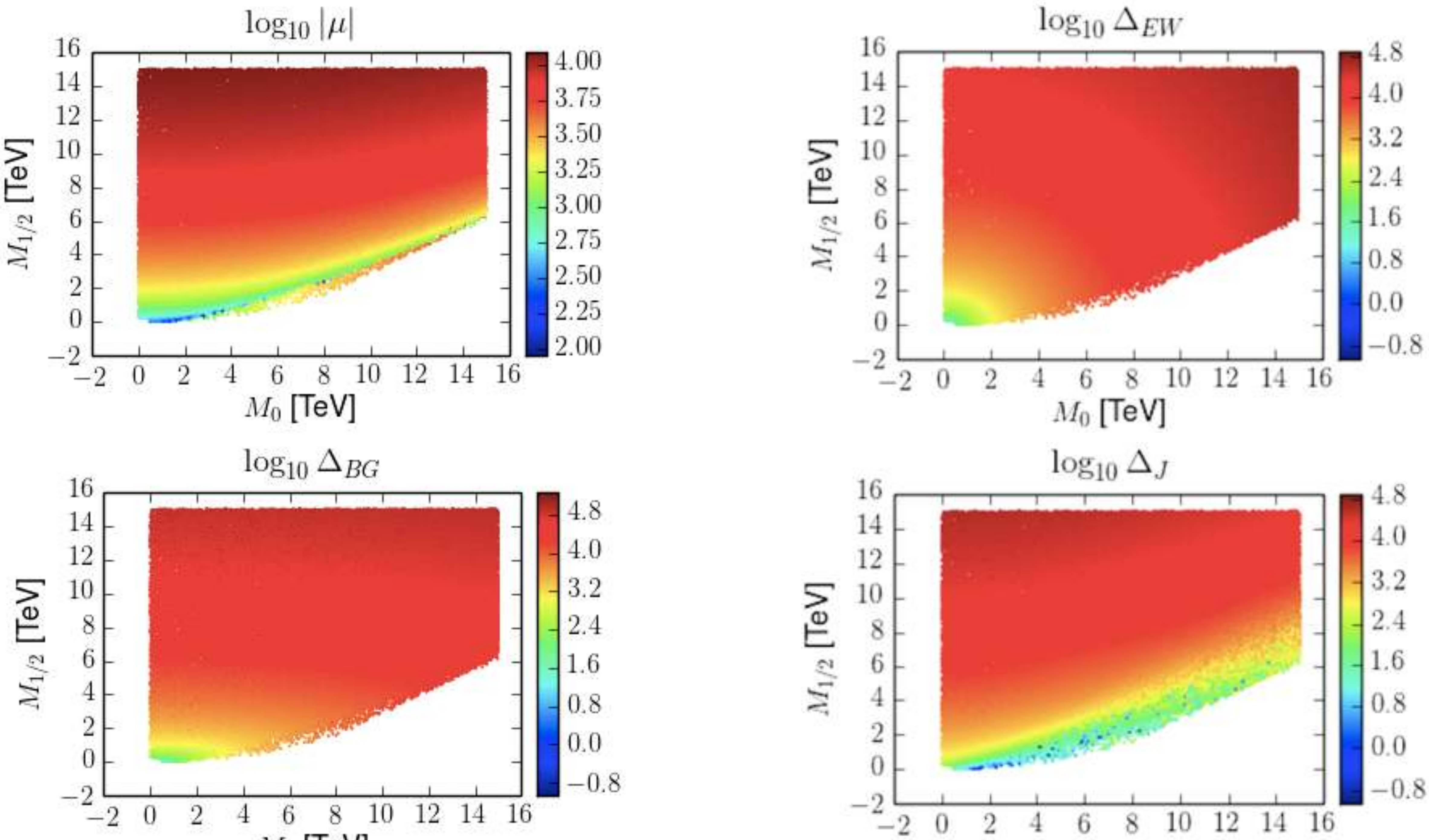
$$A_0 = -1\ \mathrm{TeV}, \tan\beta = 10$$

CMSSM w/ Yukawa



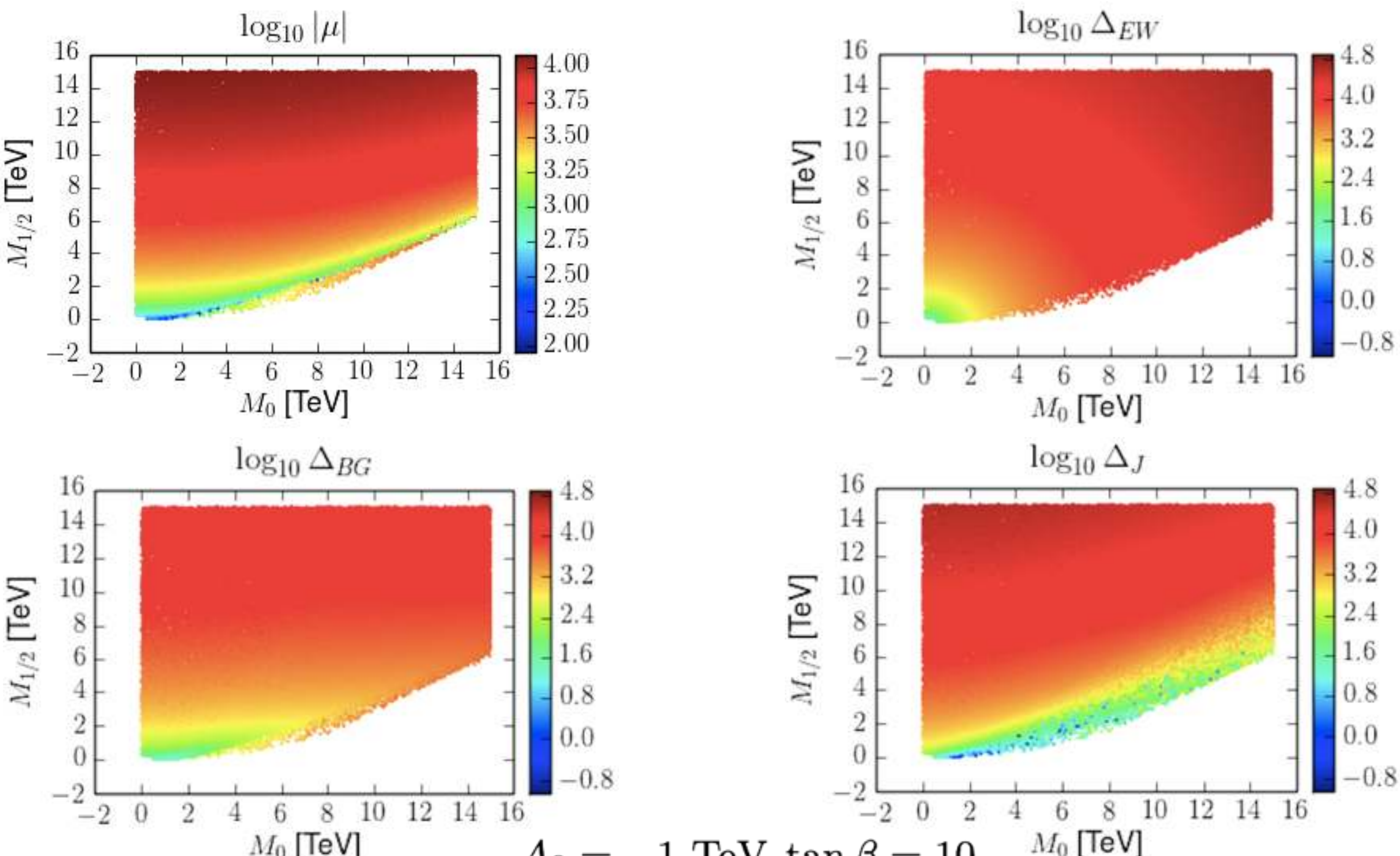
$$A_0 = -1\ \text{TeV}, \tan\beta = 10$$

CNMSSM w/o Yukawa



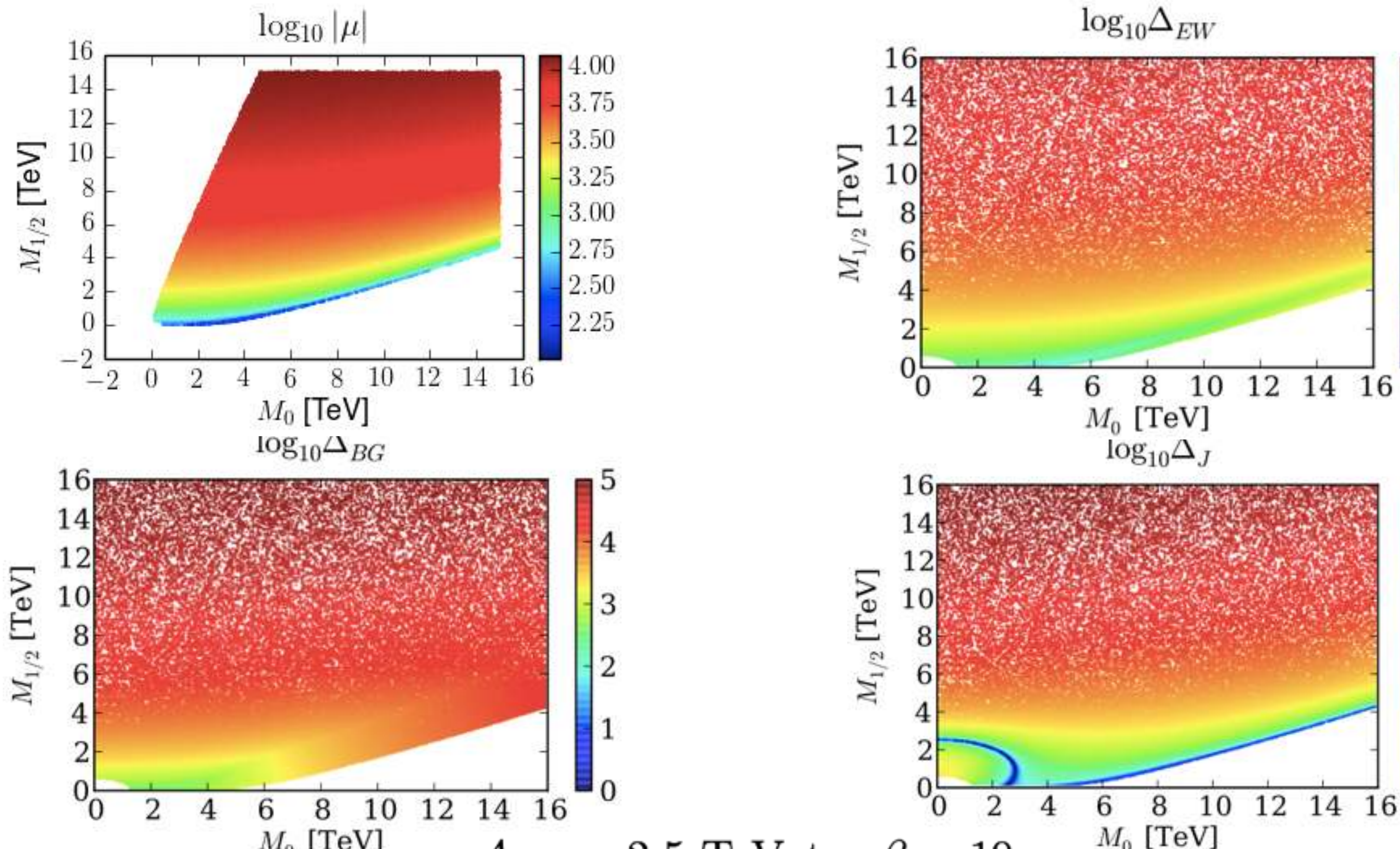
$$A_0 = -1\ \mathrm{TeV}, \tan\beta = 10$$

CNMSSM w/ Yukawa



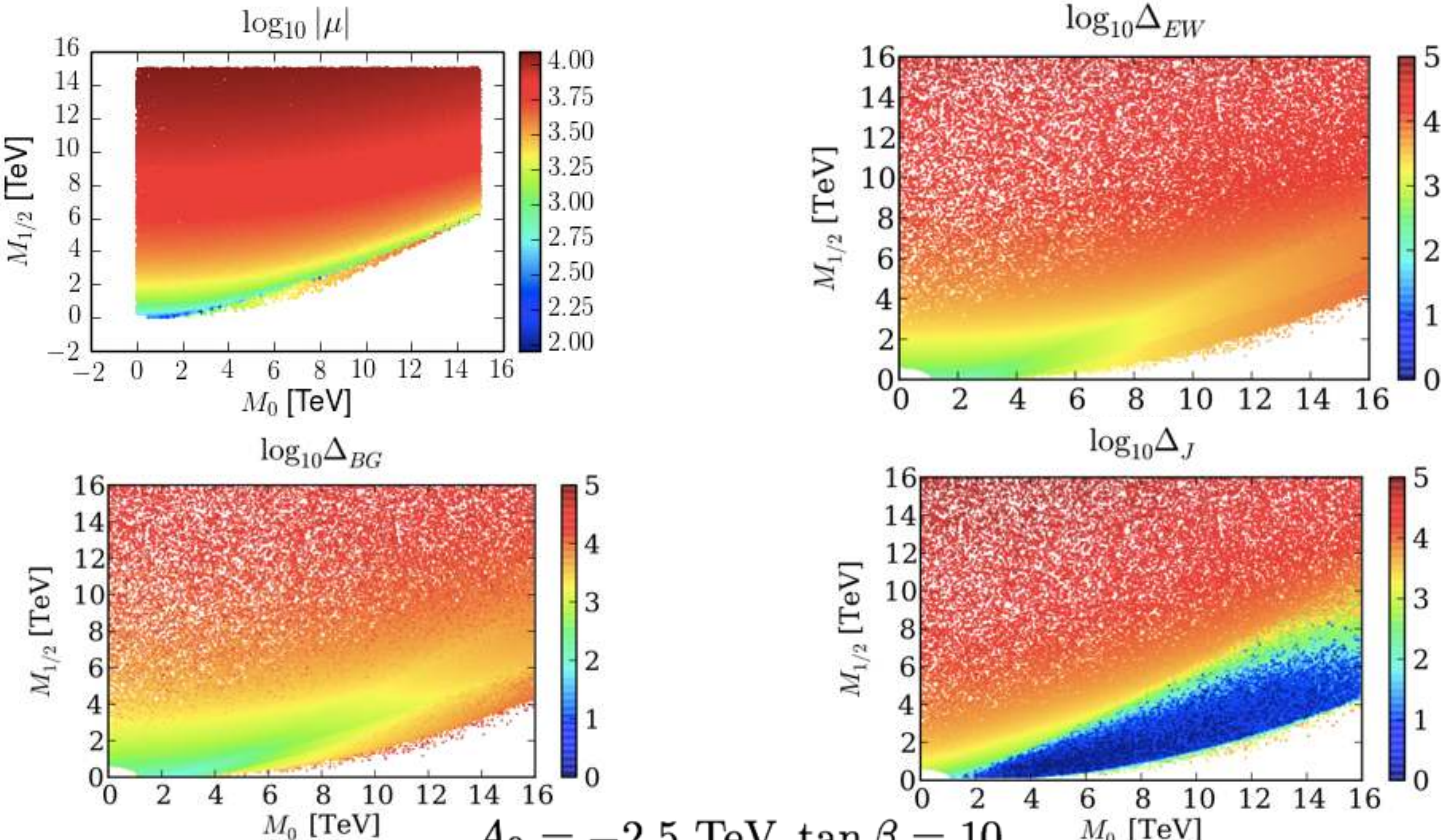
$$A_0 = -1\ \text{TeV}, \tan\beta = 10$$

CMSSM w/ Yukawa



$A_0 = -2.5$ TeV, $\tan\beta = 10$

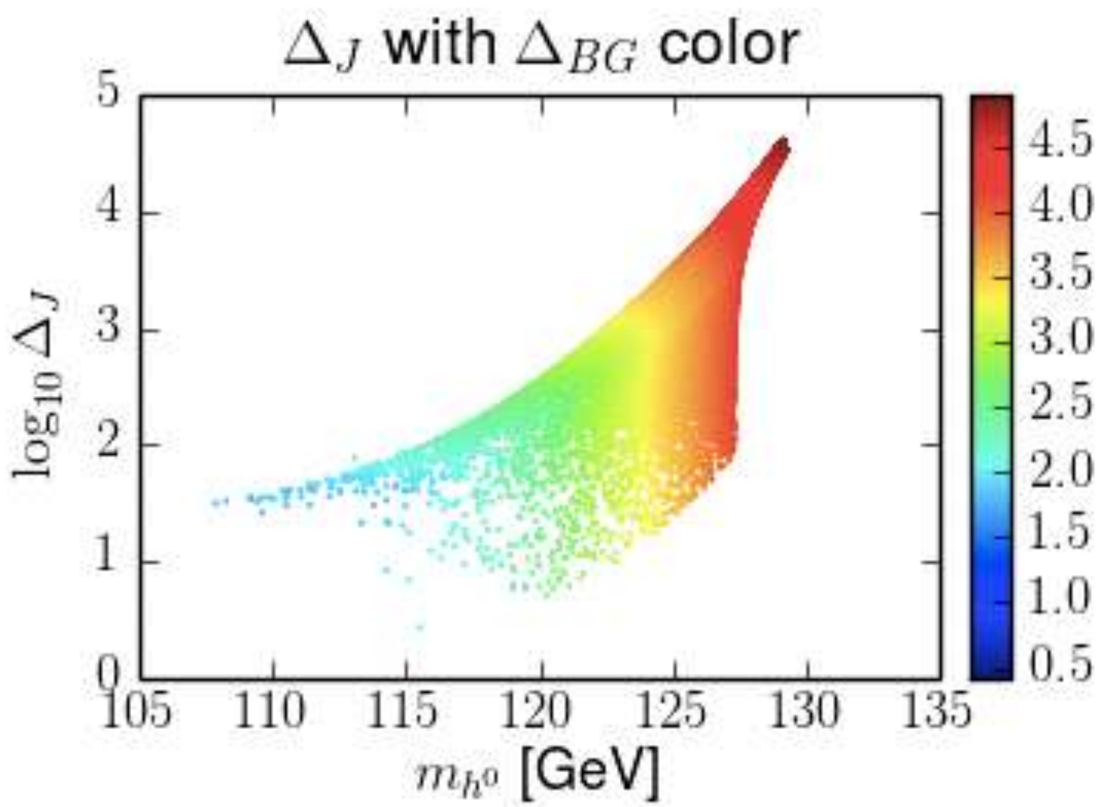
CNMSSM w/ Yukawa

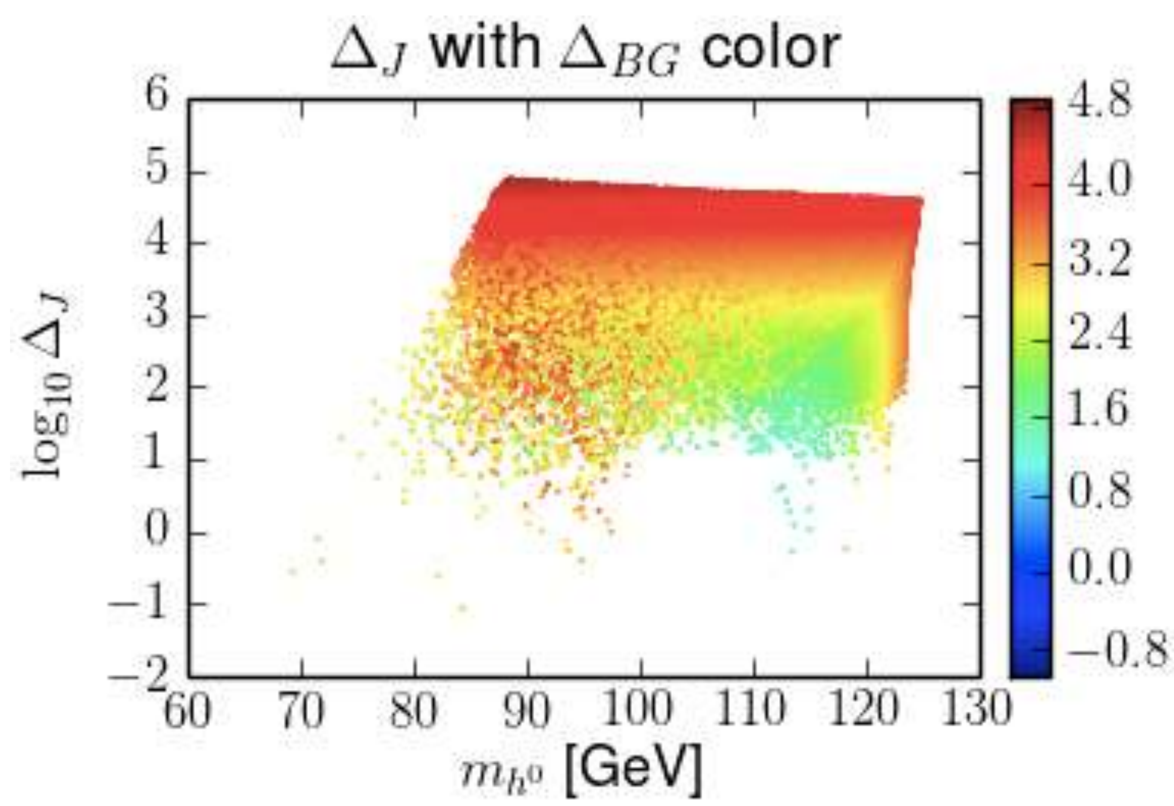


$$A_0 = -2.5 \text{ TeV}, \tan\beta = 10$$

Higgs in CMSSM and CNMSSM

$$A_0 = -1\ \mathrm{TeV}, \tan\beta = 10$$

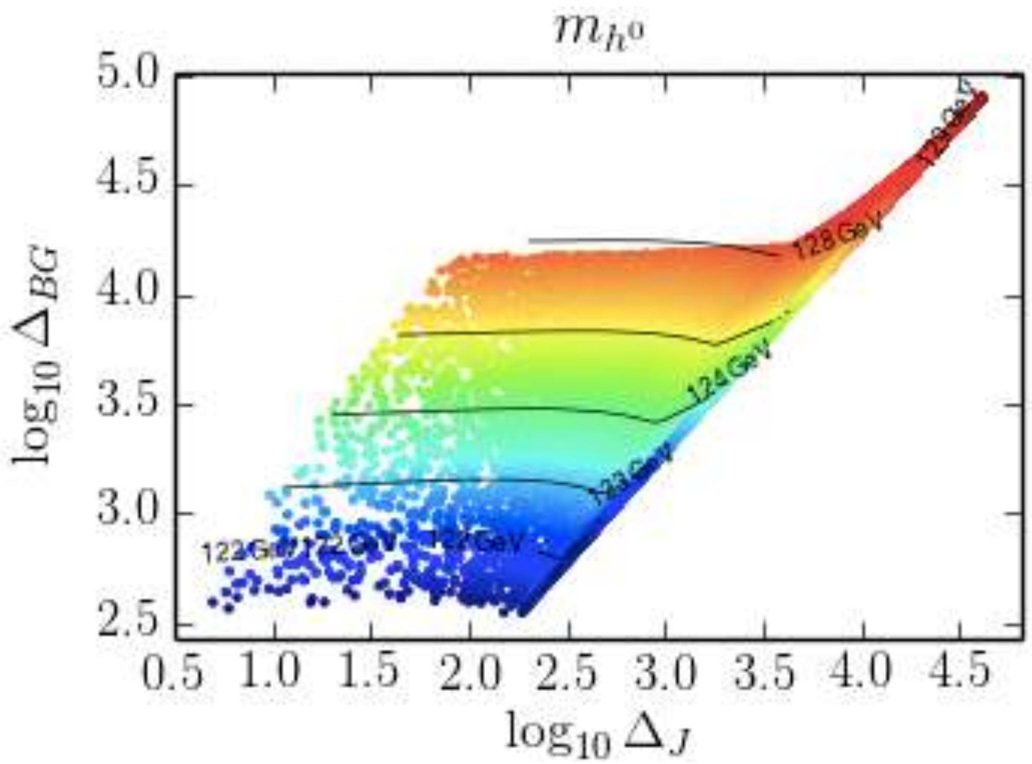


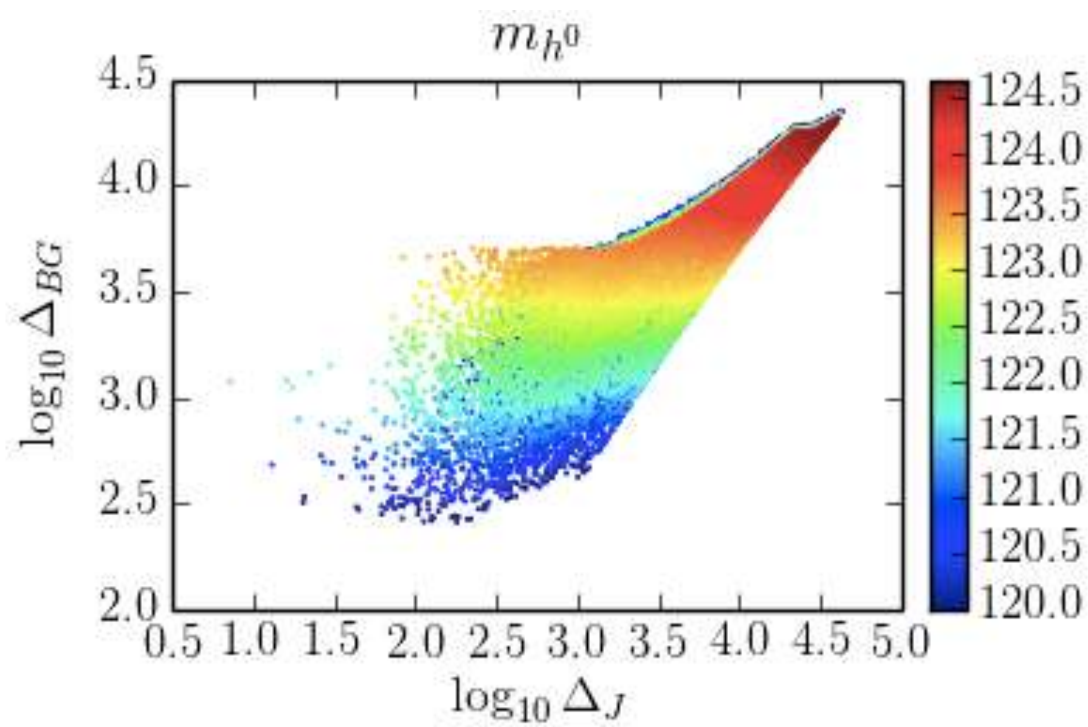


- CMSSM CNMSSM

Higgs for CMSSM and CNMSSM

$$A_0 = -1 \text{ TeV}, \tan\beta = 10$$

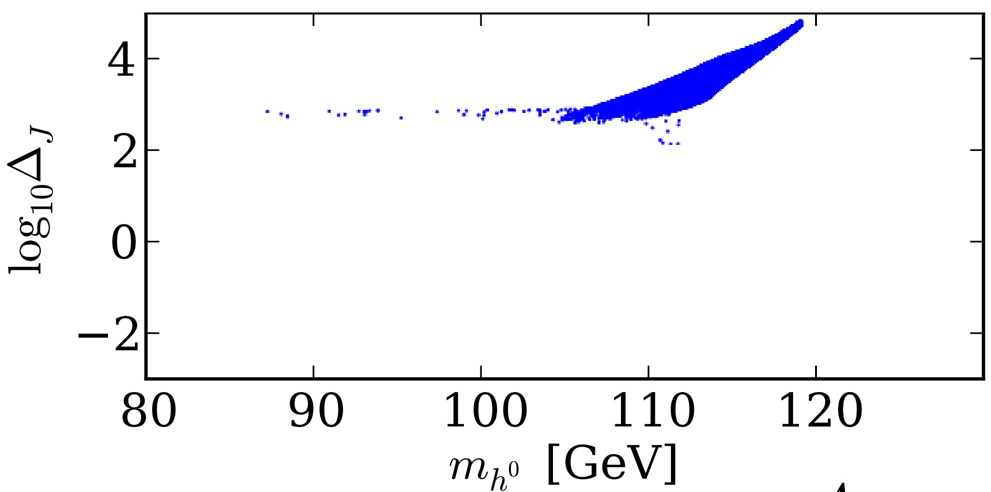


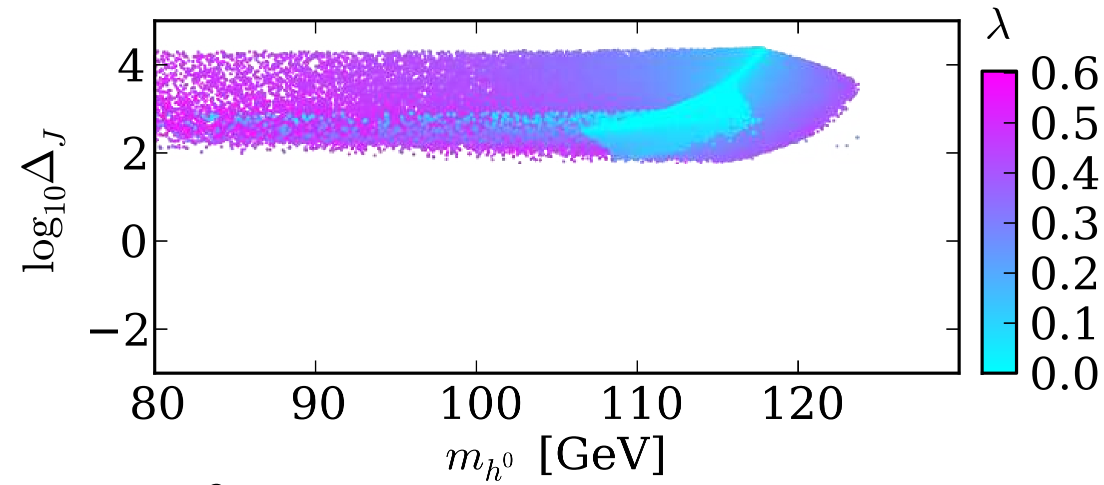


- CMSSM CNMSSM
- It is not a Good idea to simply extend CMSSM to NMSSM!

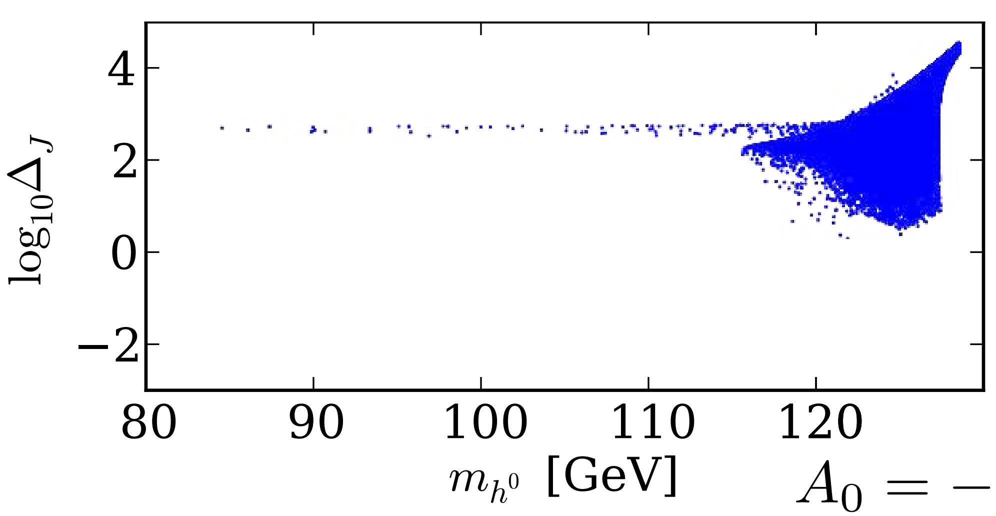
Higgs for CMSSM and CNMSSM

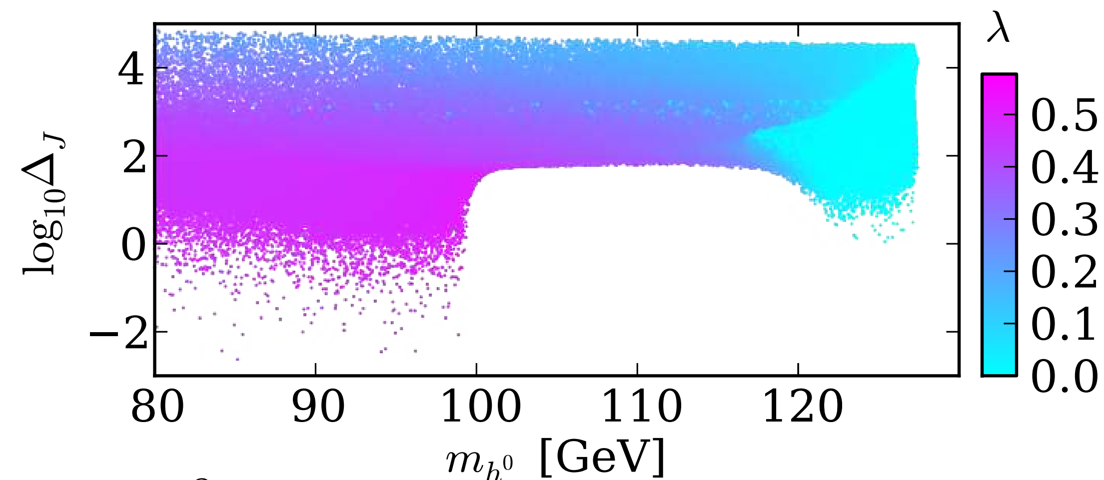
- CMSSM　　　　　　　　CNMSSM





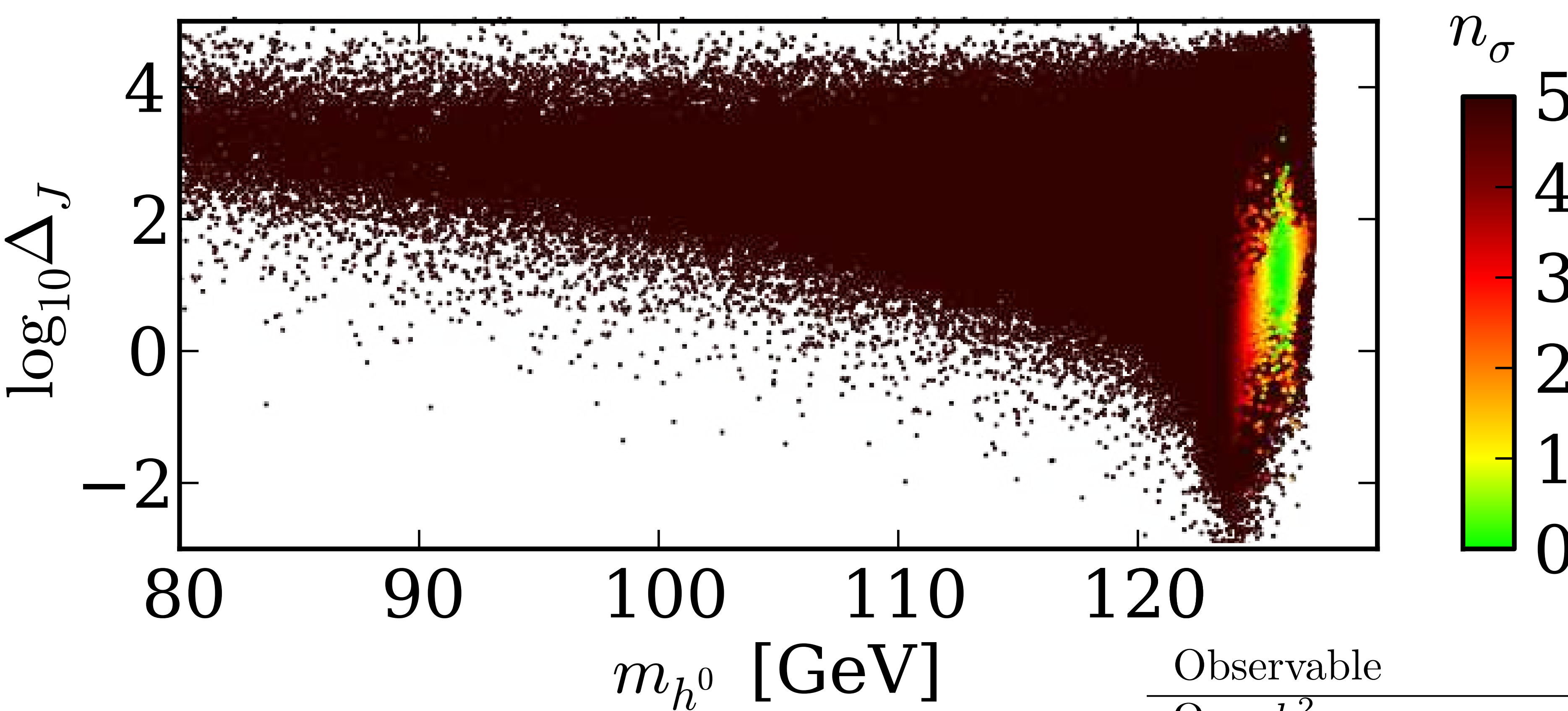
$$A_0 = -2.5\ \mathrm{TeV}, \quad \tan\beta = 3$$





$$A_0 = -2.5\ \mathrm{TeV}, \quad \tan\beta = 10$$

CNMSSM with other experimental data



- $A_0 = -2.5\ \text{TeV}, \quad \tan\beta = 10$

- A_λ, A_κ are set released

Observable	Experimental value
$\Omega_{DM}h^2$	0.1187 ± 0.0017
m_h	125.9 ± 0.4 GeV
$\mathrm{BR}\,(B_s \to \mu^+\mu^-)$	$(2.9 \pm 1.1) \times 10^{-9}$
$\mathrm{BR}\,(b \to s\gamma)$	$(343 \pm 21 \pm 7) \times 10^{-6}$
$\mathrm{BR}\,(B \to \tau\nu)$	$(114 \pm 22) \times 10^{-6}$
$m_{\tilde{\chi}_1^0}$	> 46 GeV
$m_{\tilde{\chi}_1^\pm}$	> 94 GeV if $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 3$ GeV
$m_{\tilde{q}}$	> 1.43 TeV
$m_{\tilde{g}}$	> 1.36 TeV

Summary

- Sensitive to EW scale cancellation
  Cares for the sensitivity of param. to observables
  Generic in the Bayesian analysis

- The simplest initial condition of the NMSSM must not just a simple extension of CMSSM. A-term constraints should be released in order for a flexible EWSB compatible with the Higgs mass. Then what must be the reasonable starting point for the NMSSM?

- This is a fine-tuning map for given models. For a systematic comparison of models, we need to compare Bayesian evidence.

- This is a fixed $(A_0, \tan\beta)$ slice scanning
  -> Complete scanning to appear soon

Higgs for CMSSM and CNMSSM

$$A_0 = -1\ \mathrm{TeV}, \tan\beta = 10$$