Transactions Letters

Beamforming with Limited Feedback in Amplify-and-Forward Cooperative Networks

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***Abstract*—A relay selection approach has previously been shown to outperform repetition-based scheduling for both amplify-and-forward (AF) and decode-and-forward (DF) cooperative networks. The selection method generally requires some feedback from the destination to the relays and the source, raising the issue of the interplay between performance and feedback rate. In this letter, we treat selection as an instance of limited-feedback distributed beamforming in cooperative AF networks, and highlight the differences between transmit beamforming in a traditional multi-input single-output (MISO) system and the distributed case. Specifically, Grassmannian line packing (GLP) is no longer the optimal codebook design, and orthogonal codebooks are no longer equivalent to each other. We derive the high signal-to-noise ratio expressions for outage probability and probability of symbol error for unlimited-feedback and selection schemes, which are then used for performance comparisons. The selection protocol is compared to a limited-feedback distributed beamformer that assigns codebooks based on the Generalized Lloyd Algorithm (GLA), and one that uses random beam-vectors. The main conclusion is that the performance improvement to be seen using the very complex GLA is small, and that many more feedback bits are required with random beamforming than selection for the same performance. These results indicate that the selection protocol is a very attractive protocol, with low-complexity, that provides excellent performance relative to other known methods.**

***Index Terms*—Beamforming, limited feedback, selection, amplify-and-forward, wireless cooperative networks.**

I. INTRODUCTION

COOPERATIVE diversity [1] is an important concept for achieving spatial diversity in distributed wireless networks where antenna arrays are not available at each node. Amplify-and-Forward (AF) relaying, where a relay node simply amplifies its received signal from the source node and retransmits, is perhaps the most studied cooperative diversity scheme due to its low complexity. Given multiple relay nodes in the system, retransmissions can take place in orthogonal channels (e.g., time slots) in a repetition-based (RB) protocol [1].

The bandwidth efficiency of RB scheduling is limited as it only uses $1/(m+1)$ of the available channel bandwidth for information transmission, where m is the number of relays. A selection scheme was introduced in [2]–[4] to make more efficient use of channel resources. It was proven in these works that selection outperforms RB scheduling in AF networks in terms of achieving higher average throughput, lower outage probability and lower error probability, while maintaining the same diversity order, at the cost of only $\log_2 m$ bits of feedback. One should note that in [5] the authors present an alternative selection approach, but one that may result in packet collisions at the receiver.

In fact, the selection scheme is a special case of beamforming with $\log_2 m$ bits of feedback and the columns of the $m\times m$ identity matrix used as the beamformer codebook. Optimal relay beamforming with unlimited feedback (B-UF) has been studied in [6]. Although the B-UF scheme seems impractical with its requirement for perfect phase synchronization among relays, it provides a performance upper bound for practical beamforming schemes with limited feedback. In this letter, we derive the outage probability and probability of error for B-UF, and compare these expressions with the ones corresponding to the selection scheme [3], [4] to find the performance gap between B-UF and selection.

With limited feedback, distributed beamforming over multiple relays appears equivalent to multiple input single output (MISO) beamforming. In the latter context, Grassmannian Line Packing (GLP) provides the optimal codebook design for both average received signal to noise ratio (SNR) [7] and outage probability [8]. However, because AF relays include some noise amplification, a different solution is required. A numerical solution is developed here to find the optimal beamforming codebook. Aiming for complexity reduction [9], [10] introduce codebooks of random beamforming vectors as a practical implementation of limited feedback beamforming. In this letter, we show that, to achieve the same performance, B-RC (beamforming with random codebooks) needs significantly more feedback than the selection scheme. In addition, all beamforming schemes require synchronization among all relay nodes. Therefore, selection relaying may be the most attractive currently known protocol for AF networks with limited

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feedback.

The rest of the letter is organized as follows. We introduce the system model in Section II. In Section III, we obtain the outage and error probabilities, as well as an OPA scheme for the B-UF and selection schemes. In Section IV, we compare the performance of selection scheme to B-OC and B-RC. Finally, we draw some conclusions in Section V.

II. SYSTEM MODEL

Consider a wireless system where a source node transmits to a destination with the help of m relay nodes. As in [1], each transmission block is divided into two non-overlapping phases in time. In Phase I the source node transmits the unit power signal x to the destination and relay nodes. The received signals at relay node i and the destination are

$$\begin{aligned} r_i &= \sqrt{E_s} h_{s,i} x + n_{s,i}, && (1) \\ r_{d,1} &= \sqrt{E_s} h_{s,d} x + n_{s,d}, && (2) \end{aligned}$$

where E_s is the transmitted symbol energy used at the source node. $h_{s,i}$ and $h_{s,d}$ are complex Gaussian coefficients of the source-relay and source-destination channels, respectively. $n_{s,i}$ and $n_{s,d}$ are the additive white Gaussian noise (AWGN) in the corresponding channels. We assume that all the noise contributions are independent and identically distributed (i.i.d.) with single-sided power spectral density N_0.

In Phase II, AF relays normalize and retransmit their received signals. The relay nodes jointly beamform to forward data to the destination; selection, wherein only one "best" relay node participates, is a special case of beamforming with the beamforming vector chosen from columns of the identity matrix. The unit-energy signal to be transmitted from relay i is thus

$$x_i = \frac{r_i}{\sqrt{\mathcal{E}\left\{|r_i|^2\right\}}} = \frac{\sqrt{E_s} h_{s,i} x + n_{s,i}}{\sqrt{E_s |h_{s,i}|^2 + N_0}} \qquad (3)$$

All m nodes transmit simultaneously to the destination, with the i-th relay weighting x_i with w_i, a complex beamforming weight. Thus, the received signal at the destination is

$$\begin{aligned} r_{d,2} &= \sum_{i=1}^{m} \sqrt{E_r} w_i h_{i,d} x_i + n_d \\ &= \sum_{i=1}^{m} \frac{\sqrt{E_s E_r} w_i h_{s,i} h_{i,d} x}{\sqrt{E_s |h_{s,i}|^2 + N_0}} + \sum_{i=1}^{m} \frac{\sqrt{E_r} w_i h_{i,d} n_{s,i}}{\sqrt{E_s |h_{s,i}|^2 + N_0}} + n_d \\ &= \sum_{i=1}^{m} w_i \tilde{h}_i x + \tilde{n}_d, \qquad (4) \end{aligned}$$

where E_r is the total transmitted symbol energy shared among relay nodes, $h_{i,d}$ is the complex Gaussian channel between relay i and the destination, and n_d is the AWGN at the destination node in Phase II. Here a total energy constraint is assumed among the multiple relays instead of individual constraints. This is necessary to ensure fairness when comparing the performance of the beamforming scheme with other schemes. To keep the total energy used by all relays at E_r, the beamformer weights must satisfy $\sum_{i=1}^{m} |w_i|^2 = 1$. Note that beamforming schemes (other than selection) require phase synchronization across relays, an issue of practical importance.

In (4) we defined an equivalent channel and noise term to simply the expression. The equivalent channel through relay i is

$$\tilde{h}_i = \frac{\sqrt{E_s E_r} h_{s,i} h_{i,d}}{\sqrt{E_s |h_{s,i}|^2 + N_0}}, \qquad (5)$$

and equivalent noise $\tilde{n}_d$ is circularly symmetric Gaussian distributed, i.e.

$$\tilde{n}_d \sim \mathcal{CN}\left(0, \ \left(1 + \sum_{i=1}^{m} |w_i|^2 |H_{ii}|^2\right) N_0\right), \qquad (6)$$

where

$$H_{ii} = \frac{\sqrt{E_r} h_{i,d}}{\sqrt{E_s |h_{s,i}|^2 + N_0}}. \qquad (7)$$

Maximal Ratio Combining (MRC) of the received signals over the two phases provides sufficient statistics for detection. The SNR in the decision statistic can be shown to be

$$\gamma_r = E_s |h_{s,d}|^2 \gamma_t + \frac{\left|\sum_{i=1}^{m} w_i \tilde{h}_i\right|^2 \gamma_t}{1 + \sum_{i=1}^{m} |w_i H_{ii}|^2}, \qquad (8)$$

where $\gamma_t = 1/N_0$ is the equivalent system SNR since it is proportional to all the transmit and receive SNRs at all the nodes in the system. It can also be interpreted as the transmit SNR when the signal power is one.

III. OPTIMAL AF BEAMFORMING WITH UNLIMITED FEEDBACK

In [6], the ideal scenario of zero channel estimation error and infinite feedback channel bandwidth is assumed, and hence w_i's can be calculated using exact instantaneous channel state information (CSI) at the destination and fed back to the relays without error. This yields the (unfortunately impractical) B-UF scheme and serves as a loose upper bound on performance for all beamforming schemes with limited feedback. In this section, we derive the outage and error probabilities for B-UF, at high SNR.

The first term in the total SNR expression of (8) depends on the source-destination channel alone. Hence only the SNR in Phase II (the second term in (8)) impacts the beamformer design. Stacking w_i's and $\tilde{h}_i$'s into column vectors $\mathbf{w}$ and $\mathbf{h}$, respectively, and defining a diagonal matrix $\mathbf{H}$ whose ith diagonal element is H_{ii}, we can rewrite the second term in (8) as

$$\gamma_{r,2} = \frac{\mathbf{w}^{\dagger} \mathbf{h} \mathbf{h}^{\dagger} \mathbf{w}}{\mathbf{w}^{\dagger} \left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right) \mathbf{w}} \gamma_t, \qquad (9)$$

where $(\cdot)^{\dagger}$ denotes the Hermitian or conjugate transpose.

Maximizing $\gamma_{r,2}$ over $\mathbf{w}$ (see [11]), with the energy constraint $\|\mathbf{w}\|_2 = 1$, yields the optimal beamforming vector with unlimited feedback as

$$\mathbf{w}^{*} = \frac{\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right)^{-1} \mathbf{h}}{\| \left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right)^{-1} \mathbf{h} \|_2}, \qquad (10)$$

where $\| \cdot \|_2$ denotes the 2-norm.

Note that the beamforming vector $\mathbf{w}$ not only determines the received signal power, but also contributes to noise amplification, as shown in (8). As a result, the optimal beamforming vector differs significantly from the matched filtering solution in traditional MISO systems. We can therefore expect that the beamforming codebook design with limited feedback is quite different from the Grassmannian approach for MISO systems [7].

Substituting (10) and (9) into (8), we obtain the received SNR of the optimal beamforming AF network as

$$\gamma_r^{opt} = E_s|h_{s,d}|^2\gamma_t + \sum_{i=1}^{m} \frac{E_s E_r |h_{s,i}|^2 |h_{i,d}|^2}{E_s|h_{s,i}|^2 + E_r|h_{i,d}|^2 + N_0}\gamma_t. \tag{11}$$

Note that, interestingly, this received SNR expression has the *same form* as in TDMA scheduled AF systems [4, Eq.(1)]. Therefore, the results in [3], [4] directly lead to the following approximations on the outage probability and probability of error of the optimal beamforming with unlimited feedback AF network at high SNRs:

$$\begin{aligned} P_{out}^{opt}(\gamma_t) &\simeq \frac{\zeta}{m!(m+1)}\left(\frac{2^{(m+1)R}-1}{\gamma_t}\right)^{m+1}, & (12)\\ P_e^{opt}(\gamma_t) &\simeq \frac{\zeta(2m+1)!}{m!(m+1)!(2c\gamma_t)^{m+1}}, & (13) \end{aligned}$$

where

$$\zeta = \frac{1}{E_s\sigma_{s,d}^2}\prod_{i=1}^{m}\left(\frac{1}{E_s\sigma_{s,i}^2} + \frac{1}{E_r\sigma_{i,d}^2}\right) \tag{14}$$

is a constant determined by transmission power and channel variances $\sigma_{s,d}^2$, $\sigma_{s,i}^2$ and $\sigma_{i,d}^2$, R is the target transmission rate and c is a constant determined by the modulation scheme.

The idea of an AF network with relay selection was introduced in [3] as an improvement to the conventional TDMA-based AF networks. The high-SNR approximation of the outage probability and probability of error of the selection AF network are [3], [4]

$$\begin{aligned} P_{out}^{s}(\gamma_t) &\simeq \frac{\zeta}{(m+1)}\left(\frac{2^{(m+1)R}-1}{\gamma_t}\right)^{m+1}, & (15)\\ P_e^{s}(\gamma_t) &\simeq \frac{\zeta(2m+1)!}{(m+1)!(2c\gamma_t)^{m+1}}. & (16) \end{aligned}$$

From (12), (13) and (15), (16) with the same transmit SNR, the performances of the two schemes satisfy

$$\frac{P_{out}^{s}}{P_{out}^{opt}} = \frac{P_e^{s}}{P_e^{opt}} = m! \tag{17}$$

Since the high SNR approximation of the two schemes are parallel lines with slope $-(m+1)/10$ in Log-dB scale, the SNR difference for the two schemes to achieve the same performance is therefore

$$\delta_\gamma = \frac{\log m!}{(m+1)/10} = \frac{10}{m+1}\log_{10} m! \quad (\text{dB}). \tag{18}$$

Therefore, the asymptotic SNR gap[1] between the selection AF scheme and the B-UF scheme is $\frac{10}{m+1}\log_{10} m!$ dB.

Applying Stirling's formula[2] to (18) and retaining only the most significant term, we can see that as the number of relay nodes m increases, the asymptotic SNR gap between selection AF and B-UF is $10\log_{10} m/e$ dB. This gap is a quickly increasing function of m and may seem to be severe in large networks. For instance, the loss is 1dB when $m = 2$ and 1.95dB when $m = 3$, but it grows to 5.96dB when $m = 10$. However, the optimal beamforming scheme is highly impractical for any realistic application, especially when m is large, since it involves feedback of m complex numbers in real time and strict synchronization. Therefore we treat it as a performance bound only and compare more practical schemes using their performance loss in relation to this optimal one.

IV. BEAMFORMING WITH LIMITED FEEDBACK

In Section III we treated the selection AF scheme as one special case of beamforming with limited feedback and studied its performance loss in relation to the B-UF scheme. Clearly, this loss can be reduced by increasing the amount of feedback. In this section, we study the performance of AF beamforming schemes as a function of feedback available. In such a network, a codebook whose size is determined by the amount of feedback is first established at both the destination and the relay nodes. For each transmission, the destination selects the optimal codeword based on the CSI, and feeds back its index to the relays. The relays perform beamforming using the corresponding codeword as the weight vector.

A. *Optimal Codebook Design*

Optimal transmit beamforming codebook design has been studied in the context of single-user MISO systems [7], [8], [12], where GLP provided the optimal codebook design for both average received SNR [7] and outage probability [8]. In the AF networks, however, GLP is no longer optimal due to the noise amplification by relay nodes. The Generalized Lloyd Algorithm (GLA) [13] can still be used in the optimal codebook design. In particular, assuming B bits of feedback, the algorithm starts with a set of randomly selected 2^B vectors, and repeats the following two steps until convergence:

1) For each codeword in the current codebook, find the region in $\mathbb{C}^m$ for which it is the γ_r-maximizing beamforming vector.
2) For each region, find a new codeword to replace the current one by maximizing the average γ_r over that region.

Due to the complex form of γ_r, only numerical results are available for AF relays (shown later in simulations), even for the special case of i.i.d channels among the network. We are particularly interested in $\log_2 m$ bits of feedback, for the selection scheme is a possible candidate in this case. Unlike in MISO systems, where any orthogonal basis of the m-dimensional space is an optimal codebook, selection (identity codebook) is the unique optimal for AF beamforming. The reason for the uniqueness is straightforward: although all orthogonal codebooks achieve the same maximal received signal power, selection is the one that minimizes noise amplification.

Figure 1 shows the outage probability of these schemes in a network with 3 potential relays. All the channel gains in the network are i.i.d. $\mathcal{CN}(0,1)$ random variables. From the figures we can see the 1.95 dB performance loss between selection and B-UF as predicted in Table I. It also shows that optimal beamforming with limited feedback achieves

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[1] Defined as the difference in SNR required to achieve the same asymptotic performance with selection AF and with B-UF.

[2] $\lim_{m\to\infty}\sqrt{2\pi m}e^{-m}m^m/m! = 1$

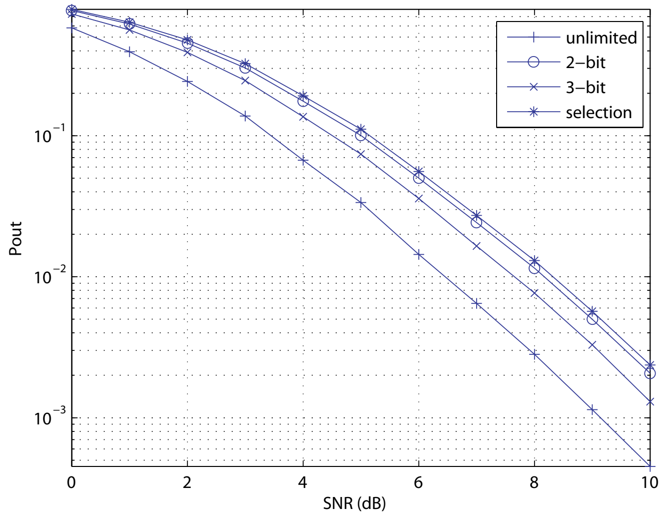

Fig. 1. Outage Probabilities for AF B-OC with $m = 3$, $R = 0.9$.

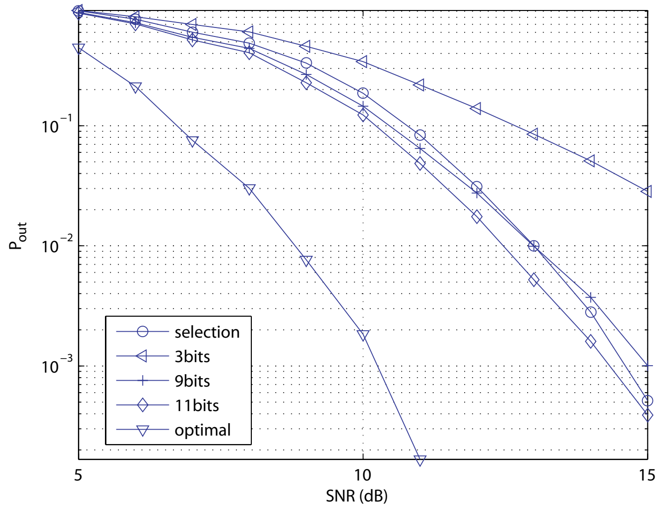

Fig. 2. Outage Probabilities for AF B-RF with $m = 8$, $R = 1.7$.

little improvement over selection, while entailing significant system-wide disadvantages such as extremely high complexity and slow convergence rate for GLA and synchronization of relays. In other words, although increasing the amount of feedback and designing a near-optimal codebook through the GLA can improve performance, this approach involves practical difficulties and yields only small performance gains.

B. Random Beamforming with Limited Feedback

The GLA in the previous section has extremely high complexity and is thus impractical. If we assume equal power allocation and i.i.d. channels among the network, a random codebook generated from the uniform distribution on the complex unit sphere can be used instead. It has been proven that in MIMO systems with random codebooks, the gap between beamforming with limited and unlimited feedback, in terms of capacity [10] and outage probability [8], can be closed by increasing the number of feedback bits. Next, we study the random codebook method in AF networks by first showing the simulation results.

Figure 2 shows the outage performances of an AF network with 8 relay nodes. It shows that B-RC performs worse than the selection AF scheme with the same amount of feedback. For instance, the AF B-RC scheme requires about 6 bits more feedback to achieve the same performance at $P_{out} = 10^{-2}$ as the selection scheme.

Furthermore, a diversity order loss can be found with the B-RC scheme. From Section IV-A we know that AF beamforming imposes more requirements on codebook design than transmit beamforming due to the noise amplification. For instance, in the case of $\log_2 m$ bits of feedback, any orthogonal codebook is optimal for transmit beamforming in minimizing the outage probability, however, only selection is optimal for AF beamforming. This may be due to the distortion of the unit sphere by the noise amplification. In particular, since the optimal beamforming vector is distorted by a factor involving $\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right)^{-1}$ compared to matched filtering with $\mathbf{h}$ in multi-antenna transmission, as shown in (10), the angle between $\mathbf{h}$ and $\mathbf{w}$ no longer determines performance. However, since this distortion factor is channel related, the average performance is hard to analyze.

Since the selection scheme outperforms the AF B-RC by requiring less feedback, without diversity loss and operates asynchronously, we can claim that the selection scheme is the sub-optimal practical scheme of choice for AF networks with limited feedback. In other words, for AF networks with limited feedback, we would choose the selection scheme over other distributed beamforming methods, if the amount of available feedback is about $\log_2 m$, for its excellent performance and the significant advantage of not requiring synchronization.

V. CONCLUSION

The relay selection scheme has been proven to outperform the repetition-based relaying scheme in multiple-relay AF networks. In this letter, we treat relay selection scheme as a special case of relay beamforming with limited feedback of $\log_2 m$ bits and beamforming vectors as columns of the identity matrix. Transmit beamforming with limited and unlimited feedback has been studied in MISO systems, where matched filtering is optimal with unlimited feedback, and Grassmanian Line Packing gives the optimal codebook design for limited feedback. In AF networks, however, these two schemes are no longer optimal due to noise amplification at the relay nodes. We find the performance loss of selection compared to optimal relay beamforming with unlimited feedback by deriving the outage probability and error probability of both schemes. We also present an optimal power allocation scheme to further improve the performance.

In the case of limited feedback, although beamforming with a codebook designed using the Generalized Lloyd Algorithm has the best performance, the codebook design is very complex and therefore impractical. By comparing selection with another practical scheme, beamforming with random codebooks, we observed that selection has stronger performance and the advantage of not requiring synchronization, while the latter results in a diversity order loss. Therefore the selection scheme is probably the most attractive scheme for AF networks with limited feedback currently known.

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