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Citation: *Applied Physics Letters* **82**, 3820 (2003); doi: 10.1063/1.1579849

View online: <http://dx.doi.org/10.1063/1.1579849>

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## Beam shaping by a periodic structure with negative refraction

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(Received 19 December 2002; accepted 2 April 2003)

We analyze transmission of a layered photonic structure (a one-dimensional photonic crystal) consisting of alternating slabs of two materials with positive and negative refractive index. For the periodic structure with zero averaged refractive index, we demonstrate a number of unique properties of the beam transmission observed in strong beam modification and reshaping. © 2003 American Institute of Physics. [DOI: 10.1063/1.1579849]

Recent experimental results confirmed the existence of composite materials possessing negative real parts of dielectric permittivity and magnetic permeability, also known as negative refraction materials (MNR) or left-handed metamaterials. Such materials were suggested theoretically a long time ago,<sup>1</sup> but they have recently attracted much attention due to their experimental realization and recent “hot” debates on the use of a MNR slab as a perfect lens to focus both propagating and evanescent waves.<sup>2</sup> The perfect lens concept was first suggested by Pendry,<sup>3</sup> who demonstrated that a slab of a lossless negative-refraction material can provide a perfect image of a point source. Although the perfect image is a result of an ideal theoretical model employed in the analysis of Ref. 3, the resolution limit was shown to be independent on the wavelength of electromagnetic wave (but can be determined by other sources such as loss, spatial dispersion, etc.), and it can be better than the resolution of a conventional lens.<sup>4</sup>

Multilayered structures containing MNR can be considered as a sequence of the perfect lenses and, therefore, they are expected to possess unique transmission properties. Such multilayered structures have been investigated theoretically for calculating the transmittance or reflectance in the Bragg regime.<sup>5,6</sup> More recently, it was shown that a one-dimensional stack of layers with alternating dielectric and MNR with zero averaged refractive index displays a narrow spectral gap in the transmission,<sup>7,8</sup> which is quite different from a Bragg reflection gap.

In this letter, we study the properties of layered photonic structures consisting of alternating slabs of positive and negative refractive index materials (see Fig. 1), and demonstrate unusual angular dependencies for the transmission of such slabs when the averaged refractive index is close to zero. We demonstrate how these properties can be employed for strong beam reshaping.

We consider a one-dimensional photonic crystal formed by alternative slabs, as schematically shown in Fig. 1. We assume that the periodic structure is made from the materials of two types: the slabs of the width  $b$  are made of a MNR, and they are separated by the dielectric layers of the width  $a$ . Such a structure can be treated as a periodic array of Pend-

ry’s perfect lenses with the variation of the refractive index in one structural period

$$n(z) = \begin{cases} n_r = \sqrt{\epsilon_r \mu_r}, & 0 < z < a; \\ n_l = -\sqrt{\epsilon_l \mu_l}, & a < z < a + b = \Lambda, \end{cases} \quad (1)$$

where  $n_r$  and  $n_l$  are the positive and negative indices of refraction of the dielectric and MNR, respectively. First, we consider waves propagating in the  $(x, z)$  plane with the wave vectors  $\mathbf{k} = (k_x, 0, k_z)$ , which are TE polarized, i.e., they have the only component  $E = E_y$  described by the Helmholtz-type equation

$$\Delta E + \frac{\omega^2}{c^2} n^2(z) E - \frac{1}{\mu(x)} \frac{\partial \mu}{\partial x} E = 0, \quad (2)$$

where  $\Delta$  is the two-dimensional Laplacian. In an infinite periodic structure the propagating waves have the form of Bloch modes, which electric field envelopes satisfy the periodicity property,  $E(z + \Lambda) = E(z) \exp(iK_b)$ . Here  $K_b$  is the normalized Bloch wave number, and it can be found as a solution of the standard equation for two-layered periodic structures (see, e.g., Ref. 7 and 8);

$$2 \cos(K_b) = 2 \cos(k_{rz}a + k_{lz}b) - \left( \frac{p_r}{p_l} + \frac{p_l}{p_r} - 2 \right) \cdot \sin(k_{rz}a) \cdot \sin(k_{lz}b). \quad (3)$$

Here  $k_{rz}$  and  $k_{lz}$  are the  $z$  components of the wave vector in the dielectric and MNR, respectively, and  $p_{r,l} = \sqrt{\epsilon_{r,l} / \mu_{r,l}}$ ,  $\cos(\theta_{r,l})$ , where  $\theta_{r,l} = \tan^{-1}[k_{(r,l)x} / k_{(r,l)z}]$  are the propagation angles in the corresponding media. For real  $K_b$  the Bloch waves are propagating; complex  $K_b$  indicate the presence of band gaps, where the wave propagation is inhibited.

In comparison with the conventional case of the periodic structure made of two different dielectric materials, the struc-

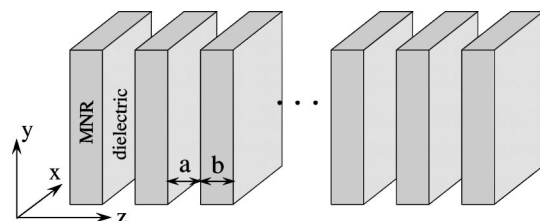


FIG. 1. Schematic of a multilayered structure consisting of slabs with alternating dielectric and material with negative refraction.

<sup>a)</sup>Web address: <http://www.rphysse.anu.edu.au/nonlinear>

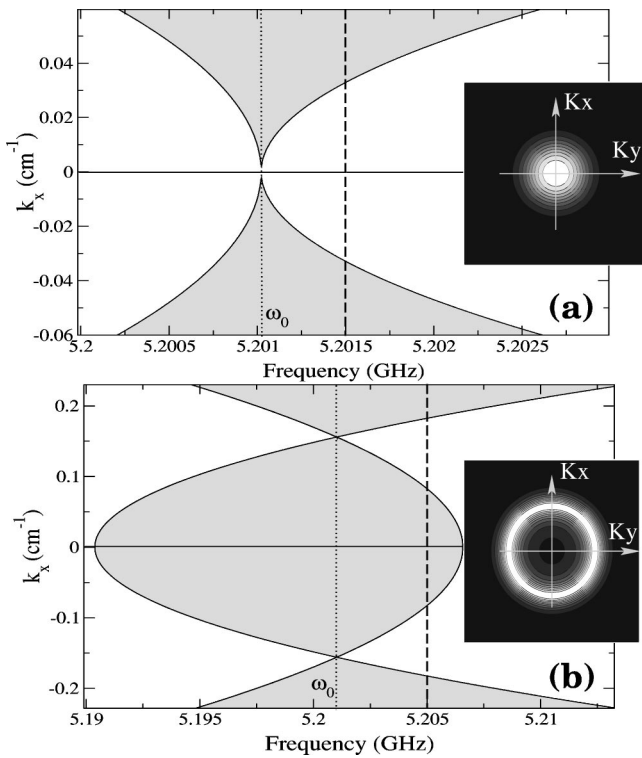


FIG. 2. Band gap structure on the parameter plane  $(\omega, k_x)$  with gaps shaded. (a) Transmission band corresponds to a normal incidence. (b) Transmission band corresponds to an oblique incidence. A dotted line is the frequency  $\omega_0$ . Insets show the beam transmission coefficients at the frequencies marked by dashed lines.

ture with MNR layers shown in Fig. 1 possesses a distinctive band gap of a different type when  $|k_{r_z}a| = |k_{l_z}b|$ , the property recently revealed in Refs. 7 and 8. The inherent feature of the materials with negative refraction is their frequency dispersion, so that in most of the cases we can find the characteristic frequency  $\omega_0$  such that  $n_r = |n_l|$ . Then, a periodic structure consisting of the alternating MNR and dielectric slabs of the same thickness ( $a = b$ ) will formally possess a band gap for all angles of incidence, with the only exception of the 100% transmission resonances, which appear when the arguments of the sinus functions in Eq. (3) are equal to the whole multiple of  $\pi$ . Such highly unusual transmission can happen because the “zero-index” band gap does not depend on the optical period of the structure, whereas usual Bragg-reflection and transmission resonances are highly sensitive to variations of the period and the incident angle.

It is important to study the transmission properties of a realistic system when the resonant conditions may not be exactly satisfied. To be specific, let us consider a periodic structure consisting of the layers of metamaterial separated by air. We assume that the composite material possesses the negative refractive index in the microwave region with the effective dielectric permittivity and magnetic permeability<sup>9</sup>

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu(\omega) = 1 - \frac{F\omega^2}{(\omega^2 - \omega_r^2)}, \quad (4)$$

where  $\omega_p/2\pi = 10$  GHz,  $\omega_r/2\pi = 4$  GHz, and  $F = 0.56$ . The frequency range, where both  $\epsilon$  and  $\mu$  are negative is 4–6 GHz. We plot the corresponding band gap on the plane  $(\omega, k_x)$  (see Fig. 2). We reveal that for a fixed incident angle  $(k_x)$  there appears a single frequency gap inside the trans-

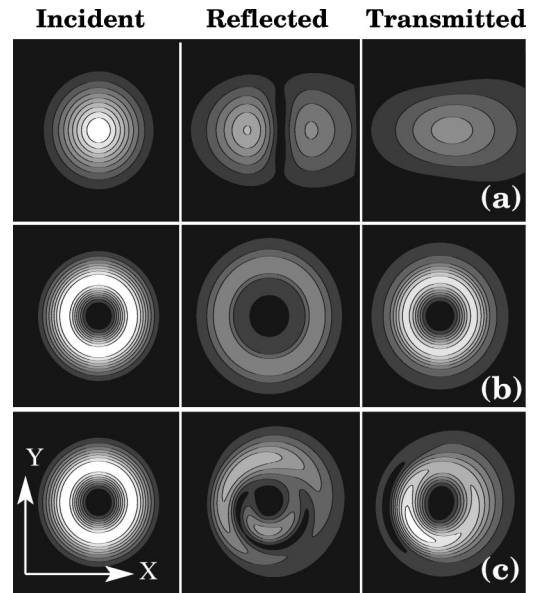


FIG. 3. Cross-section intensity profiles of the incident (left), reflected (middle), and transmitted (right) beams of various shapes: (a) Gaussian and (b)–(c) Gaussian with the vortex (topological) charge four at (b) the normal and (c) oblique incident angles.

mission region. Quite a different situation occurs for the angular dependence of the transmission coefficient at a given frequency, when there exists a narrow transmission band inside the bandgap.

In Fig. 2(a), the period is chosen to satisfy the condition  $k_r a_0 = \pi$  (in our case,  $a_0 = b_0 \approx 2.88$  cm), and the resonant transmission of electromagnetic waves of the frequency  $\omega_0$  occurs at the normal incidence. In contrast, Fig. 2(b) corresponds to the case  $a = b = a_0 + 0.03$  cm when the transmission resonance occurs for the waves at the incidence angle  $\theta$ :  $k_r \cos \theta a = \pi$ . When the period is further increased, multiple transmission bands are observed. The width of the transmission band in the  $k$  space can be controlled by tuning the frequency in the vicinity of  $\omega_0$ . The insets in Fig. 2 show the cross sections of the transmission coefficients for the frequencies marked by dashed lines. Thus, one can make a periodic structure with the transmission band at the desired angle of incidence by a proper choice of the system period, while the width of the transmission band can be adjusted by tuning the frequency. Similarly, the structures with more complicated transmission properties which include multiple rings in the transmission coefficient versus wave vector can be obtained. Most importantly, we find that such narrow-band angular filters can have almost 100% transparency in the transmission region.

We now use the standard transfer matrix method to analyze beam shaping by a finite composite structure consisting of 100 layers with the parameters corresponding to the angular transmission coefficient shown in the inset of Fig. 2(b). We consider the transmission of (2 + 1)-dimensional beams in the paraxial approximation when the angles of incidence are small and polarization effects are negligible. For a Gaussian beam incident at a normal angle, we observe almost complete reflection if the the angular spectrum width is comparable to the width of the transmission band. Transmission is only possible at the incident angles when the spectra overlap, and under such conditions the reflected beam has a

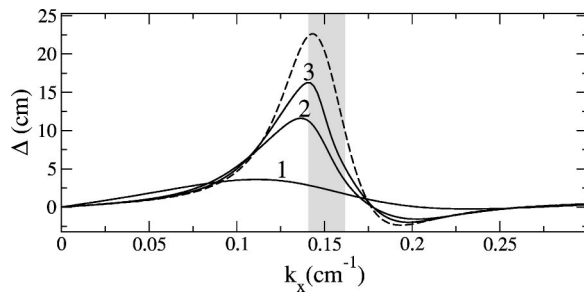


FIG. 4. The Goos–Hänchen shift  $\Delta$  of the reflected beam vs  $k_x$ . Solid curves 1,2,3—numerically calculated shifts for the incident Gaussian beams 20, 60, and 100 cm wide, respectively. Dashed curve—the asymptotic result for wide beams. Shaded area corresponds to the transmission band of the infinite periodic structure.

two-humped shape, see Fig. 3(a). On the other hand, vortex beams have a ring structure of the angular spectrum and can therefore be very effectively transmitted even at the normal incidence [Fig. 3(b)]. Similar to the case of a Gaussian beam, we observe that the weak reflected beam has a two-hump vortex structure. Small variations of the angle of incidence from the normal destroy the structure of the reflected and transmitted vortex beams and the results depend on the vortex topological charge [see Fig. 3(c)].

A beam reflected by an interface experiences a shift of its center relative to the center of the incident beam, and this classical effect is known as the Goos–Hänchen shift (see, e.g., Refs. 10 and 11). Reflection from a single MNR slab results in a negative Goos–Hänchen shift,<sup>12</sup> and it is interesting to calculate the beam shift for the layered structure. The centers of the incident and reflected beams are found as  $\mathbf{X}_{i,r} = \int_{-\infty}^{\infty} \mathbf{r}_{\perp} |E_{i,r}|^2 d\mathbf{r}_{\perp} / \int_{-\infty}^{\infty} |E_{i,r}|^2 d\mathbf{r}_{\perp}$ , where  $\mathbf{r}_{\perp}$  is the transverse coordinate in the plane  $(x, y)$ . For wide beams (i.e., the beams with a narrow spectrum), the shift  $\Delta = \mathbf{X}_r - \mathbf{X}_i$  is determined by the gradient of the phase  $\phi$  of the reflection coefficient,  $\Delta = -\nabla_{\mathbf{k}_{\perp}} \phi$ . We calculate the Goos–

Hänchen shift for the beam reflected by the periodic structure described earlier, using the parameters corresponding to Fig. 2(b). With no loss of generality, we assume that the  $(x, z)$  plane is the plane of incidence, and therefore the beam is only shifted along the  $x$  axis. We plot the shifts experienced by the Gaussian beams of various widths (see the curves 1–3 in Fig. 4). We see that the shift of wide beams indeed approaches the asymptotic result shown with dashed line in Fig. 4. The shift can be either positive or negative, and the absolute value increases dramatically for the angles of incidence close to the transmission band.

In conclusion, we have analyzed the transmission properties of a one-dimensional photonic crystal composed of two materials with positive and negative refractive indices. We have demonstrated unusual angular dependencies of the beam transmission through such composite structures which can be used for efficient beam reshaping. We have calculated the Goos–Hänchen shift of the reflected beam which is shown to increase for the angles of incidence close to the transmission band.

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