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Citation for published version:
Ojha, R, Ghadge, A, Tiwari, MK & Bititci, US 2018, 'Bayesian network modelling for supply chain risk propagation', *International Journal of Production Research*. https://doi.org/10.1080/00207543.2018.1467059

Digital Object Identifier (DOI):
10.1080/00207543.2018.1467059

Published In:
International Journal of Production Research

Ojha, R., Ghadge, A., Tiwari, MK. and Bititci, U. (2018), "**Bayesian network modelling for supply chain risk propagation**", *International Journal of Production Research*, Accepted.

Bayesian network modelling for supply chain risk propagation

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Abstract:

Supply chain risk propagation is a cascading effect of risks on global supply chain networks. The paper attempts to measure the behaviour of risks following the assessment of supply chain risk propagation. Bayesian network theory is used to analyse the multi-echelon network faced with simultaneous disruptions. The ripple effect of node disruption is evaluated using metrics like fragility, service level, inventory cost and lost sales. Developed risk exposure and resilience indices support in assessing the vulnerability and adaptability of each node in the supply chain network. The research provides a holistic measurement approach for predicting the complex behaviour of risk propagation for improved supply chain risk management.

Keywords:

Supply chain risk management, Risk propagation, Ripple effect, Risk modelling, Bayesian network, Supply chain disruptions, Uncertainty modelling

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1. Introduction

Supply Chain Risk Management (SCRM) follows a systematic approach for managing risks by identifying, quantifying and mitigating the risks. Current supply chains accentuate efficiency rather than resilience (Kim *et al.* 2015), thus making them vulnerable to disruptions with increased exposure points (Stecke and Kumar 2009). Global supply chain networks face unforeseen and uncontrollable disruptions and there is substantial evidence that catastrophic events are on the rise, with an increased frequency (Samvedi *et al.* 2013). The average cost of disruptive events has grown more than 1000% since the 1960s (Hasani and Khosrojerdi 2016). Due to this increased complexity and the inter-relationship of modern supply chains, the impact of uncertainty has become difficult to predict (Heckmann *et al*. 2015). Supply chain management practices like outsourcing, decentralisation and product customisation have amplified the number of risks in global supply chain networks (Jüttner *et al*. 2003; Ho *et al.* 2015). Therefore, proactive mitigation approaches are required to identify and manage the nodes of failure in the current volatile business environment. Decisions regarding the choice of mitigation strategy are driven by robust risk assessment outcomes. Thus, supply chains could benefit from developing predictive models that could estimate the impact of risks from a holistic perspective (Stecke and Kumar 2009; Tang and Musa 2011). To evaluate such a complex web of interconnected nodes together, Supply Chain (SC) systems must be holistically studied to identify the fracture points and level of 'risk propagation' across the network.

Risks originate at one node of the supply chain and create a '*ripple effect'* generating further risks across the network with an amplified impact. We define this cascading phenomenon across the inter-connected networks as risk propagation. Failure due to risk at a node in the network can cause an entire supply chain system to collapse. Supply chain disruptions propagate along the supply chain network in a similar manner to the lifecycle of the product (Ghadge *et al.* 2013; Snyder *et al.* 2016). The risk impact can propagate not only along the supply delivery direction, but may also cause a backlash on the upstream supply chain network due to the dependence of different stakeholders in the network (Keow Cheng and Hon Kam 2008). Bueno-Solano and Cedillo-Campos (2014) validate this '*reverse bullwhip effect*' while analysing the dynamic impact of disruption propagation on global supply chain networks. Previous academic work on SCRM has attempted to model risk/loss propagation within SC networks quantitatively (e.g. Wu *et al.* 2007; Mizgier *et al*. 2012, Ghadge *et al*. 2013; Bueno-Solano and Cedillo-Campos 2014; Mizgier *et al*. 2015; Scheibe and Blackhurst, 2017). The above studies contribute significantly to the advancement of research in supply chain risk propagation. Disruption network analysis, agent-based modelling, systems dynamics and simulation modelling have been used to model the risk propagation phenomenon. However, most of the above research considers limited nodes or variables and is, thus, unable to capture the overall cascading impact on the SC network. Complex relationships and dependencies (impacting factors) significantly contribute to the disturbance propagation in the network (Hallikas *et al*. 2004). The holistic measurement of the overall behaviour of risks in the SC network is found to be a missing link. Hence, the research aims to measure risk propagation in the supply chain network holistically. To achieve the aim, the following research objectives are set:

1. To capture the risk propagation behaviour at each node and across the supply chain network.
2. To assess the total fragility of a supply chain network due to disruption propagation within the nodes and between nodes.
3. To study the simultaneous disruptions and develop Risk Exposure Indices (REI) and Resilience Index (RSI) for multiple nodes within supply chain networks.

The amount of uncertainty and subjectivity inherent in a supply chain network makes it difficult to analytically examine the risk scenario (Samvedi *et al.* 2013). In the context of the phrase "*You can't manage, what you don't measure*" (McAfee *et al.* 2012), supply chain managers require further insights into quantifying the supply chain risk exposure of a firm in order to, consequently, determine the effectiveness of the SCRM (Wagner and Neshat 2010; Scheibe and Blackhurst, 2017). This study is an effort towards bridging the obvious research gap.

The remaining part of the paper is structured as follows. A literature review on SC risk modelling, risk propagation and holistic measurement is provided in section 2. In section 3, the theory associated with the Bayesian network is explained along with the development and formulation of the model. Section 4 discusses the numerical analysis extensively. Theoretical and managerial implications of the research are discussed in section 5. The paper concludes with discussion on the key findings, limitations and future research directions.

2. Literature review

2.1. Supply chain risk modelling

Qualitative and quantitative approaches have been widely used to study supply chain disruptions in the context of SCRM. Conceptual as well as empirical methods are used, with case study being the most preferred approach (Ghadge *et al.* 2012). Limited studies have used modelling and simulation techniques to understand the intricacies of SCRM (Tang and Musa 2011, Aqlan and Lam 2015). A collaborative approach for mitigating operational risks focused on supply and demand risks is presented by Chen *et al.* (2013). Hendricks and Singhal (2005) quantify the adverse effects of supply chain disruption following empirical analysis. Mizgier *et al*. (2012) and Mizgier *et al*. (2013) follow an agent-based modelling and simulation approach respectively for understanding the loss propagation in SC networks. Chong *et al*. (2014) extend Mizgier *et al.'s* (2012) model to a dynamic setup and study risk propagation and diversification during financial crises.

Simulation is a useful tool to visualise the stochastic nature and uncertainty of supply chain risks. Discrete Event Simulation (DES) and Monte Carlo simulation have been widely used to quantify supply chain disruption risks (e.g., Schmitt and Singh 2012; Mizgier *et al*. 2015). For instance, Carvalho *et al.* (2012) use a DES approach to study the supply chain risks in an automotive three-echelon supply chain. Similarly, Schmitt and Singh (2012) utilise a DES approach to estimate the disruption risk of production and supply capacities in a multi-echelon supply chain. Bayesian Network (BN) theory is seen to be a practical modelling approach for supply chain risks (e.g., Badurdeen *et al.* 2014; Garvey *et al.* 2015; Qazi *et al*. 2018). Interestingly, most of the past research considers disruption at single or limited nodes in the network. Therefore, it is unable to capture a complete picture of the *'risk propagation'* phenomenon (Han and Shin 2016). Multi-period disruptions at multiple nodes to capture their simultaneous, long-term effects on supply chain network variables is lacking in the extant literature. Indeed, measurement of supply chain vulnerability is challenging, as it is multi-dimensional and there are no well-developed matrices for evaluating the factors on which the vulnerability depends. Given this deficiency, one of the objectives of this study is to develop a Risk Exposure Index (REI) and Resilience Index (RSI) following a BN approach for risk propagation modelling. Risk modelling for understanding the cascading impact of risks is critical for the development of a resilient supply chain network (Bueno-Solano and Cedillo-Campos 2014). The research discussed in this paper follows a quantitative modelling approach by combining a BN theory and simulation approach to develop a rich understanding of risk propagation across the supply chain network.

2.2. Supply chain risk propagation

In today's dynamic environment, a decision taken by one firm in a supply chain network has a direct influence on the performance of another firm. This leads to the disruptions caused by one level to quickly spread to other levels with an adverse effect (Samvedi *et al.* 2013). The impact of risks propagating to the complete network and beyond, flowing along the connected nodes is defined as disruption propagation (Han and Shin 2016; Tang *et al.* 2016). If not controlled proactively, the risk propagation leads to the collapse of the entire supply chain network.

Disruptive risks cause a ripple effect in the SC system and impacts heavily on the supply chain performance (Ivanov *et al.* 2014; Ivanov *et al.* 2017). It is evident through recent man-made and natural disasters that such disruptions should not be considered as isolated instances. While designing the resilient supply chain network, it is vital to consider that an isolated disruption may cause a series of cascading disruptions (termed as ripple effect) with a potential global impact (Cantor *et al.* 2014; Dolgui *et al.* 2017). Although there is no robust supply chain strategy to restrict risk propagation impact, developing robust assessment and mitigation plans to deal with the supply chain disruptions effectively has gained significant attention (Kamalahmadi and Mellat-Parast 2016). Comprehensive research on risk propagation can help to identify redundancy measures for quick recovery and SC continuity (Schmitt and Singh 2012; Snyder *et al.* 2016). It is clear that studying SC performance assessment and recovery decisions, in light of the impact of risk propagation, is necessary (Mizgier *et al*. 2015). The paper attempts to close this evident research gap following a BN modelling approach.

2.3. Holistic performance measurement and Network theory

A performance measure is a metric utilised to quantify the effectiveness and efficiency of a system (Neely *et al.* 1995, Gunasekaran and Kobu 2007). The term '*holistic*' is characterised by an interconnected, integrated system understandable only as a whole and not in parts

(Oxford Dictionary 2017). Holistic measurement is all about quantifying the performance of the system by harnessing its overall inter-linkages (Anderson *et al.* 2006). Performance measures should be designed to have a reference value that should imply corresponding consequences depending on whether the actual measurement is on, below or above the desired value (Melnyk *et al.* 2014). While considering the holistic measurement for supply chain networks, this reference value typically considers all possible inter-linkages rather than only a part of the network.

Feedback loops can capture such interactions between complex network inter-linkages (Sterman 2001). However, they tend to fail under uncertainty (Qiu *et al.* 2014; Qazi *et al.* 2017). Bayesian networks have become an increasingly popular approach for handling complexity and uncertainty in systems (Feng *et al.* 2014; Hosseini and Barker 2016). The Bayesian network follows the '*Bayes theorem'* by explicitly representing the conditional probability dependencies between different variables through feedback (Kabir *et al*. 2015). It can represent uncertain variables as nodes, with causal relationship represented by edges between the two nodes, forming an acyclic directed graph (Cooper and Herskovits 1992). Existing methods such as causal-loop/feedback loop diagramming, social network analysis and interpretive structural modelling fail to capture the power of relationships between inter-connected risks in inter-connected nodes (Qazi *et al.* 2017). A Bayesian network is an analytical tool for computing the subsequent probability distribution of un-observed variables conditioned on the observed variables. The BN has several advantages such as the ability to combine multiple information sources, structural learning possibility and explicit treatment of uncertainty (Uusitalo 2007). Moreover, BN is most effective for assessing cascading disruptive events (Badurdeen *et al.* 2014; Qiu *et al.* 2014). This unique ability to model several variables and their interconnected structure in a complex network system encourages adaptation of BN modelling as a preferred research methodology.

3. Research design

In this section, BN theory is provided before developing the SC network model and associated formulation.

3.1. Bayesian network conditional independence

The theory of BN relies on the notion of conditional independence between the variables of the network (Lemmer and Kanal 2014); thus it must be formally defined. Let us assume that, we have prior knowledge φ and A_1 and A_2 are two events of which very little is known in terms of their dependence upon each other. Depending on φ we can ascertain whether two events would be dependent or independent. Suppose we observe a third event, A_3 with which we can now conclude that the first two events are independent. This concept of independence of two events given a prior condition is known as conditional independence and is demonstrated in Eqn. (1).

$$P(A_1, A_2|A_3, \varphi) = P(A_1|A_3, \varphi). P(A_2|A_3, \varphi) \quad (1)$$

Likewise for four events A_1, A_2, A_3, B the equation can be written as shown in Eqn. (2).

$$P(A_1, A_2, B|A_3, \varphi) = P(A_1|A_3, \varphi). P(A_2|A_3, \varphi). P(B|A_3, \varphi) \quad (2)$$

3.2. Conditional probability table and joint probability distribution

BN is capable of representing dependence relationships among random variables (Feng *et al.* 2014). Let $V = (A_1,\ A_2 \ldots.A_n)$ define the set of variables with edges, whose structure defines conditional independence. If an edge is directed from A_m to A_n then $A_m$is the parent node while A_n is the child node of A_m. There are three types of nodes in BN: 1) Root nodes are nodes without a parent node, 2) Leaf nodes are nodes without child nodes and 3) Intermediary nodes have both parent and child nodes. The causal relationships between the variables of the BN are assessed from the Conditional Probability Table (CPT). The complete joint probability distribution of BN consisting of *n* variables $A_1,\ A_2 \ldots.A_n$ is shown in the following eqn. (3).

$$P(A_1,\ A_2 \ldots.A_n) = \prod_{i=1}^{n} P(A_i|Parents(A_i)) \quad (3)$$

The variables in each graph denote the risk factors associated with a facility. The individual probability values for the occurrence of a risk are obtained from expert knowledge. The causal relationship structure between these risks is determined by learning BN using a K2 algorithm and, consequently, the corresponding CPT is developed depending on the severity of the risk factors. The efficacy of a BN is in its ability to determine the possible cause of an event by bottom-up inference (Kabir *et al.* 2015). In BN analysis, for n number of mutually exclusive parameters (risk factors) $R_{i(i-1,2,\ldots,n)}$ and a given observed data (disruption of the node) Z, the updated probability of occurrence of Z due to R_i is computed by eqn. (4).

$$P(R_i/Z) = \frac{P(Z/R_i).P(R_i)}{\sum_j P(Z/R_j).P(R_j)} \tag{4}$$

Where $p(R/Z)$ represents the posterior occurrence probability of R given the condition that Z occurs, $p(R)$ denotes the prior occurrence probability of R, $p(Z)$ denotes the total occurrence probability of Z and $p(Z/R)$ refers to the conditional occurrence probability of Z given that R occurs (Kabir *et al.* 2015).

3.3. Bayesian network development

Structure and parameter learning are two types of learning in the BN. Structure learning is the estimation of links of the network, while parameter learning is the estimation of conditional probabilities in the network (Feng *et al.* 2014). Within structural learning, constraint-based and score based approaches exist. Unlike the constraint-based approach which tests the conditional independence of the data, the score based function operates on the principle of defining a scoring function that represents how well it fits with the data (Feng *et al.* 2014). The aim here is to find the highest scoring network structure. A score based structure in BN learning is utilised in this paper because it is less sensitive to errors in the individual tests. The BN is used to develop a feasible network of risk factors for the facility nodes in the supply chain.

The Bayesian score function is decomposable in the presence of complete data. Eqn. (5) and (6) represent the Bayesian score functions used in the model.

$$f(Graph) = \sum_i f(x_i, Pa(x_i)) \tag{5}$$

$$f(x_i, Pa(x_i)) = \sum_{j=1}^{q_i} (\log\left(\frac{(r_i-1)!}{(N_{ij}+r_i-1)!}\right) + \sum_{k=1}^{r_i} \log(N_{ijk}!)) \tag{6}$$

Where Graph represents a set of nodes and edges, x_i is the risk factor associated with node i, $Pa(x_i)$ are the parents of risk x_i, r_i is the number of possible values of risk x_i, q_i is the number of possible configurations of variables in $Pa(x_i)$, N_{ijk} is the number of cases in which x_i has its k^{th} value, $Pa(x_i)$ is configured to its j^{th} value and $N_{ij} = \sum_{k=1}^{r_i} N_{ijk}$ (Feng *et al.* 2014). The K2 algorithm, a heuristic search algorithm for structural learning problems, is used in the model. The logic of this search algorithm is that it adds a node to a parent set incrementally and maximises the joint probability of the structure by finding the best parent set. Details of the input and output variables for the K2 algorithm are provided below.

Inputs: Set of risk factors, ordering of risk factors, upper bound on the number of parents a risk factor can have and a database containing several cases of risk.

Output: Learned network of risk factors for each node [as shown in Figure 1 (a, b) and 2 (a, b)].

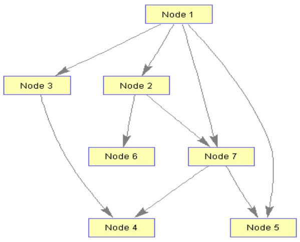


Figure 1 (a): Network of risk factors for Suppliers

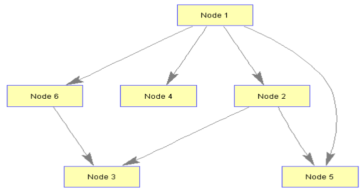


Figure 1 (b): Network of risk factors for Manufacturer

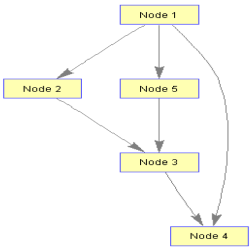


Figure 2 (a): Network of risk factors for Distributors

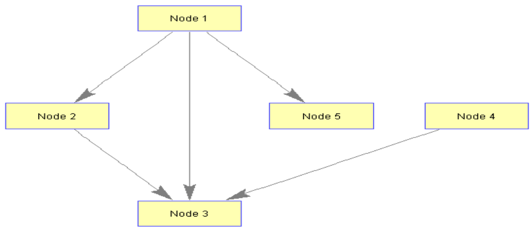


Figure 2 (b): Network of risk factors for Retailer

Figure 1 (a,b) and Figure 2 (a,b) show the dependency of several risks on each other for the identified facilities (nodes) in a supply chain network. The risk graphs are constructed using the K2 algorithm and basic characteristics of given risks associated with each facility node.

3.4. Model development

To investigate how simulation can be employed to assess risk propagation and capture product flow, a large Discrete Event Simulation (DES) model is introduced representing an automotive SC network. Leading automotive organization based in India was chosen for the study. The supply chain network of the case organization consisted of six nodes and four echelons as described below.

- First echelon: *Supplier 1* and *Supplier 2*
- Second echelon: *Manufacturer*
- Third echelon: *Distributor 1* and *Distributor 2*
- Fourth Echelon: *Retailer*

Establishing meaningful limitations for the SC network under study is critical. Since automotive SC network is very complicated, with several parts flowing from various suppliers, located in different countries, it will be a colossal task to consider all the SC entities and respective information, financial and material flows (Carvalho *et al.* 2012). Hence, it is imperative to identify the significant factors contributing to disruptions at nodes in the SC network. Semi-structured interviews with operations managers were conducted for the collection of relevant data. To evaluate the SC node performance, the simulation model was developed to evaluate SC indices/performance in both single disruption and multiple disruption scenarios.

To understand the system behaviour, simulation parameters such as demand, inventory costs, backup capacity, risk factors and their initial occurrence probabilities related data were collected from the *Risk Registers* maintained by different SC network stakeholders associated with the automotive organization. Historical data were also made available by operations managers to estimate the significant risk factors which had led to the failure of SC nodes in the past. Using this data, the simulation model was developed to test different scenarios.

In the simulation model for a SC network, the components are produced according to the end customer’s order specifications and make-to-order policy. The suppliers have a base level raw material inventory. Shocks due to disruption can wholly or partially disrupt the production process in the model and all the inputs are independently pre-set for the simulation study. The conditional probability table (CPT) of the risk factors associated with the nodes is defined using the network developed by the K2 algorithm. The BN theory provides the probability of occurrence of risk associated with a supply chain node, given the occurrence probability of the parent risk. Parent risk is a primary risk, which later propagates into other secondary risks. A scenario is defined as a combination of risk factors associated with a node.

The total number of possible scenarios for a node is 2^n, where n denotes the number of risk factors associated with the node. For example: [1,0,1,0,0] is a scenario for the retailer with five risk factors in which risks 1 and 3 have occurred. Since the risk factors are connected to each other, the occurrence of risk 1 and 3 at the retailer might induce occurrence of the remaining three risks. Consequently, the probability of occurrence of risks at the retailer maybe as follows $[1,0.4,1,0.35,0.2]$. The combination of these risks results in a partial disruption of retailer due to its risks. However, another scenario with risk factor probabilities as [1,1,1,1,1] results in total disruption of the retailer. The total disruption cost for every scenario is determined by the expected cost incurred due to each risk in the given scenario. The disruption probability of a node for a scenario is calculated using the formulation provided in section 3.5. Based on the suggestion provided by respondents, the production capacities of the supplier and demand at each node are generated using *Uniform* distribution (as shown in Table 1(b)) and the initial inventory level of each node is forecast as the mean value of the total demand in the previous year. An additional backup capacity of P % with a lead time of Q weeks is provided to every node whose inventory level falls below X % in any time period. Production capacities of the supplier and demand at each node are among the additional inputs generated randomly. A 20-week time horizon with $Q = 1\,week$ is considered for the simulation. The disruption weeks for the nodes in the single node disruption scenario are generated randomly to replicate the real world scenario. Random numbers are generated for each node and compared with the cumulative disruption probability to obtain a plausible scenario, its associated cost and the disruption probability from *risk registers* for each simulation run.

Table 1 (a): Production or supply decrement of each node depending on TTR

TTR	Cumulative Probability for TTR	Supplier 1	Supplier 2	Manufacturer	Distributor 1	Distributor 2	Retailer
2	0.20	0.20	0.20	0.10	0.10	0.15	0.10
4	0.40	0.40	0.45	0.30	0.25	0.25	0.35
7	0.70	0.60	0.75	0.60	0.50	0.50	0.50
10	1.00	0.80	1.00	0.75	0.75	0.70	0.70

Once the disruption scenario and its probability for a node have been determined, time to Recovery (TTR) for the node is estimated from Table 1(a). It is expected that the disruption at a node causes a decrement in the production or supply. The production/supply decrement in each node depends on TTR and is shown in Table 1(a). The estimation of TTR values and production decrement ratios are taken from the *risk registers*. The demand at each node and the production level at the supplier are used to determine the relative dependency between nodes of the adjacent layers (β_{ji}). β_{ji} plays a major role in determining the disruption propagation between the adjacent layers. The unit inventory and backup cost is taken to be same for a node and is shown in Table 1(b).

Table 1(b): Demand and unit inventory cost at each node

Node	Demand	Production	Unit inventory cost (in pounds)
Supplier 1	Uniform(30000,32000)	Uniform(32000,34000)	0.15*30
Supplier 2	Uniform(12000,14000)	Uniform(13000,15000)	0.15*30
Manufacturer	Uniform(42000,44000)	-	0.20*30
Distributor 1	Uniform(10000,12000)	-	0.25*30
Distributor 2	Uniform(32000,34000)	-	0.25*30
Retailer	Uniform(42000,44000)	-	0.40*30

The simulation code was developed using Matlab, a commercial programming platform to depict a product flow in a multi-node supply chain network for a disruption scenario; 3000 simulation runs were conducted to capture the overall behaviour of risks. The performance variables analysed through the simulation are customer service level and risk exposure index of the nodes based on fragility and lost sales. These output variables enable us to capture the long-term service level and REI at each node. Figure 3 provides a step-by-step approach for the analysis of the model. Table 2 shows the supply chain nodes and associated risk with each node.

Table 2: Risk factors for supply chain nodes

Sl. No.	Facility Nodes	Risk Factors
1.	Supplier	• Natural disaster • Supplier bankruptcy • Machine breakdown • Product quality issues • Inaccurate forecast • Labor strikes • Information/Infrastructure breakdown
2.	Manufacturer	• Natural disaster • Machine breakdown • Product quality issues • Labor strikes • Excess pollution and emissions

3.	Distributor	• Information/Infrastructure Breakdown • Natural disaster • Labor strikes • Product damage/Obsolesce risk
4.	Retailer	• Information/Infrastructure Breakdown • Natural disaster • Labor strikes • Inaccurate forecast • Competition • Delivery delays

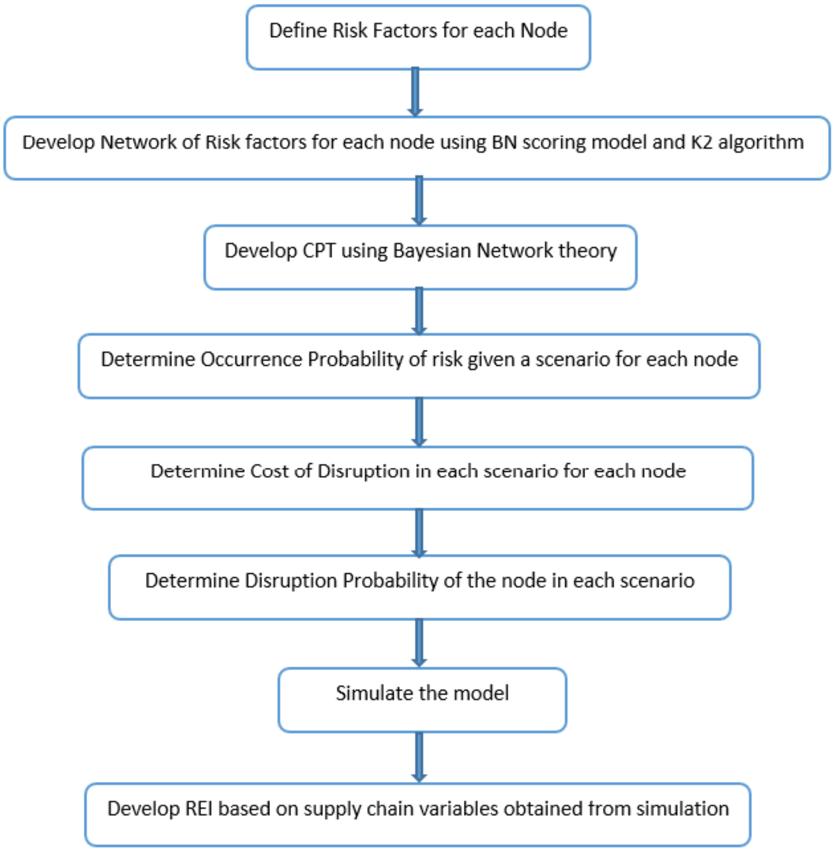


Figure 3: Research methodology

The following *assumptions* were made to overcome the complexity inherent in the modelling and simulation of the SC under study:

1. Each node in the supply chain network has multiple risks associated with it.

2. Probabilities in CPT have been allocated on the basis of managerial advice and network of risk associated with each node.
3. Disruption propagates only from upstream to downstream nodes in the supply chain network.

3.5. Model formulation

The disruption probability of a node n_i in layer l due to the occurrence of risk k is defined as:

$$P_{dis}\left(r_{kl}^{n_i}\right) = P\left(r_{kl}^{n_i}\right).\, g\left(r_{kl}^{n_i}, n_i\right) \qquad k \in \{1,2,\dots,N\left(R_{n_i}\right)\} \tag{7}$$

$$P_W\left(R_l^{n_i}\right) = \emptyset\left(P_{dis}\left(r_{kl}^{n_i}\right)\right) \tag{8}$$

The disruption probability of all nodes in the supply chain network, except supplier layer, is a function of disruption due to its risk scenario and the propagated impact of disruption from nodes in the previous layers.

$$P_F\left(R_l^{n_i}\right) = f\left(P_W\left(R_l^{n_i}\right), P_B\left(R_l^{n_j n_i}\right)\right) \tag{9}$$

$$P_B\left(R_l^{n_j n_i}\right) = P_F\left(R_{l-1}^{n_j}\right).\, \beta_{n_j n_i} \,.\, g\left(n_j, n_i\right) \qquad n_j, n_i \in N \tag{10}$$

n_i: node in layer l

n_j: node in layer $l-1$

$$\beta_{n_j n_i} = \frac{cap_{n_j n_i}}{\sum_{n_j} cap_{n_j n_i}} \tag{11}$$

$$g\left(n_j, n_i\right) \in [0,1] \qquad \forall\, n_j, n_i \in N \tag{12}$$

$$g\left(r_{kl}^{n_i}, n_i\right) \in [0,1] \tag{13}$$

Eqn. (8), Eqn. (9), Eqn. (10) and Eqn. (11) are adapted from Han *et al.* (2016) to solve the model developed in this paper. A summary of the parameters used in this paper are presented in Table 3.

Table 3: Summary of the model parameters

Parameters	Description
S	Set of Suppliers
M	Set of Manufacturers
D	Set of Distributors
R	Set of Retailers
N	Set of all nodes in the network, $N \in \{S, D, M, R\}$
L	Total number of layers (in our case study $L = 4$)
n_i	Node element from the supply network, $n \in N$, if n represents S then n_i represents supplier i i.e. $S(i)$
$R_l^{n_i}$	Set of risk factors associated with node n_i in layer l also referred

	to as scenario of n_i , $R_l^{n_i} = \{r_{1l}^{n_i}, r_{2l}^{n_i}, \ldots, r_{N(R_{n_i})l}^{n_i}\}$
$r_j^{n_i}$	Risk factor j associated with n_i in layer l
$P(r_{kl}^{n_i})$	Probability of occurrence of risk k associated with node n_i in layer l
$P_W(R_l^{n_i})$	Disruption probability of node n_i in layer l due to risks associated with it
$P_B(R_l^{n_j n_i})$	Disruption probability of node n_i in layer l due to risks propagated from node n_j of previous layer
l	layer index $l \epsilon \{1,2,3,4\}$
$g(r_{kl}^{n_i}, n_i)$	function to determine the impact of occurrence of risk k on node n_i
$g(n_j, n_i)$	function to determine the impact of disruption propagated from node n_j to node n_i
$\emptyset(.)$	**Average function** to determine the disruption probability of a scenario at a node due to risks associated with it.
$f(.)$	**Maximum function** for disruption due to risks within the node and the disruption due to risk propagated from nodes in preceding layer
$h(.)$	**Average function** to determine overall disruption probability of the supply chain network
$\beta_{n_j n_i}$	Demonstrates the relative dependency of node n_i on node n_j
$cap_{n_j n_i}$	capacity allowed on link connecting n_i and n_j
F_i	Fragility of a node for option i inventory and backup level
N	Total number of simulations
DC_{in}	Disruption cost for option i inventory and backup level at a node in simulation n ($n \in N$)
DP_{in}	Disruption probability for option i inventory and backup level at a node in simulation n
SL_i	Service level of a node for option i inventory and backup level
SL_{k0}	Service level of node k when there is no disruption
SL_{kw}	Service level of node k in week w
LS_{kw}	Lost sales of node k in week w
Dem_{kw}	Demand of node k in week w ($w \in W$)
W	Number of weeks in each simulation
REI	Risk Exposure Index
RSI_k	Resilience index of node k for a given inventory and backup level

4. Numerical analysis

Based on the multi-echelon inventory literature, it is understood that inventory at downstream nodes (finished goods inventory) incur higher holding costs than inventory at upstream nodes (raw material/WIP inventory) (Chopra and Meindl 2007; Schmitt and Singh 2012). In the model, this knowledge is used to determine the holding cost at the nodes (Table 1b). Three options were tested for inventory and backup levels at each node as shown in Table 4. The following parameters are measured for the holistic understanding of risk propagation within the SC network:

- Fragility of each node
- Lost sales
- Service level
- Total inventory and backup cost

Table 4: Percentage inventory and backup level at each Layer

Option	Supplier	Manufacturer	Distributor	Retailer
1	20%	20%	20%	40%
2	20%	40%	20%	20%
3	25%	25%	25%	25%

4.1. Fragility

Fragility at a node is defined as the expected increase in the cost of a supply chain in the event of a disruption on a given node (Chopra and Sodhi 2004). Supply chain networks, where the disruption impact could propagate across the entire network, often have higher fragility than networks where the disruption impact is more localised. In this model, it is measured in terms of million pounds. The variation in fragility is depicted in Figure 4. Here, F_i: Fragility for option i inventory and backup level at a node.

$$F_i = \left(\frac{1}{N}\right) * \sum_n DC_{in} * DP_{in} \tag{14}$$

The fragility at each node is almost the same for all three options of inventory and backup level (Figure 4). Since fragility is dependent on the disruption probability and cost, it does not vary much with the variation in the inventory level. Suppliers 1 and 2 have a greater number of risks associated with them as compared to other nodes. Therefore, the disruption probability and cost of impact is higher at these suppliers. Fragility is higher for the manufacturer compared to all other nodes in the supply chain, although it has less number of risks associated with it. This is attributed to the fact that the manufacturer is the bottleneck in

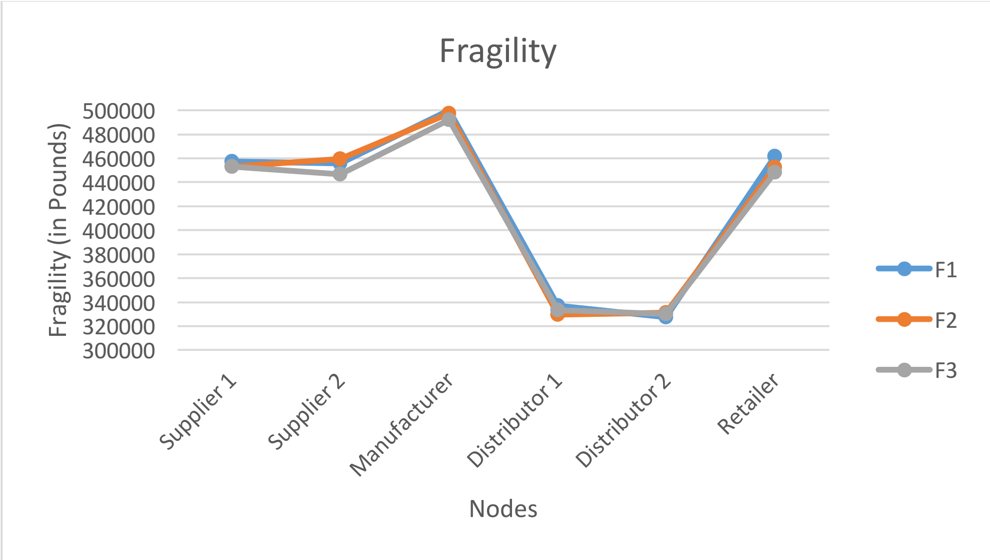


Figure 4: Fragility at the nodes in a supply chain network

the above supply chain network. Any disruption occurring at the manufacturer or propagated from the supply side gets amplified and can significantly impact the performance of the manufacturer. The number of risks and cost impact associated with distributors is less and hence the fragility value is minimal. Moreover, the disruption propagated from the manufacturer is shared by the distributors in this supply chain network structure. Despite the small number of risks associated with it, the retailer seems to be highly fragile. This is because any disruption at the retailer directly affects customer demand. Thus, the cost impact of any disruption is significantly higher. Hence, the disruption cost at the retailer is higher than at the suppliers. To contain fragility, reducing disruption risk at a node is not sufficient. It is equally important to reduce the propagated effects of disruption from upstream nodes because the final disruption probability at a node is a function of disruption probability due to the propagated effects of disruption from upstream nodes and disruption probability due to risks within the node.

4.2. Lost sales

Loss in sales is mainly due to individual disruption at the respective nodes and the propagated effects of disruption at the nodes in the downstream supply chain network. From the graph in Figure 5(a), it is evident that using higher inventory and backup with a small lead time is very effective in reducing the after-effects of disruption. The manufacturer's graph shows that $LS_1 > LS_3 > LS_2$, an apparent conclusion as inventory and backup, is maximum for option 2 and minimum for option 1. Any disruption at the manufacturer can reduce its production

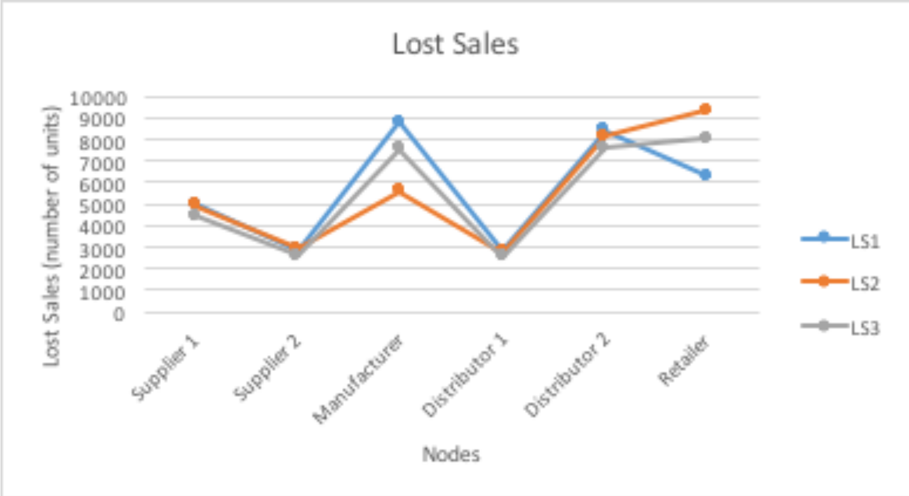


Figure 5(a): Lost sales at the nodes in supply chain network

drastically which, in turn, increases lost sales. Lost sales are high for distributor 2 and retailer, as they manage the transfer of a large volume of products. Any disruption propagated from the upstream nodes propagates and significantly reduces the supply to downstream nodes. The higher volume of inventory and backup is the most effective way to reduce the effects of disruption. Considering the entire supply chain network as a single entity option 3 leads to a minimum average in lost sales. Increasing the inventory and backup at all the nodes by a small percentage is more effective in containing the propagated effect of the disruption**.**

4.3. Service level

Service level at a node is defined in Equation 15 as the average fill rate of customers. Here, SL_i is a service level for option i inventory and backup level. Service level at the retailer is

$$SL_i = \left(\frac{1}{N}\right) * \sum_n \left(\left(\frac{1}{W}\right) * \sum_w (1 - \frac{LS_{iw}}{Dem_{iw}})\right) \tag{15}$$

maximum in option 3, although retailer's inventory increment is 40% in option 1 (Figure 5(b)). Increasing inventory and backup to reduce the effects of disruption at the downstream node is costly as it increases the finished goods inventory**.** However, this is not the best method to overcome disruption effects. To contain the effects of disruption at the upstream nodes and thereby reduce amplification, a small increase in inventory level at all the nodes of a supply chain network is recommended.

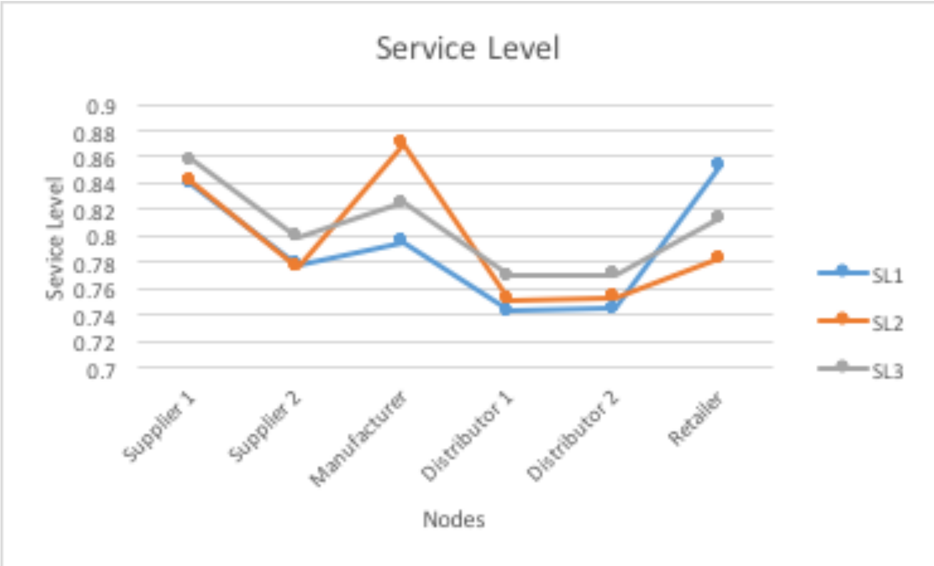


Figure 5(b): Service level at the nodes in supply chain network

4.4. Total cost

In this section, the total cost of SC network is calculated. Here, INV_i: inventory cost for option i inventory and backup level. The total inventory and backup cost for the retailer is minimum for option 2 because the bottleneck node (manufacturer) has a higher backup and therefore the disruption propagated from upstream nodes is least in option 2 (Figure 6). Since disruption propagation to the subsequent downstream nodes is minimal, the requirement of the inventory is low. In option 1, this cost is higher because of a large amount of finished goods inventory and backup being maintained at the retailer (Figure 7). A major proportion

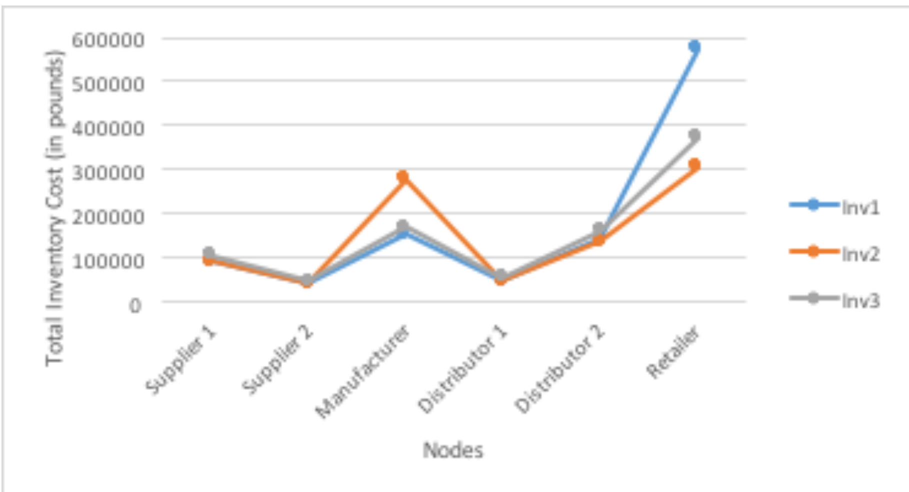


Figure 6: Total inventory and backup cost at the nodes in supply chain network

of the cost is due to the occurrence of the risk. However, in the case of the retailer, since the cost of finished goods inventory is very high, fragility is comparable to the inventory and backup cost. Retailer's fragility is comparable to that of supplier despite having a smaller number of risk factors associated with it. Simultaneous occurrence of a disruption at both nodes (supplier and retailer) causes a higher cost impact at the retailer.

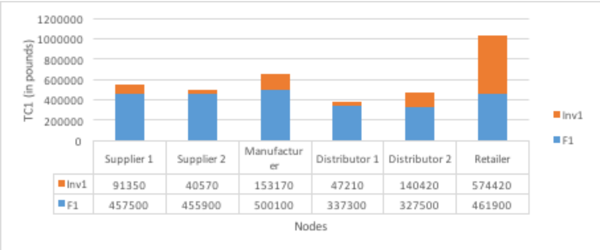


Figure 7(a): Total cost for option 1

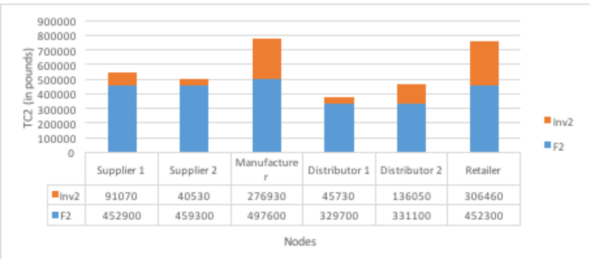


Figure 7(b): Total cost for option 2

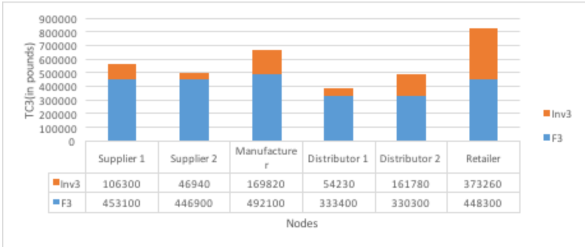


Figure 7(c): Total cost for option 3

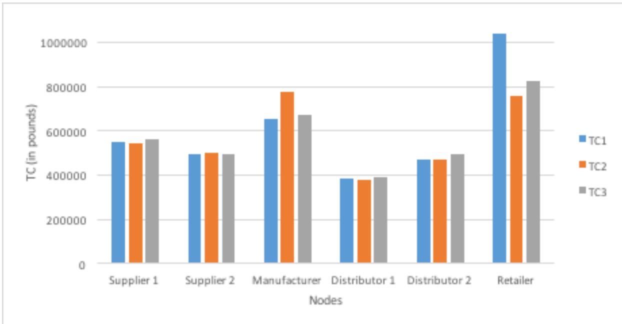


Figure 7(d): Total cost at the nodes in supply chain network

It is evident from the above results that option 3 incurs a minimum cost. The inventory and backup cost for option 3 is more than the respective cost for option 2. However, due to less fragility in the case of option 3, the total cost is less. Option 1 is not a good choice because not only is its cost high, but also the service level is low.

4.5. Risk exposure index

The REI helps a firm to identify the nodes that need attention from the operations and risk manager. In the model, REI is assessed based on two performance impact factors: Fragility and lost sales. The node with the largest performance impact value is assigned with a value of

1, while REI for other nodes is calculated relative to this node. REI value 1 implies that the node is most vulnerable to disruption (Simchi-Levi *et al*. 2014).

REI based on fragility

In the context of fragility, the manufacturer is the most vulnerable node of all three options. This is because, for the same disruption at all nodes, the manufacturer has a higher cost impact than suppliers and distributors. Although it has a lesser cost impact than retailers, the greater number of risks associated with the manufacturer elevates its REI. A small disruption at the manufacturer reduces productivity significantly. Not only does this reduce the supply towards the downstream layer, it also backpressures upstream layer with high inventory levels at suppliers. Table 5(a) and 5(b) shows the REI values based on fragility and lost sales for each node.

Table 5(a): REI (PI: Fragility) for each node in supply chain network

Nodes /REI	REI_1	REI_2	REI_3
Supplier 1	0.914817	0.910169	0.920748
Supplier 2	0.911618	0.923031	0.908149
Manufacturer	1	1	1
Distributor 1	0.674465	0.66258	0.677505
Distributor 2	0.654869	0.665394	0.671205
Retailer	0.923615	0.908963	0.910994

Table 5(b): REI (PI: Lost Sales) for each node in supply chain network

Nodes /REI	REI_1	REI_2	REI_3
Supplier 1	0.566413	0.528775	0.550542
Supplier 2	0.328485	0.311649	0.325813
Manufacturer	1	0.601222	0.937555
Distributor 1	0.321668	0.293109	0.316465
Distributor 2	0.955914	0.872575	0.943537
Retailer	0.713555	1	1

REI based on lost sales

According to the results obtained for REI based on lost sales, the retailer is the most vulnerable since it is the last node in the supply chain network. Customers receive products directly from the retailer. They do not have any alternative source to mitigate the propagation. Thus, any disruption at this node or the upstream nodes significantly reduces supply to the retailer, reducing the fill rate to customers.

4.6. Resilience Index

This section attempts to measure the resilience of supply chain nodes with the help of a *Resilience Index (RSI)*. Hosseini *et al*. (2016) define RSI as a function of the quality of community infrastructure. The *RSI* of SC nodes in each week is calculated as the performance impact parameter for the SC network (adapted from Barroso *et al*. 2015). w_0 is the week in which disruption occurs at the SC node, while w_n is the time when disruption ends plus time to recover from the negative effect of disruption. Operations managers could not provide (due to non-availability) the service level at each node in the absence of disruption (i.e., SL_{k0}). Hence, a value of $SL_{k0} = 1$ is assumed which implies that every node satisfies demand in the absence of disruption. When disruption occurs, the service level versus weeks' graph shows a triangular pattern (Figure 8). It can be concluded from the formulation that, the higher the SL of a node in the event of a disruption, the higher is its *RSI*.

$$RSI_k = 1 - \frac{\int_{w=w_0}^{w_n}(SL_{k0} - SL_{kw})dw}{SL_{k0}(w_n - w_0)} \cong 1 - \frac{\sum_{w=w_0}^{w_n}(SL_{k0} - SL_{kw})}{SL_{k0}(w_n - w_0)} = 1 - \frac{\sum_{w=w_0}^{w_n}\left(1 - {SL_{kw}}/{SL_{k0}}\right)}{(w_n - w_0)} \quad (16)$$

The RSI value of each node ranges from 0 to 1. The RSI index of a node tending to 0 implies that node k is less resilient to disruption; whereas, if it is tending to 1, it implies the node is resilient to the disruption and can sustain its performance. It is evident from Table 6 that option 3's inventory and backup capacity is most effective in reducing the propagated effects of disruption, as it ensures the highest average RSI for the SC network.

Table 6: RSI for each node in supply chain network

Nodes /RSI	RSI_1	RSI_2	RSI_3
Supplier 1	0.8405	0.8414	0.8564
Supplier 2	0.7808	0.7794	0.7956
Manufacturer	0.7688	0.8704	0.8275

Distributor 1	0.7446	0.7522	0.7677
Distributor 2	0.7389	0.7531	0.7812
Retailer	0.8554	0.7827	0.8185
Average RSI	0.7882	0.7965	0.8078

It must be noted that REI is a relative index as opposed to RSI, which is an absolute value. RSI is more suited to ascertain the resilience of each node and, more importantly, the entire SC network. It shows the positive aspects of the proactive measures taken to improve the ability of the system to recover from disruption. REI, on the other hand, helps us to understand how vulnerable a node is relative to the most vulnerable node.

4.7. Impact of single node disruption on SC network

To assess the impact of single node disruption on the SC network, simulation runs were conducted for every disrupted node. Based on the finding discussed in the paper, option 3's inventory and backup level was found to be the ideal input for this analysis. The backup supply with a lead time of 1 week is engaged when the inventory level of the node falls below X% (say 5%) of the demand at a single node in a given time period. It is interesting to observe TTR of retailer for disruption at supplier-1, supplier-2, manufacturer, distributor-1 and distributor-2 are 5, 2, 10, 1, 10 days respectively.

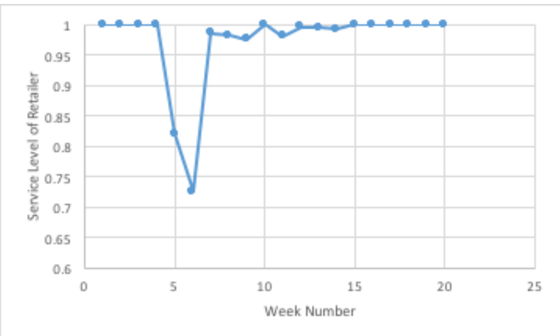


Figure 8(a): SL when Supplier 1 disrupts in week 5

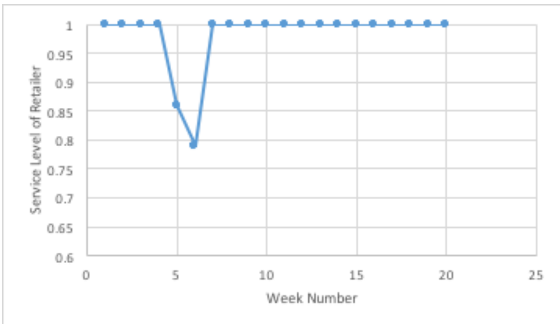


Figure 8(b): SL when Supplier 2 disrupts in week 5

When the disruption occurs at the supplier node, the fill rate curve drops due to a subsequent decrease in the supply from the supplier through the intermediate nodes up to the retailer. The TTR is small due to the equal proportion of inventory and backup at each layer which reduces the propagated effects of the disruption before it reaches the retailer. The demand at supplier 2 is less than that at supplier 1, thus decrement in supply due to disruption at supplier 2 is less significant. This factor enables the retailer to quickly recover from the disruption due to supplier 2, compared to that from supplier 1 (Figure 8(b)). It is for the same reason that the minimum service level during supplier 2 disruption is greater than that during supplier 1 disruption. The ripples in Figure 8(a) from weeks 7 to 14 show that impact time exceeds the disruption duration and takes relatively more time to recover compared to the other case. This is known as the amplification of the disruption propagation from the upstream nodes.

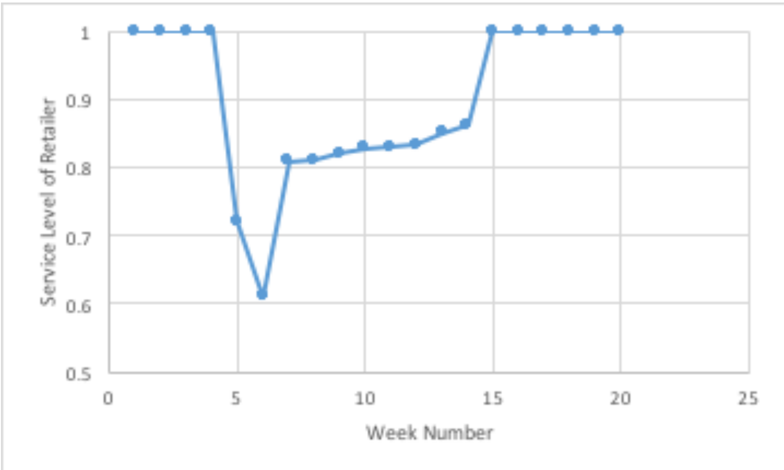


Figure 8(c): SL when Manufacturer disrupts in week 5

Disruption at the manufacturer node results in a significant drop in the service level at the retailer as the production is reduced considerably. As soon as inventory at the retailer falls below 5% of the demand, backup supply is ordered at the end of the fifth week. Since the lead time is one week, the backup supply is received at the end of the sixth week, which justifies the rise in SL in Figure 8(c) from 0.60 (approx. value) to 0.81(approx. value). The reduced supply from upstream nodes is fractionally compensated by the backup capacity. The graph fluctuates between 0.81 to 0.85 from weeks 7 to 14 and the system recovers from the disruption in week 15.

As the volume of goods handled by distributor 1 is small, it has a negligible impact on the retailer's SL in week 5. The backup capacity of the retailer received in week 5 is sufficient to compensate for the reduced supply from distributor 1. Thus, the retailer recovers towards the end of the sixth week. It can be inferred from Figure 8(d) that some amount of inventory is always left (because backup quantity is significantly higher than the production decrement), which compensates for the reduced supply from week 6 to week 15, thereby keeping fluctuations to a minimum. When distributor 2 is disrupted in week 5, the service level decreases sharply to 0.75 (approx. value) as shown in Figure 8(e). As soon as the backup is received at the end of sixth week, the service level rises. From weeks 7-14 the graph gradually increases and the system recovers at the end of week 15. Since distributor 2 handles a large volume of goods, the decrement in supply due to disruption is high. Thus, the inventory is exhausted during the recovery period and the backup supply is used to support the demand. Since the production decrement is significantly higher than the backup capacity, the retailer takes time to recover from the disruption.

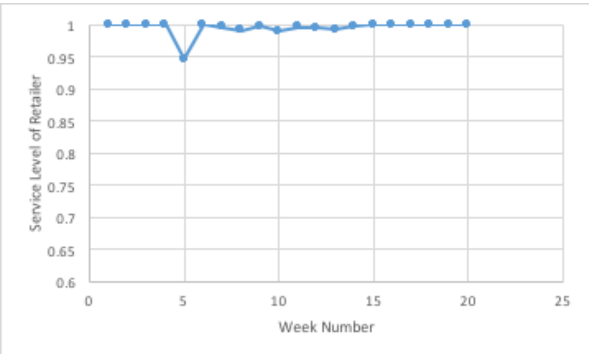


Figure 8(d): SL when Distributor 1 disrupts in week 5

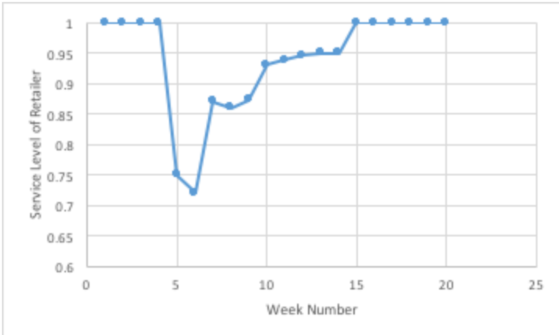
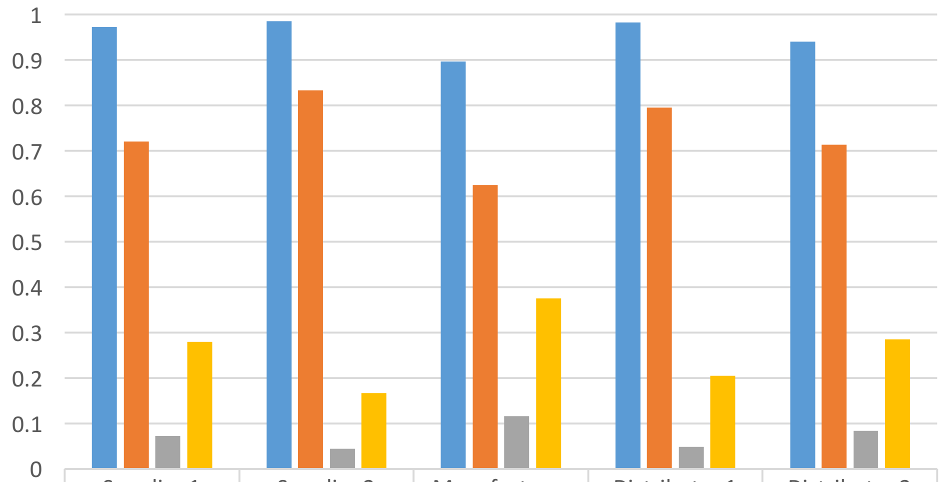


Figure 8 (e): SL when Distributor 2 disrupts in week 5

The mean SL of the retailer is at a maximum when disruption occurs at supplier 2. Since the node is located upstream in the supply chain network, the disruption propagated by it is reduced by the inventory and backup at the intermediate node, before it reaches the retailer. The small deviation in SL from the mean in the case of disruption at supplier 2 and distributor 1 is attributed to the low volume of goods handled by these nodes. The mean SL is at a minimum in the case of the manufacturer because any disruption at the manufacturer has a significant impact on its production and propagates to the retailer. Even though a small number of risks are associated with distributor 2, disruption at this node causes a significant drop in the supply to the retailer due to its proximity (Figure 9).

It can be inferred from the above analysis that, in order to reduce the fragility and increase the service level at the downstream supply chain network (i.e., retailer) the firm can opt to locate another manufacturing facility. In several SC systems such as food and electronics, it may be difficult to depend totally on the inventory as it is expensive in the long run and may not be a feasible solution due to technological changes. For these critical systems, adding flexibility through redundancy increases systems resilience and thus reduces the risk of failure.



	Supplier 1	Supplier 2	Manufacturer	Distributor 1	Distributor 2
mean SL	0.973	0.9858	0.8966	0.9835	0.9411
SL min	0.7206	0.8333	0.6243	0.7952	0.7142
std	0.07257	0.04433	0.116	0.04884	0.08357
Range of Dip	0.2794	0.1667	0.3757	0.2048	0.2858

Disrupted node

Figure 9: Retailer's SL during single node disruption

4.8. System behaviour with increasing number of suppliers and tiers of suppliers

The adapted equation from Bimpikis *et al.* (2017) imply that the Coefficient of Variation (CV) of retailer R decreases, as the number of nodes in any tier increases. This is because when the number of nodes increases, the tier becomes more reliable and each disruption has a smaller impact on the final output. Similarly, CV increases if the disruption probability increases, since having disruptive events more frequently implies that the output fluctuates more around its expected value.

$$CV_R = \left(\frac{1}{(1-DP_R)} \prod_{l \in L-\{R\}} \left(\frac{1}{N(l)}\right) \left(N(l) - 1 + \frac{1}{(1-DP_l)}\right) - 1\right)^{\frac{1}{2}} \tag{17}$$

$N(l)$: number of nodes in layer l

DP_l: average disruption probability of nodes in layer l

DP_R: average disruption probability of retailer

L: set of layers or tiers in supply chain network

For a fixed disruption profile and network structure, the coefficient of variation is also a measure of how balanced the supply chain network is, i.e., a measure of how evenly distributed firms are in different tiers. For instance, if there are 'bottleneck' tiers in the network, i.e., tiers with only a few nodes, then the corresponding multiplier in Eqn. 17 and, consequently, CV take large values. Conversely, if all tiers have a comparable number of nodes (hence the network is more balanced), then CV takes a smaller value.

5. Theoretical and managerial implications

Based on the above findings, there are three contributions to the theory. First, following a systematic approach, different performance variables such as fragility, lost sales, service level and inventory cost are captured for the holistic understanding of risk propagation within the SC network. Second, the research develops novel indices for measuring the risk behaviour following the development of a risk exposure index and resilience index. The third contribution may be associated with the use of BN methodology. A robust methodology for probabilistic inter-dependency modelling to capture complexity and uncertainty in supply chain networks is proposed in the research design. This modelling approach provides a unique capability to model inter-connected risks in the inter-connected network. The sensitivity of risk exposure at different nodes with varying inventory and backup levels further assists in understanding the complex behaviour of the risk.

In terms of contribution to practice, the REI values can assist risk managers in quantifying the risk exposure of each facility node relative to other prior and after the implementation of risk mitigation measures in the SC network. The RSI index will provide further insights into the adaptability of each node in the SC network. The study shows that vulnerability at a facility depends upon the factors impacting performance. Existing risk propagation research does not capture the holistic network-wide impact of risks (Qazi *et al.* 2017). SC managers can use the 'risk propagation' model to quantify risks based on the performance of the impacting factors. The study on the impact of single node disruption on downstream SC network service level is expected to help managers in making robust SC decisions. Foremost, the research shows that facility nodes which handle large volumes of products are more exposed to the adverse impact of disruption than nodes with small volumes. SC managers could limit the tedious and costly supply chain proactive measures for facility nodes with low REI to improve cost efficiency.

In the context of SCRM, practitioners should avoid sourcing and backup supply from regions prone to disruptions (e.g., Japan due to frequent earthquakes), as this could make the SC more vulnerable. SC managers should also reduce dependency on the finished goods inventory. Using the developed model, practitioners can quantify the financial impact on the SC network due to the disrupted facility node. SC analysts could use the model to drill down into detail and account for disruptions of varying severity. This could be achieved by running scenarios using TTR of different durations for the facility nodes.

6. Conclusion and future research

The research successfully demonstrates the quantitative and holistic modelling approach for understanding risk propagation within the SC network. Risk analysis of extended supply chain networks, considering multiple levels of stakeholders and hazards, is critical for the success of SCRM (Mizgier 2017). The findings are believed to contribute to the growing field of SCRM by providing a comprehensive understanding of the complex behaviour of risks. The holistic measurement of risk propagation was found to be lacking in the SCRM literature and the paper has attempted to close that evident research gap. The research delivers the first objective of capturing the risk propagation behaviour at each node and across the supply chain network by conceptualising and modelling the phenomenon. The research investigates a multi-period SC model faced with varied risks comprising of organisational, network and environmental risks. The quantitative approach combines Bayesian networks with the K2 algorithm to develop the network of risks associated with a node and CPT. Similar techniques can be effectively used to handle large networks such as global supply chains to identify bottlenecks (Mizgier *et al.* 2013). The SC network model lays important groundwork for quantitative approaches to measuring supply chain disruptions. The second objective of the research was to assess the total fragility of SC network due to disruption propagation. The simulation results demonstrate the vulnerability of nodes due to disruption propagation by calculating different influential parameters such as lost sales, service level and total cost. The measures adopted to represent disruption risks and impact on the SC can be easily adapted to risk assessment in the financial markets and humanitarian disaster management. The measurement of risk exposure and resilience indices for a given supply chain network captures the third research objective. Development of such indices is crucial for the SCRM and the paper contributes to the growing interest in creating adaptable supply chains in the future (Hohenstein *et al.* 2015).

Like any other research, the study has some limitations, which pave the way for future research opportunities. An automotive SC network was considered for measuring risk propagation. However, each SC network is unique and differs in terms of performance and exposure points, thus making it difficult to generalise the findings. We have incorporated the significant, recurrent risk factors and their occurrence probability as collected from the risk registers of the participating firms. One of the most important limitations is the static approach proposed by the BN model to develop the network of risk factors. The model considered a small number of risk factors and future research can look into multiple risk factors disrupting multiple nodes and links. Developed CPT are based on manager's perception of the risk and are likely to vary depending on different scenarios due to bounded rationality of the decision maker. Impact of bounded rationality on risk propagation in SC network is another avenue for research.

The model focuses on quantifying the risk propagation impact on the facility nodes of a SC network. We have not fully considered the link failure at the nodes due to unavailability of product-specific processes and attributes required to quantify link failures. The disruption impact is captured in terms of cost and service level. However, the study does not consider how to reduce SC risk exposure without losing financial performance. Future research can include a rigorous study following optimisation techniques to demonstrate the trade-off between SC financial performance and risk mitigation. To keep the complexity of the model to a minimum, the research imposed certain constraints. For example, the model did not consider logistics risks (lengths of arcs between nodes) in the network and lead time due to different node layers. Future study can include logistical network failures between the nodes. The limited number of nodes in the considered SC network may limit the generalizability of the findings. Future research can incorporate the application of this model to more complex SC networks. The probabilistic theory has been used to demonstrate disruption propagation in the network. However, other theories such as utility theory or stakeholder theory have the potential to provide insights into the behaviour of risks. It could be insightful to evaluate the effectiveness of these methods in the context of our model. The paper contributes to the growing SCRM research by providing a holistic measurement approach for predicting the complex behaviour of risk propagation.

Acknowledgment: The authors would like to thank two anonymous reviewers and AE for their constructive recommendations for improving the overall quality of the manuscript.

Appendix

1. Service Level

Fraction of unfulfilled demand = $\frac{LS_{iw}}{Dem_w}$

Fraction of fulfilled demand in week w = $1 - \frac{LS_{iw}}{Dem_w}$

Average fill rate in W weeks = $\frac{\sum_{w=1}^{W}\left(1-\frac{LS_{iw}}{Dem_w}\right)}{W}$

Average service level over N simulations =$\left(\frac{1}{N}\right) * \sum_{n=1}^{N}\left(\frac{\sum_{w=1}^{W}\left(1-\frac{LS_{iw}}{Dem_w}\right)}{W}\right)$

2. Resilience index

Service level in absence of disruption = SL_{k0}

Expected degraded service level during disruption in week w = SL_{kw}

Change in service level during disruption = $\Delta SL = SL_{k0} - SL_{kw}$

ΔSL increases during the disruption phase and decreases during the recovery phase to 0, hence, resulting in a triangle pattern.

When time is *continuous*, aggregating ΔSL over $[w_0, w_n]$ we get,

Total $\Delta SL = \int_{w=w_0}^{w_n}(SL_{k0} - SL_{kw})dw$

We consider time at a *discrete* weekly level we get,

Total $\Delta SL = \sum_{w=w_0}^{w_n}(SL_{k0} - SL_{kw})$

In ideal scenario total SL = $\sum_{w=w_0}^{w_n}(SL_{k0}) = SL_{k0}(w_n - w_0)$

Overall fractional loss in SL = $\frac{\sum_{w=w_0}^{w_n}(SL_{k0}-SL_{kw})}{SL_{k0}(w_n-w_0)}$.

We can conclude, intuitively, that if the overall fractional loss in SL is high then the supply chain node will take time to recover from the disrupted phase and will be less resilient. However, lower fractional loss implies that the node can spring back to its potential when disruption ends. Consequently, we define Resilience Index (RSI) in Equation 16 such that *higher service level in a week*, during disruption, contributes to *higher resilience index*

$$RSI_k = 1 - \frac{\int_{w=w_0}^{w_n}(SL_{k0}-SL_{kw})dw}{SL_{k0}(w_n-w_0)} \cong 1 - \frac{\sum_{w=w_0}^{w_n}(SL_{k0}-SL_{kw})}{SL_{k0}(w_n-w_0)} = 1 - \frac{\sum_{w=w_0}^{w_n}\left(1-{SL_{kw}}/{SL_{k0}}\right)}{(w_n-w_0)}$$

3. Coefficient of Variation (*Adapted from Bimpikis et al. 2017*)

$N(l)$: number of nodes in layer l
$C(l)$: Cost of processing and production in layer l
DP_l: average disruption probability of nodes in layer l
DP_R: average disruption probability of retailer
L: set of layers or tiers in supply chain network
y_j: realised output of node j
m_{ij}: ratio of demand of node i to realised output of node j
$m_{ij}(y_j)$: Demand of node i
p_i: Price at node i
s: supply of raw material by $(L+1)^{th}$ tier supplier
$E(.)$: Expected value of a variable
Total realised output of a node = $y_i = \sum_{j\epsilon P(i)} y_j\, m_{ij}$
The expected output at retailer is given by the following equation:

$$\mu(R) = E(y_R) = \frac{1 - DP_R}{N(R)} \prod_{l \in L - \{R\}} (1 - DP_l)$$

Profit at retailer $\varphi_i = E(Revenue_i - cost\ of\ buying\ from_j - cost\ of\ processing_i)$
Where $P(i)$ is the set of nodes to which node i supplies products

$$\varphi_i = E\left[p_i \sum_{l\epsilon P(i)} m_{il} y_l - \sum_{l\epsilon P(i)} p_l m_{il} y_l - C(i)\left(\sum_l m_{il} y_l\right)^2\right]$$

Differentiating with respect to m_{ij} i.e. with respect to sourcing decisions of i to get first order optimality criteria for node i:

$$\frac{\partial \varphi_i}{\partial m_{ij}} = \frac{\partial}{\partial m_{ij}} E\left[p_i \sum_{l\epsilon P(i)} m_{il} y_l - \sum_{l\epsilon P(i)} p_l m_{il} y_l - C(l)\left(\sum_l m_{il} y_l\right)^2\right] = 0$$

$$E\left[p_i y_j - p_j y_j - 2C(l)\left(\sum_l m_{il} y_l\right) y_j\right] = 0$$

$$(1 - DP_l)p(l)\mu(l+1) - p(l+1)\mu(l+1) - 2C(l)\left(\sum_{l \neq j | l\epsilon P(i)} m_{il}\pi(l+1) + m_{ij}\hat{\pi}(L+1)\right) = 0$$

$$\pi(l) = E(y_{i1}, y_{i2}) = E\left[\left(\sum_{j\epsilon P(i1)} y_j m_{i1j}\right)\left(\sum_{j\epsilon P(i2)} y_j m_{i2j}\right)\right]$$

$$\pi(l) = \left(\frac{s(1 - DP_l)}{N(l)}\right)^2 \prod_{l=l+1}^{L} \left(\frac{(1 - DP_l)^2}{N(l)}\right)\left(N(l) - 1 + \frac{1}{(1 - DP_l)}\right)\ for\ \ N(l) > 1$$

$$\hat{\pi}(l) = \frac{\pi(l)}{(1 - DP_l)}$$

Using the above equations for finding variation in output at the retailer we write CV_R as:

$$CV_R = \sqrt{\frac{Var(R)}{E(R)^2}} = \sqrt{\frac{E(R^2) - E(R)^2}{E(R)^2}} = \sqrt{\frac{\hat{\pi}(R)}{(s\prod_{l=1}^{L}(1 - DP_l))^2} - 1}$$

$$CV_R = \left(\frac{1}{(1-DP_R)}\prod_{l\in L-\{R\}}\left(\frac{1}{N(l)}\right)\left(N(l) - 1 + \frac{1}{(1-DP_l)}\right) - 1\right)^{\frac{1}{2}}$$

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