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<table>
<thead>
<tr>
<th>Journal:</th>
<th>Canadian Geotechnical Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript ID</td>
<td>cgj-2020-0109.R1</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>16-May-2020</td>
</tr>
<tr>
<td>Complete List of Authors:</td>
<td>Wu, Jinbiao; The University of Newcastle, Faculty of Built and Environment Kouretzis, George; The University of Newcastle, Faculty of Engineering and Built Environment Suwal, Laxmi; The University of Newcastle, Engineering and Built Environment</td>
</tr>
<tr>
<td>Keyword:</td>
<td>pipelines, physical modelling, bearing capacity</td>
</tr>
<tr>
<td>Is the invited manuscript for consideration in a Special Issue?:</td>
<td>Not applicable (regular submission)</td>
</tr>
</tbody>
</table>
Bearing capacity mechanisms for pipes buried in sand

Jinbiao Wu, George Kouretzis and Laxmi Suwal

ABSTRACT

This paper presents results of scaled physical model tests performed to measure the reaction developing on a rigid pipe buried in dry sand, when the pipe is subjected to vertical downwards movement relative to its surrounding soil. The aim of this experimental study is to evaluate the efficacy of methods used to determine the properties of vertical bearing springs, an integral part of beam-on-nonlinear Winkler spring models used for the analysis of buried pipelines subjected to permanent ground displacements. We show that bearing capacity formulas used in practice to estimate the ultimate reaction developing on buried pipes may provide reasonably accurate estimates, provided that they are used together with sand friction angle values that account for the fact that granular materials do not obey an associative flow rule, and with bearing capacity factors compatible with the mode of sand failure observed in the tests. We also provide evidence suggesting that laying pipes in loose sand backfills does not have a beneficial effect on the reaction developing on the pipe, compared to medium dense sand, and we recommend against using loose sand material properties for the estimation of the properties of vertical bearing springs.

Key words: pipelines; physical modelling; bearing capacity

1. INTRODUCTION

Estimating the resistance to vertical downwards movement of a rigid pipe buried in soil is a key part of the analysis of buried pipelines traversing areas where permanent ground displacements are expected, such as crossings with normal, reverse or oblique faults, landslide zones or areas where near-surface tunnelling works are planned, to name a few. To calculated pipe strains or joint displacements due to ground displacements in such problematic areas, pipeline engineers commonly use numerical (ALA 2005, NEN3650 2003) or analytical (e.g. Trifonov and Cherniy 2010; Karamitros et al. 2011; Kouretzis et al. 2015) beam-on-nonlinear Winkler spring analysis models. Determining the parameters of vertical bearing springs used in such models requires estimating the soil reaction to vertical downwards relative soil-pipe movement. In fact, the structural response of the pipe model is particularly sensitive to the properties of the vertical bearing Winkler springs in cases where the permanent ground displacement under consideration has a vertical component. The reason is that the reaction developing on the pipe from soil during vertical downwards movement is considerably higher compared to uplift or lateral movement, leading to higher pipe strains/joint displacements due to the prescribed displacement nature of the problem. Despite that, much less attention has been paid in the past to estimating the properties of vertical bearing springs, compared to lateral and vertical uplift springs. This is perhaps due to the focus on the analysis of pipes crossing strike-slip faults, imposing only lateral relative soil-pipe movements, and the fact that vertical downwards movement of a buried rigid pipe can be considered equivalent to the problem of bearing capacity of an infinitely long strip footing embedded in soil. Actually, current codes and guidelines for the analysis of pipes subjected to permanent ground displacements, such as ALA (2005), NEN3650 (2003), DNVGL-RP-F114 (2019) and PRCI (2009), all recommend using the classical bearing capacity formulas for shallow foundations to estimate the peak reaction that will develop on a pipe during vertical downwards relative movement. For example, ALA (2005) and PRCI (2009) recommend using the classic bearing capacity formula to determine
the ultimate vertical resistance acting on the pipe $Q_b$, which for the case of a pipe in dry cohesionless sand is:

$$Q_b = N_q \gamma HD + N_\gamma 0.5\gamma D^2$$

where $D$ is the diameter of the pipe, $H$ is the embedment depth measured from the pipe springline, $\gamma$ is the dry unit weight of sand and $N_q$, $N_\gamma$ are the bearing capacity factors for a strip footing with flat surface in contact with soil:

$$N_q = e^{\tan \phi \cdot \tan^2 \left(45^\circ + \phi/2\right)}$$

$$N_\gamma = e^{(0.18 \phi - 2.5)}$$

and $\phi$ (in degrees) is the friction angle of sand. The vertical pipe displacement required for $Q_b$ to fully develop is, according to ALA (2005) and PRCI (2009), equal to $0.1D$ for pipes in sand.

Some more recent studies in the literature, namely Kouretzis et al. (2014), Jung et al. (2016) and Qin et al. (2019) challenge the use of bearing capacity formulas for strip footings for the estimation of the ultimate resistance of vertical bearing springs. The main reason for doing so is that bearing capacity formulas proposed for footings with a flat bearing surface do not consider the effect of the cylindrical pipe geometry on the developing resistance. In fact, Xie et al. (2013) measured by means of centrifuge tests the ultimate resistance developing on a pipe during rupture of a normal fault to be between 1/3 and 1/10 of that predicted from conventional bearing capacity formulas (Jung et al., 2016), in particular Eqs. (1-3) recommended by ALA (2005). The effect of the shape of the surface of the footing in contact with soil on its bearing capacity is well-known, and also underlined in studies relevant to the resistance developing during the installation of spudcans in offshore sand deposits (see Cassidy and Houlsby 2002; White et al. 2008).

However, to the authors’ knowledge, there is no study in the literature that systematically investigates the mechanisms of development of bearing resistance on buried rigid pipes by means of physical modelling experiments, similar to those used to determine the reaction developing on pipes during uplift and lateral relative movements. Given the challenges in numerically modelling this large deformation problem (see Kouretzis and Bouckovalas 2019), physical model tests of vertical penetration of rigid buried pipes are invaluable for exploring the efficacy of using bearing capacity formulas to estimate bearing spring properties for pipe analyses in practice, as well as for benchmarking relevant numerical models. To cover this gap in the literature, we introduced certain modifications in the 1-g physical modelling rig presented in Ansari et al. (2018), Ansari et al. (2019) and Wu et al. (2019) that allow us to model vertical penetration of a pipe buried in dry sandbeds of different initial densities. The testing rig was reinforced to account for the increased forces developing during pipe penetration (compared to uplift and lateral drag tests), and additional sensors were installed to monitor parasitic friction effects and ensure that test results are not compromised by limitations of the testing equipment.

Here we present the results of 18 small scale tests performed in the mentioned rig, covering pipes buried in relatively shallow depths (relevant to pipelines laid in trenches) in loose to very dense dry sand. The paper is structured as follows: First we present the testing rig, the properties of the sand used in the tests and the techniques used to correct measurements for friction effects in brief, as these are discussed in detail in previous publications. More emphasis is placed on boundary and scale effects, the latter being associated with limitations of this study. Accordingly we present the reaction-displacement response measured in each test and the approach we adopted to interpret measurements, particularly to define a nominal value of the bearing capacity and associated yield pipe displacement, and facilitate comparison with existing methods for determining vertical bearing spring properties. Next, we attempt to assess the efficacy of bearing capacity formulas in capturing the observed response and failure mechanisms in sand, depicted by means of analysis of images captured during the tests with the Particle Image Velocimetry (PIV) method using the software GeoPIV-RG (Stanier et al. 2016). We conclude with certain recommendations for determining pipe analysis model parameters in practice.

2. PHYSICAL MODELLING FACILITY AND MATERIAL PROPERTIES

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The 1-g physical modelling rig is presented in detail in previous publications (Ansari et al. 2018, 2019), therefore to avoid repetition emphasis is placed here upon the modifications introduced to perform the presented tests. The rig (Fig. 1) comprises a testing chamber with dimensions 1050 mm (length) x 750 mm (height) x 75 mm (width) fitted on a steel frame anchored on the laboratory strongfloor. The sidewalls of the chamber are made of annealed glass to allow capturing images of the pipe and the sandbed continuously during tests. The pouring and raining sand deposition systems developed in-house allow preparing sandbeds inside the chamber at a very wide range of initial relative densities $D_r$ ranging from $D_r=19\%$ to $D_r=92\%$. The excellent uniformity of the sandbeds, irrespective of their density, was documented by means of mini cone penetration tests presented by Ansari et al. (2018). The sand used in the tests is the Stockton Beach Sand (STK), a uniform silica sand with mean grain size $D_{50}=0.36$mm. The physical and the (stress-dependent) mechanical properties of STK sand are presented in detail in other publications (Ajallooeian et al. 1996; Ansari et al. 2018; Kouretzis and Bouckovalas 2019) thus here we only summarise results of the characterisation program in Table 1, for the three sandbed densities used in the tests at hand.

A rigid pipe made of high-density PVC, with length approximately $L=75$mm to fit firmly inside the chamber, is embedded in the sandbed during deposition and connected to a 2mm-diameter steel cable by means of a “D-shackle”, to prevent imposing any kinematic constraints on pipe movement. A winch is used to pull the pipe by means of the “pulley-clamp” mechanism at a constant rate of 1mm/min, thus modelling relative sand-pipe movement along the desired direction. The diameter $D$ of pipes used in the study at hand are $D=37.5$mm and $D=75$mm, and their length to diameter ratio ranges from $L/D=2$ to 1 respectively i.e. is smaller than the diameter of most common pipes. A discussion on scale effects is provided in the sections that follow. Both pipes featured endcaps surfaced with felt, to minimise pipe-sidewall friction and prevent sand grains from becoming trapped between the pipe and the glass during the tests. Finally the thickness and material of the two pipes ensured that longitudinal bending and ovalisation effects during testing are negligible, and that the pipe behaves as rigid body. This setup mimics plane-strain conditions, therefore the measured reaction at the load cell as function of relative downwards pipe movement measured with the string potentiometer will match the force-displacement response that needs to be assigned to bearing Winkler springs of a beam-spring analysis model.

**Figure 1.** Schematic of the physical modelling rig and photos of modifications introduced to physically model vertical pipe penetration.

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Table 1. Summary of test sandbed densities and corresponding mechanical properties of STK sand (after Wu et al. 2019).

<table>
<thead>
<tr>
<th>Density</th>
<th>Average dry unit weight, ( \gamma ) (kN/m(^3))</th>
<th>Relative density, ( D) (%)</th>
<th>Effective dry unit weight, ( \gamma_{eff} = \sigma_{v,measured}/z ) (kN/m(^3))</th>
<th>Peak plane strain friction angle(\psi_{p,\phi}) (deg)</th>
<th>Peak dilation angle(\psi_{d}) (deg)</th>
<th>Critical state friction angle(\psi_{c}) (deg)</th>
<th>Coefficient of volume compressibility(\alpha), 1/m, (kPa(^{-1}))</th>
</tr>
</thead>
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<tr>
<td>Loose</td>
<td>14.9</td>
<td>19</td>
<td>14.9(-0.044z(^{-2})-0.6034z(^{-2})+0.921)(^{1})</td>
<td>-3.06ln((\sigma_{n}))+41.9</td>
<td>-3.6</td>
<td>32</td>
<td>2000((\sigma_{n}/4.47)^{0.85})</td>
</tr>
<tr>
<td>Medium</td>
<td>16.0</td>
<td>58</td>
<td>16.0(-0.044z(^{-2})-0.6034z(^{-2})+0.921)(^{1})</td>
<td>-4.1ln((\sigma_{n}))+51.75</td>
<td>-0.687ln((\sigma_{n}))+7.85</td>
<td>Not measured</td>
<td>4700((\sigma_{n}/3.85)^{0.85})</td>
</tr>
<tr>
<td>Dense</td>
<td>17.0</td>
<td>92</td>
<td>17.0(-0.044z(^{-2})-0.6034z(^{-2})+0.921)(^{1})</td>
<td>-6.06ln((\sigma_{n}))+62.69</td>
<td>-1.575ln((\sigma_{n}))+16.13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{1}\) \(z = z/1m\) is the dimensionless depth, measured from the surface of the sand bed (z in m)

\(^{2}\) Measured from direct shear tests under normal stresses \(\sigma_{n} = 1.25kPa\) to \(100kPa\) (Ansari et al. 2018). \(\sigma_{n}\) is normalised against a reference stress \(\sigma_{n} = 1kPa\) (\(\sigma_{n}\) in kPa)

\(^{3}\) From direct shear tests (Ansari et al. 2018) and isotropically consolidated drained and undrained triaxial tests (Ajalloelian et al. 1996)

\(^{4}\) From incremental loading oedometer tests. \(\sigma_{n}\) is normalised against a reference stress \(\sigma_{n} = 1kPa\) (\(\sigma_{n}\) in kPa)

The setup of the winch and pulley-clamp system is modified as shown in Fig. 1 to allow pulling the pipe downward through a 20mm hole drilled at the centre of the chamber. The hole is fitted with a PVC cap that prevents flow of sand, while also minimising friction at the cable-cap interface. In addition, the chamber is reinforced by four steel 50x50mm SHS beams, with two bars fitted symmetrically on each side of the chamber. This is required as pilot tests revealed that penetration of the pipe results in out-of-plane earth pressures developing in the mobilised sand prism below the pipe, leading to deformation of the glass sidewalls and sand grains becoming trapped between the pipe endcaps and the glass. The drawback of using braces to minimise glass deflection is that the field of view of the camera used to capture images of the tests is reduced, therefore the failure mechanism developing in sand can be tracked only at the vicinity of the pipe, as discussed later.

In addition, two pressure sensors P1 and P2 (type KDF-200KPA, manufactured by Tokyo Measuring Instruments Lab) of capacity 200kPa are installed at the bottom of the chamber, at the locations shown in Fig. 1. The rationale behind installing these pressure sensors is twofold: First, the pressure sensors are used to monitor normal stresses acting at the bottom of the chamber during pipe penetration tests, and assessing boundary effects. In addition, pressure measurements during sand deposition are used to validate measurements of geostatic stresses developing at the bottom of the chamber as deposition of sand progresses, obtained with the setup described in Kouretzis and Bouckovalas (2019). As explained in detail in Ansari et al. (2018) and Kouretzis and Bouckovalas (2019), vertical geostatic stresses inside the chamber are lower than the theoretical free-field vertical stresses \(\sigma_{v,free\,field}/\gamma z\), where \(\gamma\) is the dry unit weight of the sandbed measured by means of a set of external button load cells (see Ansari et al. 2018) and \(z\) is the distance from the surface of the sandbed. This is due to part of the weight of sand being transferred to the sidewalls through sand-glass friction. Therefore an effective dry unit weight \(\gamma_{eff}\) needs to be considered for the interpretation of the test results (Table 1), equal to \(\gamma_{eff} = \sigma_{v,measured}/z\), where \(\sigma_{v,measured}\) is the vertical stress measured at the pressure sensors. Figure 2 compares the vertical total stress measured with the load cell setup presented in Kouretzis and Bouckovalas (2019), from which the expressions for \(\gamma_{eff}\) of Table 1 resulted from, against measurements from the pressure sensor P1 obtained during deposition of loose and dense sandbeds. Excellent agreement is observed, providing confidence in using \(\gamma_{eff}\) to analyse the physical model test results.
3. TESTING PROCEDURES

The main round of tests was performed with the $D=37.5$mm pipe used by Wu et al. (2019), while one additional test was performed with the $D=75$mm pipe to assess repeatability and scale effects. The small pipe was chosen to avoid boundary effects associated with interaction of the mobilised sand prism with the bottom of the chamber, and to limit the reaction at the cable within the capacity of the 1.5 t winch and gearbox to maintain a constant pull rate. Each test commenced by passing the steel cable through the hole and PVC cap and depositing in the chamber a layer of sand of thickness 450mm ($12D$ for the $D=37.5$mm pipe), using the pouring or the rainer method, depending on the desired density. Details on the custom deposition methods are presented by Ansari et al. (2018). Next, the pipe was connected to the steel cable and placed at the centre of the chamber using the as-laid method described in Ansari et al. (2019) i.e. the pipe was carefully laid on the surface of the sandbed and was not pre-embedded in sand to minimise disturbance. The distance of the pipe from the left and right walls of the chamber was 525mm ($14D$), measured from each pipe spingline. Finally, deposition continued until the desired pipe embedment was achieved, which ranged nominally between $H=1.5D$ and $H=4D$, where $H$ is the distance from the pipe springline to the sandbed surface. The tested embedment depths cover a wide range of mid-to-large diameter pipelines laid in trenches. The average dry density of the sandbed was calculated from the button load cell measurements that continuously monitor the weight of the chamber, and if it deviated more than 0.25kN/m$^3$ from the target values listed in Table 1, the test was cancelled and deposition was repeated.

We should note here that selection of the distance between the pipe and the bottom of the chamber was based on the results of the numerical study of Limnaiou et al. (2019), who have shown that the reaction developing on a pipe during vertical downwards movement $z_d$ of magnitude up to $z_d=1D$ is not affected by the existence of a rigid boundary, if the distance between the pipe and the rigid boundary is larger than $10D$. This was confirmed by means of independent finite element analyses performed as part of this study, not presented here for brevity. Further evidence on the capacity of the $12D$ sand layer to mitigate boundary effects is drawn from measurements of earth pressure developing at the bottom of the chamber during a test from the deepest pipe embedment depth tested in this study $H/D=4$. Observe in Fig. 3 that the pressure developing on the sensor P1, which is located closer to the pipe centreline (Fig. 1) is considerably higher, compared to the pressure at the sensor P2. However, there is no indication of an abrupt change in the rate of pressure increase during the test, or in the rate of increase of the reaction force, suggesting that there is no interaction with the bottom boundary even at vertical pipe displacement $z_d=0.75D$ when the test was stopped. These measurements, although not directly associated with the reaction measured on the pipe, are also potentially useful for benchmarking numerical models simulating vertical downwards movement of a pipe in sand.
After deposition of the last layer of sand was completed, the test commenced and the pipe was pulled downwards up to a maximum displacement of about \( z_d = 0.75D \), which is sufficient to reach the steady-state part of the test, where the rate of increase of the resistance developing on the pipe \( \Delta Q_d/\Delta z_d \) remains constant with increasing pipe penetration. An exception was the test with the \( D = 75 \)mm pipe which was stopped when the displacement reached \( z_d = 0.35D \), to avoid damaging the equipment. Following the completion of each test, sand was vacuumed along the trajectory of the pipe to perform the so-called “friction test” described in Ansari et al. (2018, 2019), where we measure the friction developing at the glass-pipe interface \( F_{\text{pipe-glass}} \) and at the pulley-cable system \( F_{\text{pulley-cable}} \), to subtract it from the measured reaction force. Friction between the 2mm-diameter cable and sand was measured to be of the order of 6 to 10N, depending on the density of the sandbed, and was ignored in this study, as the net (corrected) reaction force was considerably higher. However, owing to the dimensions of the chamber, the measured reaction force also needs to be corrected for the friction that develops at the mobilised sand prism and the glass walls \( F_{\text{glass-sand}} \), as discussed extensively by Ansari et al. (2018, 2019). To do so, first the friction coefficient at the sand-glass interface was determined by means of direct shear tests to be \( \mu = \tan \phi_i \) with \( \phi_i \) ranging between \( \phi_i = 17^\circ \) and \( 18^\circ \) for loose to dense sand, respectively (Kouretzis and Bouckovalas 2019). The same direct shear tests revealed that the shear displacement required to mobilise the peak interface strength is of the order of 0.1mm to 0.5mm. Using the approach followed in Ansari et al. (2018, 2019) and Wu et al. (2019) we determine the dimensions and geometry of the mobilised sand prism from images captured during the tests, which are analysed with the PIV method to infer the area of the sandbed where the vertical displacement exceeded 0.1mm (Fig. 4). Accordingly a simplified trapezoidal geometry is assigned to the mobilised prism referred to as influence zone, as depicted in Fig. 4, and the total friction force that resists pipe movement is calculated by integrating interface shear stresses \( \tau_f \) along the area of the trapezoid, as:

\[
F_{\text{glass-sand}} = 2 \mu \int_0^{n_{\text{inf}}} \left[ A(z) \cdot \sigma_n(z) \right] \cdot dz
\]

The considered distribution of normal stresses acting on the glass sidewalls \( \sigma_n(z) \) is depicted in Fig. 4, and is based on measurements of the peak normal stress acting on the pipe during lateral dragging by Palmer et al. (2009) and the distribution considered by Ansari et al. (2019) to calculate \( F_{\text{glass-sand}} \) for lateral drag tests in the same chamber. Indicatively, the distribution of stresses shown in Fig. 4 suggests that the additional normal stress acting at the bottom of the chamber at the end of the test in dense sand shown in Fig. 3 is \( \sigma_n = K_p \gamma_{\text{eff}} (4D + 0.45) \gamma_{\text{eff}} (4D + 0.45) = 21.2 \)kPa. This is reasonably close to the value measured at sensor P1 at the end of the test (≈18.7kPa, Fig. 3), if we further consider that the mobilised prism does not reach the bottom of the chamber.

Figure 3. Left: Measurements of additional earth pressures developing on the sensors P1 and P2 during penetration of the \( D = 37.5 \)mm pipe in dense sand (initial pipe embedment depth \( H/D = 4 \)); Right: Net reaction force measured on the pipe as function of the normalised downward displacement \( z_d/D \) from the same test.
Figure 4. Geometry of the sand prism mobilised during vertical penetration of the pipe, determined by means of PIV analysis. The considered distribution of normal stresses acting on the glass walls is depicted on the left. $K_0 = 1 - \sin \phi_{cs}$ is the coefficient of earth pressure at-rest and $K_p$ is Coulomb’s passive earth pressure coefficient.

4. SCALE EFFECTS

Scale effects on the estimation of the bearing capacity of foundations on sand is a multi-faceted problem discussed in extend in the relevant literature. One of the phenomena resulting in the bearing capacity factors being dependent on the dimensions of the foundation is the so-called “particle size effect” (Tatsuoka et al. 1991; Cerato and Lutenegger 2007; White et al. 2008). This refers to the fact that dilation developing in shear bands depends on the size of sand grains. Therefore, the smaller the foundation, or as the ratio of the diameter of the foundation over the mean grain size $D/D_{50}$ decreases, the more prominent the effect of dilation. This phenomenon is more relevant to centrifuge tests. In the case of the small pipe tests in STK sand the ratio $D/D_{50}$ exceeds 100, therefore is higher than the most conservative thresholds proposed in the literature (e.g. Kusakabe 1995).

Another issue acknowledged widely in the literature is that the mechanical properties of sand depend on the mean stress level, therefore the bearing capacity factors will decrease as the size of the foundation increases. The dependence of the friction angle, dilation angle and compressibility of STK sand on the mean stress level has been documented extensively during the laboratory characterisation study (see Table 1). To approximately consider this in the estimation of the reaction developing on pipes during uplift and lateral relative movements, Ansari et al. (2018, 2019) proposed to normalise the measured reaction $Q$ per unit meter against the (stress-dependent) peak shear strength of sand at the pipe springline $\gamma_{eff}HD \tan \phi_{ps,p}$, instead of the vertical geostatic stress at the springline $\gamma_{eff}H$ as per standard practice. This simplistic approach needs to be applied with caution when attempting to extrapolate small scale test results to larger pipes, as when the development of reaction to pipe movements is dominated by progressive failure on localised slip planes, the response will also depend on the actual length of the slip plane. In other words, the peak strength may be mobilised almost simultaneously along a short slip plane developing around a small pipe, while progressive failure effects will be more prominent for a large pipe, which movement will mobilise longer slip planes (White et al. 2008). Using the modified reaction factor $N_{mod} = Q/\gamma_{eff}HD \tan \phi_{ps,p}$ proposed by Ansari et al. to normalise tests results cannot account for this, hence the approach is only approximate.

To assess to some extent the above phenomena for the problem of vertical downwards pipe movement, we performed two tests on pipes with diameters $D=37.5$mm and $D=75$mm embedded at the same nominal dimensionless depth $H/D=3$, in dense sand. The reaction-displacement response measured during these two tests is depicted in Fig. 5, where we present the measured reaction as function of the dimensionless vertical
downwards pipe displacement \( z_d/D \) using the reaction factors \( N_{vd} = Q_d/\gamma_{eff}HDL \) and \( N_{vd-mod} = Q_d/\gamma_{eff}HDL \tan \phi_{ps,p} \). In these expressions \( Q_d \) is the net force measured during the tests, corrected for friction effects. The expressions listed in Table 1 are used to calculate \( \tan \phi_{ps,p} \) as function of the vertical geostatic stress at the pipe springline. Observe that while the \( N_{vd}-z_d/D \) curves for the two pipes match quite well, this is not the case for the \( N_{vd-mod}-z_d/D \) curves. This suggests that the dimensionless parameter \( N_{vd} \), which is equivalent to the bearing capacity factor \( N_q \) (Eq. 2) used to determine the bearing capacity of a footing embedded in weightless soil, can be used to normalise results of the performed tests. We will thereafter present test measurements in terms of the dimensionless reaction \( N_{vd} = Q_d/\gamma_{eff}HDL \), acknowledging that extrapolating the measurements to directly obtain the reaction that would develop in large diameter pipes is questionable. However, the test results presented in the following allow assessing the efficacy of methods for predicting \( N_{vd} \), provided that the stress-level dependency of the sand mechanical properties is considered in the calculations.

Figure 5. Pipe diameter effects on normalised reaction-pipe displacement measurements during penetration tests. Tests on pipes nominally embedded at \( H/D=3 \) in dense sand.

5. TEST RESULTS

As mentioned in the introduction, we performed a total of 18 tests with the \( D=37.5\text{mm} \) pipe buried in loose to dense dry STK sand, with the initial embedment depth of the pipe ranging nominally from \( H/D=1.5 \) to 4. The average dry density of the sandbed inferred from the button load cell measurements and the initial position of the pipe corresponding to each test are listed in Table 2.

<table>
<thead>
<tr>
<th>Test No</th>
<th>Sandbed density</th>
<th>Measured dry unit weight ( \gamma ) of the sandbed (kN/m(^3))</th>
<th>As-tested ( H/D )</th>
<th>Net reaction force at failure threshold ( Q_{df} ) (N)</th>
<th>Reaction factor at failure threshold ( N_{vd} = Q_{df}/\gamma_{eff}HDL )</th>
<th>( z_{df}/D )</th>
<th>( \gamma_{eff} ) relevant to the stress level at pipe springline (deg)</th>
<th>( \phi^* ) calculated according to Davis 1968 (deg)</th>
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<tbody>
<tr>
<td>1</td>
<td>Dense</td>
<td>16.95</td>
<td>1.54</td>
<td>730</td>
<td>274.8</td>
<td>0.28</td>
<td>16.36</td>
<td>63.1</td>
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<tr>
<td>2</td>
<td>Medium</td>
<td>16.87</td>
<td>2.12</td>
<td>825</td>
<td>229.8</td>
<td>0.32</td>
<td>16.06</td>
<td>61.2</td>
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<tr>
<td>3</td>
<td>Dense</td>
<td>16.99</td>
<td>2.52</td>
<td>870</td>
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<td>0.29</td>
<td>16.01</td>
<td>60.1</td>
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<tr>
<td>4</td>
<td>Medium</td>
<td>17.04</td>
<td>3.10</td>
<td>890</td>
<td>171.9</td>
<td>0.3</td>
<td>15.83</td>
<td>58.9</td>
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<tr>
<td>5</td>
<td>Dense</td>
<td>16.90</td>
<td>3.58</td>
<td>960</td>
<td>163.8</td>
<td>0.31</td>
<td>15.52</td>
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<tr>
<td>6</td>
<td>Medium</td>
<td>16.91</td>
<td>4.02</td>
<td>1030</td>
<td>158.2</td>
<td>0.32</td>
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<tr>
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<td>Dense</td>
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<td>180</td>
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<td>15.89</td>
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<tr>
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<td>Dense</td>
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<td>2.53</td>
<td>235</td>
<td>58.6</td>
<td>0.21</td>
<td>15.02</td>
<td>50.2</td>
</tr>
<tr>
<td>10</td>
<td>Medium</td>
<td>15.96</td>
<td>3.05</td>
<td>250</td>
<td>52.3</td>
<td>0.2</td>
<td>14.85</td>
<td>49.5</td>
</tr>
</tbody>
</table>
Measurements of the dimensionless reaction developing on the pipe $N_i = Q_i / \gamma_d H DL$ as function of the normalised vertical pipe displacement $z_d/D$ are depicted in Fig. 6. There are several things worth noting here, however first we focus on the fact that the reaction-displacement response is characterised by an initial nonlinear (parabolic) part, associated with the development of the failure mechanism. At larger displacements the reaction appears to increase linearly as the penetration of the pipe increases, regardless of the embedment of the pipe or the density of the sandbed. The initially parabolic increase in the reaction is similar to that observed by White et al. (2008) for penetration of a conical footing with sharp cone angle 130°, which resembles most a circular pipe, as well as other tests for embedded foundations in sand (e.g. Vesić 1963). Only in the test in dense sand from the shallowest embedment depth $H/D = 1.54$ the increase in the resistance past the initial nonlinear part appears to be less sharp, indicating a more significant contribution of post-peak softening of sand in the overall reaction-displacement response. This of course suggests that considering elastic-perfectly plastic bearing springs in pipe stress analysis models will result in underestimating the soil reaction developing on the pipe if these springs reach their yield displacement. Underestimating soil reaction will result in un-conservative estimates of strains or joint displacements developing on pipes called to accommodate large vertical movements of their surrounding soil, due to the prescribed displacement nature of the problem.

To understand what leads to the response changing with increasing penetration of the pipe, we will examine the failure mechanism that develops in sand as the tests progress. The failure mechanism is depicted via incremental shear strain contours obtained from the analysis of images captured during the tests with the PIV method (Stanier et al. 2015, Ansari et al. 2018). The evolution of the mechanism with increasing pipe penetration is presented in Fig. 7 for the tests #1 and #4 (Table 1) in dense sand, for two different initial pipe embedment depths. Incremental shear strain contours from different stages of each test suggest that during the initial parabolic part of the reaction-displacement response (Stage 1) two failure wedges are being formed above and below the pipe. As the displacement increases to the point where the rate of increase of the reaction on the pipe $\Delta Q_d/\Delta z_d$ becomes constant with increasing displacement $z_d$ (Stage 2), log-spiral wedges have already been formed, indicating a general failure mechanism. As pipe displacements continue to increase further (Stage 3), the geometry of the slip planes remains unchanged and a steady state is reached. However, the resistance continues to increase as the pipe penetrates deeper into the sandbed, as the overburden stress acting on the slip planes increases, similar to the continuous increase in the resistance developing at the tip of a CPT cone penetrating uniform sand.

However, these observations do not explain why the dimensionless reaction curves of the tests performed in loose and medium sand appear to insensitive to sand density in Fig. 6, as this is not compatible with general bearing capacity theory. To shed some light on this, we plot in Fig. 8 the incremental shear strain and incremental displacements contours from three tests in loose (test #13), medium (test #7) and dense (test #1) sand, corresponding to the steady-state stage of the test where the failure mechanism has already been formed and $\Delta Q_d/\Delta z_d$ remains constant with increasing displacement $z_d$. Notice that densification of sand takes place in a wedge below the pipe during the tests performed in the loose and medium sandbed, and that the extend of this densified zone is similar in both tests. This suggests that the properties of sand in the zone offering resistance to pipe movement are altered due to movement of the pipe, and its shear strength is similar regardless of the initial sandbed relative density. We attribute that to the fact that the absolute decrease in the void ratio of the loose sandbed will be larger than the absolute decrease in the void ratio of the medium dense sandbed, for the same amount of pipe movement under the same test conditions. Still the absolute value of the resistance developing on the pipe in medium sand tests is higher than the absolute value of the resistance in loose sand tests. The difference in absolute values is associated with the increased

| 11 | Loose | 16.10 | 3.53 | 260 | 47.2 | 0.21 | 14.80 | 48.7 | 39.8 |
| 12 | 15.85 | 4.04 | 275 | 44.9 | 0.2 | 14.39 | 48.2 | 39.5 |
| 13 | 15.05 | 1.58 | 160 | 66.2 | 0.17 | 14.51 | 42.3 | 35.2 |
| 14 | 15.09 | 2.02 | 170 | 55.4 | 0.18 | 14.40 | 41.7 | 34.8 |
| 15 | 15.04 | 2.55 | 205 | 53.8 | 0.21 | 14.17 | 41.0 | 34.5 |
| 16 | 14.97 | 3.05 | 220 | 49.1 | 0.19 | 13.93 | 40.4 | 34.2 |
| 17 | 15.10 | 3.55 | 225 | 43.3 | 0.18 | 13.88 | 40.0 | 34.0 |
| 18 | 14.95 | 4.12 | 245 | 41.6 | 0.2 | 13.54 | 39.7 | 33.7 |
| 19 | Dense | 16.85 | 3.00 N/A | N/A | N/A | 14.52 | 55.6 | N/A |

(i) Not calculated, test #19 was performed only to assess pipe diameter effects.
overburden stress (increased \( \gamma_{\text{eff}} \)) in medium sand. This is not the case for the dense sandbed test where further densification of sand is not possible, therefore the conditions of the experiment are closer to the assumption of rigid-plastic sand response, allowing the formation of a general failure mechanism. Similar observations regarding the development of localised failure mechanisms below foundations are of course reported elsewhere in the literature, however we must state that plasticity solutions developed for local or punching shear failure (e.g. Vesić 1963) are still based on the assumption of rigid-plastic response, therefore cannot take into account the change in sand state during the test.

Figure 6. Dimensionless reaction-pipe displacement curves obtained from the tests with the \( D=37.5 \text{mm} \) pipe. Reported embedment depths are nominal, and the actual embedment depth of each test is listed in Table 2.
Figure 7. Reaction-displacement response measured during tests in dense sand with the pipe initially embedded at $H/D=1.54$ (test #1, top left) and at $H/D=3.10$ (test#4, top right). Incremental shear strain contours developing in sand during different stages of the tests #1 and #4 are presented in the bottom.
Figure 8. Incremental shear strain contours and displacement vectors (left) and incremental displacement contours and vectors (right) obtained at the steady-state stage of tests #1, #7 and #13. Results for nominal initial pipe embedment $H/D=1.5$ and loose (test#13, top), medium (test#7, mid) and dense (test#1, bottom) sandbed.

Another observation worth discussing here is that the initial parabolic part of the reaction-displacement curve extends to larger displacements during tests in dense sand, a phenomenon also observed in the deep foundation tests of Vesić (1963). This is a consequence of the different failure mechanisms developing in dense and in loose/medium sand (Fig. 8). Particularly in the case of dense sand the pipe displacement required to reach the steady-state part of the reaction-displacement curve is considerably higher than the $0.1D$ yield displacement associated in the ALA (2005) and PRCI (2009) guidelines with the ultimate resistance developing on the pipe.

Finally, for the evaluation of the efficacy of different methods to predict the response observed in the experiments, it is necessary to define a nominal bearing capacity of the pipe. Here we will use the failure threshold (Akbas and Kulhawy 2009; Qin et al. 2019), defined as the resistance acting on the pipe when the reaction-displacement response changes from nonlinear to steady-state ($\Delta Q_d/\Delta z_d$ constant), suggesting that the failure mechanism has been fully formed, and the increase in the resistance as pipe penetration continues is associated with the increase in overburden stress. For that we follow the approach used by Qin et al. (2019), who plotted the ratio of $\Delta Q_d/\Delta z_d$ as function of the pipe displacement $z_d$ to determine the displacement $z_{df}$ where the resistance increases linearly with any further increase of pipe displacement (Fig. 9). The resistance
corresponding to this displacement \( Q_{df} \) is termed failure threshold, and the relevant \( Q_{df} \) and \( z_{df} \) values, together with the factor \( N_{vd,f} \) corresponding to the failure threshold are listed in Table 2.

6. EVALUATION OF METHODS FOR PREDICTING THE BEARING CAPACITY OF PIPES

To evaluate the efficacy of bearing capacity theory to predict the value of the failure threshold \( Q_{df} \) measured from each test, we plot in Fig. 10 the variation of the capacity factor \( N_{vd} \) corresponding to the failure threshold \( N_{vd,f} = Q_{df} / \gamma_{eff} HDL \) with the initial embedment of the pipe \( H/D \) and the relative density of the sandbed. In addition, we plot the factor \( N_{vd,f} \) calculated with Eq. 1 for three characteristic sand friction angle values \( \phi \), considered as representative of dense, medium and loose sand in ALA (2005). We do not use the peak plane-strain friction angle of STK sand corresponding to the stress level at the pipe springline measured from direct shear tests (Table 2) with the bearing capacity formula, as this would result in excessively high \( N_{vd,f} \) values. This is in line with the findings of Xie et al. (2013) as well as Roy et al. (2016) and is somewhat expected, as Eq. (1) was derived while assuming general shear failure but also that the response of soil can be described with an associative coaxial flow rule (sand friction angle \( \phi \) equal to the sand dilation angle \( \psi \)), which is not the case for real granular materials (see Table 2). Notice that the factor \( N_{vd,f} \) corresponding to the experimental measurements decreases as \( H/D \) increases, as the analytically calculated values, a finding which is in line with the numerical results of Qin et al. (2019) and Kouretzis et al. (2014).

![Figure 9](https://example.com/figure9.png)

**Figure 9.** Determination of the failure threshold \( Q_{df} \) and associated displacement \( z_{df} \) from the rate of increase of the resistance on the pipe \( \Delta Q_{d} / \Delta z_{d} \). Indicative results from test#1.

![Figure 10](https://example.com/figure10.png)

**Figure 10.** Variation of the \( N_{vd,f} \) values corresponding to the failure threshold \( Q_{df} \) measured from each test with the initial embedment of the pipe \( H/D \), for different sandbed densities. Predictions of Eq. (1) for three characteristic friction angle values are also presented.
Results of the analysis of images captured during the tests with the PIV method suggest that a general failure mode will develop only when the pipe is embedded in dense sandbed, while a localised failure mode will develop when the pipe is embedded in medium and loose sandbed. So the question that arises is which is the appropriate method for estimating $Q_{df}$, and which sand parameters should be considered in the calculation, given that we have shown that the native sandbed properties may be altered due to vertical downwards movement of the pipe, depending on the initial sand density.

To answer this question, we back-calculated the friction angle of sand $\phi_{bc}$ that needs to be used as input to predict the failure threshold value $Q_{df}$. For the tests in dense sand we used the numerical methodology of Kouretzis et al. (2014), which is based on the finite element limit analysis (FELA) method (Sloan 2013). The computer software OptumG2 (OptumG2 2019) was used to model penetration of a rigid cylindrical pipe in sand, with the friction angle of the latter varying until the resulting ultimate reaction on the pipe $Q_{df}$ matched the measured resistance reported in Table 2. A similar approach was followed by White et al. (2008) to back-analyse measurements obtained during centrifuge tests modelling spudcan footing penetration in dense and medium sand, and its advantage compared to using Eq. (1) is that it accounts for the effect of the shape of the pipe on the resistance. As shown in Fig. 11 the failure mode observed in the numerical simulations matches well with the failure mode observed in the experiments in dense sand, underpinning the validity of this back-analysis approach. However, simulation of this problem with the FELA method requires modelling sand as associative material. To be able to compare the back-calculated friction angles with the measured mechanical properties of STK sand, we will calculate an equivalent pressure-dependent friction angle $\phi^*$ following the approach proposed by Davies (1968). According to Davies, modelling a material the follows a non-associative coaxial flow rule with friction angle $\phi_{ps,p}$ and dilation angle $\psi$ is equivalent to modelling an associative material with friction angle $\phi^*$ calculated as:

$$\tan \phi^* = n \tan \phi_{ps,p}$$

where

$$n = \frac{\cos \psi \cos \phi_{ps,p}}{1 - \sin \psi \sin \phi_{ps,p}}$$

Pressure-dependent equivalent friction angle values for STK sand calculated with Eqs. (5-6) are listed in Table 2.

This numerical approach is appropriate for back-calculating the friction angle for dense sand however, as the results of Fig. 8 suggest, this is not the case for medium or loose sand, as the failure mechanism is different. To account for this we will back-calculate the friction angle required to predict the capacity measured in medium and loose sand tests with Eq. (1), but while using the factor $N_{q,l}$ proposed by Vesić (1963) for local or punching shear failure of deep foundations, instead of the bearing capacity factor $N_q$ for general shear failure (Eq. 2):

$$N_{q,l} = e^{3.8 \tan \phi} \cdot \tan^2 (45^\circ + \phi/2)$$

Figure 11. Comparison of incremental shear strain contours obtained from PIV analysis of images captured during the tests (left) and from FELA (right). Results of test/simulation for dense sand with the pipe initially embedded at $H/D=3.10$. 

https://mc06.manuscriptcentral.com/cgj-pubs
The back-calculated friction angles obtained with the FELA method for dense sand, and Eqs. (1), (3) and (7) for loose and medium sand are presented in Fig. 12 as function of the sandbed density and initial embedment of the pipe. In addition we compare back-calculated friction angles against the equivalent friction angle $\phi^*$ values resulting from direct shear test results and Eqs. (5-6), that would be used in a “blind” prediction of the failure threshold. Observe that the ratio $\phi^*/\phi_{bc}$ is close to unity for tests in medium sand, suggesting that using the Eq. (1) with the bearing capacity factor $N_{q,l}$ and the equivalent friction angle of STK sand results in reasonably accurate predictions of the threshold resistance on the pipe. For tests in dense sand, the back-calculated friction angle is lower than the equivalent friction angle. This is attributed to effects of progressive failure (Roy et al. 2018a, b), that cannot be captured with the FELA method, as the modelled shear strength is the same along the entire length of the slip planes. However, as the length of the slip planes for the problem at hand is relatively short, failing to capture this phenomenon does not result in large discrepancies with the experimental measurements. On the other hand, there is considerable difference between the back-calculated friction angle and the equivalent friction angle for loose sand. In fact, as expected, the former is practically equal to the friction angle that resulted from the back-analysis of medium sand tests. This is line with the loose sandbed becoming densified during penetration of the pipe, therefore considering sand properties from element tests for predicting the bearing capacity of pipes embedded in loose sand is not realistic.

![Figure 12](https://mc06.manuscriptcentral.com/cgj-pubs)

**Figure 12.** Left: Variation of the back-calculated sand friction angle $\phi_{bc}$ with the embedment of the pipe, for different sandbed densities. Right: Variation of the equivalent over the back-calculated friction angle ratio $\phi^*/\phi_{bc}$ with the embedment of the pipe, for different sandbed densities.

Given the above observations, it is worth investigating what is the error in predicting the bearing capacity of pipes using the pressure-dependent equivalent friction angle $\phi^*$ from direct shear tests (Table 2) together with Eq. (1) and the bearing capacity factor $N_q$ (Eq. 2) for tests in dense sand, or the bearing capacity factor $N_{q,l}$ (Eq. 6) for tests in medium and loose sand. Predictions obtained with the mentioned formulas are compared against the measured failure threshold values $Q_{df}$ in Fig. 13. Although the analytical formulas were derived for foundations with a flat surface, therefore the effect of the shape of the pipe on the bearing capacity factors is not considered, they provide reasonably accurate predictions of pipe capacities for dense and medium sand, with the former being consistently over-predicted due to the effects of progressive failure not taken into account. This is not the case for loose sand tests, where the bearing capacity is systematically under-predicted due to the more prominent effect of densification on the sandbed properties, as discussed earlier.
Figure 13. Comparison of predicted bearing capacity values using Eqs. (1-3) for dense sand and Eqs. (1), (2) and (6) for medium and loose sand together with the equivalent friction angle $\phi^*$ of STK sand, against measurements of the failure threshold $Q_{df}$.

Finally, since definition of the properties of elasto-plastic Winkler springs requires estimating their yield displacement, we present in Fig. 14 the variation of the displacement $z_{df}$ associated with the failure threshold with the initial embedment of the pipe $H/D$ and sand density. Notice first that the displacement $z_{df}$ appears to be insensitive to pipe embedment $H/D$, unlike results of e.g. lateral drag tests (Ansari et al. 2019). We attribute that to the fact that the size of the failure mechanism does not increase much as $H/D$ increases, as it is the case in lateral tests. As mentioned earlier the pipe displacement required to reach the steady-state part of the reaction-displacement curve is increased for dense sand, and is consistently higher than the yield displacement $0.1D$ recommended in the ALA (2005) and PRCI (2009) guidelines. Given though the parabolic shape of the initial nonlinear part of the reaction-displacement curves, considering the resistance on the pipe linearly increasing to $Q_{df}$ as the displacement increases from zero to $z_{df}$ will result in underestimating the reaction on the pipe at relative displacements lower than $z_{df}$. Therefore, if the analyst wishes to use elasto-plastic soil springs in their pipe stress analysis model, it is considered prudent to consider a yield displacement of $0.2D$ for dense sand and $0.1D$ for medium sand.

Figure 14. Variation of the displacement $z_{df}$ corresponding to the failure threshold $Q_{df}$ with the initial embedment of the pipe $H/D$, for different sandbed densities.

7. RECOMMENDATIONS FOR PRACTICE

Outcomes of this experimental study can be summarised into the following recommendations for estimating the properties of vertical bearing springs used in pipe stress analysis models:
The bearing capacity formula (Eq. 1) can provide realistic estimates of the ultimate resistance of elasto-plastic vertical bearing springs, if used together with the bearing capacity factor for general failure $N_q$ (Eq. 2) for pipes in dense sand, and together with the bearing capacity factor for local shear failure $N_{q,l}$ (Eq. 5) for pipes in medium sand. This approach will result in somewhat conservative resistance values for pipes in dense sand, as the reduction in resistance due to progressive failure in the forming slip lines is not accounted for. Use of relative simple numerical methods to determine the reaction on the pipe, such as the FELA method, will allow accounting for the effect of the shape of the contact area between pipe and sand, thus provide more accurate results, at the expense of increased analysis cost. However, the FELA method cannot take into account the effect of progressive failure on the developing resistance, unlike other numerical methods which can explicitly model sand softening.

The sand friction angle used as input in the abovementioned bearing capacity formula and factors needs to be the equivalent friction angle of an ideal material that obeys an associated flow rule (Eqs. 5-6) and not the peak friction angle of sand measured in direct shear tests. Using the peak plane-strain friction angle of sand at low confining stress levels pertinent to pipe backfills will result in unrealistically high estimates of the reaction developing on the pipe, as the effects of sand dilation are overestimated. On the other hand, using the peak plane-strain friction angle measured in tests performed at higher confining stresses will probably result in more reasonable estimates of the reaction, however this is a crude approach that ignores the mechanics of the problem. Certainly, considering the stress-dependency of the friction and dilation angles of sand will allow directly accounting for pipe diameter effects on the measured reaction.

It is not recommended to consider loose backfill mechanical properties when estimating the ultimate resistance of vertical bearing springs, as this will result in significantly under-predicting the reaction developing on the pipe, thus is un-conservative. In fact, placing the pipe in loose sand backfill will have little effect on reducing the force developing on the pipe during downwards relative movement, as movement of the pipe will densify the loose backfill during a ground movement episode due to the increased confinement.

If the analyst wishes to use elasto-plastic bearing springs, they should keep in mind that the yield displacement is function of sand density. It can be taken equal to $0.1D$ for pipes in medium sand, as per ALA (2005) or PRCI (2009) recommendations, but should be increased to $0.2D$ for pipes in dense sand.

Using elasto-plastic bearing springs will result in underestimating the reaction developing at large relative movements, exceeding $0.3D$, due to the increase in overburden stress. Bilinear or parabolic springs are more appropriate for modelling the continuous increase in the resistance at pipe displacements larger than the yield displacement, if such large relative movements are expected.

8. ACKNOWLEDGMENTS

The work presented in this paper is supported by the Australian Research Council (Project DP180103497).

9. REFERENCES


ALA 2005. Guidelines for the design of buried steel pipes. American Lifelines Alliance (ALA), ASCE.


PRCI 2009. Guidelines for constructing natural gas and liquid hydrocarbon pipelines in areas subject to landslide and subsidence hazards. RPCI Pipeline Research Council International Inc.


