BCH Code Selection and Iterative Decoding for BCH and LDPC Concatenated Coding System

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Introduction

Introduction

- In the last decade, Bose-Chaudhuri-Hocquenghem (BCH) codes have been widely used for data protection on flash memory.
- Algebraic decoding algorithms for BCH codes are then employed when errors are detected.
- As the rapid development of flash memory, the required bit error rate (BER) is more strict than before.
- For flash memory industries, a favorable coding scheme is perhaps to concatenate systematic BCH codes with quasi-cyclic low-density parity-check (QC-LDPC) codes.

This coding scheme have several advantages apart from its powerful correcting capability.

- Owing to the systematic characteristic, the decoding complexity is often not large.
- The data read operation can be directly performed by the outer BCH code decoder if the quality of read channel is good. Only when the outer code decoding fails, the inner LDPC code decoder is activated to produce more reliable estimates of the original data.
- The conventional hardware implementation of BCH code encoder and decoder can be retained.
- The special parity-check matrix structure of the QC-LDPC code makes fast encoding and low complexity decoding feasible.

- Based on the concatenated coding scheme, the QC-LDPC code can be decoded by the sum-product algorithm (SPA) which is proposed by Gallager in his 1963 thesis.
- It is observed from simulations that an error floor at high signal-to-noise power ratio (SNR) due to some particular error patterns seems unavoidable.
- Although the outer BCH code can improve this error floor, increasing the error correcting capability of the outer code will be at the expense of code rate loss.
- In this thesis, we propose a selection method of BCH codes and a feedback strategy for iterative decoding.

Preliminaries

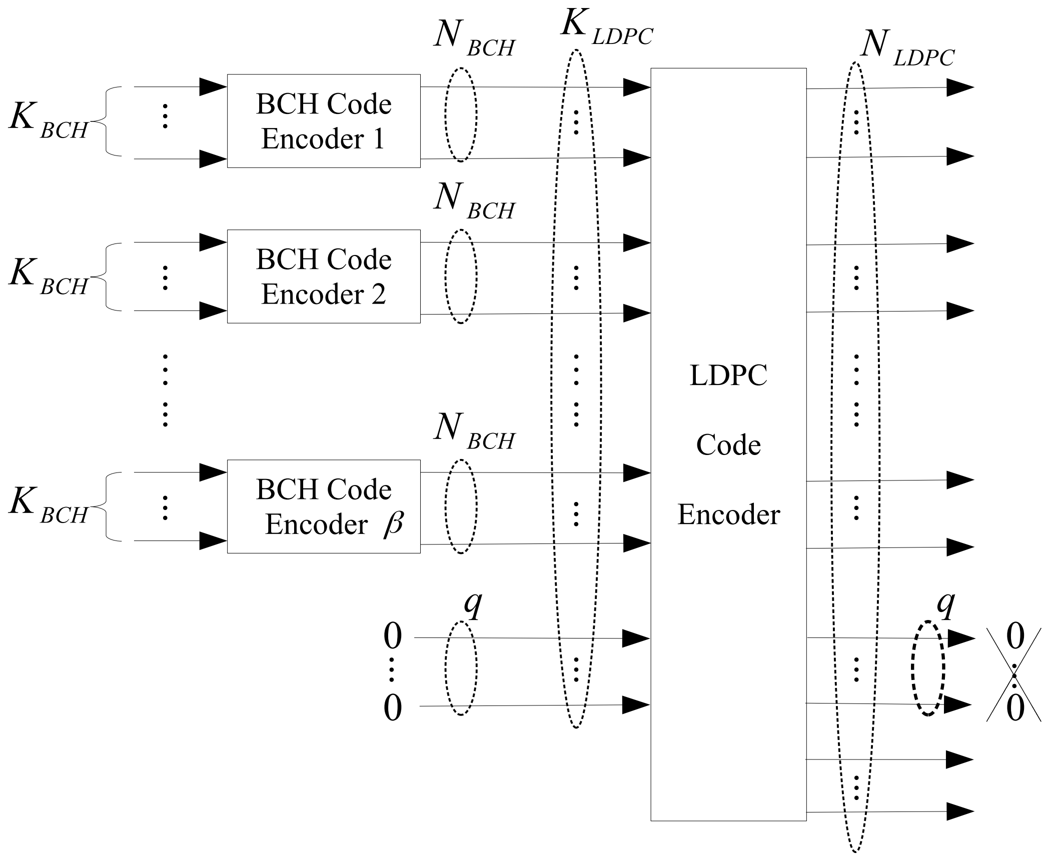

Figure 1: Block diagram of the concatenated coding system consisting of β $(N_{\mathrm{BCH}}, K_{\mathrm{BCH}})$ BCH codes and one $(N_{\mathrm{LDPC}}, K_{\mathrm{LDPC}})$ LDPC code. q zeros will be added at the end of these β BCH codewords, where $q = K_{\mathrm{LDPC}} - \beta \cdot N_{\mathrm{BCH}}$.

System Model

- We suppose that except for the q extra zeros, the coded bits are BPSK-modulated and transmitted over an additive white Gaussian noise (AWGN) channel.
- For notational convenience, let $\mathcal{P}$ be the set of indices corresponding to these q extra zeros in an LDPC codeword.
- The received signal at time i can be represented as

$$y_i = x_i + w_i, \;\; \forall \, i \notin \mathcal{P} \tag{1}$$

where $x_i \in \{-1, +1\}$ is the ith modulated symbol, and w_i denotes a zero-mean Gaussian noise sample with variance σ^2.

Receiver

- At the receiver side, the sum-product algorithm (respectively, bit-flipping algorithm) is first employed to decode the LDPC code. The hard-decision decoding result is denoted by $\hat{\boldsymbol{z}}$.
- Upon reception of this hard-decision sequence $\hat{\boldsymbol{z}}$, an algebraic decoding algorithm then performs the decoding of BCH codes and outputs the estimates of the information bits. These decoded information bits are denoted by $\hat{\boldsymbol{d}}$.

Decoding Algorithm for LDPC Code

For $(N_{\text{LDPC}}, K_{\text{LDPC}})$ QC-LDPC code:

- Denote by $\boldsymbol{H}$ the parity-check matrix of a $(N_{\text{LDPC}}, K_{\text{LDPC}})$ QC-LDPC code.
- The Tanner graph corresponding to $\boldsymbol{H}$ contains N_{LDPC} variable nodes and M check nodes.
- Let v_i and $\mathcal{M}(i)$ be the ith variable node and the set of check nodes connecting to v_i in the Tanner graph, respectively.
- Let c_j and $\mathcal{N}(j)$ be respectively the jth check node and the set of variable nodes connecting to c_j.
- $E_{i,j}$ represents the extrinsic information passing from check node c_j to variable node v_i.
- Denote by $L_{i,j}$ the extrinsic information passing from variable node v_i to check node c_j.

Step 0. For $1 \le i \le N_{LDPC}$, *compute the LLR*

$$R_i \triangleq \begin{cases} 2y_i/\sigma^2, & \text{if } i \notin \mathcal{P} \\ \infty, & \text{if } i \in \mathcal{P} \end{cases} \tag{2}$$

For $1 \le i \le N_{LDPC}$ *and* $1 \le j \le M$, *initialize* $E_{i,j} = 0$.

Step 1. Bit to Check Message Update: For $1 \le i \le N_{LDPC}$, *and for* $j \in \mathcal{M}(i)$ *for a given* i,

$$L_{i,j} = \begin{cases} \sum_{j' \in \mathcal{M}(i), j' \neq j} E_{i,j'} + R_i, & \text{if } i \notin \mathcal{P} \\ R_i, & \text{if } i \in \mathcal{P} \end{cases} \tag{3}$$

Step 2. Check to Bit Message Update: For $1 \le j \le M$, *and for* $i \in \mathcal{N}(j)$ *for a given* j,

$$E_{i,j} = \ln \left(\frac{1 + \prod_{i' \in \mathcal{N}(j), i' \neq i} \tanh\left(\frac{L_{i',j}}{2}\right)}{1 - \prod_{i' \in \mathcal{N}(j), i' \neq i} \tanh\left(\frac{L_{i',j}}{2}\right)} \right). \tag{4}$$

Step 3. Codeword Test: Compute the reliability D_i of the ith bit as

$$D_i = \begin{cases} \sum_{j \in \mathcal{M}(i)} E_{i,j} + R_i & \text{if } i \notin \mathcal{P} \\ R_i & \text{if } i \in \mathcal{P} \end{cases} \tag{5}$$

For $1 \leq i \leq N_{LDPC}$, obtain the hard-decision result:

$$\hat{z}_i = \begin{cases} 1 & \text{if } D_i \leq 0 \\ 0 & \text{if } D_i > 0 \end{cases} \tag{6}$$

If $\boldsymbol{H}\hat{\boldsymbol{z}}^T = 0$, output $\hat{\boldsymbol{z}}$ as the decoded result and stop the algorithm; else if the maximum number of iterations is reached, stop the algorithm; else go to Step 1.

Dominated Failure Patterns: Trapping Sets

As widely reported in many research documents, decoding LDPC codes by the SPA can result in relatively large BERs at high SNR in AWGN channels. These relatively large BERs usually appear in the form of error floor, and this error floor phenomenon is more evident especially for short block length LDPC codes.

From a series of studies, a major failure pattern that causes the error floor phenomenon is the "trapping set."

- We say that a sub-graph of a Tanner graph is an (a, b) trapping set if it contains a variable nodes and b odd degree neighboring check nodes that are connected to these variable nodes.
- When these a variable nodes are erroneous while the remaining bits are correct, it can be observed that only these b check nodes can possibly send correct messages to these a variable nodes.
- Hence, the trapping sets, in particular with smaller values of a and b, are the dominant error patterns in the error floor region.

BCH Code Selection and Iterative Decoding

BCH Code Selection

For an LDPC code, it has been known that the error floor is majorly contributed by the dominant trapping sets. Our initial idea is therefore that by concatenating with a proper outer code, this problem can be alleviated.

- A search of the dominated trapping sets must be first performed for a given inner QC-LDPC code.
- Under the premise that the codeword length of the outer BCH code has been determined, and a LDPC code codeword comprises β BCH code codewords, if an (a, b) trapping set with $a = t$ occurs, those t errors might simultaneously fall into a single BCH code block in the worst case.
- To ensure that such an error pattern can be removed by the BCH code decoder, a t-error correctable BCH code must be chosen.

Iterative Decoding for BCH and LDPC Codes

Simulations are performed and show that our choice does improve the BER in the error floor region. However, a little performance loss can also be observed in the waterfall region due to the rate loss.

- To compensate this BER performance degradation in the waterfall region, and also inspired by the success of iterative decoding, we propose a feedback-based iterative decoding for BCH and LDPC concatenated coding system.
- It is worth mentioning that one can of course employ a soft-decision decoding algorithm for BCH codes and result in an iterative decoding naturally; however, such approach will evidently lose the advantage of the fully developed hardware implementation of algebraic decoders.

In order to maintain this hardware superiority of hard-decision BCH decoders, we propose an alternative yet simple solution.

- Our proposal relies on a premise that owing to the superior error correcting capability of the inner LDPC code, the noisy received signals are mostly corrected back to its original transmission values after the LDPC code decoding.
- In addition, by our selection of BCH codes, it is reasonable to expect that most of the residual errors can be removed by the outer BCH code decoders.
- Therefore, our main idea is to feedback these decoding results to the LDPC code decoder and re-do the LDPC decoding. This may make the LDPC code decoder re-generating even more reliable outputs, and hence form a positive interaction cycle between the inner and outer decoders.
- We design a novel iterative decoding strategy for our concatenated coding system: namely, to feedback information from the β algebraic BCH code decoders to the LDPC code decoder.

Iterative Decoding for BCH and LDPC Codes

- Let $\mathcal{S}$ denote the set of indices in an LDPC codeword of length N_{LDPC} that correspond to those BCH decoder inputs, out of which valid BCH codewords can be obtained by the BCH decoder.
- Denote by $\hat{d}_i$ the estimate of the ith bit in an LDPC codeword obtained from the previous outer[1] iteration, where $1 \leq i \leq N_{\mathrm{LDPC}}$.
- Notably, the direct correspondences between the $\beta \cdot K_{\mathrm{BCH}}$ input bits of the BCH encoders and the LDPC codeword are well defined since both the BCH codes and the LDPC code are systematic.

[1]There are two kinds of iterations in our decision feedback-aided iterative decoding algorithm. They are the internal iterations of the LDPC decoder and the external iterations between the BCH decoders and the LDPC decoder. For convenience, the former will be referred to as *inner* iterations, while the latter will be named *outer* iterations.

Iteration between Algebraic Algorithm and SPA

In the first outer iteration of the decision feedback-aided iterative decoding, the LDPC code is decoded by the original sum-product algorithm as have been described in equations (2)-(6). Starting from the second outer iteration, the decoding procedure of the LDPC code is changed to the following:

Step 0. For $1 \le i \le N_{LDPC}$*, compute the LLR*

$$R_i \triangleq \begin{cases} \frac{2\left(1-2\hat{d}_i\right)}{\sigma^2}, & \text{if } i \in \mathcal{S} \\ \infty, & \text{if } i \in \mathcal{P} \\ 2y_i/\sigma^2, & \text{otherwise} \end{cases} \tag{7}$$

For $1 \le i \le N_{LDPC}$ *and* $1 \le j \le M$*, initialize* $E_{i,j} = 0$*.*

Step 1. Bit to Check Message Update: For $1 \le i \le N_{LDPC}$*, and for* $j \in \mathcal{M}(i)$ *for a given* i*,*

$$L_{i,j} = \begin{cases} R_i, & \text{if } i \in \mathcal{P} \cup \mathcal{S} \\ \sum_{j' \in \mathcal{M}(i), j' \neq j} E_{i,j'} + R_i, & \text{otherwise} \end{cases} \tag{8}$$

Step 2. Check to Bit Message Update: For $1 \leq j \leq M$, and for $i \in \mathcal{N}(j)$ for a given j,

$$E_{i,j} = \ln\left(\frac{1 + \prod_{i' \in \mathcal{N}(j), i' \neq i} \tanh\left(\frac{L_{i',j}}{2}\right)}{1 - \prod_{i' \in \mathcal{N}(j), i' \neq i} \tanh\left(\frac{L_{i',j}}{2}\right)}\right). \tag{9}$$

Step 3. Codeword Test: Compute the reliability D_i of the ith bit as

$$D_i = \begin{cases} R_i & \text{if } i \in \mathcal{P} \cup \mathcal{S} \\ \sum_{j \in \mathcal{M}(i)} E_{i,j} + R_i & \text{otherwise} \end{cases} \tag{10}$$

For $1 \leq i \leq N_{LDPC}$, obtain the hard-decision result:

$$\hat{z}_i = \begin{cases} 1 & \text{if } D_i \leq 0 \\ 0 & \text{if } D_i > 0 \end{cases} \tag{11}$$

If $\boldsymbol{H}\hat{\boldsymbol{z}}^T = 0$, output $\hat{\boldsymbol{z}}$ as the decoded result and stop the algorithm; else if the maximum number of (inner) iterations is reached, stop the algorithm; else go to Step 1.

Additional Modifications on Decision Feedback-Aided Iterative Decoding

Modifications on Iterative Decoding

- From simulations, we sense that the outer BCH code decoder may feedback incorrect information to the LDPC code decoder when the error correcting capability of the BCH code is too small.
- This occurs especially when the BCH code decoder outputs a valid but wrong codeword.
- In principle, this probability is roughly inversely proportional to the error correcting capability of the BCH code.
- In order to ensure the exactness of the feedback information from the BCH code decoder to the LDPC code decoder, we have experimented three strategies.

Strategy 1: SPC-enhanced concatenated scheme

- To further secure the exactness of the feedback information, we add an extra single parity-check (SPC) code to double-check the validity of the output BCH codewords.
- This strategy will only result in a small rate loss, and the additional system complexity is almost minimized.

Block Diagram of Strategy 1

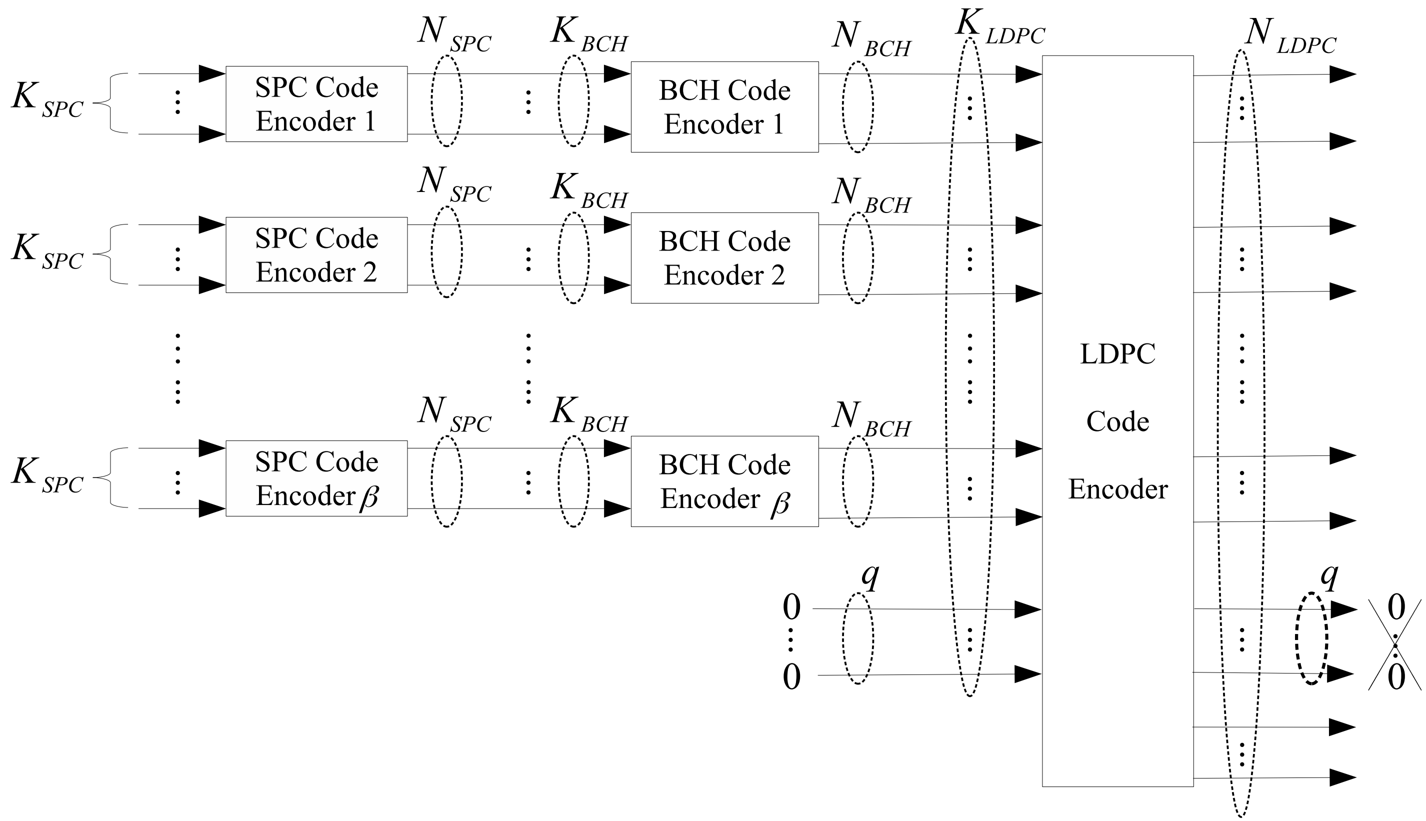

Figure 2: Block diagram of concatenation of the SPC, BCH and LDPC codes.

Decoding Procedure of Strategy 1

As aforementioned, the SPC code is used to detect the errors that the BCH code decoders make.

- In the first outer iteration, the LDPC code decoder produces decoding outputs $\hat{\boldsymbol{z}}$ of length N_{LDPC}.
- Based upon this hard-decision sequence $\hat{\boldsymbol{z}}$, the BCH decoders then output the estimates $\hat{\boldsymbol{d}}$ of the information bits of BCH codes by using an algebraic decoding algorithm.
- Additionally, the results $\hat{\boldsymbol{d}}$ are double-checked to see whether they form a valid codeword of the SPC code.
- As a result, the set $\mathcal{S}$ is re-defined as the set of indices in an LDPC codeword of length N_{LDPC} that correspond to those SPC decoder inputs, out of which valid SPC codewords and also valid BCH codewords can be obtained.
- By the re-defined $\mathcal{S}$, the second outer iteration can be performed, following the steps described in equations (7)-(11).

Strategy 2: Feedback Condition on Euclidean Distance

- In this strategy, we add an explicit condition under which the information bits in $\mathcal{S}$ can be feedbacked.
- The system structure in Figure 1 remains and no rate loss is resulted.

Decoding Procedure of Strategy 2

The decoding procedure is modified as follows. In the first outer iteration, the LDPC code decoder generates hard-decision sequence $\hat{\boldsymbol{z}}$. Based on this hard-decision sequence $\hat{\boldsymbol{z}}$, the BCH code decoders then output the estimates $\hat{\boldsymbol{d}}$ of the information bits of BCH codes by using an algebraic decoding algorithm. We then generate β BCH codewords with respect to the estimated information sequence $\hat{\boldsymbol{d}}$.

- Afterwards, for the ith BCH codeword just generated, where $1 \leq i \leq \beta$, we compute the Euclidean distance $u_{\mathrm{BCH}}^{(i)}$ between this BCH codeword and the elements in its corresponding positions in channel output $\boldsymbol{y}$.
- We also compute the Euclidean distance $u_{\mathrm{LDPC}}^{(i)}$ between $\hat{\boldsymbol{z}}$ and $\boldsymbol{y}$ by taking into consideration only the elements corresponding to the the respective positions about the ith BCH codeword.
- Again, the correspondences between BCH codewords and $\boldsymbol{y}$ are well defined since the LDPC code is systematic.

Decoding Procedure of Strategy 2

From our simulations, we notice that the portion of the decoded results $\hat{\boldsymbol{d}}$ that generates the ith BCH codeword is correct with higher probability if $u_{\text{BCH}}^{(i)}$ is smaller than $u_{\text{LDPC}}^{(i)}$.

Therefore, we redefine the set $\mathcal{S}$ as follows:

- $\mathcal{S}$ is the set of indices in an LDPC codeword of length N_{LDPC} that correspond to those BCH decoder inputs, out of which valid BCH codewords can be obtained by the BCH decoder and also its respective $u_{\text{BCH}}^{(i)}$ is smaller than $u_{\text{LDPC}}^{(i)}$.

By the re-defined $\mathcal{S}$, the second outer iteration can be performed, following the steps described in equations (7)-(11).

Strategy 3: Shortening the Outer BCH Code

- It is obvious that if some of the information bits transmitted are known to the receiver, then the outer iterations can evidently improve the error performances by broadcasting these correct values.
- We propose to fix λ information bits as zeros for each BCH coder so that they can serve as the known information bits at the receiver to help improving the error performance.

Block Diagram of Strategy 3

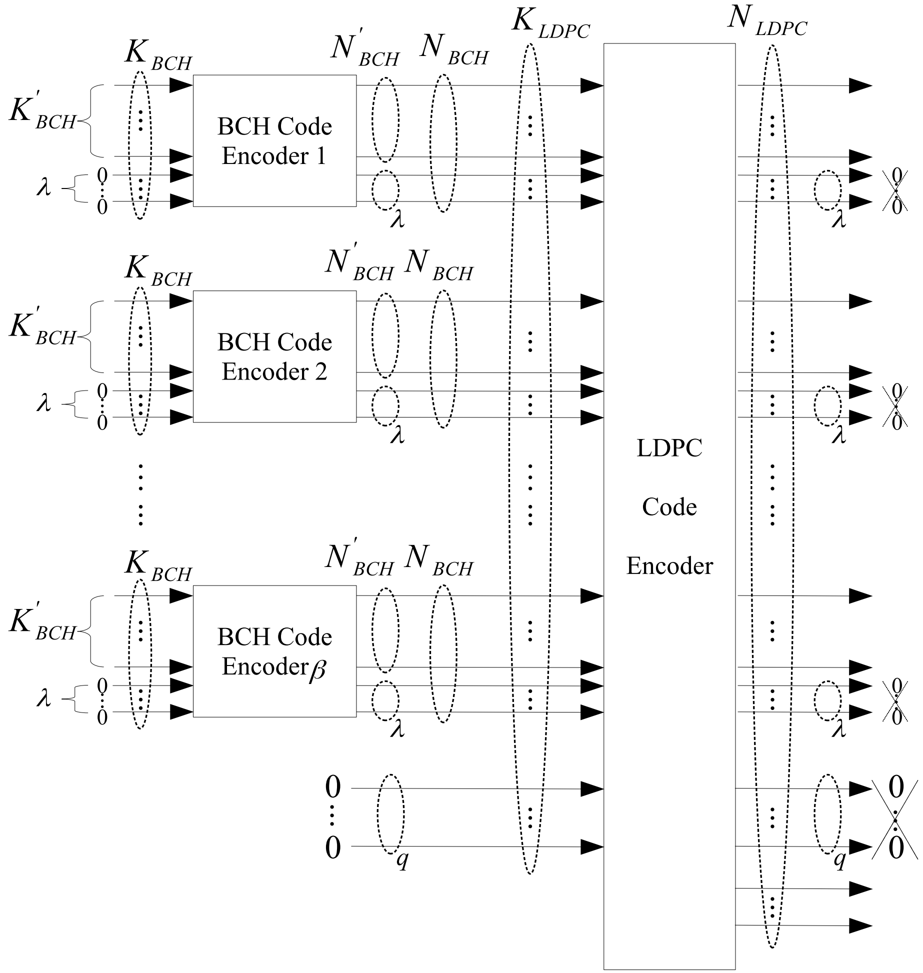

Figure 3: Block diagram of concatenated coding system with λ information bits being fixed as zeros.

Decoding Procedure of Strategy 3

Note that the $q + \beta\lambda$ extra zeros are not transmitted since the values of these bits are prior known at the receiver. Therefore, this strategy will only cause a small rate loss and the new code rate is given by:

$$\frac{\beta K'_{\text{BCH}}}{N_{\text{LDPC}} - \beta\lambda - q} = \frac{\beta K_{\text{BCH}} - \beta\lambda}{N_{\text{LDPC}} - \beta\lambda - q}. \tag{12}$$

- The set $\mathcal{P}$ is redefined as those indices corresponding to these $\beta\lambda + q$ zeros.
- These λ zeros can be used to double-check whether the decoding outputs of the BCH decoder are valid BCH codewords or not. Notably, with this setting, a valid BCH codeword must be one, of which the component bits equal zeros in these λ specific positions.

The detail of the decision feedback-aided iterative decoding procedure is then the same as that described in equations (7)-(11).

Simulation Results

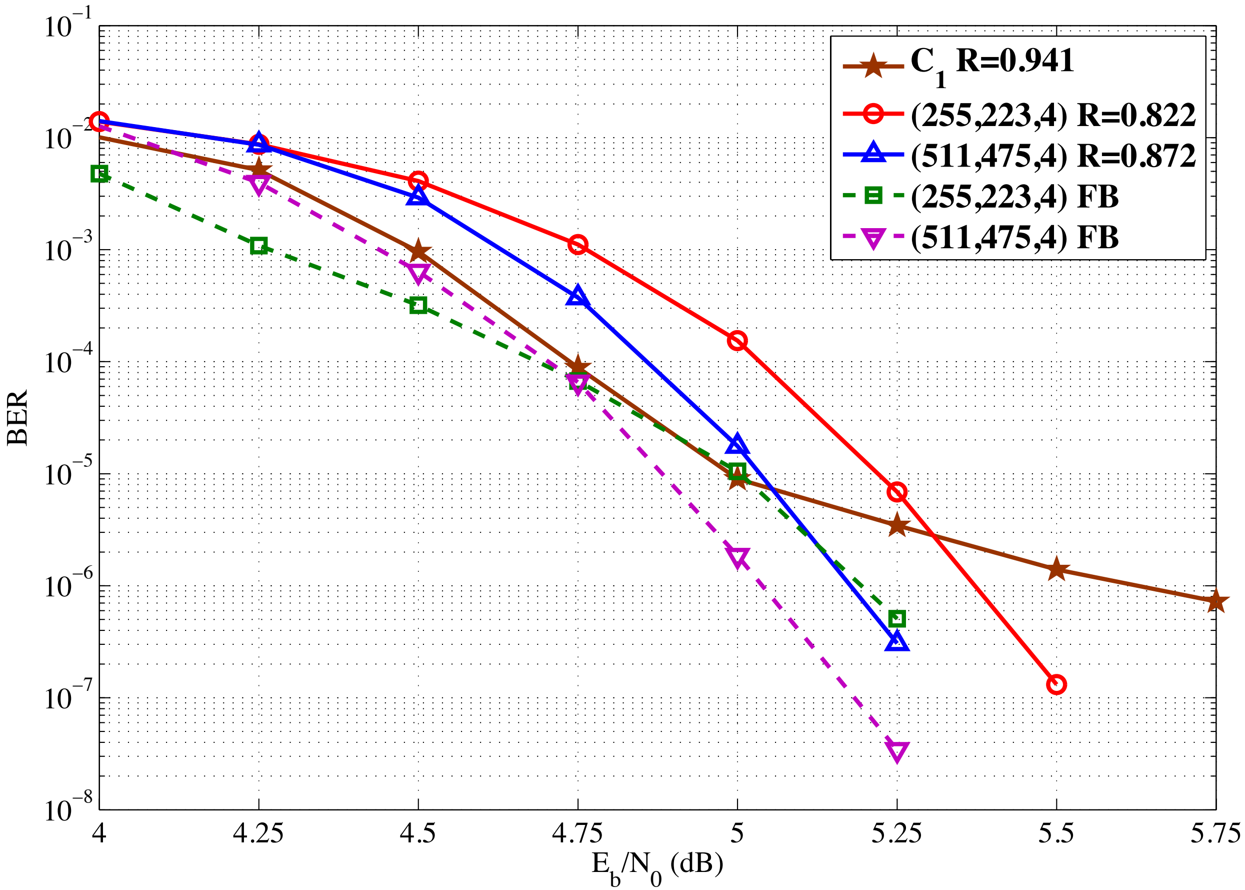

Figure 4: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

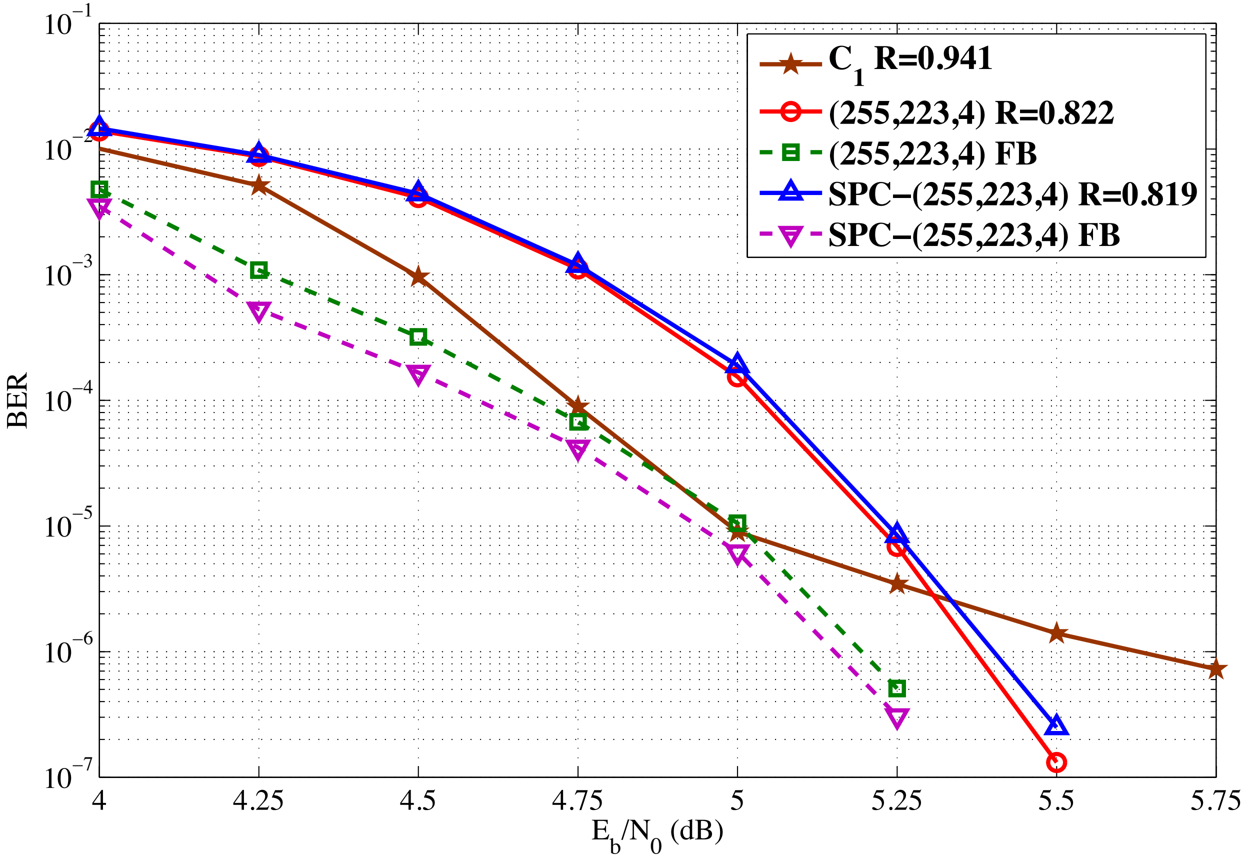

Figure 5: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding SPC-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

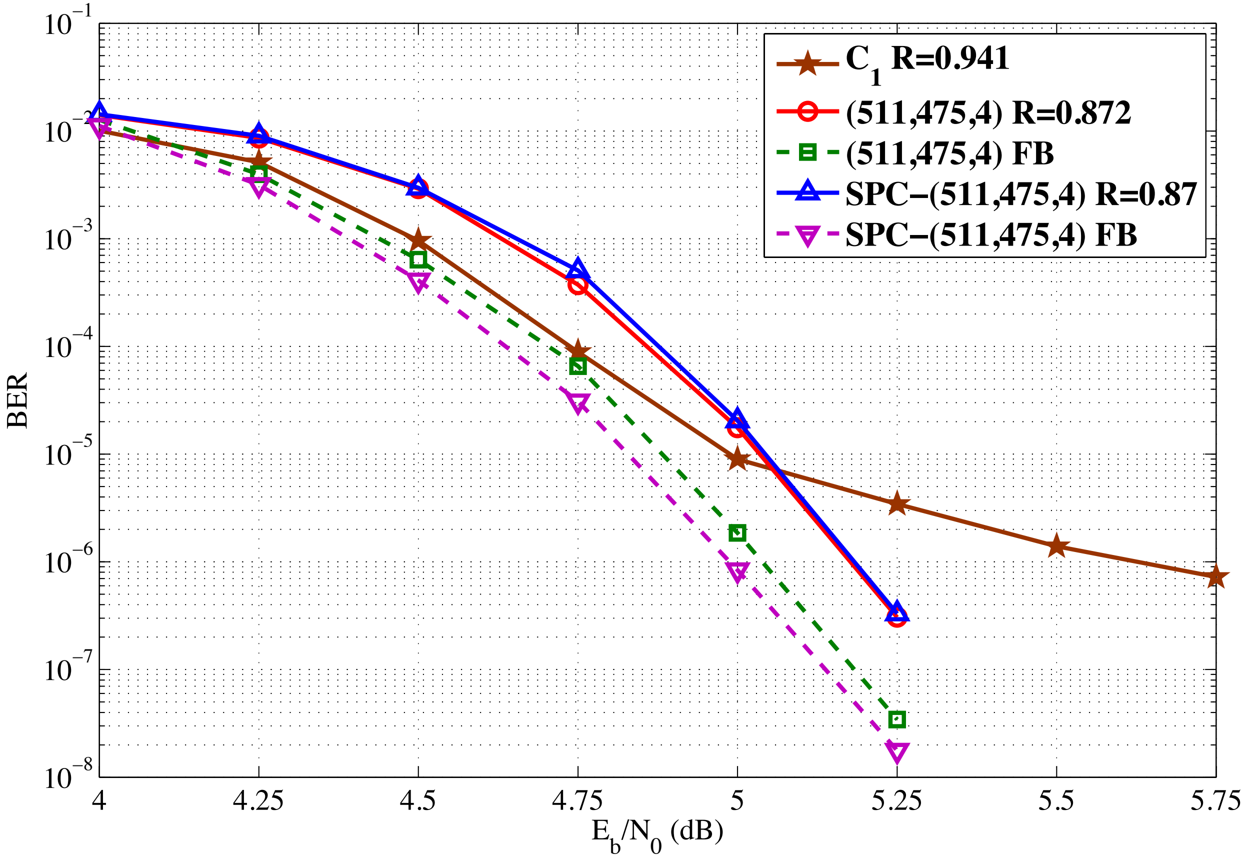

Figure 6: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding SPC-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

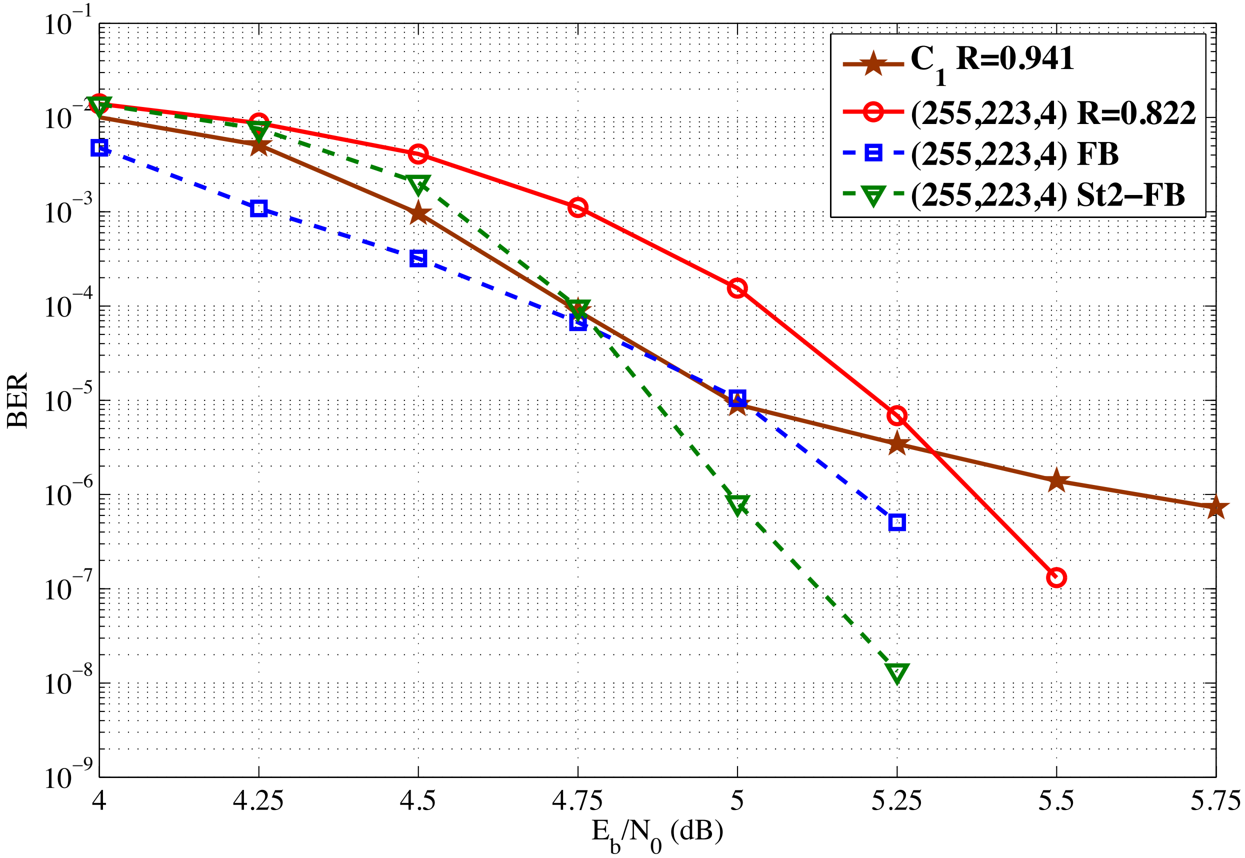

Figure 7: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-2-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with feedback loop. The LDPC code is decoded by the SPA.

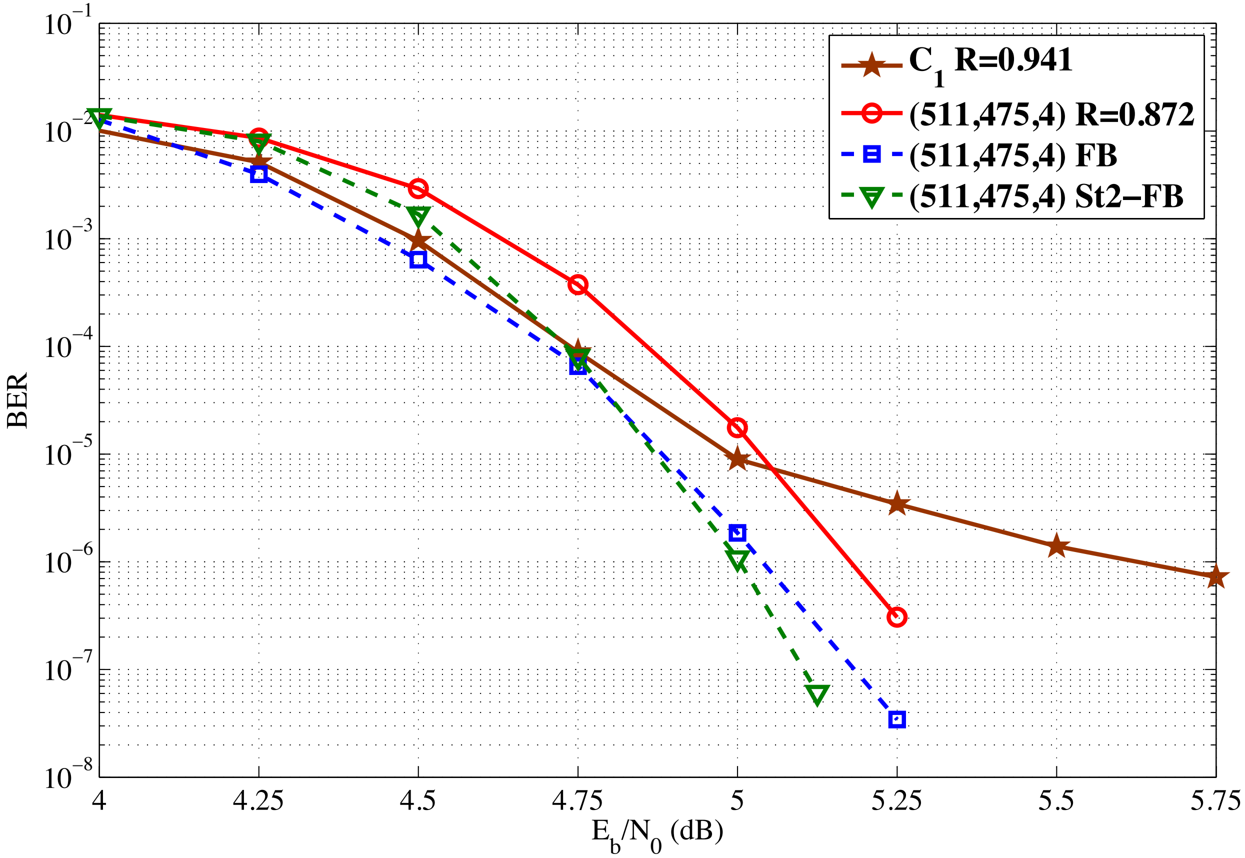

Figure 8: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-2-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with feedback loop. The LDPC code is decoded by the SPA.

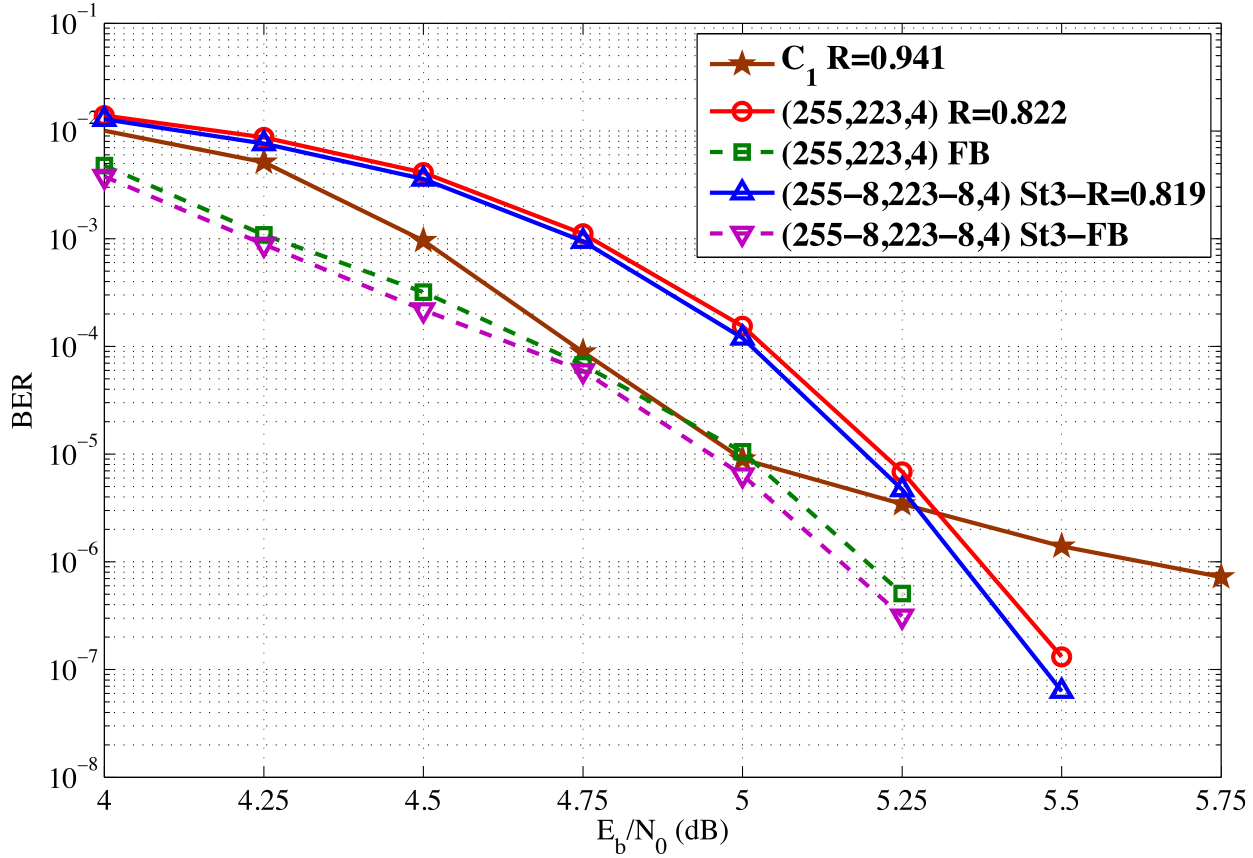

Figure 9: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-3-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop. The parameter λ chosen in Strategy 3 is 8, and the LDPC code is decoded by the SPA.

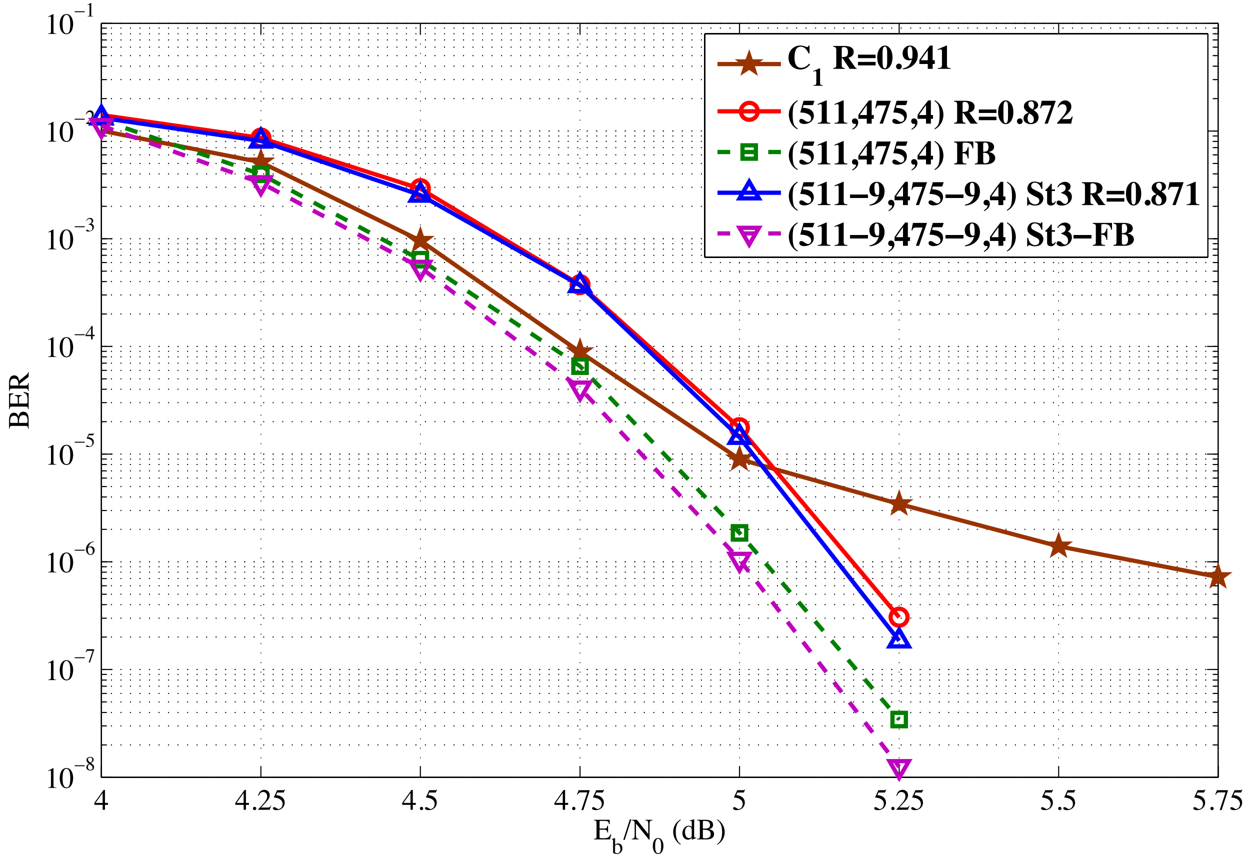

Figure 10: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-3-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The parameter λ chosen in Strategy 3 is 9, and the LDPC code is decoded by the SPA.

Table 1: Effective indices of feedback deeding for the concatenation of $(255, 223, 4)$ BCH codes and the $(6350, 5978)$ QC-LDPC code. The index located at (x, y) indicates the number of occurrences during our simulations for initial (outer) iteration to have y BCH successfully decoding blocks but the final outer iteration to improve up to x BCH successfully decoding blocks. The SNR simulated is 5 dB.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0	0
12	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	0	0
13	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0	1
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0	0
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	0	0
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	0	0	6
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	0	0	6	26
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1	6	57
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	0	1	16	153
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	2	58	423
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8	85	1120
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	113	2962
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	47043

Table 2: Effective indices of feedback deeding for the concatenation of $(511, 475, 4)$ BCH codes and the $(6350, 5978)$ QC-LDPC code. The index located at (x, y) indicates the number of occurrences during our simulations for initial (outer) iteration to have y BCH successfully decoding blocks but the final outer iteration to improve up to x BCH successfully decoding blocks. The SNR simulated is 5 dB.

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	—	0	0	0	0	0	0	0	0	0	0	0
2	—	—	0	0	0	0	0	0	0	0	0	1
3	—	—	—	0	0	1	2	0	0	0	2	4
4	—	—	—	—	0	3	0	0	1	1	0	30
5	—	—	—	—	—	1	1	3	1	4	4	52
6	—	—	—	—	—	—	20	9	2	4	13	115
7	—	—	—	—	—	—	—	14	9	4	12	219
8	—	—	—	—	—	—	—	—	18	9	32	477
9	—	—	—	—	—	—	—	—	—	18	45	906
10	—	—	—	—	—	—	—	—	—	—	67	1410
11	—	—	—	—	—	—	—	—	—	—	—	430298

Table 3: Statistics of the number of outer iterations required in our simulations. The concatenated system consists of $(255, 223, 4)$ BCH codes and an $(6350, 5978)$ LDPC code. The SNR simulated is 5 dB.

Number of outer iterations	1	2	3	4	5	6	7
Number of occurrences	47157	4919	14	0	1	0	0

Table 4: Statistics of the number of outer iterations required in our simulations. The concatenated system consists of $(511, 475, 4)$ BCH codes and an $(6350, 5978)$ LDPC code. The SNR simulated is 5 dB.

Number of outer iterations	1	2	3	4	5	6	7
Number of occurrences	430365	3355	69	20	3	0	0

Conclusion

Conclusion

- For a given inner LDPC code and a given codeword length of outer BCH codes, we suggest a method to determine the error correcting capability of the outer BCH codes based on the knowledge of the dominant trapping sets of the inner LDPC code.

- We present a feedback-based iterative decoding scheme for the BCH and LDPC concatenated coding system. Due to the algebraic decoding characteristic of the outer BCH code decoders, generating soft outputs for the outer code decoders to fit the need of the iterative decoding between inner and outer codes is unnatural. Thus, the feedback-based iterative decoding scheme requires an elaborate design.

- We additionally propose three strategies to improve the reliability of the feedback information and hence the performance of the feedback-based iterative decoding can be further enhanced.

Thank You for Your Attention.

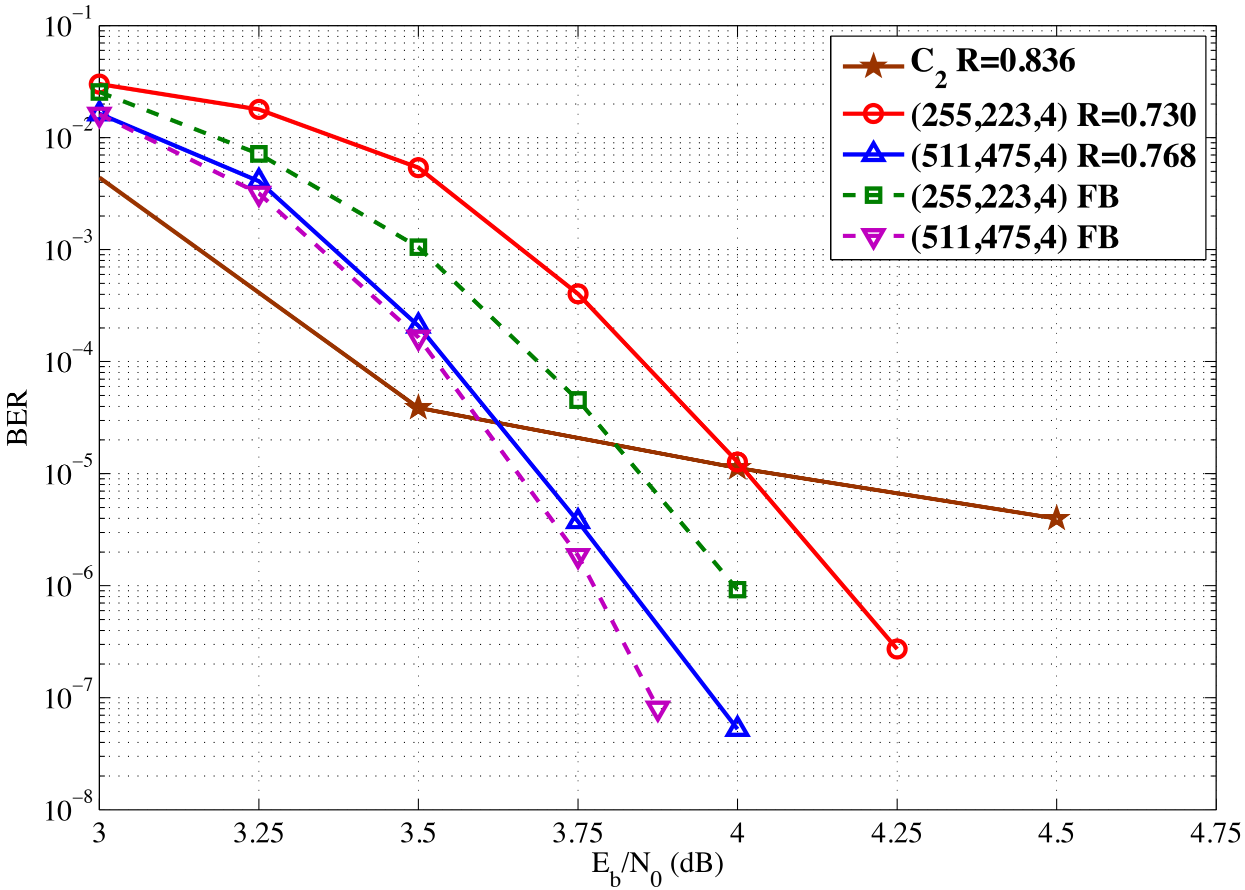

Figure 11: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

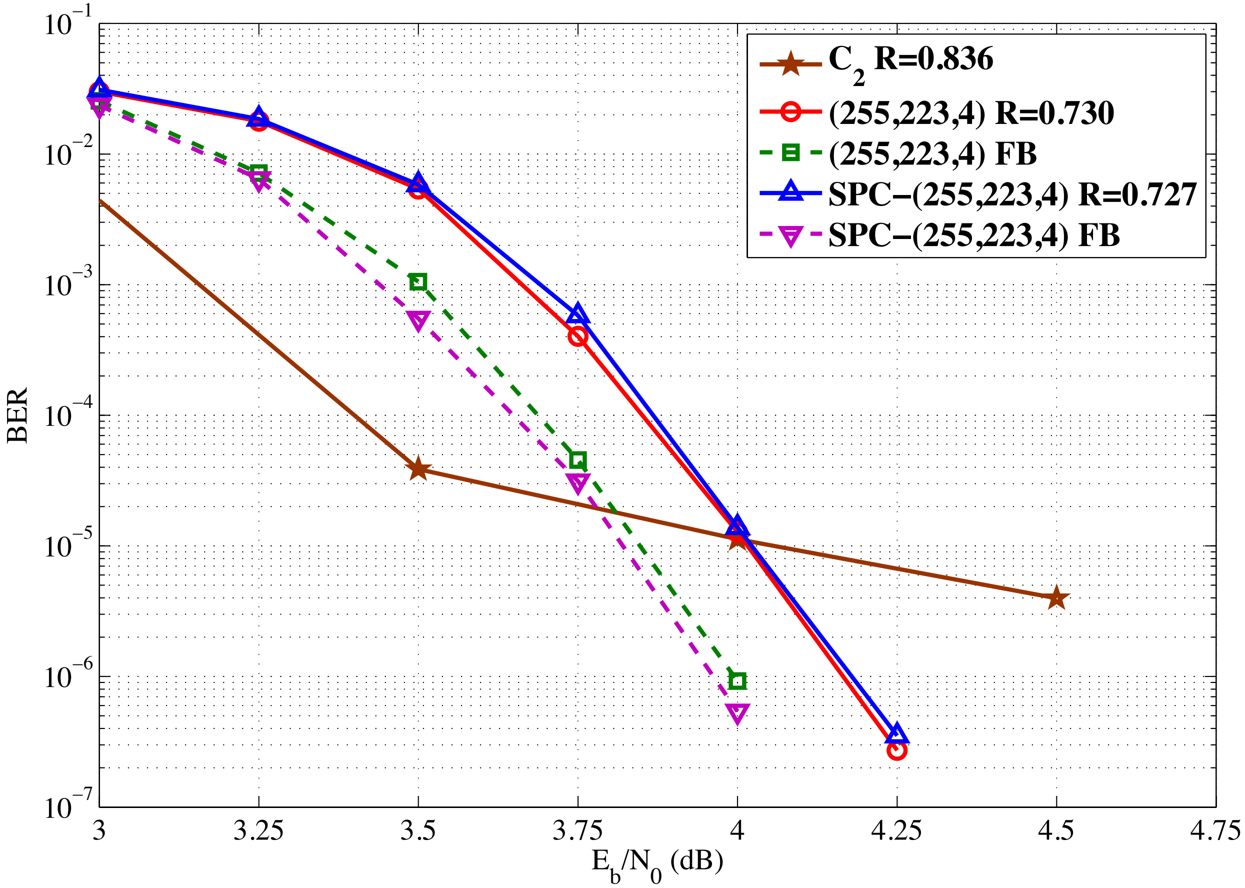

Figure 12: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding SPC-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

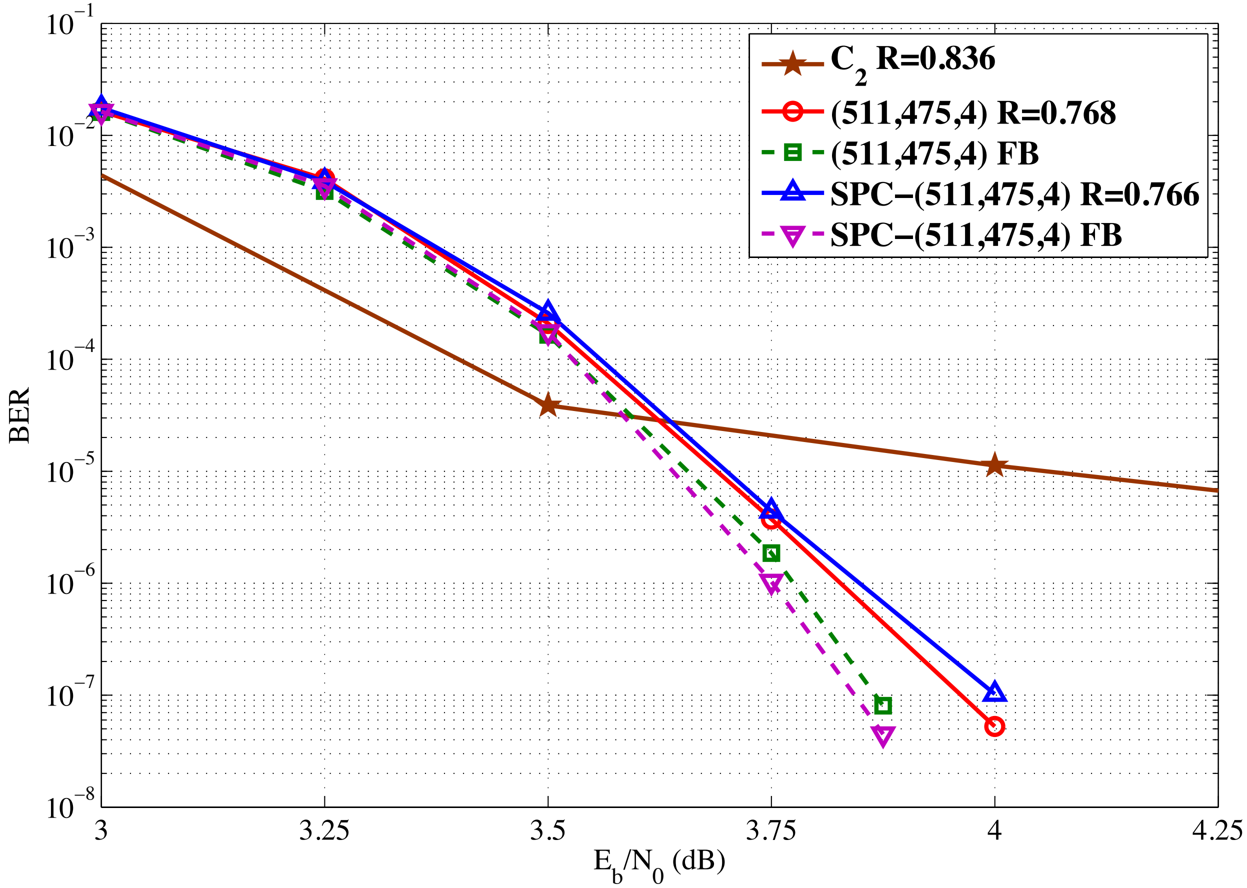

Figure 13: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding SPC-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the SPA.

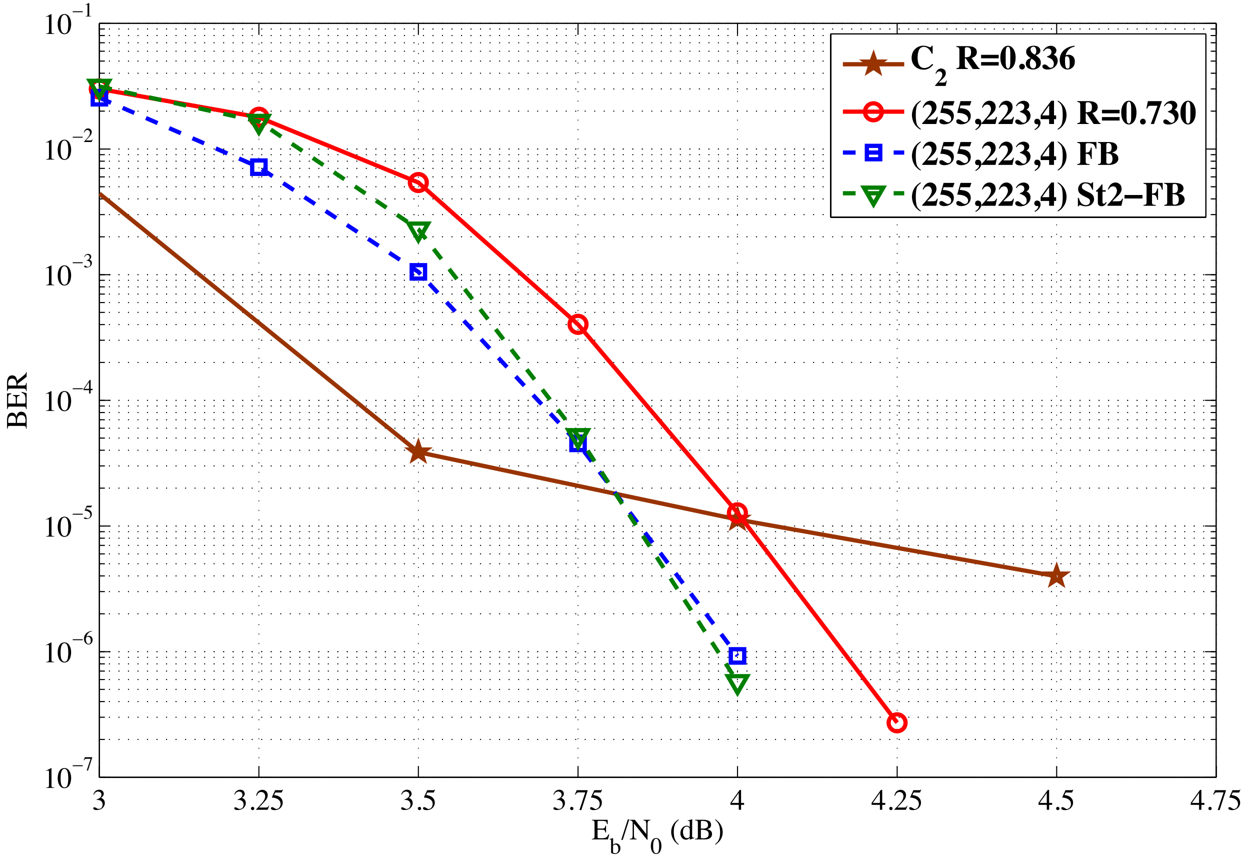

Figure 14: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-2-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with feedback loop. The LDPC code is decoded by the SPA.

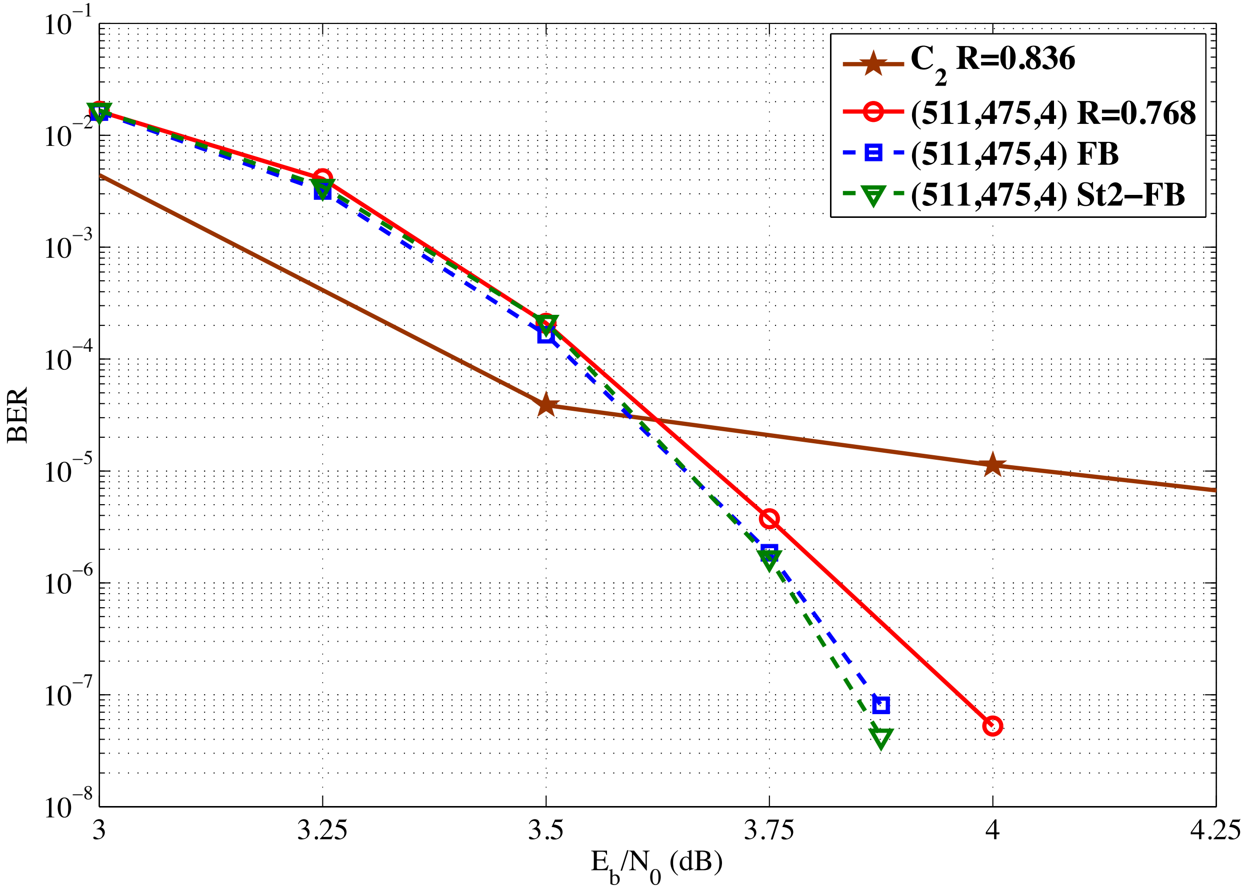

Figure 15: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-2-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with feedback loop. The LDPC code is decoded by the SPA.

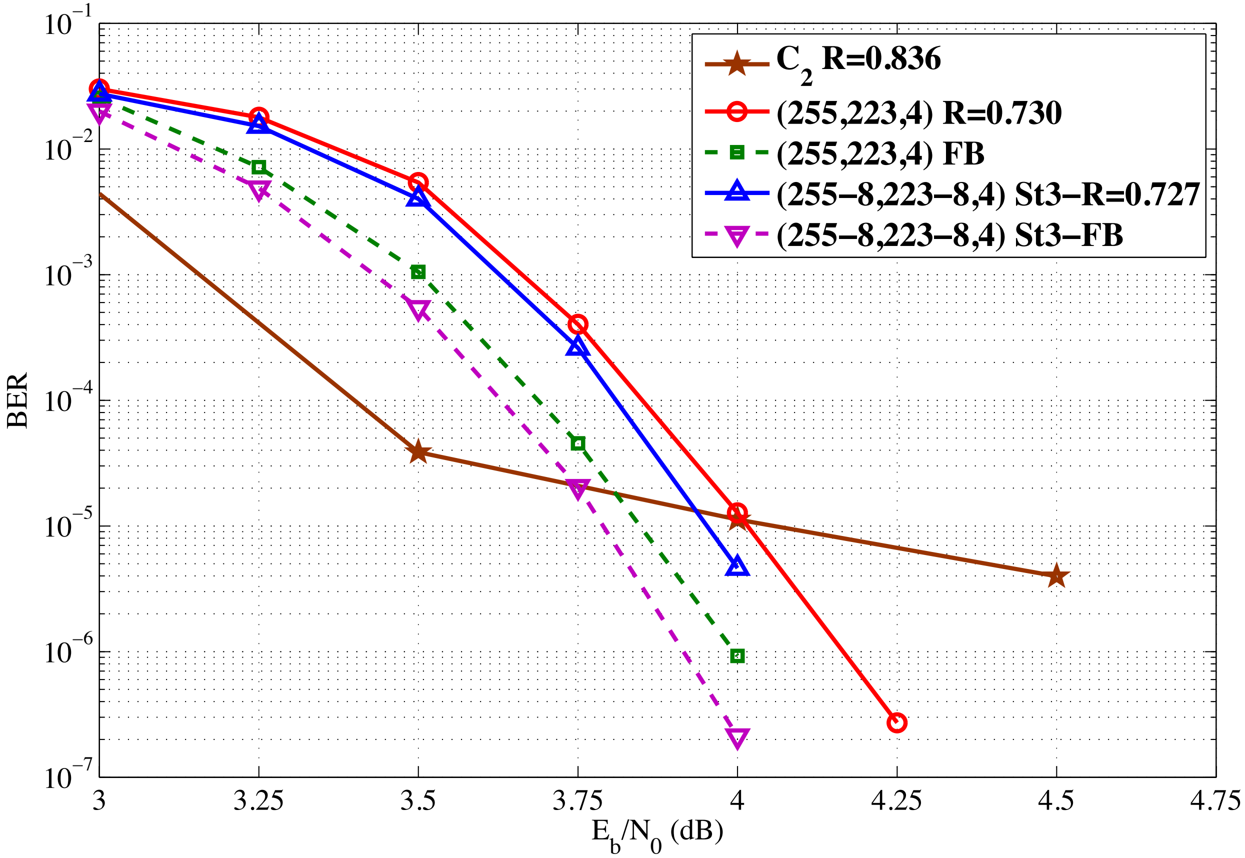

Figure 16: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-3-enhanced concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop. The parameter λ chosen in Strategy 3 is 8, and the LDPC code is decoded by the SPA.

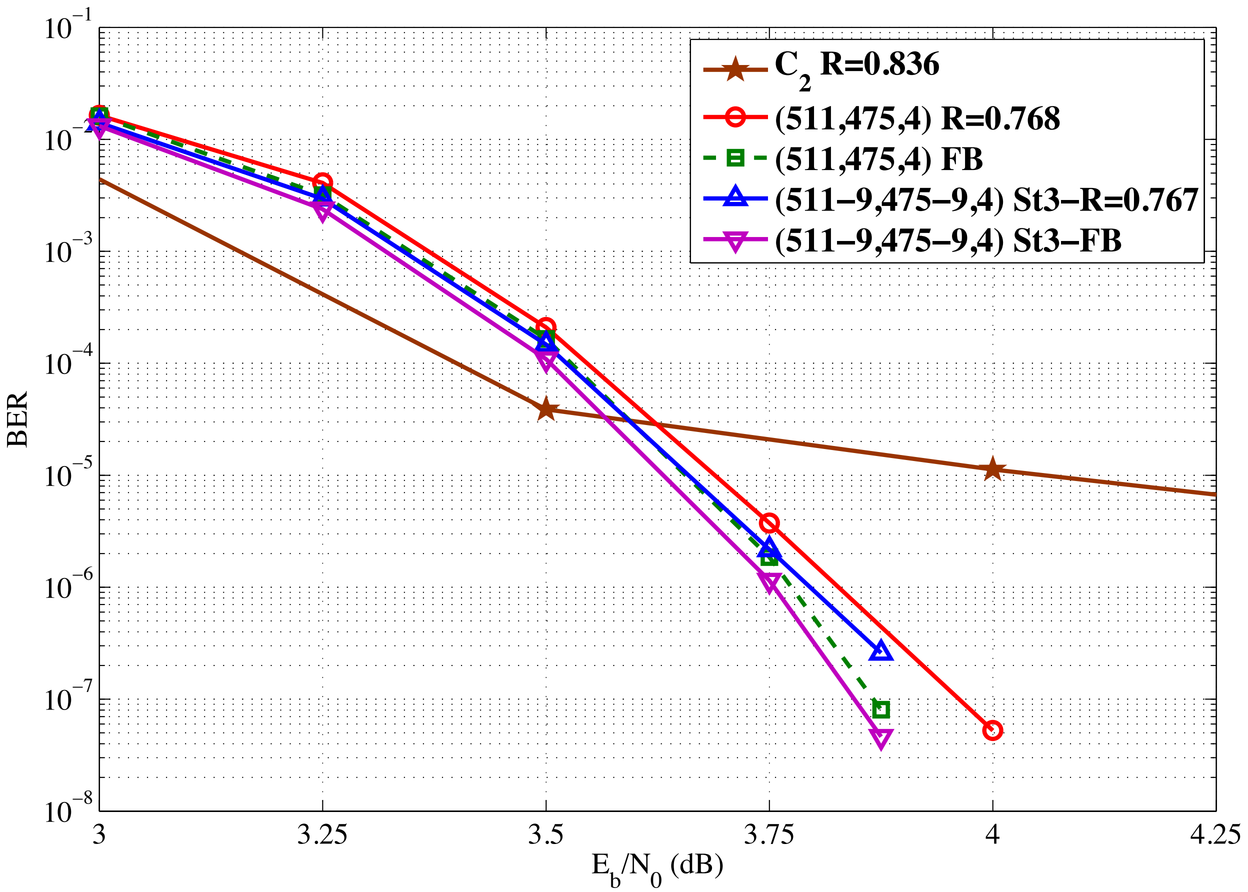

Figure 17: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop, and its corresponding Strategy-3-enhanced concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The parameter λ chosen in Strategy 3 is 9, and the LDPC code is decoded by the SPA.

Table 5: Effective indices of feedback deeding for the concatenation of $(255, 223, 4)$ BCH codes and the $(4590, 3835)$ QC-LDPC code. The index located at (x, y) indicates the number of occurrences during our simulations for initial (outer) iteration to have y BCH successfully decoding blocks but the final outer iteration to improve up to x BCH successfully decoding blocks. The SNR simulated is 3.5 dB.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	—	7	0	1	1	1	0	1	0	0	0	0	0	2	5	8
2	—	—	0	4	1	2	1	1	0	0	1	0	0	2	4	21
3	—	—	—	11	3	2	3	2	1	1	0	0	0	2	10	40
4	—	—	—	—	4	1	0	1	1	0	0	2	1	2	16	48
5	—	—	—	—	—	2	1	2	0	1	0	0	0	3	18	55
6	—	—	—	—	—	—	4	0	0	0	0	0	0	2	30	78
7	—	—	—	—	—	—	—	0	2	0	0	1	1	2	29	80
8	—	—	—	—	—	—	—	—	0	0	0	1	1	2	23	97
9	—	—	—	—	—	—	—	—	—	0	2	1	0	1	18	78
10	—	—	—	—	—	—	—	—	—	—	0	0	0	1	15	80
11	—	—	—	—	—	—	—	—	—	—	—	0	0	0	16	84
12	—	—	—	—	—	—	—	—	—	—	—	—	0	0	10	73
13	—	—	—	—	—	—	—	—	—	—	—	—	—	0	6	79
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	59
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1556

Appendix I: Simulation Results of $\mathcal{C}_2$

Table 6: Effective indices of feedback deeding for the concatenation of $(511, 475, 4)$ BCH codes and the $(4590, 3835)$ QC-LDPC code. The index located at (x, y) indicates the number of occurrences during our simulations for initial (outer) iteration to have y BCH successfully decoding blocks but the final outer iteration to improve up to x BCH successfully decoding blocks. The SNR simulated is 3.5 dB.

	0	1	2	3	4	5	6	7
0	245	0	0	0	0	0	0	0
1	—	23	2	1	0	0	0	86
2	—	—	9	0	1	1	2	64
3	—	—	—	2	0	0	3	41
4	—	—	—	—	0	0	2	24
5	—	—	—	—	—	0	2	12
6	—	—	—	—	—	—	3	23
7	—	—	—	—	—	—	—	35007

Table 7: Statistics of the number of outer iterations required in our simulations. The concatenated system consists of $(255, 223, 4)$ BCH codes and an $(4590, 3835)$ LDPC code. The SNR simulated is 3.5 dB.

Number of outer iterations	1	2	3	4	5	6	7
Number of occurrences	1558	969	130	35	30	8	6

Table 8: Statistics of the number of outer iterations required in our simulations. The concatenated system consists of $(511, 475, 4)$ BCH codes and an $(4590, 3835)$ LDPC code. The SNR simulated is 3.5 dB.

Number of outer iterations	1	2	3	4	5	6	7
Number of occurrences	35009	511	30	6	1	0	0

In the first outer iteration of the decision feedback-aided iterative decoding, the LDPC code is decoded by the original bit-flipping algorithm. Starting from the second outer iteration, the decoding procedure of the LDPC code is changed to the following:

Step 0. Initialization: With the availability of $\mathcal{S}$ *and* $\{\hat{d}_i\}$ *obtained from the previous outer iteration, the value of the* i*th variable node,* $\hat{z}_i$*, is obtained from the* i*th received value* y_i *via hard-decision as:*

$$\hat{z}_i = \begin{cases} 1 & \text{if } y_i \leq 0 \text{ and } i \notin \mathcal{P} \cup \mathcal{S} \\ \hat{d}_i & \text{if } i \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases}, \ \forall\, 1 \leq i \leq N_{LDPC}. \tag{13}$$

Step 1. Bit to Check Message Update: Each variable node sends its value to its neighboring check nodes. Afterwards, each check node calculates whether the parity-check is satisfied or not in terms of the messages obtained from the variable nodes. Set $Q_j = 1$ *if the* j*-th parity-check equation is satisfied; otherwise, set* $Q_j = -1$*. In other words,*

$$Q_j = \prod_{i \in \mathcal{N}(j)} (1 - 2\hat{z}_i)\,, \ \forall\, 1 \leq j \leq M \tag{14}$$

If all $Q_j = 1$*, then output* $\hat{\boldsymbol{z}}$ *and stop the algorithm.*

Step 2. Check to Bit Message Update: Each check node sends its Q-value to its neighboring variable nodes. Set $F_i = 1$ *if all check equations associated to check nodes that are connected to the* ith *variable node are "not satisfied;" otherwise, set* $F_i = 0$*. Flip the value of the variable node if* $F_i = 1$*. Specifically,*

$$F_i = \prod_{j \in \mathcal{M}(i)} \frac{1 - Q_j}{2}, \ \forall\ 1 \leq i \leq N_{LDPC} \text{ and } i \notin \mathcal{P} \cup \mathcal{S}. \tag{15}$$

$$\hat{z}_i = \begin{cases} (\hat{z}_i + 1) \bmod 2, & \text{if } F_i = 1 \\ \hat{z}_i, & \text{if } F_i = 0 \end{cases}, \ \forall\ 1 \leq i \leq N_{LDPC} \text{ and } i \notin \mathcal{P} \cup \mathcal{S}. \tag{16}$$

If $\hat{\boldsymbol{z}}$ *is a valid LDPC codeword or the maximum number of (inner) iterations is reached, output* $\hat{\boldsymbol{z}}$ *and stop the algorithm;*
else go to Step 1.

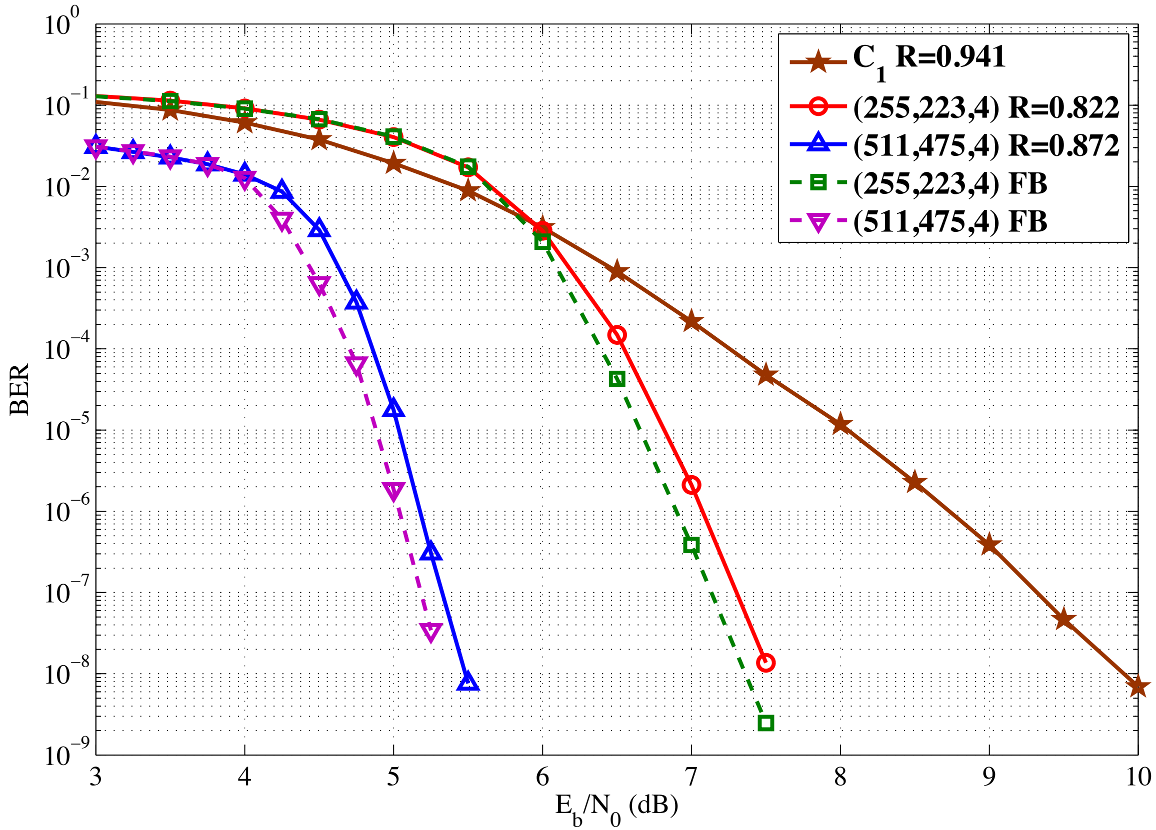

Figure 18: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the BFA.

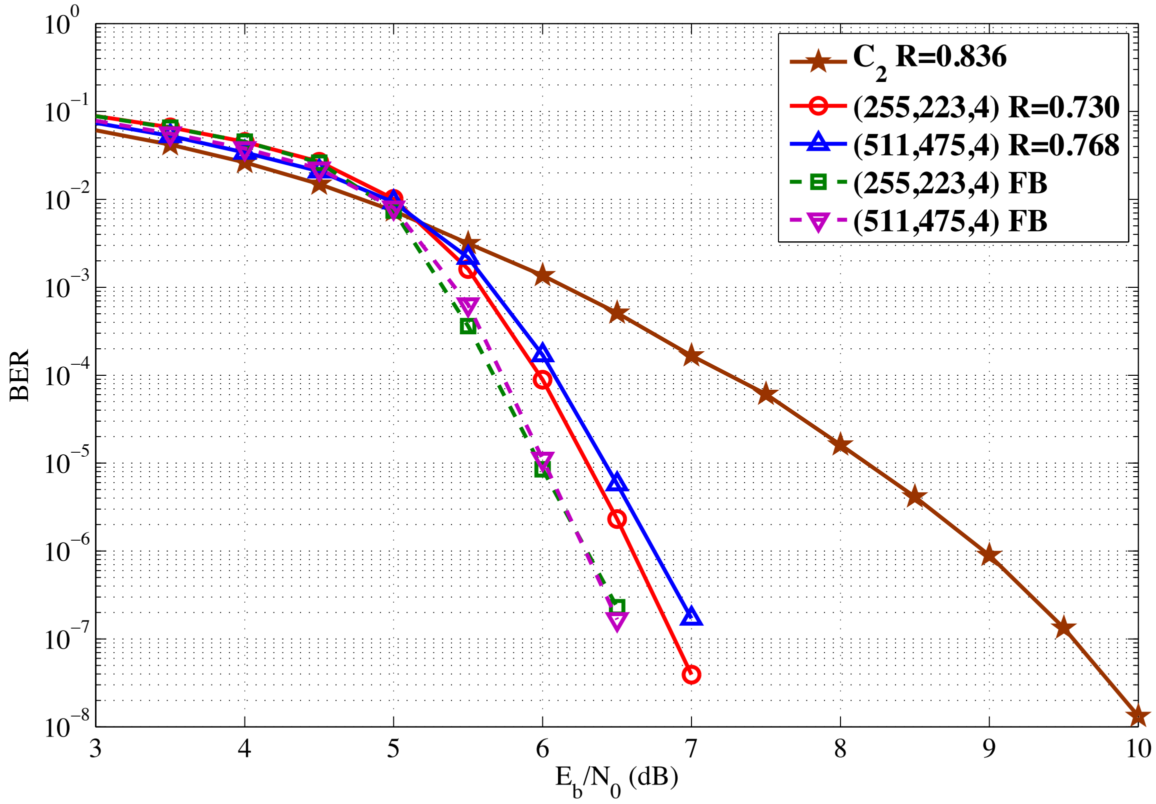

Figure 19: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding concatenated system with $(255, 223, 4)$ BCH codes decoded with and without feedback loop, and its corresponding concatenated system with $(511, 475, 4)$ BCH codes decoded with and without feedback loop. The LDPC code is decoded by the BFA.

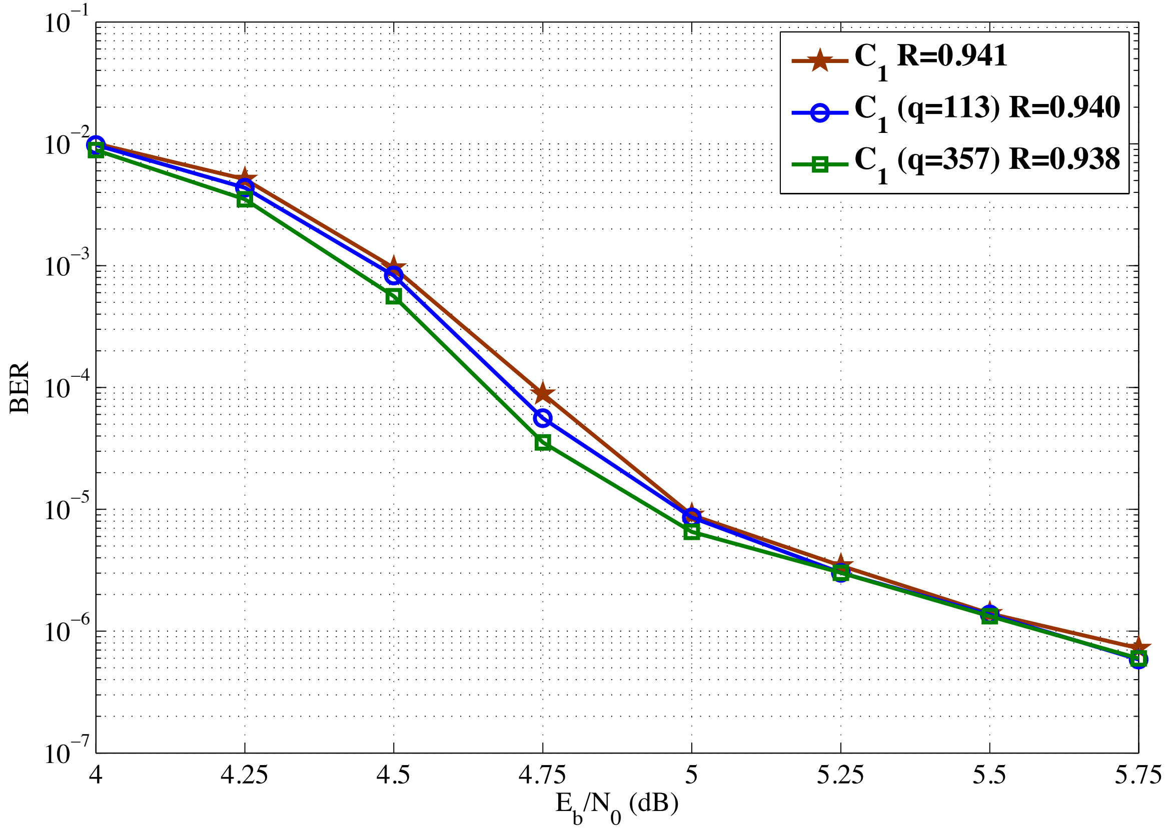

Figure 20: Performances of the rate-0.941 $(6350, 5978)$ QC-LDPC code $\mathcal{C}_1$, its corresponding zero-padded code with $q = 113$, and its corresponding zero-padded code with $q = 357$. The LDPC code is decoded by the SPA.

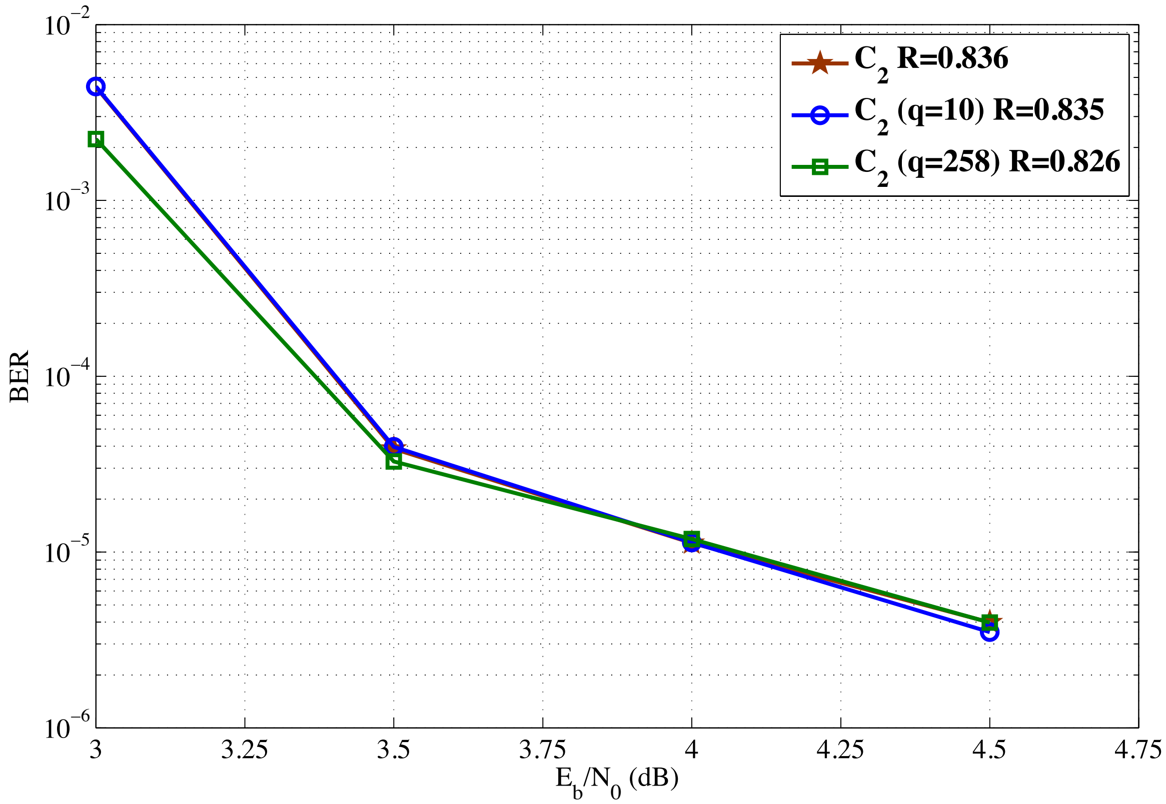

Figure 21: Performances of the rate-0.836 $(4590, 3835)$ QC-LDPC code $\mathcal{C}_2$, its corresponding zero-padded code with $q = 10$, and its corresponding zero-padded code with $q = 258$. The LDPC code is decoded by the SPA.