Available online at www.CivileJournal.org

Civil Engineering Journal

(E-ISSN: 2476-3055; ISSN: 2676-6957)

Vol. 7, No. 07, July, 2021

Bearing Capacity of Interfered Adjacent Strip Footings on Granular Bed Overlying Soft Clay: An Analytical Approach

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Received 12 February 2021; Revised 23 May 2021; Accepted 04 June 2021; Published 01 July 2021

Abstract

In the present paper, the interference effects on bearing capacity of two and three closely spaced strip footings resting on granular bed overlying clay are being studied. A simple analytical model is proposed to predict the load-carrying capacity and the interference factor of an interfered footing, when adjacent strip footings are optimally placed on the surface of a Granular Bed (GB) overlying clay and both the footings are simultaneously loaded. A punching shear failure mechanism is envisaged in the analytical model. The load-carrying capacity of the footing is taken as the sum of total shearing resistances along the two vertical planes through the edges of the strip footing in the upper granular layer and the load-carrying capacity of the soft clay beneath the GB. Insights gained from finite element simulations are used to develop the new modified punching shear model for interfering footing. Bearing capacity can be easily calculated by using the proposed punching shear model for interfering footing. The analytical model is validated with numerical analyses and previous experimental results and found to be in reasonably good agreement. The influence of different parameters such as granular bed thickness, width of footing, number of footings are carried out in this study.

Keywords: Interference Effect; Adjacent Strip Footings; Bearing Capacity; Layered Soil; Analytical Model; Interference Factor.

1. Introduction

Closely spaced adjacent footings undergo the phenomenon of interference. Interference alters the bearing capacity, settlement, rotational, and failure mechanism of footings. Most of the interference studies on shallow footings are carried out in homogeneous soils. But quite often, geotechnical engineers come across layered soil profiles. Sometimes low-lying areas with clayey soil (or weak soil) are provided with granular fill (granular bed) on top. The granular Bed (GB) has two objectives: it adjusts the ground level to the adjacent road and enhances the permitted load of the superstructure on the backfilled ground. To evaluate the effect of interference, especially in terms of bearing capacity, different approaches such as experimental, numerical, and analytical are adopted. Calculation of bearing capacity of a footing in presence of adjacent footing is difficult to determine by using experiments or numerical studies since they are time-consuming or expensive.

In the present study, an analytical model is proposed to predict the load-carrying capacity of an interfered footing, and the interference factor, when adjacent strip footings are optimally spaced on the surface of a Granular Bed (GB) overlying clay and both the footings are simultaneously loaded. The proposed model is an extension/modification of

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http://dx.doi.org/10.28991/cej-2021-03091723

the punching shear model proposed earlier to predict the load-carrying capacity of a single independent strip footing on a granular bed overlying clay [1, 2]. The proposed analytical model is validated with results of numerical and previous experimental studies and is found to be in reasonably good agreement.

2. Background of the Study

2.1. Single Footing on Sand Over Clay

Several researchers have estimated the bearing capacity of a strong layer overlying a weak layer by different methods. One such method is assuming that the upper layer spreads the footing load over a wider region on the lower layer surface, thereby minimizing the stress on the lower layer [3]. The load-carrying capacity (q_u) of a surface footing is determined by Terzaghi and Peck's [3] projected area model which is expressed as shown in Equation 1.

$$q_u = q_c[1 + 2(H/B)tan\ \alpha] \leq q_s \tag{1}$$

where q_c is ultimate carrying capacity of clay, q_s is ultimate carrying capacity of the top sand layer, H is thickness of upper layer, B is width of footing, α is load spread angle.

A punching shear model was proposed by Meyerhof [4] for estimating the ultimate bearing capacity of footings on sand layers overlying clay. In the punching shear model, footing and the upper sand block punch down into the clay. For surface strip footings on layered soil, Meyerhof and Hanna's [5] ultimate bearing capacity of dense sand on soft clay is expressed as shown below in Equation 2.

$$q_u = cN_c + \Upsilon_s\ H^2 K_s \frac{\tan \Phi_{sand}}{B} \leq \frac{1}{2}\Upsilon_s\ BN_\gamma \tag{2}$$

where c is undrained cohesion of clay, N_c = 5.14, N_c and N_γ is bearing capacity factors, B is width of footing, Υ_s is unit weight of sand, K_s is punching shear coefficient, H is thickness of the top sand bed, Φ_{sand} is angle of internal friction of sand.

Shivashankar et al. [1] considered both the footing and portion of reinforced granular bed directly beneath the footing to work in tandem to punch through the soft soil underneath the granular bed. In their punching shear model, they considered the total shearing resistances along the vertical planes through the edges of the footing in the upper granular layer i.e., they considered both the shear layer effect and confinement effects. Also, an additional surcharge effect was considered to contribute to the increase in the bearing capacity of the footing.

Thus, according to Shivashankar et al. [1], the three effects which are responsible for the increased bearing capacity of a reinforced granular bed overlying soft clay are shear layer effect, confinement effect and surcharge effect. The three effects are expressed as:

$$q_u+\Delta q_R=c_uNc+\Delta q_{SL}+\Delta q_{CE}+\Delta q_{SE} \tag{3}$$

where $q_u = c_uNc$, (Δq_R, Δq_{SL}, Δq_{CE}, Δq_{SE}) is denote the improvements in bearing capacity due to reinforcement, shear layer effect, confinement effect, and additional surcharge effect, respectively.

The ultimate bearing capacity of a footing resting on reinforced granular bed overlying soft soil is given by:

$$q_u = c_u Nc + \frac{K_p \gamma_s H^2 \tan \phi_s}{B} + \frac{2T_R\ tan\phi_S}{B} + 0.84\left(\Delta q_{SL} + \Delta q_{CE}\right) \tag{4}$$

where $K_p$is coefficient of lateral passive earth pressure, T_R is reinforcement force, ϕ_s is friction angle of the granular soil.

Okamura et al. [6, 7] expanded the punching shear model of Meyerhof [4] based on experimental test results. Rethaliya and Verma [2] conducted experimental and mathematical modeling of strip, rectangular and square footings on reinforced sand layer overlying soft clay. The optimum thickness of the sand layer was found to be much higher in the unreinforced case compared to 0.8 times the width of footing in the reinforced case. Kumar and Chakraborty [8] studied the bearing capacity of a circular footing on sand overlying clay layer by lower bound limit analysis with finite element and linear optimization. A non-dimensional efficiency factor (η) is defined as the ratio of bearing capacity in the presence of sand layer, to that for a footing placed directly over clayey strata was calculated. The efficiency factors were found to increase with an increase in ϕ and $q/(\gamma_b)$ and a decrease in $c_u/(\gamma_b)$. The failure pattern indicated that the inclusion of sand layer below footing generally leads to a wider spread of the plastic zone. The dispersion angle is close to the prescribed dilation angle in analysis. Experimental studies on rectangular footings on sand overlying soft soil with or without a layer of geogrid at the interface were performed by Saha Roy and Deb [9]. An analytical model was proposed to calculate the ultimate bearing capacity for an isolated rectangular foundation resting on sand fill underlying soft clay as;

$$q_u=q_b+\Delta q_1+\Delta q_2 \tag{5}$$

where q_b is ultimate bearing capacity of clay layer (CuNc), Δq_1 is bearing capacity contribution due to the passive earth pressure developed at the side surfaces of the sand block, Δq_2 is contribution due to the bearing capacity due to the load spreading mechanism;

$$\Delta q_1 = \frac{\gamma B k^2 K_p}{m cos\varphi}\left[\frac{\gamma B k^2 K_p}{m cos\varphi}\left(\frac{\sin(\varphi-\beta_x)(m+ktan\beta_y)}{\cos\beta_x} + \frac{\sin(\varphi-\beta_y)(1+ktan\beta_x)}{\cos\beta_y}\right)\right] \tag{6}$$

$$\Delta q_2 = q_b \quad \left[\frac{2k}{m}\left(tan\beta_y + mtan\beta_x + 2ktan\beta_x tan\beta_y\right)\right] \tag{7}$$

where β_x is load spreading angle along the width direction of the footing, β_y - load spreading angle along the length direction of the footing, k is sand thickness to footing width ratio (H: B), m is footing length to width ratio (L:B).

Salimi et al. [10] used finite element limit analysis (FELA) to estimate the undrained bearing capacity of a rigid strip footing resting on the surface of a finite thickness sand layer overlying clay. The 'Ksr' coefficient can be used to indirectly account for the impact of the complex form of the failure planes in the sand.

$$q_{ult}B = \gamma H^2 K_{sr} \tan\varphi' + (N_c c_u)\ [B + 2H \tan(\pm\Theta)] + \gamma\ H^2 \tan(\pm\Theta) \leq q_t B \tag{8}$$

where q_{ult} is ultimate bearing capacity of the footing on layered soil, q_t is bearing capacity of the strip footing on uniform sand, φ' is friction angle of sand.

$$Ksr = C(c_u/\gamma H) + 2 \tag{9}$$

$$C = -3.48(\tan\varphi') + 8.693 \tag{10}$$

$$\Theta(rad) = A\ In\ (Cu/\gamma H) + B \tag{11}$$

$$A = 0.039\ In(\tan\varphi') - 0.164 \tag{12}$$

$$B = 0.594 In\ (\tan\varphi') - 0.051 \tag{13}$$

Using finite element limit analysis, Yang et al. [11] calculated the bearing capacity of ring foundations resting on a sand layer overlying clay. According to Yang et al. [11] punching shear failure occured in the sand layer for $H/R_0 <$ Hc/R_0, (H-thickness of sand layer, R_0-external radius) with log-spiral rupture lines extending from the clayey strata to the upper sand layer.

Kumar and Chakraborthy [12] computed the bearing capacity of strip and circular footings resting on two-layered clays. The strength of bottom clay layer did not affect beyond a certain top clay layer thickness (t_{opt}). The t_{opt}/b value was found to vary depending on the foundation type and c_{u1}/c_{u2} ratio (where t_{opt}-optimum top layer thickness, b-diameter/width of foundation, c_{u1} and c_{u2} are undrained cohesion values of the top and bottom clay layers respectively). Studies by Panwar and Dutta [13] found that the ultimate bearing capacity increased up to a H/W value of 1.75, and beyond this value of H/W of 1.75 the increase was only marginal. They studied rectangular footings on upper dense sand layer overlying loose sand layer.

2.2. Interference Effects of Adjacent Strip Footings

To evaluate the effect of interference, especially in terms of bearing capacity, different approaches are adopted such as experimental, numerical, and analytical. Interference effects of adjacent strip footings were first studied by Stuart [14] on homogeneous sand. Most of the interference studies on shallow footings are carried out in homogeneous soils [14-21]. Das et al. [22] conducted experimental studies with two adjacent footings on dense sand over soft clay but a limited range of affecting parameters were considered. Ultimate bearing capacity was found to increase as dense sand thickness increased until it reached critical depth (Hcr), after which it remained constant. Model experiments were used by Ghosh and Kumar [23] to investigate the impact of strip footings resting on layered cohesionless soil. It was found that the bearing capacity of neighbouring footings reaches a limit at a certain critical spacing between them. Srinivasan and Ghosh [24] carried out experimental studies on circular and rectangular footings on layered cohesionless soil. Saha Roy and Deb [25] studied interference effects on settlement and load-carrying capacity of angular footings (square and rectangular) resting on granular bed over soft clay through model tests. Their analytical solution considers the bearing strength as the sum of the bearing capacity due to passive earth pressure generated at the sides of the sand block and the load-bearing capacity due to the mechanism of load spread.

Using an upper-bound limit state plasticity method known as discontinuity layout optimization, Zheng et al. [26] calculated the ultimate bearing capacity of two interfering strip footings on sand overlying clay. The ultimate bearing capacity of two interacting strip footings on sand overlying clay is found to be affected by geometric patterns and soil characteristics. Increasing the angle of internal friction or decreasing $c_u/(\gamma B)$ was found to increase the value of critical spacing.

2.3. Justification for the Necessity of Doing this Research

Most previous studies in literature which studied interference effects of adjacent strip footings on granular bed overlying weak soil, have not provided an analytical model. This research work attempted to provide an analytical model to estimate the ultimate bearing capacity of two and three adjacent strip footings resting on granular bed overlying weak soil, with a fair and acceptable degree of accuracy. The accuracy of the proposed model is verified with finite element simulations and the percentage error is about 13%. In several situations, a granular bed (GB) is laid over weak soil as a simple ground improvement method and for other practical reasons. In this study, granular bed overlying soft clay is being considered. The parameters varied are the clear spacing between the adjacent strip footings, width of footings, thickness of the top granular bed, and number of footings.

From the insights gained from finite element simulations, a simple analytical model has been proposed to estimate the ultimate load carrying capacity and interference factor of adjacent strip footings resting on granular bed overlying weak soil. From finite element simulations, it is seen that punching shear failure of footing/s is the dominant failure mechanism for two or three adjacent footings on granular bed over clay. The earlier model proposed by Shivashankar et al. [1] for an isolated footing resting on a reinforced granular bed overlying clay has been extended for adjacent and interfering footings. A punching shear failure mechanism, similar to Shivashankar et al. [1] is envisaged in the present analytical model as well. It is assumed that rigid surface footings are resting on granular fill overlying weak soil. The surcharge effect is neglected due to the interference phenomenon in upper granular layer. The adjacent strip footings are assumed to be simultaneously loaded.

3. Numerical Analysis

3.1. Methodology

Rigid strip footings each of width B are considered to be resting on medium dense sand of finite thickness adjacent to each other, overlying soft clay extending to a large extent. The top layer thickness of medium dense sand (H) is varied as 0.75B, 1.0B, 1.5B, and 2.0B (Figure 1a). This top layer thickness represents the depth of the granular bed. Surface strip footings of widths 1 and 2 m are considered. Two and three adjacent footings are considered. The strip footings are loaded simultaneously up to failure. The clear spacing between the footings is represented as 'S' (Figure 1 b-c). The distance between the footings is represented in a normalized manner, as the spacing ratio, S/B. Spacing ratio, S/B values considered here are 1.0, 1.5, 2.0, 2.5 and 3.0.

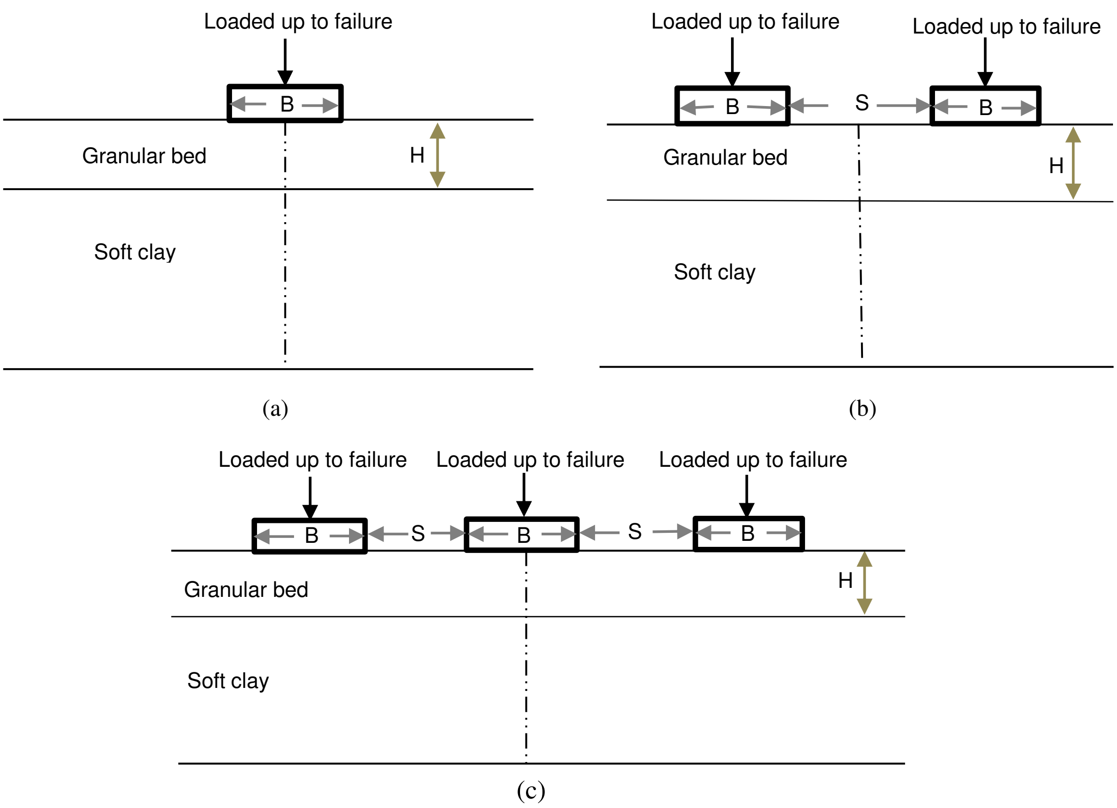

Figure 1. Footing/s on the top of the granular bed (GB) overlying weak soil layer (a-c)

Numerical analysis is conducted by using the finite element-based program PLAXIS 2D. The soils are assumed to be elastic-perfectly plastic material obeying the Mohr-Coulomb model failure criterion in conjunction with a nonassociated flow rule. The dilatancy angle is assumed to be 2/3 of the angle of friction [18]. The geotechnical properties of soils considered for the analysis are shown in Table 1. Fifteen noded triangular elements with plane strain conditions are used. The rough surface footing is simulated by using plate elements with concrete properties, modulus of elasticity, E as 25×10^6 kN/m^2, and Poisson's ratio, ν is considered as 0.15. The medium dense sand and soft clay are considered as the top granular bed (GB) and weak soil layer, respectively. The footing width B is taken to be 1 and 2m, and the soil domain is 10B away from the footing's edge on either side, and 10B in depth to reduce the possible boundary effects. The bottom horizontal boundary is fixed in both the vertical and horizontal directions, and the side boundary is restricted only along the horizontal direction. Generating finer mesh led to a satisfying result in the numerical analyses when compared with the experimental result of Das et al. [22] in the verification study. The methodology adopted in this study is shown in flowchart (Figure 2).

Table 1. Properties of soils considered

Properties	Soft clay	Medium dense sand
Material type	Undrained	Drained
Unit weight, γ (kN/m^3)	16	18.20
Young's modulus[a], E (kN/m^2)	6000	30000
Poisson's ratio[a], ν	0.35	0.28
Cohesion (c) (kN/m^2)	20*	0
Angle of internal friction[b], ϕ (degrees)	0	30°

[a] After Bowles [27]; [b] Referenced from [28]; * Undrained shear strength, Su (kN/m^2)

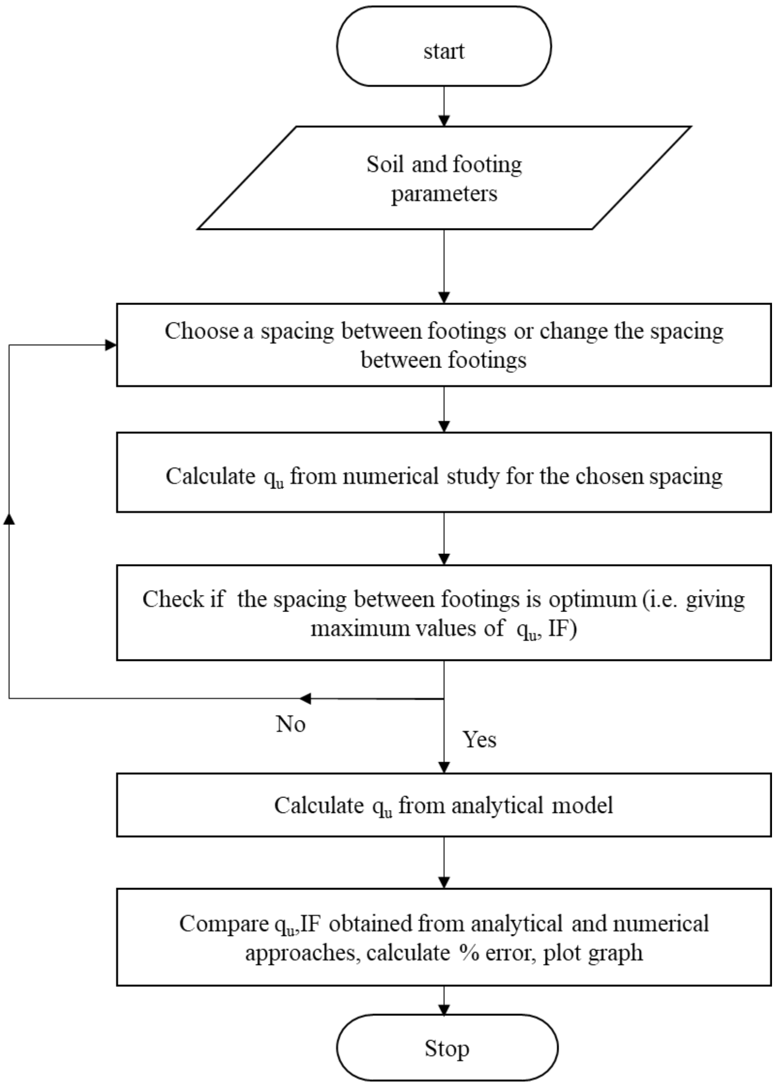

Figure 2. Flowchart of the research methodology

4. Results and Discussion

4.1. Single Strip Footing on GB Overlying Clay

4.1.1. Validation of the Numerical Model

A single strip footing is placed on a granular bed overlying weak soil. The width of footing and thickness of the granular bed are varied. Figure 3 shows a comparison between bearing capacities obtained by numerical analyses from this study and theoretical bearing capacities as obtained by the punching shear approach suggested by Meyerhof and Hanna [5]. The value of bearing capacity initially increases with an increase in H/B ratio up to a maximum and then remains constant [6, 22]. Thus, when H/B < Hopt/B, the failure surface goes beyond the upper sand layer into the clay layer beneath. However, when H/B≥ Hopt/B, the failure surface at ultimate load is entirely located in the top sand layer [22, 29, 30]. The bearing capacity values obtained by numerical analysis and theoretical approach, in the case of a single strip footing on granular bed overlying clay, are in good agreement.

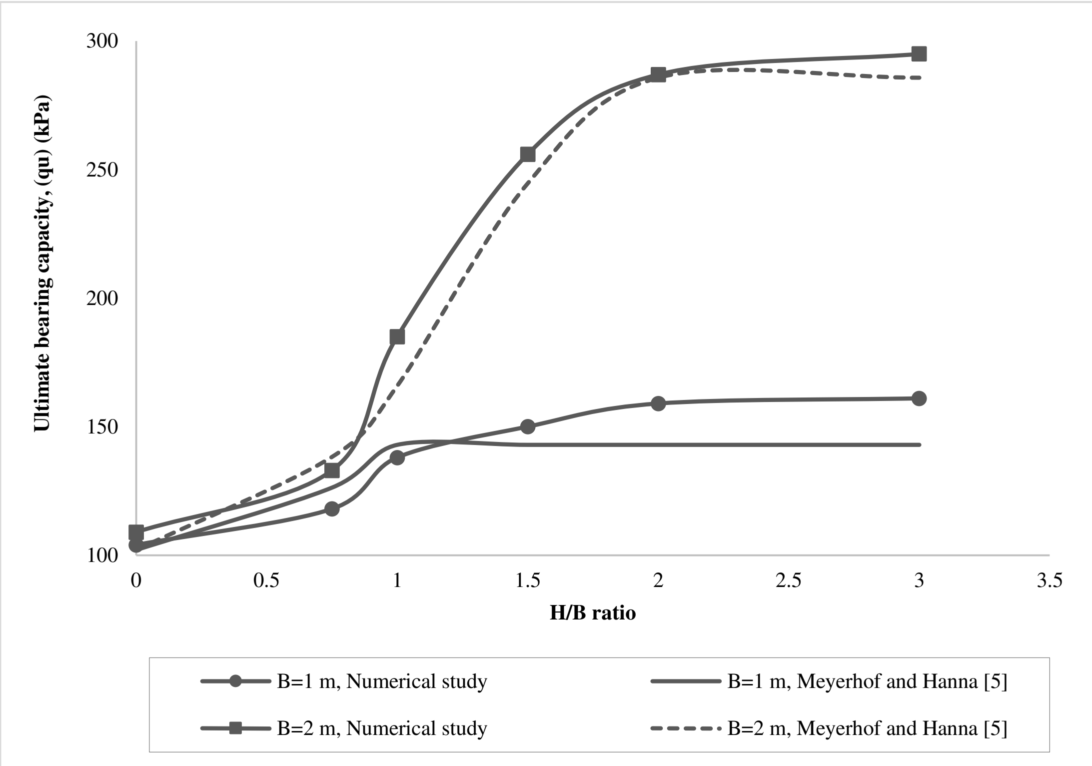

Figure 3. Variation of the bearing capacity with the thickness of granular bed expressed as H/B ratio, for single strip footing on GB over clay

4.2. Two Adjacent Strip Footings on GB Overlying Clay

To study the interference effect of two adjacent strip footings on granular bed overlying weak soil/clay, analyses were performed for different H/B ratios of 0.75, 1.0 and 1.5. The spacing ratio, S/B values between the footings are varied as 1.0, 1.5, 2.0, 2.5, and 3.0. The effect of interference of two adjacently spaced strip footings on the bearing capacity of the soil is expressed in terms of the interference factor of bearing capacity (IF). Ultimate bearing capacities are taken as peak values. Whenever the curves had not peaked ultimate bearing capacities are obtained by the tangential intersection of load settlement curves. The interference factor of load carrying capacity (IF) is defined as follows in Equation 14.

$$\mathrm{IF} = \frac{\text{Load carrying capacity of the footing in question in the presence of an adjacent footing on GB overlying weak soil}}{\text{Load carrying capacity of single independent strip footing on GB overlying weak soil}} \tag{14}$$

The interference factor initially increases with the increase in spacing up to a maximum value and then decreases with the further increase in spacing [23, 25, 29, 30] (Figure 4). The spacing at which the highest interference factor for bearing capacity is observed is considered as the optimum spacing between the footings. In this case, 1.5B is obtained as optimum spacing.

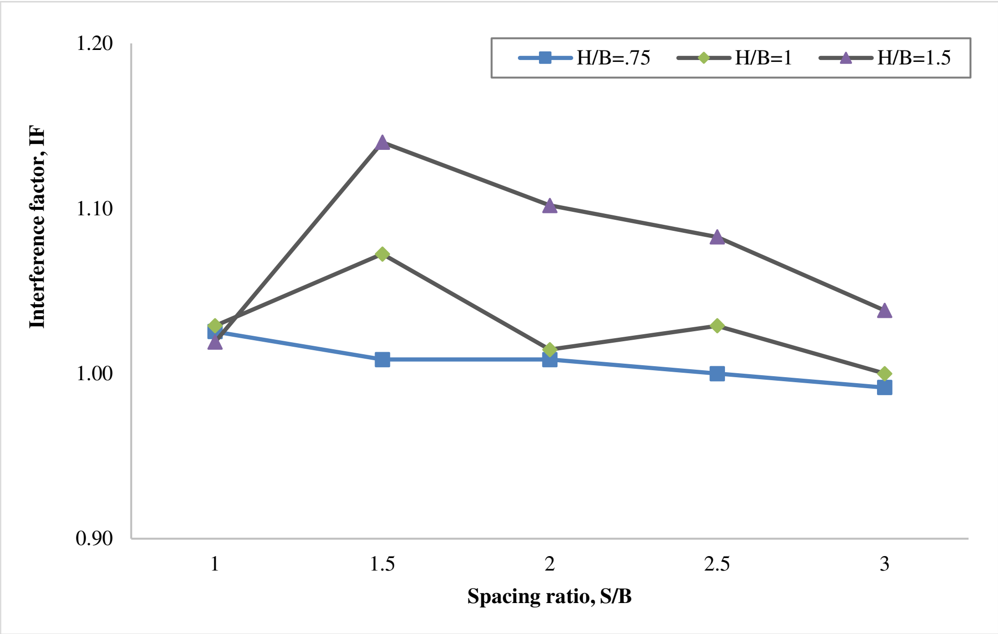

Figure 4. Variation of interference factor of bearing capacity (IF) with spacing ratio, S/B, for footing width of 1 m, for different thickness of granular bed (H/B) overlying weak soil

4.3. 'Punching Shear Analytical Model': Analytical Model for A Single Strip Footing on Granular Bed Overlying Clay

In this study, the analytical model proposed is based on the model developed and used by earlier researchers [1, 2] based on the punching shear failure mechanism, which hereinafter will be referred to as 'punching shear analytical model'. Both the footing and the portion of the granular bed (GB) directly beneath the footing are envisaged to act in unison to punch through the soft soil underneath. The load-carrying capacity of the footing is taken as the sum of total shearing resistances along the two vertical planes through the edges of the strip footing in the upper granular layer and the load-carrying capacity of the soft clay beneath the GB (Figure 5).

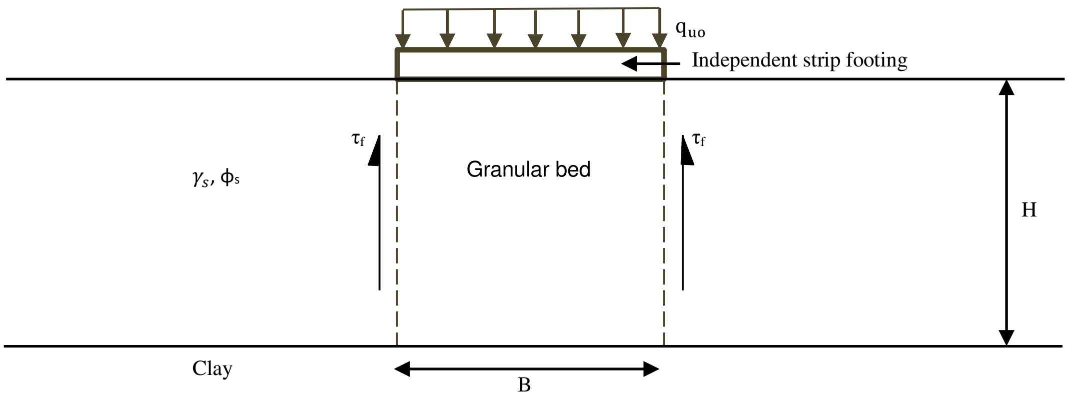

Figure 5. 'Punching shear analytical model' for a single strip footing on granular bed overlying clay

Therefore, the improvement in bearing capacity of a strip footing on a granular bed (GB) overlying clay is attributed solely to the shear layer effect in the upper granular layer. The shear layer effect considered is similar to the one considered by Shivashankar et al. [1] "while studying the bearing capacity of footings on reinforced granular bed (RGB) overlying soft clay". It can be mathematically represented as shown below in Equation 15.

$$q_{uo} = c_u N c + \Delta q_{SL} \quad (15)$$

where $q_{uc} = c_u Nc$ is bearing capacity of clay ground, Δq_{SL} is improvement in bearing capacity due to shear layer effect, q_{uo} is Bearing capacity of an independent strip footing on the composite/layered ground with no interference.

The improvement in bearing capacity is quantified in terms of the Bearing Capacity Ratio (BCR). BCR is defined as the ratio of the bearing capacity of the improved ground (q_{uo}) to the bearing capacity of the unimproved clay ground (q_{uc}). BCR can be expressed as:

$$BCR = \frac{q_{uo}}{q_{uc}} = 1 + \Delta BCR_{SL} \quad (16)$$

where ΔBCR_{SL} is improvement in bearing capacity ratio due to the shear layer effect $\Delta BCR_{SL} = \frac{\Delta q_{SL}}{q_{uc}}$.

Previous experimental and numerical studies by Das et al. [22], Anaswara and Shivashankar [30, 31] have proved that if the thickness of the upper granular bed is more than a critical thickness, then the entire failure surface beneath the footing will be within the granular layer. If the thickness of the upper granular bed is less than the critical thickness, then only the failure surface will reach up to the lower weaker clay layer, and punching shear failure is likely to occur.

$$Q_{u0} = q_{u0}B = cNc\, B + (2\,\tau_f)H \le q_sB \quad (17)$$

where q_s is bearing capacity of footing on the sand layer.

In the shear layer effect [1], the shearing resistances mobilized along the vertical planes at the two edges of the strip footing due to the passive pressure developed in granular soil are considered. The equations given for strip footings are;

$$\tau_f = \frac{k_p \gamma_{s\,H^2}}{2} tan\phi_s \quad (18)$$

$$\Delta q_{SL} = \frac{2\tau_f}{B} \quad (19)$$

$$\Delta BCR_{SL} = \frac{2\tau_f}{BN_C C_u} \quad (20)$$

where $N_C C_u$ is bearing capacity of underlying weak soil (q_{uc}), τ_f is punching shear resistance along a vertical plane due to Shear Layer Effect, k_p is Coefficient of passive earth pressure, ϕ_s is angle of internal friction of the granular material.

4.3.1. Validation of the 'Punching Shear Analytical Model' for Single Independent Strip Footing

In the present study, the parameters considered are as follows: γ_{sand}=18.2 kN/m^3, ϕ_s= 30^0, $k_P = \frac{1+\sin\phi_s}{1-\sin\phi s}$=3, B=1m, H=1 m , c=20 kN/m^2 , $N_c = 5.14$

$$\tau_f = \frac{3\,X18.2X1X1X0.5773}{2} = 15.76 \text{ kN} \quad (21)$$

$$q_{uc} = 20\times5.14 = 102.8 \text{ kN/m}^2 \quad (22)$$

Substituting 21 and 22 in 17,

$$Q_{u0}/B = q_{u0} = 102.8 + (2\times15.76) = 134.32 \text{ kN/m}^2 \quad (23)$$

The corresponding bearing capacity of a single strip footing on GB overlying clay obtained from finite element analysis is 137 kN/m^2.

To verify the veracity of the analytical method, one of the most relevant case studies has been numerically simulated and the results obtained are compared. Das et al. [22] conducted some experiments to study the load-carrying capacity of a strip footing on dense sand overlying soft clay. The experiments were conducted in a box measuring 1.22 m length × 0.305 m width × 0.915 m height. A top layer of dense sand with a unit weight of 17.29 kN/m^3 and friction angle of 39.8^0; and lower soft clay with undrained shear strength as 5.51 kPa were used. The width of the model strip footing used was 101.6mm. The thickness of dense sand was varied.

Figure 6 shows the comparison of the ultimate bearing capacities of single strip footings on the granular bed, of varying thicknesses, i.e. (H/B) values "H/B varying from 1 to 5", overlying weak soil obtained by experimental studies of Das et al. [22] with the 'punching shear analytical model' and results of numerical studies from this research study. The bearing capacities obtained by the 'punching shear analytical model' are in good agreement with the results of the numerical analysis of the present study and the experimental results of Das et al. [22].

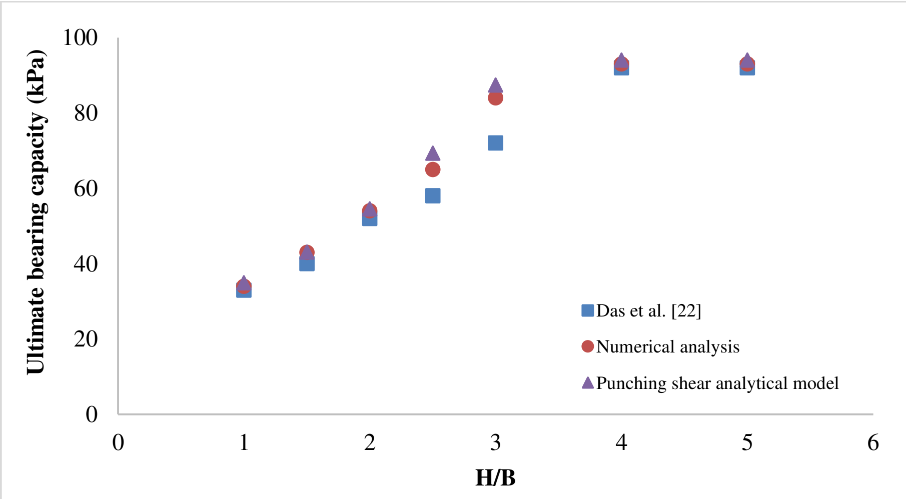

Figure 6. Comparison of results of the 'punching shear analytical model' for single independent strip footing (for varying H/B values) with the experimental results of Das et al. [22] and numerical analysis from the present study

4.3.1.1. Comparison between Results of 'Punching Shear Analytical Model' with those of Experimental and Numerical Studies

The values of bearing capacities of single strip footings predicted by the 'punching shear analytical model' on GB overlying clay is compared with the results obtained from finite element analyses and some experimental results available in the literature. Table 2 shows the predicted and numerical ultimate bearing capacity values for single strip footing on granular bed overlying weak soil. Figure 7 shows a comparison between predicted values of the ultimate bearing capacities with results of experimental and numerical studies. Figure 8 shows a comparison between predicted values with experimental and numerical studies of bearing capacity ratios (BCR). It can be observed that the 'punching shear analytical model' predicts the values of bearing capacities and bearing capacity ratios reasonably well.

Table 2. Predicted and numerical ultimate bearing capacity values for single strip footing on granular bed overlying weak soil

B (m)	H/B	H (m)	q_c (kPa) (1)	τ_f	Δq_{SL} (2)	Punching shear analytical model q_{u0} (kPa) (1) + (2)	Numerical analysis q_u (kPa)	Percentage error
1	0.75	0.75	102.8	8.87	17.73	120.53	118	2.14
1	1	1	102.8	15.76	31.52	134.32	137	-1.96
1	1.5	1.5	102.8	35.46	70.92	173.72	157	10.65
2	0.75	1.5	102.8	35.46	35.46	138.26	145	-4.65
2	1	2	102.8	63.04	63.04	165.84	169	-1.87
2	1.5	3	102.8	141.84	141.84	244.64	218	12.22

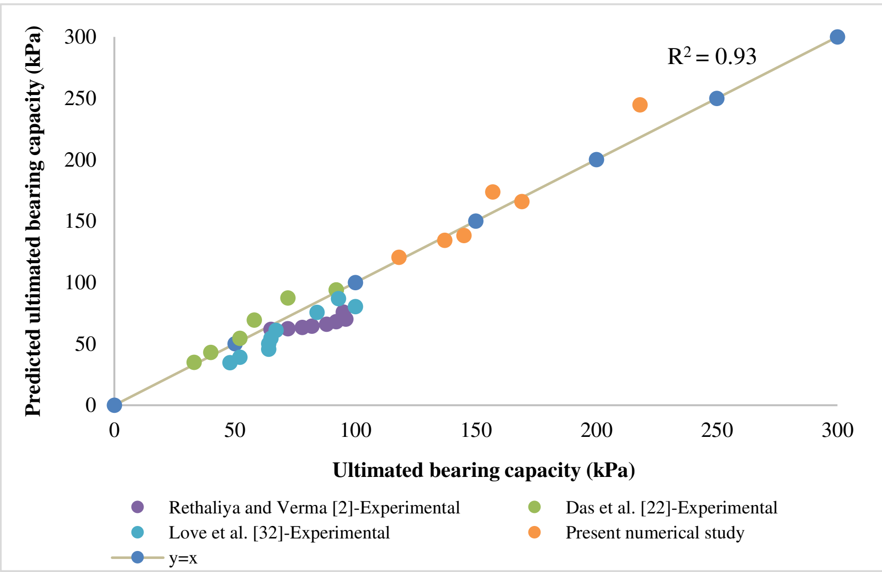

Figure 7. Comparison between predicted values of ultimate bearing capacity from 'punching shear analytical model' for single strip footing with those of experimental and numerical studies

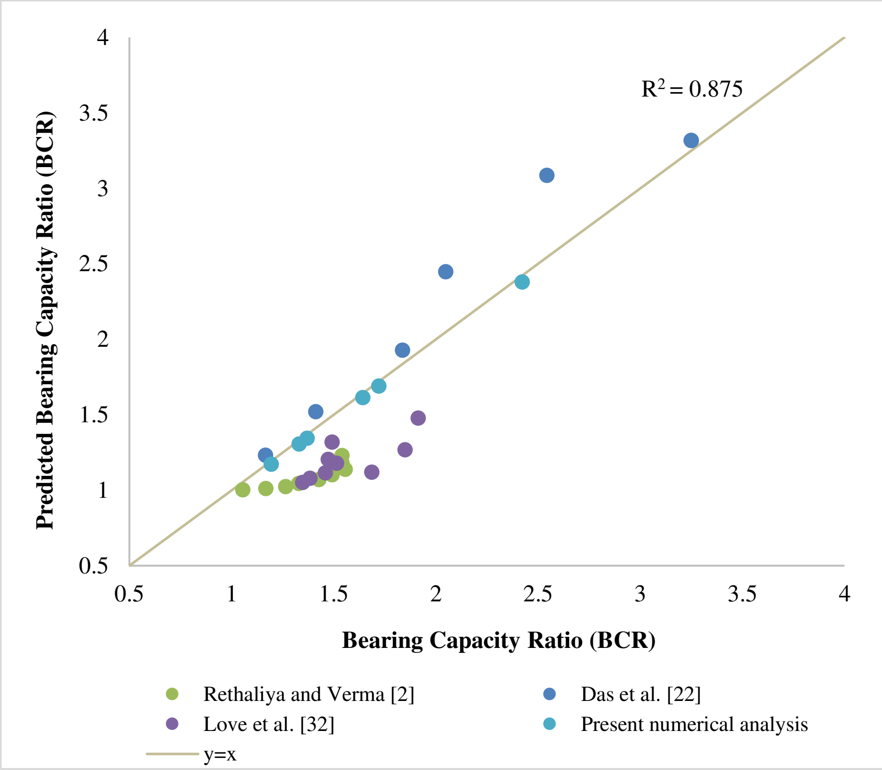

Figure 8. Comparison between predicted values of bearing capacity ratio (BCR) from 'punching shear analytical model' for single strip footing with those of experimental and numerical studies

4.4. Analytical Model to Predict the Load-Carrying Capacity of The Interfered Footing and Interference Factor, In Case of Two Adjacent Strip Footings on Granular Bed (GB) Overlying Clay (Simultaneously Loaded)

When two footings are placed adjacent to each other on GB, or GB overlying clay, there will be an 'interference effect'. In the case of two adjacent strip footings on GB overlying clay and simultaneously loaded (Figure 9), an analytical model is proposed to predict the ultimate bearing capacity of the interfered footing and the interference factor. The proposed model is again based on the philosophy of the punching shear mechanism. It is an extension/modification of the punching shear model for a single strip footing on granular bed overlying clay, as explained earlier in Section 4.3. This proposed model will hereinafter be referred to as 'proposed analytical model for interfered footing'. The proposed model is applicable at the optimum spacing between the adjacent footings (i.e., S/B=1.5, Figure 4). The granular bed thickness H/B is to be equal to or less than optimum thickness for punching action of footing along with sand block in to the clay layer to occur.

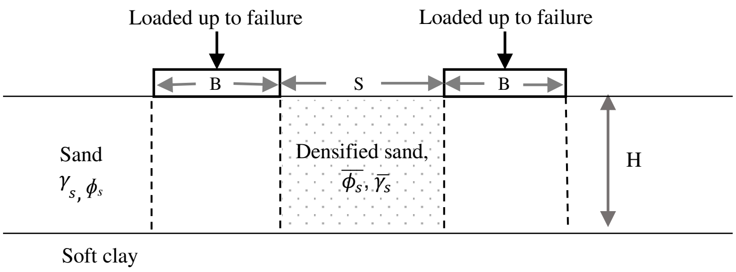

Figure 9. Adjacent strip footings on granular bed overlying weak soil

Both the footings are interfered footings in the case of two adjacent surface strip footings. Similar Equations as 18 and 19 are adopted for the shear layer effect, but the passive lateral pressure coefficients (kp) are not taken the same on the two vertical shearing surfaces on either side of the strip footing. The lateral passive pressure and the shearing resistance in the interfered zone, i.e., the zone of granular material between the two footings, will be more due to the lateral compression of the granular soil due to the lateral confinement stresses developed due to the vertical loads on the two footings. Maximum lateral compression of the granular soil due to interference effect is seen (from numerical studies) to occur when the footings are optimally spaced (maximum interference factor from numerical analysis) (Figure 10).

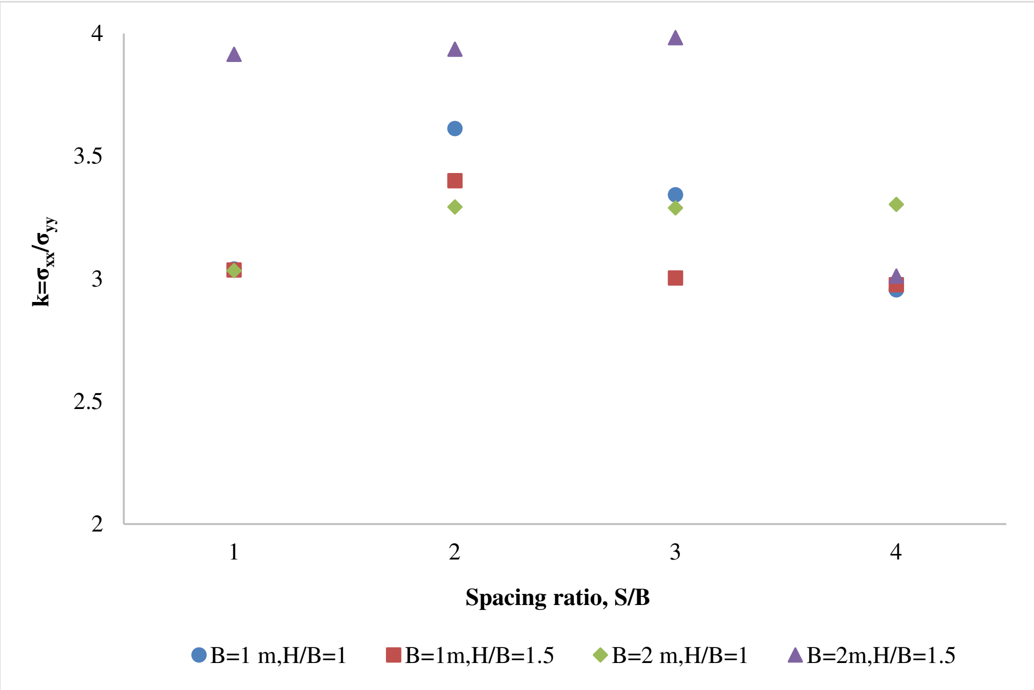

Figure 10. Variation of lateral earth pressure coefficient in the interfered zone, with spacing ratio, S/B ('S' is spacing between the footings) [from numerical studies]

4.4.1. Interference Factor

To quantify the effect of interference of two adjacently spaced strip footings on the bearing capacity of soil; the interference factor of load carrying capacity (IF) is defined as follows in Equation 24 below.

$$\text{IF} = \frac{\text{Load carrying capacity of the footing in question in the presence of an adjacent footing on GB overlying weak soil}}{\text{Load carrying capacity of single independent strip footing on GB overlying weak soil}} = \frac{q_{ui}}{q_{uo}} \tag{24}$$

The load-carrying capacity of interfered footing (q_{ui}) and that of single independent strip footing (q_{uo}) on GB overlying weak soil (no interference) are calculated by the 'proposed analytical model for interfered footing' and 'punching shear analytical model' respectively. It is observed from experimental and numerical studies that in the case of two strip footings on GB or GB overlying clay, the interference factor increases at first as the spacing between the footings is increased, and thereafter, the IF of bearing capacity value decreases beyond the optimum spacing [30]. The maximum bearing capacity value is noted at the optimum spacing. At the optimum spacing between the adjacent footing, maximum confinement pressure (coefficient of lateral earth pressure 'k') is observed (Figure 10).

4.4.2. Validation of the 'Proposed Analytical Model for Interfered Footing'

The densified granular soil mass between the two adjacent footings is assumed to be densified to the maximum with increased density $\overline{\gamma_s}$ and increased friction angle $\overline{\phi_s}$ (Figure 9). Maximum value of $\overline{\gamma_s}$ as determined from laboratory experiments [21] is 20kN/m^3. Maximum value of $\overline{\phi_s}$ is got from the analogy drawn from compaction of sand (and increase of angle of internal friction of sand) below pile tip in case of driven piles (while determining the load-carrying capacity of a pile in bearing in granular material). According to Kishida [31], the maximum friction angle beneath the pile will be;

$$\overline{\phi_s} = \frac{\phi_s + 40}{2} \tag{25}$$

Substituting ϕs=30 (angle of internal friction of medium dense sand considered in the present study) in Equation 25, we get $\overline{\phi_s} = \frac{30+40}{2} = 35^0$ as the increased angle of internal friction of sand due to compression in the interference zone.

Results of numerical analysis in the present study also gave maximum k value of soil between footings around 3.7 which corresponds to about $\overline{\phi_s} = 35^0$ (Figure 10). With the modified density and friction angle, mobilized shear resistance developed at the compacted soil side is estimated as τ'_f (Figure 11).

$$\text{Qui} = q_{ui}B = q_{uc}B + (\tau'_f + \tau_f)H \leq q_s B \tag{26}$$

$$\tau_f' = \frac{\overline{k_p}\,\overline{\gamma_s}\,H^2}{2} \tan \overline{\phi_s} \tag{27}$$

$$\overline{k_p} = \frac{1+\sin\overline{\phi_s}}{1-\sin\overline{\phi_s}} \tag{28}$$

Where, $\overline{\gamma_s} = 20\text{kN/m}^3$, $\overline{\phi_s} = 35^0$, $\overline{k_p} = 3.69$, B=1m, H=1m, $\tau_f = 15.76\text{kN/m}^2$, c=20kN/m^2, $\tau_f' = 25.83\ \text{kN/m}^2$

Qui/B= q_{ui} =20×5.14+(25.83+ 15.76)=144.39 kN/m^2

The corresponding bearing capacity of interfered strip footing on GB overlying clay obtained from finite element analysis is 147 kN/m^2.

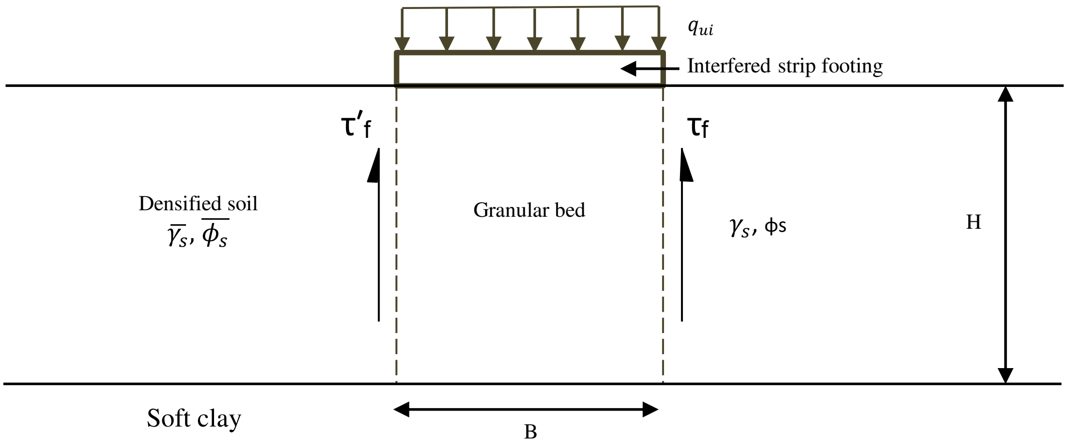

Figure 11. Proposed analytical model for interfered footing for adjacent strip footings on granular bed overlying weak soil

4.4.2.1. Comparison between Results of 'Proposed Analytical Model for Interfered Footing' and Numerical Analysis

The values of bearing capacity of interfered strip footing predicted by the 'proposed analytical model for interfered footing' are compared with those obtained from finite element analyses (Table 3). Figure 12 shows a comparison between the values of bearing capacity predicted by the proposed analytical model and finite element analyses for interfered footing on GB overlying weak soil.

The comparison between the values of interference factor, IF predicted by the 'proposed analytical model for interfered footing', and finite element analyses for strip footing on GB overlying weak soil are presented in Table 4 and Figure 13. The ultimate bearing capacity values estimated by the proposed analytical model for interfered footing are in good agreement with the values obtained from numerical analysis, with a maximum variation of 12%. Even the Interference factor (IF) values from both analytical and numerical approaches show a maximum variation of 12.7%. The average variation in bearing capacity prediction is about 6% and average variation in prediction of IF values are 5 to 6%. The coefficients of determination, R^2, are respectively 0.959 for bearing capacity (Figure 12) and 0.904 for interference factor (Figure 13) which are reasonably good.

Table 3. Predicted and numerical ultimate bearing capacity values for interfered strip footing on granular bed overlying weak soil

B (m)	H/B	H (m)	q_c (kPa) (1)	τ_f	τ'_f	Δq_{SL} (2)	Proposed analytical model q_{ui} (kPa)(1)+ (2)	Numerical analysis q_{ui} (kPa)	Percentage error
1	0.75	0.75	102.8	8.87	14.53	23.40	126.20	119	6.05
1	1	1	102.8	15.76	25.84	41.60	144.40	147	-1.77
1	1.5	1.5	102.8	35.46	58.13	93.59	196.39	185	6.16
2	0.75	1.5	102.8	35.46	58.13	46.80	149.60	170	-12.00
2	1	2	102.8	63.04	103.35	83.20	186.00	201	-7.47
2	1.5	3	102.8	141.84	232.54	187.19	289.99	296	-2.03

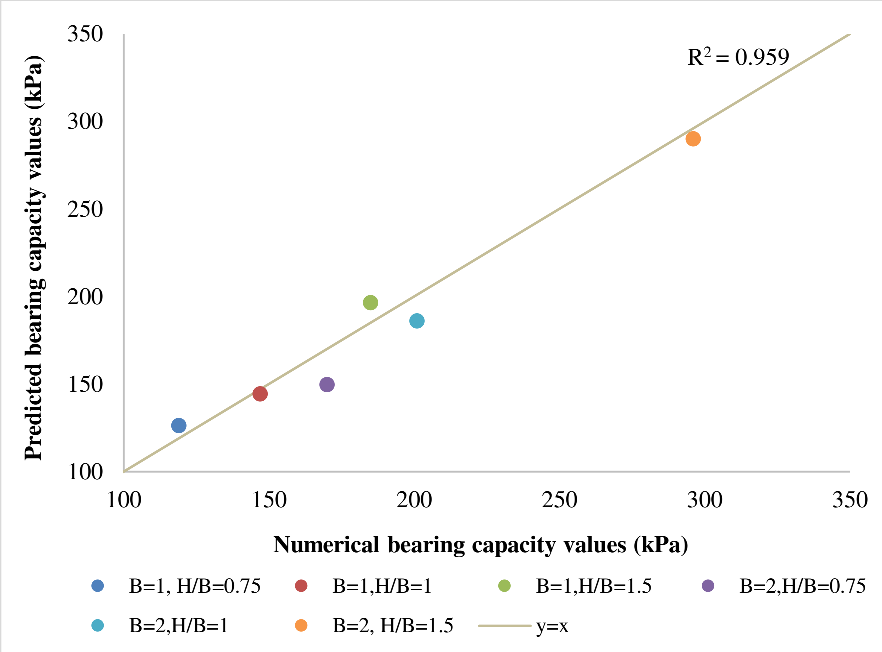

Figure 12. Comparison between predicted values of ultimate bearing capacity from "proposed analytical model for interfered footing' and *numerical* studies for interfered footing

Table 4. Predicted and numerical interference values for interfered strip footing on granular bed overlying weak soil

Specification		Proposed analytical model, q_u (kPa)		Numerical analysis, q_u (kPa)		Interference factor, IF		Percentage error
B (m)	H (m)	Single footing, q_{u0}	Interfered Footing, q_{ui}	Single footing, q_{u0}	Interfered Footing, q_{ui}	Proposed model	Numerical analysis	
1	0.75	120.53	126.20	118	119	1.05	1.01	3.82
1	1	134.32	144.40	137	147	1.08	1.07	0.19
1	1.5	173.72	196.39	157	185	1.13	1.18	-4.06
2	1.5	138.26	149.60	145	170	1.08	1.17	-7.71
2	2	165.84	186.00	169	201	1.12	1.19	-5.70
2	3	244.64	289.99	218	296	1.19	1.36	-12.70

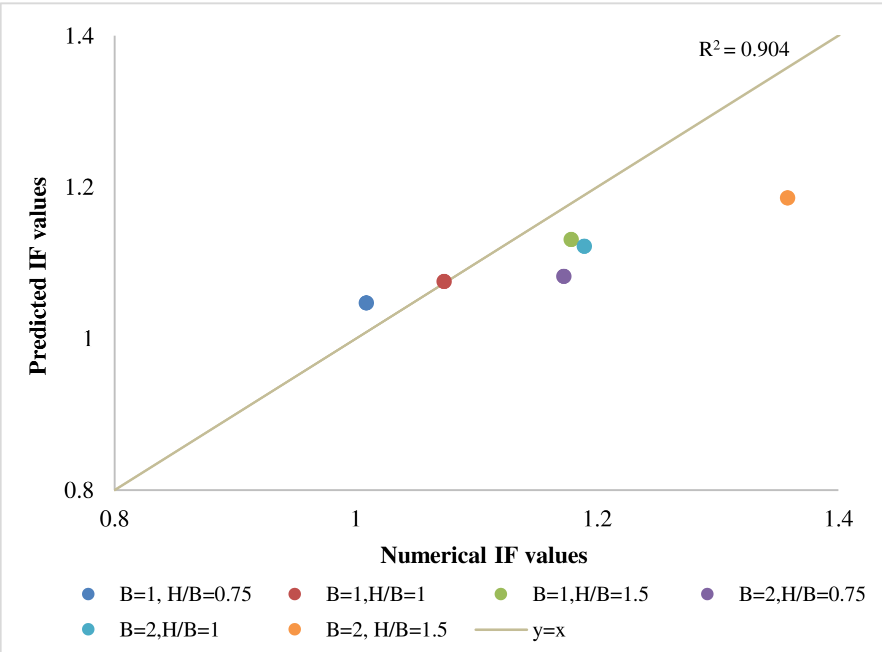

Figure 13. Comparison between predicted values of interference factor (IF) using 'proposed analytical model for interfered footing' and numerical analysis for interfered footing

4.5. Analytical Model of Three Adjacent Strip Footings on the Granular Bed (GB) Overlying Clay (Simultaneously Loaded)

4.5.1. Analytical Model to Predict the Load-carrying Capacity of the Middle-interfered Footing and Interference Factor

In the case of three adjacent strip footings on GB overlying clay and simultaneously loaded (Figure 14), an analytical model is proposed to predict the ultimate bearing capacity of the middle-interfered footing at optimum spacing (S/B=1.5). The footing at the center is under the interference effect from both the footings on either side. The bearing capacity behaviour of middle footing is being studied. It is a further extension/modification of the punching shear model for a two-strip footing on granular bed overlying clay, as explained earlier section.

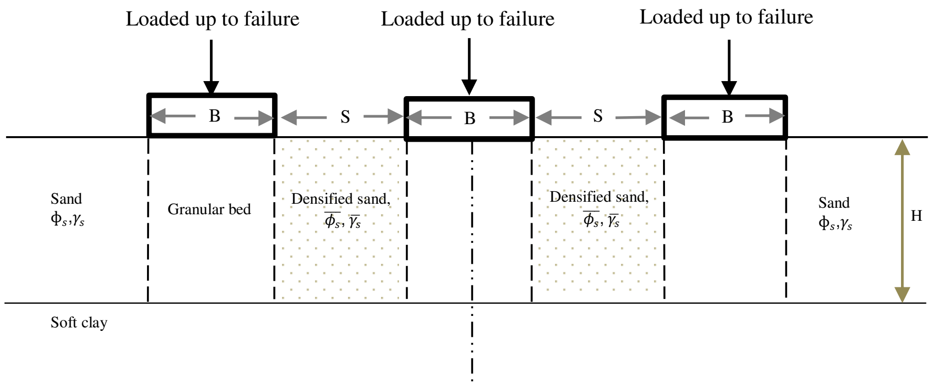

Figure 14. Adjacent three-strip footings on granular bed overlying weak soil

The granular soil mass that is present on either side of the middle footing are assumed to be densified to the maximum with increased density $\overline{\gamma_s}$ and increased friction angle $\overline{\phi_s}$ (Figure 15). With this modified density and friction angle, mobilized shear resistances, τ'_f developed on the two vertical planes, on either side of the strip footing, are of the same magnitude and are estimated similar to Equation 27. Thus, the bearing capacity of middle interfered footing in case of three adjacent strip footings on granular bed overlying weak soil is calculated by Equation 29.

$$Qui = q_{ui}B = q_{uc}B + (2\,\tau'_f)\,H \leq q_s B \tag{29}$$

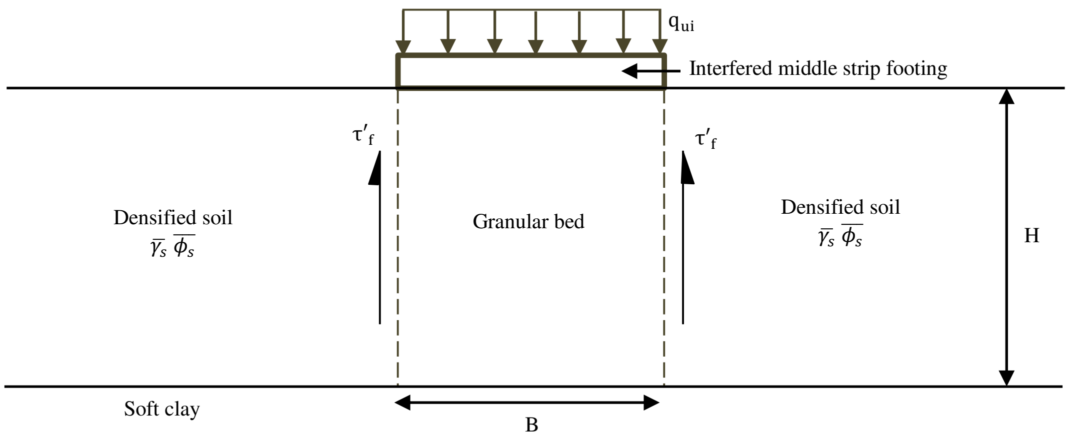

Figure 15. Proposed analytical model for the middle interfered footing for three adjacent strip footings on granular bed overlying weak soil

The comparison between the values of bearing capacity and interference factor predicted by the proposed analytical model, and finite element analyses for middle interfered strip footing on GB overlying weak soil are presented in Tables 5 and 6 and Figures 16 and 17. The ultimate bearing capacity values estimated by the analytical model are in very good agreement with results of numerical analysis (maximum error of 13.29% and average error of about 5.3%). Even the IF values agree reasonably well.

Table 5. Predicted and numerical ultimate bearing capacity values for the middle-interfered strip footings on granular bed overlying weak soil (Three footings case)

B (m)	H/B	H (m)	q_c (1) (kPa)	τ'_f	Δq_{SL} (2)	Proposed analytical model q_{ui} (kPa) (1) + (2)	Numerical analysis q_{ui} (kPa)	Percentage error
1	0.75	0.75	102.8	14.53	29.07	131.87	130	1.44
1	1	1	102.8	25.84	51.67	154.47	148	4.37
1	1.5	1.5	102.8	58.13	116.27	203.84	197	3.47
2	0.75	1.5	102.8	58.13	58.13	160.93	160	0.49
2	1	2	102.8	103.35	103.35	206.15	190	8.50
2	1.5	3	102.8	232.54	232.54	335.34	296	13.29

Table 6. Predicted and numerical interference values for middle interfered strip footing on granular bed overlying weak soil

Specification		Proposed analytical model, q_u (kPa)		Numerical analysis, q_u (kPa)		Interference factor, IF		Percentage error
B (m)	H (m)	Single footing q_{u0}	Interfered Footing, q_{ui}	Single footing, q_{u0}	Interfered Footing, q_{ui}	Proposed model	Numerical analysis	
1	0.75	120.53	131.87	118	130	1.09	1.10	-0.69
1	1	134.32	154.47	137	148	1.15	1.08	6.46
1	1.5	173.72	203.84	157	197	1.17	1.25	-6.49
2	1.5	138.26	160.93	145	160	1.16	1.10	5.39
2	2	165.84	206.15	169	190	1.24	1.12	10.57
2	3	244.64	335.34	218	296	1.37	1.36	0.95

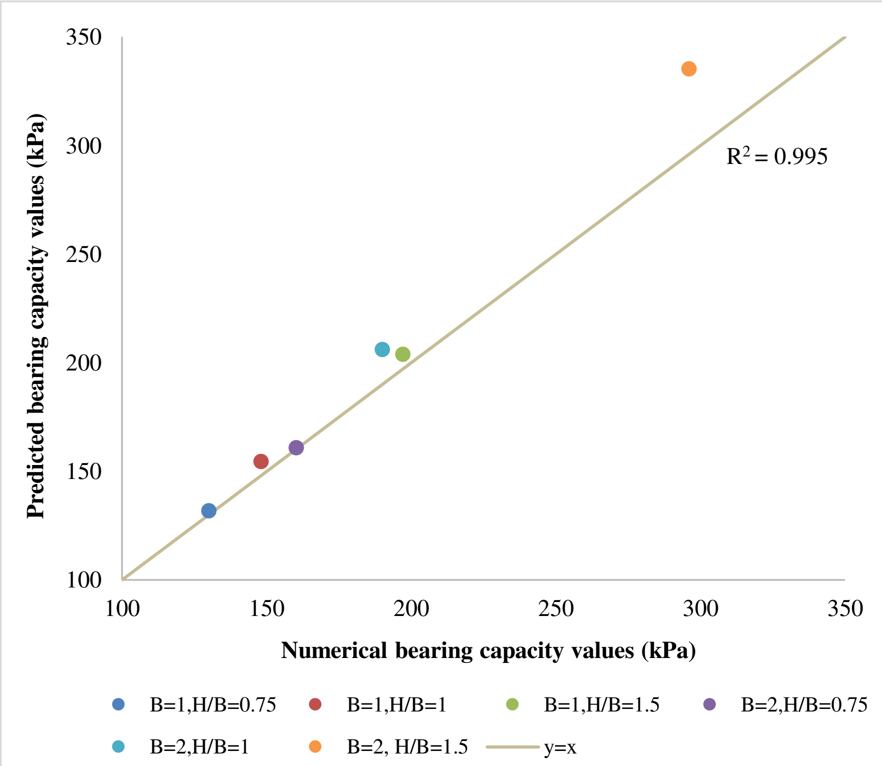

Figure 16. Comparison between predicted values of ultimate bearing capacity from 'proposed analytical model for middle interfered footing with numerical studies

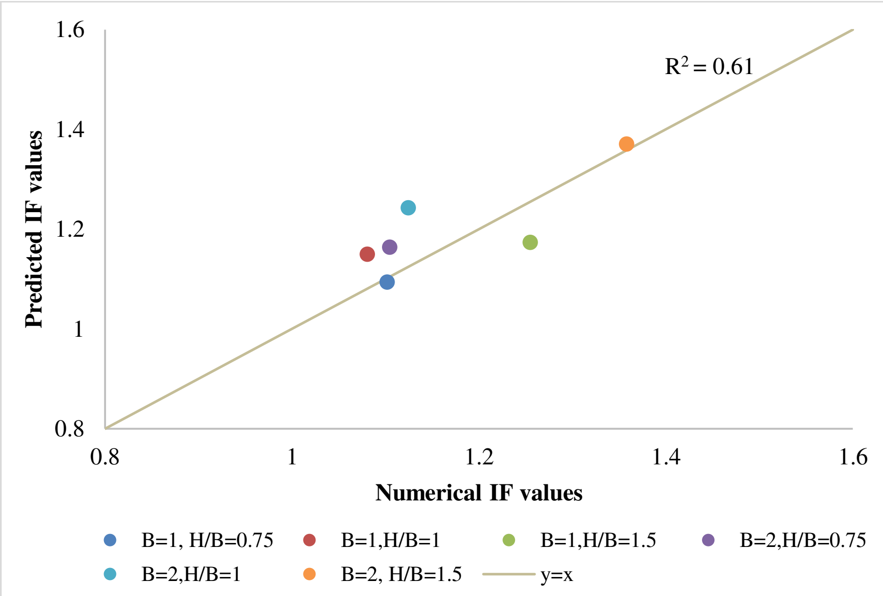

Figure 17. Comparison between predicted values of interference factor (IF) using a proposed analytical model and numerical analysis for middle interfered footing

4.5.2. Analytical Model to Predict the Load-carrying Capacity of the Outer Interfered Footing and Interference Factor

Footings that are located to the left and right of the middle strip footing are considered as outer interfered footings (Figures 18 and 19). The bearing capacity of these footings can be calculated by the 'proposed analytical model for interfered footing' (similar to two adjacent strip footings). Predicted and numerically evaluated ultimate bearing capacity values and interference factor values for the left and right-side interfered strip footings on granular bed overlying weak soil for the three footings case, are shown in Tables 7 and 8 and Figures 20 and 21. There is good agreement between the two sets of values.

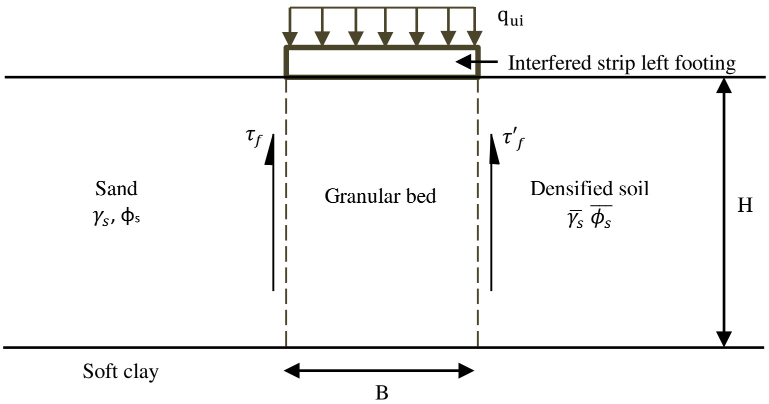

Figure 18. Proposed analytical model for the outer left side interfered footing when there are three adjacent strip footings on granular bed overlying weak soil

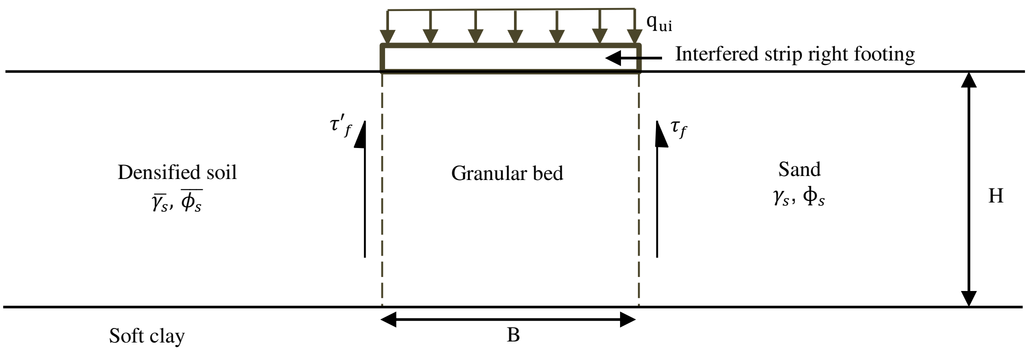

Figure 19. Proposed analytical model for the outer right side interfered footing when there are three adjacent strip footings on granular bed overlying weak soil

Table 7. Predicted and numerical ultimate bearing capacity values for the outer interfered strip footings on granular bed overlying weak soil (Three footings case)

B (m)	H/B	H (m)	q_c (1) (kPa)	τ_f	τ'_f	Δq_{SL} (2)	Proposed analytical model q_{ui} (kPa) (1)+(2)	Numerical analysis q_{ui} (kPa)	Percentage error
1	0.75	0.75	102.8	8.87	14.53	23.40	126.20	128	-1.41
1	1	1	102.8	15.76	25.84	41.60	144.40	147	-1.77
1	1.5	1.5	102.8	35.46	58.13	93.59	196.39	198	-0.81
2	0.75	1.5	102.8	35.46	58.13	46.80	149.60	161	-6.81
2	1	2	102.8	63.04	103.35	83.20	186.00	180	3.48
2	1.5	3	102.8	141.84	232.54	187.19	289.99	298	-2.69

Table 8. Predicted and numerical interference values for outer interfered strip footings on granular bed overlying weak soil (Three footings case)

Specification		Proposed analytical model, q_u (kPa)		Numerical analysis, q_u (kPa)		Interference factor, IF		Percentage error
B (m)	H (m)	Single footing q_{u0}	Interfered Footing, q_{ui}	Single footing, q_{u0}	Interfered Footing, q_{ui}	Proposed model	Numerical analysis	
1	0.75	120.53	126.20	118	128	1.05	1.08	-3.48
1	1	134.32	144.40	137	147	1.08	1.07	0.19
1	1.5	173.72	196.39	157	198	1.13	1.26	-10.36
2	1.5	138.26	149.60	145	161	1.08	1.11	-2.27
2	2	165.84	186.00	169	180	1.12	1.06	5.45
2	3	244.64	289.99	218	296	1.19	1.36	-12.70

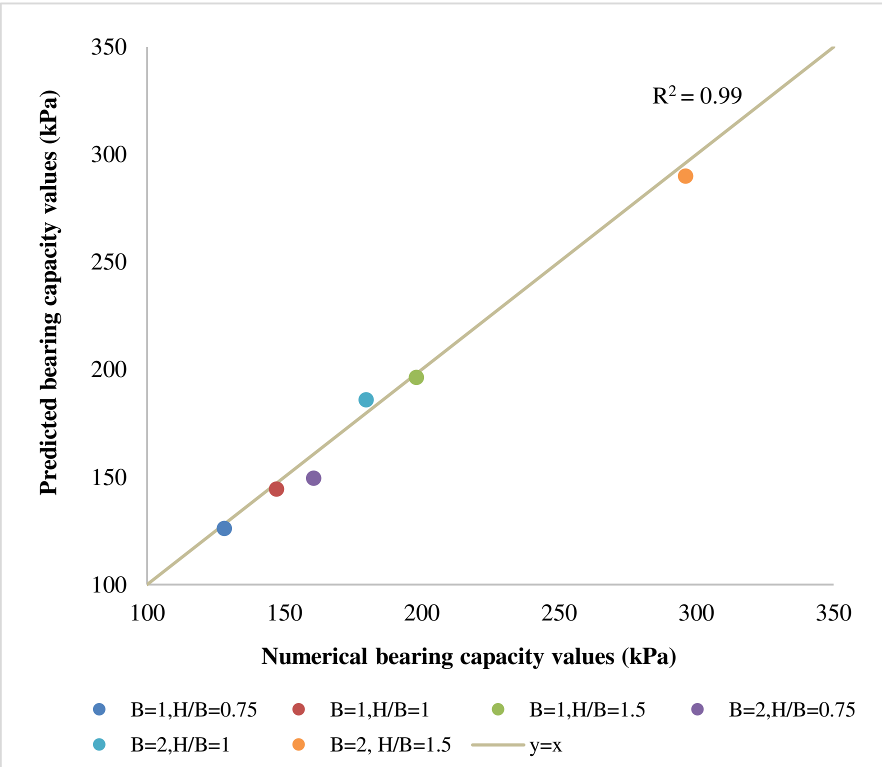

Figure 20. Comparison between predicted values of ultimate bearing capacity from 'proposed analytical model' for outer interfered footing with numerical studies

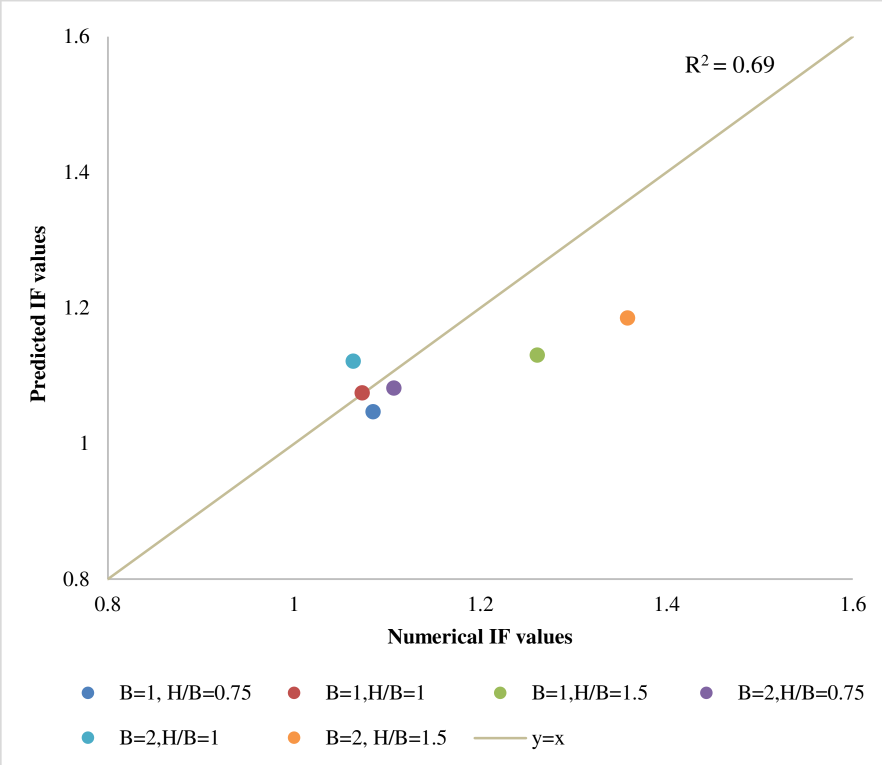

Figure 21. Comparison between predicted values of bearing capacity factors from 'proposed analytical model' for outer interfered footing with numerical studies

5. Conclusions

In the present study, an analytical model is proposed to predict the load-carrying capacity and the interference factor of an interfered footing, when adjacent strip footings are placed on the surface of a Granular Bed (GB) overlying clay, and the footings are simultaneously loaded. A punching shear failure mechanism is envisaged in the analytical model.

The conclusions drawn from this present study are given below:

- The improvement in bearing capacity is attributed to the shear layer effect of the granular bed;
- The values of bearing capacity and interference factor predicted by the proposed analytical model are in reasonably good agreement with those obtained from the finite element method and previous experimental studies;
- The punching shear model developed for interfering footings on granular bed overlying clay gives good results up to the optimum granular bed thickness. If the thickness of granular bed thickness exceeds the optimum thickness, the modified punching shear model somewhat over predicts the value;
- The modified punching shear model gives better results in the case of both two adjacent strip footing and three adjacent footing cases.

6. Declarations

6.1. Author Contributions

Conceptualization, R.S., and S.A.; methodology, R.S.; software, S.A.; validation, S.A; writing—original draft preparation, S.A.; writing—original draft preparation, R.S., and S.A.; writing—review and editing, R.S.; All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available in article.

6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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