Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis

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Answers to Exercises

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Answer to Exercise 10.1: We assume $\mathsf{P}(\mathsf{Rain}=\mathsf{no}) = \mathsf{P}(\mathsf{Sprinkler}=\mathsf{no}) = 0.9$.

(a) $\varepsilon = \{\mathsf{Holmes'\ lawn} = \mathsf{wet}, \mathsf{Gibbon's\ lawn} = \mathsf{dry}, \mathsf{Watson's\ lawn} = \mathsf{dry}\}$.

(b) We compute normalized likelihoods of the hypothesis given each subset of the evidence

Gibbon's lawn = dry	Holmes' lawn = wet	Watson's lawn = dry	Sprinkler = yes
			1
		+	1
	+		5.05
	+	+	9.88
+			1
+		+	1
+	+		9.88
+	+	+	10.0

(c) We compute Bayes' factor for each subset of the evidence

Gibbon's lawn = dry	Holmes' lawn = wet	Watson's lawn = dry	Sprinkler = yes
			1
		+	81.1
	+		0.92
	+	+	73.66
+			81.1
+		+	7290.1
+	+		73.66
+	+	+	6620.69

(d) We perform a what-if analysis on each evidence node

Evidence change	P(Sprinkler = yes)
Gibbon's lawn = wet	0.89
Holmes' lawn = dry	0.01
Watson's lawn = wet	0.89

(e) For each finding ε_X, we compute $\mathsf{P}(\mathsf{Sprinkler}=\mathsf{yes})$, $\mathsf{P}(\mathsf{Sprinkler}=\mathsf{yes} \mid \varepsilon \setminus \{\varepsilon_X\})$ and $\mathsf{P}(\mathsf{Sprinkler}=\mathsf{yes} \mid \varepsilon)$

Finding	P(Sprinkler = yes)	$\mathsf{P}(\mathsf{Sprinkler}=\mathsf{yes} \mid \varepsilon \setminus \{\varepsilon_X\})$	$\mathsf{P}(\mathsf{Sprinkler}=\mathsf{yes} \mid \varepsilon)$
Gibbon's lawn = dry	0.1	0.9879	0.9999
Holmes' lawn = wet	0.1	0.1	0.9999
Watson's lawn = dry	0.1	0.9879	0.9999

Answer to Exercise 10.2:

(a) $\varepsilon = \{\mathsf{Smoker} = \mathsf{yes}, \mathsf{Asia} = \mathsf{yes}, \mathsf{Dyspnea} = \mathsf{yes}\}$.

(b) We compute normalized likelihoods of the hypothesis given each subset of the evidence

$\mathsf{Smoker} = \mathsf{yes}$	$\mathsf{Asia} = \mathsf{yes}$	$\mathsf{Dyspnoa} = \mathsf{yes}$	$\mathsf{Bronchitis} = \mathsf{yes}$
			1
		+	1.85
	+		1
	+	+	1.80
+			1.33
+		+	1.96
+	+		1.33
+	+	+	1.93

(c) We compute Bayes' factor for each subset of the evidence

$\mathsf{Smoker} = \mathsf{yes}$	$\mathsf{Asia} = \mathsf{yes}$	$\mathsf{Dyspnoa} = \mathsf{yes}$	$\mathsf{Bronchitis} = \mathsf{yes}$
			1
		+	0.99
	+		1
	+	+	1.0
+			0.73
+		+	0.73
+	+		0.73
+	+	+	0.73

(d) We perform a what-if analysis on each evidence node

Evidence change	$\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes})$
$\mathsf{Smoker} = \mathsf{no}$	0.72
$\mathsf{Asia} = \mathsf{no}$	0.88
$\mathsf{Dyspnoa} = \mathsf{no}$	0.26

(e) For each finding ε_X, we compute $\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes})$, $\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes} \mid \varepsilon \setminus \{\varepsilon_X\})$ and $\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes} \mid \varepsilon)$

Finding	$\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes})$	$\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes} \mid \varepsilon \setminus \{\varepsilon_X\})$	$\mathsf{P}(\mathsf{Bronchitis} = \mathsf{yes} \mid \varepsilon)$
$\mathsf{Smoker} = \mathsf{yes}$	0.45	0.81	0.87
$\mathsf{Asia} = \mathsf{yes}$	0.45	0.88	0.87
$\mathsf{Dyspnoa} = \mathsf{yes}$	0.45	0.60	0.87

Answer to Exercise 10.3:

(a) $\mathsf{P}(\mathsf{Disease} = \mathsf{true} \mid \mathsf{Test} = \mathsf{true}) = 0.0196$.

(b) $f(t) = \frac{19.6*t}{18.6*t+0.98}$.

(c) $f'(t_0) = 19.3$.

(d) $(-\infty, 0.046)$.

Answer to Exercise 10.4:

(a) $f(t) = \frac{-1.086*t+1.099}{-0.986*t+1.099}$ computed for initial value $\mathsf{P}(\mathsf{Rain} = \mathsf{yes}) = 0.1$.

(b) $f'(t_0) = -0.135$.

(c) $f(t) = \frac{0.112*t}{-0.986*t+1.099}$ computed for initial value $\mathsf{P}(\mathsf{Rain} = \mathsf{yes}) = 0.1$.

(d) $(-\infty, 0.81667)$ computed for initial value $\mathsf{P}(\mathsf{Rain} = \mathsf{yes}) = 0.1$.