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A NEOCLASSICAL APPROACH**

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# Beauty Contests and Irrational Exuberance: a Neoclassical Approach\*

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## Abstract

The arrival of new, unfamiliar, investment opportunities is often associated with “exuberant” movements in asset prices and real economic activity. During these episodes of high uncertainty, financial markets look at the real sector for signals about the profitability of the new investment opportunities, and vice versa. In this paper, we study how such information spillovers impact the incentives that agents face when making their real economic decisions. On the positive front, we find that the sensitivity of equilibrium outcomes to noise and to higher-order uncertainty is amplified, exacerbating the disconnect from fundamentals. On the normative front, we find that these effects are symptoms of constrained inefficiency; we then identify policies that can improve welfare without requiring the government to have any informational advantage vis-a-vis the market. At the heart of these results is a distortion that induces a conventional neoclassical economy to behave as a Keynesian “beauty contest” and to exhibit fluctuations that may look like “irrational exuberance” to an outside observer.

*Keywords:* mispricing, heterogeneous information, information-driven complementarities, volatility, inefficiency, beauty contests.

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# 1 Introduction

The arrival of new, unfamiliar, investment opportunities—e.g., a novel technology like the Internet in the late 90s, or new markets in emerging economies—is often associated with large joint movements in asset prices and real economic activity. Many observers find these movements hard to reconcile with fundamentals. Instead, they interpret them as temporary waves of “exuberance” which appear to get reinforced as market participants look at one another’s behavior for clues regarding the profitability of the new investments. Furthermore, financial markets are blamed for playing a destabilizing role, as the agents in charge of real investment decisions become “overly” concerned about the short-run valuation of their capital instead of paying attention to the fundamentals—a popular argument that can be traced back to Keynes’ famous “beauty contest” metaphor.

Understanding these episodes, while also capturing the aforementioned ideas, requires moving away from the neoclassical paradigm of efficient markets: within this paradigm, there is no room for “exuberance,” “sentiments,” and the like; asset prices only reflect the underlying fundamentals; and it is irrelevant whether real investment decisions are driven by fundamentals or by asset prices.<sup>1</sup> A simple alternative is to assume that the observed phenomena are driven, to a large extent, by the beliefs and the behavior of irrational agents.<sup>2</sup> We do not go in that direction here. Instead, we maintain the axiom of rationality; we depart from the neoclassical paradigm only by introducing dispersed information; and we focus on the information spillovers that emerge when financial markets look at real economic activity as a signal of the underlying fundamentals.

This modeling approach reflects, in part, a matter of preferred methodology. We are uncomfortable with policy prescriptions that rely heavily on the presumption that the government has a superior ability to evaluate the economy’s fundamentals, relative to the market mechanism. Most importantly, it helps us capture two important aspects of the phenomena of interest: that information is likely to be particularly dispersed because of the absence of previous social learning; and that market participants seem anxious to look at one another’s behavior, and at various indicators of economic activity, for clues about the underlying profitability.

Our contribution is then to study how such information spillovers impact the incentives that agents face when making their real investment decisions. First, we find that the response of the economy to noise is amplified, exacerbating the disconnect from fundamentals. Second, we find that these effects are symptoms of inefficiency even relative to a constrained planning problem that respects the diversity of beliefs and the dispersion of information. Combined, these results provide a theory of *rational* exuberance and uncover a mechanism that induces an otherwise conventional neoclassical economy to behave like a Keynesian beauty contest.

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<sup>1</sup>See, e.g., Lucas (1978) and Abel and Blanchard (1986).

<sup>2</sup>Cecchetti et al. (2000), Shiller (2000), Bernanke and Gertler (2001), Dupor (2005), Akerlof and Shiller (2009).

**Preview of model and results.** The real sector of our model features a large number of “entrepreneurs” who each must decide how much to invest in a new technology on the basis of imperfect, and heterogeneous, information about the profitability of this technology. In a subsequent stage, but before uncertainty is resolved, these entrepreneurs may sell their capital in a competitive financial market. The “traders” in this market are also imperfectly informed, but get to observe a signal of the entrepreneurs’ early investment activity.

At the core of this model is a two-way interaction between financial markets and the real economy. On the one hand, entrepreneurs base their initial investment decisions on their expectations of the price at which they may sell their capital. This captures more broadly the idea that the agents in charge of real investment decisions—be they the CEO of a public corporation, or the owner of a start-up—are concerned about the future market valuation of their capital. On the other hand, traders look at the entrepreneurs’ activity as a signal of the profitability of the new investment opportunity. This captures more broadly the idea that financial markets follow closely the release of macroeconomic and sectoral data, and constantly monitor corporate outcomes, looking for clues about the underlying economic fundamentals. The first effect represents a pecuniary externality that, as in any Walrasian setting, is not by itself the source of any distortion. The second effect identifies an information spillover that is at the heart of our positive and normative results.

On the positive side, we identify a mechanism that amplifies the response of the economy to noise relative to fundamentals. By “fundamentals” we mean the profitability of the new investment opportunity (or, more broadly, technologies, preferences and endowments). By “noise” we mean any source of variation in equilibrium outcomes that is orthogonal to the fundamentals. In our model, such non-fundamental variation can originate from correlated errors in information about the fundamentals; from higher-order uncertainty; and, in certain cases, from sunspots.

To understand this result, consider the case where “noise” originates from correlated errors in the entrepreneurs’ information; this is the case we concentrate on for the bulk of our analysis. Suppose for a moment that the entrepreneurs’ decisions were driven merely by their opinions about the fundamentals. In equilibrium, aggregate investment would then depend on the entrepreneurs’ average opinion and would therefore send a signal to the financial market about the underlying fundamentals. In general, this signal is going to be noisy: any given agent—be he a trader or an entrepreneur—may not be able to tell whether high aggregate investment is caused by a positive shock to fundamentals, or by a positive correlated error in the entrepreneurs’ opinions. Nevertheless, relative to the typical trader, the typical entrepreneur is bound to have superior information about the noise in this signal. This is because the signal itself is the collective choice of the entrepreneurs; the noise in this signal thus originates in the entrepreneurs’ own private information. In effect, the entrepreneurs, as a group, are playing a signaling game vis-a-vis the traders.

This observation is crucial, for it implies that the (rational) pricing errors that occur in the financial market are partly predictable by the typical entrepreneur. In particular, whenever there is a positive correlated error in the entrepreneurs' information about the fundamentals, each entrepreneur will expect the average opinion of the other entrepreneurs—and hence aggregate investment—to increase more than his own opinion. But then the entrepreneur will also expect the financial market to overprice his capital. This in turn creates a speculative incentive for the entrepreneur to invest more than what warranted from his expectation of the fundamentals—and thereby to engage in what may look like “exuberant” investment to an outside observer. As all entrepreneurs do the same, their collective “exuberance” may trigger asset prices to inflate, because the traders will perceive this exuberance in part as a signal of good fundamentals. The anticipation of inflated prices can feed back to further exuberance in real economic activity, and so on.

This argument highlights more generally the role of higher-order uncertainty between the agents participating in the real and in the financial sector of the economy. Within the context of our model, this means the following. When an entrepreneur decides how much to invest, he must form beliefs about the traders' beliefs about the fundamentals in order to predict the price at which he will be able to sell his capital in the financial market. A trader, in turn, must form beliefs about the entrepreneurs' beliefs in order to interpret the signal conveyed by aggregate investment. It follows that investment and asset prices are driven by the higher-order beliefs of both these two groups. The key contribution of our analysis is in highlighting how the information spillovers between the two sectors of the economy may heighten the impact of such higher-order uncertainty on investment and asset prices, thereby exacerbating their disconnect from fundamentals, while also providing a micro-foundation to Keynes' “beauty contest.”

Interestingly, these effects obtain without any of the agents being strategic, in the sense that they are all infinitesimal and take prices and aggregate outcomes as exogenous to their choices. Thus, despite a certain similarity in flavor, our results are distinct from those in the financial microstructure literature, which, in the tradition of Kyle (1985), focuses on how large players can manipulate asset prices.<sup>3</sup> Rather, the manipulation effects in our model are the by-product of the “invisible hand”, that is, of the general-equilibrium interaction of multiple small players.

Turning to the normative side, the question of interest is whether the aforementioned positive results are also symptoms of inefficiency. This question is central to our formalization of “exuberance” and “beauty contests,” for these ideas typically involve a, more or less explicit, judgement that something is going “wrong” in the market and that the government should intervene.

To address this question, we consider the problem faced by a planner who has full power on the agents' incentives but has no informational advantage vis-a-vis the market—either in the form of

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<sup>3</sup>See, e.g., Goldstein and Guembel (2008), which emphasizes how this manipulation could distort real investment.

additional information, or in the form of the power to centralize the information that is dispersed in the economy.<sup>4</sup> We then show that this planner would dictate to the entrepreneurs to ignore the expected mispricing in the financial market and all the consequent higher-order uncertainty, and instead base their investment decisions solely on their expectations of the fundamentals. This is because any gain that the entrepreneurs can make by exploiting any predictable mispricing is only a private rent—a zero-sum transfer from one group of agents to the other, which only creates a wedge between the private and social return to investment. It follows that our positive results have a clear normative counterpart.

We conclude our analysis by identifying policies that improve welfare even when the government is restricted to use only information that is already in the public domain. We first show how simple policies that stabilize asset prices, like those often advocated in practice, can lead to higher welfare; but we also identify some important limitations of such policies. We then discuss how certain more sophisticated policies can do better, possibly restoring full efficiency.

**Related literature.** Morris and Shin (2002) recently put forth the idea that models that combine strategic complementarity with dispersed information can capture the role of higher-order uncertainty in Keynes’ beauty contest metaphor, spawning a rich literature. However, by lacking specific micro-foundations, this earlier work did not address the *positive* question of what is the origin of strategic complementarity, and the *normative* question of what is the cause of the inefficiency of the equilibrium, if any. Subsequent work by Allen, Morris and Shin (2006), Bacchetta and Wincoop (2005), and Cespa and Vives (2009) has addressed the positive question within the context of dynamic asset-pricing models, but has abstracted from real economic activity and has left aside the normative question. Relative to this literature, our contribution is to address the aforementioned questions within a micro-founded, neoclassical framework of the interaction between real and financial activity.

What opens the door to non-trivial effects from higher-order uncertainty in our setting is the combination of trading and information spillovers among different groups of agents (the entrepreneurs and the traders). In this respect, our formalization of “beauty contests” is connected to Townsend (1984), who was the first to highlight the role of higher-order beliefs in settings with dispersed information and endogenous learning. But while Townsend studied a framework with no trade and no other payoff links across agents, our results rest on the presence of trading opportunities: if the entrepreneurs never sold their capital in the financial market, both the amplification and the inefficiency would vanish.

Our paper also connects to the voluminous literature on herding and social learning (e.g., Amador and Weill, 2008; Banerjee, 1992; Chamley, 2004; Vives, 2008). We share with this liter-

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<sup>4</sup>This planning problem builds on the notion of constrained efficiency studied in Angeletos and Pavan (2007, 2009) for a class of environments with dispersed information.



ature the broader idea that the economy may feature heightened sensitivity to certain sources of information, leading to increased non-fundamental volatility; see especially Chari and Kehoe (2003) for an application to financial markets and Loisel, Pommeret and Portier (2009) for an application to investment booms. However, the mechanism we identify is distinct. The key distortion featured in this literature is the failure of individual agents to internalize the impact of their own actions on the information available to other agents, which in turn affects the efficiency of the decisions taken by the latter. Instead, in our model, the key distortion is the one that rests on how the *anticipation* of the signaling role of investment affects the entrepreneurs' incentives. In this regard, the mechanics are more closely related to those in the signaling literature (e.g., Spence, 1973) than to those in the herding literature. Note, though, that the “senders” in our model (the entrepreneurs) are non-strategic in the sense that the actions of each one alone do not affect the beliefs of the “receivers” (the traders); it is only the collective behavior of the former, coordinated by an “invisible hand”, that affects the beliefs of the latter.

Our paper also adds to the growing macroeconomic literature on dispersed information.<sup>5</sup> Our contribution in this regard is to identify novel positive and normative implications of the two-way interaction between real and financial activity. Closely related in this regard is the recent paper by Goldstein, Ozdenoren and Yuan (2009). This paper focuses on the opposite information spillover, namely from the financial market to the real sector. In so doing, it complements our paper, but it does not consider the amplification and inefficiency effects that are at the core of our contribution. Tinn (2009) considers an informational spillover similar to the one in our paper, but within a setting that does not feature our amplification and inefficiency results. La'O (2010) studies a model where the two-way interaction between real and financial activity rests on collateral constraints, as in Kiyotaki and Moore (1997), rather than information spillovers, as in our paper.

Finally, by touching on the broader themes of heterogeneous beliefs, speculation, and mispricing, our paper connects to Scheinkman and Xiong (2003), Panageas (2005), and Geanakoplos (2009). This line of work abstracts from asymmetric information and instead models the heterogeneity of beliefs with heterogeneous priors. By abstracting from asymmetric information, this literature rules out the information spillovers that are at the core of our positive and normative results. Our results, on the other hand, are driven by the asymmetry of information, but do not hinge on the assumption of a common prior: they can be extended to settings with multiple priors, provided that one allows for endogenous learning stemming from the presence of dispersed information.

**Layout.** The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium and delivers the key positive results. Section 4 discusses how our results provide a neoclassical formalization of “exuberance” and “beauty contests”. Section 5

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<sup>5</sup>See, e.g., Amador and Weill (2009), Angeletos and La'O (2009a), Hellwig (2006), Lorenzoni (2010), Mackowiak and Wiederholt (2009), Mankiw and Reis (2009), Veldkamp (2009), and the references therein.

characterizes the constrained efficient allocation, it contrasts it to the equilibrium, and discusses policy implications. Section 6 discusses possible extensions that may help reinforce the message of the paper. Section 7 concludes. Appendix A contains all proofs, while Appendix B proves the robustness of our results to richer payoff and information structures.

## 2 The model

Our model features a single round of real investment followed by a single round of financial trading, with information flowing from the former to the latter. The economy is populated by two types of agents, “entrepreneurs” and “traders.” Each type is of measure  $1/2$ ; we index entrepreneurs by  $i \in [0, 1/2]$  and traders by  $i \in (1/2, 1]$ .

**Timing and key choices.** Time is divided in four periods,  $t \in \{0, 1, 2, 3\}$ . At  $t = 0$ , a new investment opportunity, or “technology,” becomes available. The profitability of this technology is determined by a random variable  $\tilde{\theta}$ . This random variable defines the “fundamentals” in our model. It is drawn from a Normal distribution with mean  $\mu > 0$  and variance  $1/\pi_\theta$ , which defines the common prior (with  $\pi_\theta$  being the precision). The realization  $\theta$  of this random variable is unknown to all agents.<sup>6</sup>

At  $t = 1$ , the “real sector” of the economy operates: each entrepreneur gets the opportunity to invest in the new technology. Let  $k_i$  denote the investment of entrepreneur  $i$ . The cost of this investment in terms of the consumption good is  $k_i^2/2$ .<sup>7</sup> When choosing investment, entrepreneurs have access to various sources of information (signals) that are not directly available to the traders. The noise in some of these signals may be mostly idiosyncratic, while for other signals the noise may be correlated across entrepreneurs. We consider a general information structure along these lines in Appendix B. Here, to simplify, we let the entrepreneurs observe only two signals. The one has purely idiosyncratic noise: it is given by  $x_i = \theta + \xi_i$ , where  $\xi_i$  is Gaussian noise, independently and identically distributed across agents, independent of  $\tilde{\theta}$ , with variance  $1/\pi_x$ . The other has perfectly correlated noise: it is given by  $y = \theta + \varepsilon$ , where  $\varepsilon$  is Gaussian noise, common across entrepreneurs, independent of  $\tilde{\theta}$  and of  $\{\tilde{\xi}_i\}_{i \in [0, 1/2]}$ , with variance  $1/\pi_y$ . The key role of the correlated error  $\varepsilon$  is to introduce a source of non-fundamental movements in aggregate investment.<sup>8</sup>

<sup>6</sup>Throughout, we use “tildes” to denote random variables and drop them when denoting realizations.

<sup>7</sup>One can easily reinterpret this cost as the disutility of effort necessary to produce  $k_i$ .

<sup>8</sup>Such a correlated error, in turn, can have various origins. As discussed in Section 6.3, private signals about the actions of agents that moved in the past may lead in equilibrium to signals about  $\theta$  with correlated errors. More broadly, network effects, social learning, and information cascades may also explain this correlation. Alternatively, as emphasized in Hellwig and Veldkamp (2009) and Myatt and Wallace (2009), strategic complementarity—like the one that, as we will show, emerges endogenously in our economy—by itself generates an incentive for the agents to collect correlated sources of information. See also Dow, Goldstein, and Guembel (2010), Froot, Scharfstein, and Stein (1992), and Veldkamp (2006) for complementary justifications.

At  $t = 2$ , the “financial market” operates: some entrepreneurs transfer their capital to the traders.<sup>9</sup> These trades may be motivated by a variety of reasons unrelated to private information. To keep the analysis tractable, we model these trades as follows. Each entrepreneur is hit by an idiosyncratic shock with probability  $\lambda \in [0, 1]$ . Entrepreneurs hit by this shock do not value consumption at  $t = 3$  and have no choice but to sell all their capital at  $t = 2$ . For simplicity, the entrepreneurs not hit by the shock are precluded from trading; this last assumption can be relaxed provided that the equilibrium price does not become perfectly revealing (see the discussion in Section 6.3). From now on, we refer to this shock as a “liquidity shock”; but we think of it more broadly as a modeling device that helps us capture the concern that the agents in charge of real investment have about future equity prices.<sup>10</sup>

The financial market is competitive and the market-clearing price is denoted by  $p$ . When the traders meet the entrepreneurs hit by the liquidity shocks in the financial market, they observe the quantity of capital that these entrepreneurs bring to the market. Since  $\lambda$  is known, this is equivalent to observing the aggregate level of investment,  $K \equiv \int_0^1 k_i di$ . This is meant to capture more broadly other signals that the real sector may be sending to the financial market, including preliminary production and sale data.<sup>11</sup> The traders then use this observation to update their beliefs about  $\theta$ . Any other information that the traders may have about the fundamentals is summarized in a public signal  $\omega = \theta + \eta$ , where  $\tilde{\eta}$  is Gaussian noise, independent of  $\tilde{\theta}$ ,  $\tilde{\varepsilon}$  and  $\{\tilde{\xi}_i\}_{i \in [0, 1/2]}$ , with variance  $1/\pi_\omega$ . While  $\omega$  is modeled here as an exogenous signal, it is straightforward to reinterpret it as the outcome of the aggregation of information that may take place in the financial market when the traders have dispersed private signals about  $\theta$ .

Finally, at  $t = 3$ , the fundamental  $\theta$  is publicly revealed and production takes place using the new technology and the installed capital. To simplify the exposition, we assume that each unit of capital delivers  $\theta$  units of the consumption good, irrespective of whether it is held by an entrepreneur or by a trader. (See Section 6.2 and Appendix B for extensions that allow for richer technologies and for the marginal product of capital to depend on its ownership.)

**Preferences, endowments, and markets.** All agents receive an exogenous endowment  $e$  of the (nonstorable) consumption good in each period. Moreover, they are risk neutral and their discount rate is zero: preferences are given by  $u_i = c_{i1} + c_{i2} + s_i c_{i3}$ , where  $c_{it}$  denotes agent  $i$ 's consumption in period  $t$ , while  $\tilde{s}_i$  is a random variable that takes value 0 if the agent

<sup>9</sup>Throughout, we do not explicitly model the distinction between trading financial claims over the installed capital and trading the capital goods themselves; in our framework, this distinction is irrelevant.

<sup>10</sup>An alternative way to introduce this concern within the context of start-ups rests on the presence of efficiency gains from transferring capital ownership from the agents who have a comparative advantage in starting a new company to the ones who have a comparative advantage in running it at later stages of development (e.g., Holmes and Schmitz, 1990).

<sup>11</sup>In equilibrium, the cross-sectional distribution of  $k_i$  is Normal with known variance; observing the mean level,  $K$ , is thus informationally equivalent to observing the entire cross-sectional distribution of investment.

is an entrepreneur hit by a liquidity shock and value 1 otherwise. Finally, in addition to the aforementioned financial market, we allow the following markets to operate in each period: a market for the consumption good (which is also the numeraire); a market for a riskless bond; and a market for insurance contracts on the entrepreneurs' idiosyncratic liquidity shocks. Given the assumption of risk neutrality, these additional markets will be irrelevant for either investment decisions or asset prices in equilibrium. Rather, they are introduced so as to clarify that the *only* essential market imperfection we impose is the one that limits the aggregation of information about the fundamentals—our results will *not* be driven by borrowing constraints, incomplete risk-sharing, and the like.

### 3 Equilibrium

Because of the assumptions of linear preferences and zero discounting, the equilibrium risk-free rate is zero in all periods and all states; the consumption allocations and the trades of bonds and insurance contracts are indeterminate; and the agents' expected utility reduces to the expected present value of their net income flows. For entrepreneurs hit by the liquidity shock, net income flows sum up to  $3e + pk_i - k_i^2/2$ , while for entrepreneurs not hit by the shock (henceforth also referred to as "surviving entrepreneurs"), they sum up to  $3e + \theta k_i - k_i^2/2$ . Therefore, each entrepreneur's expected utility at the time of investment is given, up to a constant, by  $\mathbb{E}[\tilde{u}_i | x_i, y] = \mathbb{E}[(1 - \lambda)\tilde{\theta} + \lambda\tilde{p} - \frac{1}{2}k_i^2 | x_i, y]$ . Since this objective is strictly concave, the investment choice of an entrepreneur can be expressed as a function of  $x$  and  $y$ , the two signals observed by the entrepreneur. Aggregate investment is then a function of two aggregate shocks, the fundamental  $\theta$  and the correlated error  $\varepsilon$ .

A trader's net income flow, on the other hand, is given by  $3e + \theta q_i - pq_i$ , where  $q_i$  denotes the position he takes in the financial market. Since the trader observes the exogenous signal  $\omega$  and the aggregate capital  $K$ , his expected utility at the time of trading is, up to a constant,  $\mathbb{E}[\tilde{u}_i | K, \omega] = (\mathbb{E}[\tilde{\theta} | K, \omega] - p)q_i$ . It follows that the market-clearing price in the financial market is pinned down by the traders' expectation of the fundamental:  $p = \mathbb{E}[\tilde{\theta} | K, \omega]$ .<sup>12</sup> Since  $K$  is a function of  $(\theta, \varepsilon)$  and  $\omega = \theta + \eta$ , the equilibrium price can be expressed as a function of  $(\theta, \varepsilon, \eta)$ . With all these observations in mind, we define our equilibrium concept as follows.

**Definition 1** *A (linear rational-expectations) equilibrium is an individual investment strategy  $k(x, y)$ , an aggregate investment function  $K(\theta, \varepsilon)$ , and a price function  $p(\theta, \varepsilon, \eta)$  that jointly satisfy the following conditions:*

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<sup>12</sup>Since no trader has private information and the entrepreneurs who sell their capital have perfectly inelastic supplies, the market-clearing price does not reveal any information, which explains why we omit conditioning on  $p$  when describing the traders' expectations. Also, any value  $K \in \mathbb{R}$  can be observed in equilibrium, which explains why we do not have to worry about describing out-of-equilibrium beliefs.

(i) for all  $(x, y)$ ,

$$k(x, y) \in \arg \max_k \mathbb{E} \left[ (1 - \lambda)\tilde{\theta}k + \lambda p(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\eta})k - \frac{1}{2}k^2 \mid x, y \right];$$

(ii) for all  $(\theta, \varepsilon)$ ,

$$K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y | \theta, \varepsilon),$$

where  $\Phi(x, y | \theta, \varepsilon)$  denotes the joint cumulative distribution function of  $x$  and  $y$ , given  $\theta$  and  $\varepsilon$ ;

(iii) for all  $(\theta, \varepsilon, \eta)$ ,

$$p(\theta, \varepsilon, \eta) = \mathbb{E} \left[ \tilde{\theta} \mid K, \omega \right],$$

where  $K = K(\theta, \varepsilon)$  and  $\omega = \theta + \eta$ ;

(iv) there exist scalars  $\beta_0$ ,  $\beta_\theta$  and  $\beta_\varepsilon$  such that, for all  $(\theta, \varepsilon)$ ,

$$K(\theta, \varepsilon) = \beta_0 + \beta_\theta \theta + \beta_\varepsilon \varepsilon.$$

Condition (i) requires that the entrepreneurs' investment strategy be individually rational, taking as given the equilibrium price function. Condition (ii) is just the definition of aggregate investment. Condition (iii) requires that the equilibrium price be consistent with market clearing and rational expectations on the traders' side, taking as given the collective behavior of the entrepreneurs. Finally, condition (iv) imposes linearity; as usual in the rational-expectations literature, this linearity is necessary for maintaining tractability. (In our setting, linearity of the price function implies linearity of the aggregate investment, and vice versa.) To simplify the language, we henceforth drop the qualifications "linear" and "rational-expectations" and refer to our equilibrium concept simply as "equilibrium". Also, we refer to  $\theta$  and  $\varepsilon$  as, respectively, the "fundamental shock" and the "noise shock", and to the coefficients  $\beta_\theta$  and  $\beta_\varepsilon$  as the "responses" of aggregate investment to these shocks.

### 3.1 A benchmark with no information spillovers

For comparison purposes, we now consider a case in which the information spillover between the real and the financial sector is absent. In particular, suppose that the noise in the traders' signal  $\omega$  vanishes ( $\pi_\omega \rightarrow \infty$ ), so that  $\theta$  is known at the time of trading and the signaling role of  $K$  vanishes. The financial market then clears if and only if  $p = \theta$  and, by implication, the expected payoff of an entrepreneur who receives signals  $x$  and  $y$  is simply  $\mathbb{E}[\tilde{\theta} | x, y]k - k^2/2$ . It follows that the entrepreneur's optimal investment is pinned down by his expectation of  $\theta$ :

$$k(x, y) = \mathbb{E}[\tilde{\theta} | x, y] = \delta_0 + \delta_x x + \delta_y y$$

where  $\delta_0 \equiv \frac{\pi_0}{\pi} \mu$ ,  $\delta_x \equiv \frac{\pi_x}{\pi}$ ,  $\delta_y \equiv \frac{\pi_y}{\pi}$ , and  $\pi \equiv \pi_0 + \pi_x + \pi_y$ . By implication, aggregate investment is given by  $K(\theta, \varepsilon) = \delta_0 + \delta_\theta \theta + \delta_\varepsilon \varepsilon$ , where  $\delta_\theta \equiv \delta_x + \delta_y$  and  $\delta_\varepsilon \equiv \delta_y$ .

**Proposition 1** *In the absence of information spillovers, the equilibrium is unique and investment is pinned down merely by the entrepreneurs' expectations of the fundamentals.*

This result establishes that, in the absence of information spillovers, it is irrelevant for equilibrium outcomes whether investment is driven by the entrepreneurs' expectations of the fundamental or by their expectations of the asset price. In this respect, our economy behaves like any conventional neoclassical economy, leaving no room for the features of a Keynesian beauty contest. Importantly, this result does not require  $\theta$  to be known by the traders; it applies more generally as long as the information that the traders possess about  $\theta$  is a sufficient statistics for this information and for the one that the entrepreneurs as a group possess, in which case the entrepreneurs' behavior cannot possibly convey any additional information about  $\theta$ .<sup>13</sup> From now on, we refer to this benchmark as the case with no information spillovers.

### 3.2 Information spillovers

We now turn attention to the case of interest, namely when the real sector sends valuable signals to the financial market. Below, we first describe how any equilibrium can be understood as a fixed point between the aggregate investment function (which summarizes the collective behavior of the real sector) and the asset price function (which summarizes the collective behavior of the financial market), focusing on the situations where aggregate investment is increasing in both the fundamentals and the noise (i.e., where  $\beta_\theta > 0$  and  $\beta_\varepsilon > 0$ ). We then prove that an equilibrium with this property always exists and is unique for  $\lambda$  small enough.

Take an arbitrary linear aggregate investment function of the form  $K(\theta, \varepsilon) = \beta_0 + \beta_\theta \theta + \beta_\varepsilon \varepsilon$ , for some coefficients  $\beta_0$ ,  $\beta_\theta$  and  $\beta_\varepsilon$ . A central object in our analysis is the response of aggregate investment to noise *relative* to the fundamental, defined as the ratio

$$\varphi \equiv \frac{\beta_\varepsilon}{\beta_\theta}. \quad (1)$$

From the perspective of an outside observer, this ratio determines the fraction of the overall volatility in aggregate investment that cannot be explained by fundamentals. Formally,  $\varphi$  is inversely

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<sup>13</sup>To clarify this point, consider an arbitrary information structure. Let  $\mathcal{I}_2$  be the exogenous information of traders at  $t = 2$  (i.e., the information not inferred through  $K$ ). Next, let  $\mathcal{I}_{1i}$  be the information of entrepreneur  $i$  at  $t = 1$ . Finally, let  $\mathcal{I}_1 \equiv \cup_{i \in [0, 1/2]} \mathcal{I}_{1i}$  and assume that  $\mathcal{I}_2$  is a sufficient statistics for  $(\mathcal{I}_2, \mathcal{I}_1)$  with respect to  $\theta$ . This assumption implies that  $\mathbb{E}[\tilde{\theta} | \mathcal{I}_2, \mathcal{I}_1] = \mathbb{E}[\tilde{\theta} | \mathcal{I}_2]$ . Because  $K$  is measurable in  $\mathcal{I}_1$ , this also implies that  $p = \mathbb{E}[\tilde{\theta} | \mathcal{I}_2, K] = \mathbb{E}[\tilde{\theta} | \mathcal{I}_2]$ . By the law of iterated expectations, we then have that  $\mathbb{E}[\tilde{p} | \mathcal{I}_{1i}] = \mathbb{E}[\mathbb{E}[\tilde{\theta} | \mathcal{I}_2, \mathcal{I}_1] | \mathcal{I}_{1i}] = \mathbb{E}[\tilde{\theta} | \mathcal{I}_{1i}]$  for all  $i \in [0, 1/2]$ . It follows that every entrepreneur chooses  $k_i = \mathbb{E}[\tilde{\theta} | \mathcal{I}_{1i}]$ .

related to the R-square of the regression of the realized  $K$  on the realized  $\theta$ . From the perspective of a trader in the model, on the other hand,  $\varphi$  determines the informativeness of the signal that the financial market receives from the real sector. Indeed, as long as  $\beta_\theta \neq 0$ , observing  $K$  is informationally equivalent to observing the following Gaussian signal about  $\theta$ :

$$z \equiv \frac{K - \beta_0}{\beta_\theta} = \theta + \varphi\varepsilon. \quad (2)$$

It follows that  $\varphi$  pins down the noise-to-signal ratio in aggregate investment: this ratio is simply  $\text{Var}(\varphi\varepsilon)/\text{Var}(\theta) = \varphi^2\pi_\theta/\pi_y$ . We henceforth refer to  $\varphi$  interchangeably as the “relative response to noise” and as the “noise-to-signal ratio.”

Now, put aside for a moment the endogeneity of the aforementioned signal, assume that the traders observe a signal of the form  $z = \theta + \varphi\varepsilon$  for some arbitrary  $\varphi \in \mathbb{R}$ , and consider the determination of the asset price. Bayesian updating implies that the traders’ expectation of  $\theta$  is a weighted average of their prior mean  $\mu$  and their two signals  $\omega$  and  $z$ :

$$\mathbb{E}[\tilde{\theta}|K, \omega] = \mathbb{E}[\tilde{\theta}|z, \omega] = \frac{\pi_\theta}{\pi_\theta + \pi_\omega + \pi_z}\mu + \frac{\pi_\omega}{\pi_\theta + \pi_\omega + \pi_z}\omega + \frac{\pi_z}{\pi_\theta + \pi_\omega + \pi_z}z, \quad (3)$$

where  $\pi_\theta$ ,  $\pi_\omega$ , and  $\pi_z \equiv \pi_y/\varphi^2$  are the precisions of, respectively, the prior, the signal  $\omega$ , and the signal  $z$ . By implication, the equilibrium asset price can be expressed as follows:

$$p(\theta, \varepsilon, \eta) = \gamma_0 + \gamma_\theta\theta + \gamma_\varepsilon\varepsilon + \gamma_\eta\eta, \quad (4)$$

where  $\gamma_\theta \equiv \frac{\pi_\omega + \pi_y/\varphi^2}{\pi_\theta + \pi_\omega + \pi_y/\varphi^2}$ ,  $\gamma_\varepsilon \equiv \frac{\pi_y/\varphi^2}{\pi_\theta + \pi_\omega + \pi_y/\varphi^2}\varphi$ , and  $\gamma_\eta \equiv \frac{\pi_\omega}{\pi_\theta + \pi_\omega + \pi_y/\varphi^2}$  measure the responses of the asset price to the underlying shocks. Importantly, these responses depend on  $\varphi$ : because a higher  $\varphi$  means more noise in the signal  $z$  but also less sensitivity of the traders’ expectation of  $\theta$  to this signal, a higher  $\varphi$  necessarily reduces the response of the price to the fundamental  $\theta$  and increases its response to the noise  $\eta$ , while it has a non-monotonic effect on its response to the noise  $\varepsilon$ .

Next, consider the incentives faced by the entrepreneurs when they expect the asset price to satisfy (4). Optimality requires that individual investment satisfies the following condition for all  $x$  and  $y$ :

$$k(x, y) = \mathbb{E} \left[ (1 - \lambda)\tilde{\theta} + \lambda p(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\eta}) \middle| x, y \right]. \quad (5)$$

Substituting the asset price from (4) into the entrepreneur’s optimality condition (5), and noting that the conditional expectations of  $\theta$  and  $\varepsilon$  are linear functions of the signals  $x$  and  $y$ , while the conditional expectation of  $\eta$  is zero, we infer that individual investment can be expressed as a linear function of the two signals:  $k(x, y) = \beta'_0 + \beta'_x x + \beta'_y y$ , for some coefficients  $\beta'_0$ ,  $\beta'_x$  and  $\beta'_y$ . Importantly, these coefficients depend on  $\varphi$  through (4), capturing the impact that the

anticipated price behavior has on individual investment incentives. Finally, aggregating across the entrepreneurs gives  $K(\theta, \varepsilon) = \beta'_0 + \beta'_\theta \theta + \beta'_\varepsilon \varepsilon$ , with  $\beta'_\theta = \beta'_x + \beta'_y$  and  $\beta'_\varepsilon = \beta'_y$ . It follows that the signal sent by the real sector can be expressed as  $z' = \theta + \varphi' \varepsilon$ , with noise-to-signal ratio given by  $\varphi' = \beta'_\varepsilon / \beta'_\theta$ . The latter is pinned down by the relative response of individual investment to the two signals  $x$  and  $y$ , which in turn depends on  $\varphi$  through (4).

Putting the aforementioned arguments together, we infer that any equilibrium can be understood as a fixed point to a function  $\Gamma$  that maps each  $\varphi \in \mathbb{R}$  to some  $\varphi' \in \mathbb{R}$ . This mapping, which is formally defined in the appendix, has a simple interpretation: when the financial market receives a signal  $z = \theta + \varphi \varepsilon$  with noise-to-signal ratio given by some arbitrary  $\varphi \in \mathbb{R}$ , the real sector responds by sending a signal  $z' = \theta + \varphi' \varepsilon$  with noise-to-signal ratio given by  $\varphi' = \Gamma(\varphi)$ . Of course, in any equilibrium the signal received by the financial market must coincide with the signal sent by the real sector, which explains why the fixed points of the mapping  $\Gamma$  identify the equilibria of our economy. Studying the properties of this mapping then permits us to reach the following result, which concerns the existence and uniqueness of equilibria in our model.

**Proposition 2** *There always exists an equilibrium in which the coefficients  $\beta_x, \beta_y, \beta_\theta, \beta_\varepsilon, \gamma_\theta, \gamma_\varepsilon, \gamma_\eta$  are all positive. Furthermore, there exists a cutoff  $\bar{\lambda} > 0$ , such that, for any  $\lambda < \bar{\lambda}$ , this is the unique equilibrium.*

We conclude that there always exists an equilibrium in which individual investment responds positively to both the signals  $x$  and  $y$  and, by implication, aggregate investment responds positively to both the fundamentals  $\theta$  and the noise  $\varepsilon$ . Whenever this is the case, the equilibrium asset price also responds positively to both  $\theta$  and  $\varepsilon$ . This is because the traders (correctly) perceive high investment as “good news” about profitability, but cannot distinguish between increases in investment driven by  $\theta$  from those driven by  $\varepsilon$ .

### 3.2.1 Mispricing, speculation, and amplification

We now turn to our main positive result, regarding the relative response of equilibrium investment to noise. To this purpose, it is useful to rewrite the entrepreneur’s optimal investment as follows:

$$k_i = \mathbb{E}_i[\tilde{\theta}] + \lambda \mathbb{E}_i[p(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\eta}) - \tilde{\theta}] = \mathbb{E}_i[\tilde{\theta}] + \lambda \mathbb{E}_i[\mathbb{E}_t[\tilde{\theta}] - \tilde{\theta}], \quad (6)$$

where  $\mathbb{E}_i$  and  $\mathbb{E}_t$  are short-cuts for the conditional expectations of, respectively, entrepreneur  $i$  and the traders. This condition has a simple interpretation. The variable  $\theta$  represents the fundamental valuation of a unit of capital. The gap  $p - \theta = \mathbb{E}_t[\tilde{\theta}] - \theta$  thus identifies the traders’ forecast error of that valuation, or the “pricing error” in the market. The component of investment that is driven by the forecast of this pricing error can then be interpreted as “speculative.” For any given expectation



of  $\theta$ , an entrepreneur will invest more in response to a positive expectation of the traders' forecast error; this is because he expects to sell the extra capital in an "overpriced" market.

That entrepreneurs base their investment decisions both on their expectation of their fundamental valuation and on their expectation of the financial price should not surprise. This property holds in any environment where entrepreneurs have the option to sell their capital in a financial market. In particular, this property also applies to the benchmark with no information spillovers. What distinguishes the present case from that benchmark is that the entrepreneurs possess information that permits them to forecast the traders' pricing error.

This possibility rests on two properties: (i) that the traders look at aggregate investment as a signal of the underlying fundamental; and (ii) that the entrepreneurs possess additional information about the sources of variation in their investment choices. In particular, note that, for given  $\theta$ , a positive realization of the noise shock  $\varepsilon$  in the entrepreneurs' information causes a boom in aggregate investment. Since the traders cannot tell whether this boom was driven by a strong fundamental or by noise, they respond to this investment boom by bidding the asset price up. However, relative to the traders, the entrepreneurs have superior information about the origins of the investment boom. This explains why they can, at least in part, forecast the traders' forecast errors and hence speculate on the market mispricing.

This, in turn, crucially impacts the entrepreneurs' incentives. Because of the aforementioned speculative component, each entrepreneur bases his decision on his forecast, not only of  $\theta$ , but also of  $\varepsilon$ . When it comes to forecasting  $\theta$ , what distinguishes the two signals  $x$  and  $y$  is simply their precisions,  $\pi_x$  and  $\pi_y$ . When, instead, it comes to forecasting the noise  $\varepsilon$ , the signal  $y$ , which contains information on both  $\theta$  and  $\varepsilon$ , becomes a relatively better predictor than the signal  $x$ , which only contains information on  $\theta$ . This suggests that an entrepreneur who expects prices to increase with both the fundamental  $\theta$  and the noise  $\varepsilon$  will find it optimal to give relatively more weight to the signal  $y$  than what he would have done in the benchmark with no information spillovers (in which  $p$  does not depend on  $\varepsilon$ ). As all entrepreneurs find it optimal to do so, the impact of the noise on aggregate investment is amplified. This intuition is verified in the following proposition.

**Proposition 3** *For any of the equilibria identified in Proposition 2, the following is true:  $\beta_x < \delta_x$ ,  $\beta_y > \delta_y$ ,  $\beta_\theta < \delta_\theta$ , and  $\beta_\varepsilon > \delta_\varepsilon$ . That is, relative to the benchmark with no information spillovers, (i) individual investment responds less to the idiosyncratic signal and more to the correlated signal, and (ii) aggregate investment responds less to fundamental shocks and more to noise shocks.*

Proposition 3 illustrates the amplification mechanism generated by the interaction between real and financial decisions under dispersed information. In Appendix B we show that this amplification mechanism is quite general, in the sense of being present in variants of our model that allow for richer information and payoff structures. However, we will also see that the more robust positive

prediction is about the *relative* response to noise and fundamentals, rather than the absolute responses. For this reason, we henceforth define the *contribution of noise* to aggregate volatility as the fraction of the volatility in aggregate investment that is driven by noise, rather than by the fundamentals, and state the main positive prediction of the paper in the following form.<sup>14</sup>

**Corollary 1 (Main positive prediction)** *In the presence of informational spillovers, the contribution of noise to aggregate volatility is amplified.*

Put it slightly differently, the mechanism identified in the paper reduces the explanatory power of the fundamentals: it reduces the R-square of a regression of aggregate investment on  $\theta$ . We will discuss how this result complements our formalizations of exuberance and Keynesian beauty contests in Section 4. Before proceeding to this, however, we first study certain comparative statics and the possibility of multiple equilibria.

### 3.2.2 Comparative statics and multiplicity

In the absence of information spillovers, the strength of the entrepreneurs' concern for asset prices, as measured by  $\lambda$ , is irrelevant for equilibrium outcomes. With information spillovers, instead, it is crucial. The next result shows how a higher  $\lambda$  reinforces the amplification effect of Corollary 1.

**Proposition 4** *As long as the equilibrium remains unique, the contribution of noise to aggregate volatility increases with  $\lambda$ , the strength of the entrepreneurs' concern for asset prices.*

To get some intuition for this result, consider the following exercise. Suppose that the initial concern for asset prices is equal to  $\lambda_1$  for all entrepreneurs and let  $\varphi_1$  be the associated equilibrium value for the noise-to-signal ratio in aggregate investment. Now a new entrepreneur with concern  $\lambda_2 > \lambda_1$  joins the economy. Since this entrepreneur is infinitesimal, aggregate investment and the asset price remain unchanged. From (5), it is easy to see that, relative to any other entrepreneur, this entrepreneur's investment strategy will be tilted in favor of the correlated signal  $y$ . The reason is the one discussed before. Relative to the idiosyncratic signal  $x$  which contains information only about  $\theta$ , the correlated signal  $y$  contains information also about the common error  $\varepsilon$ . Because the latter impacts the asset price, a higher concern for the latter induces the entrepreneur to respond

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<sup>14</sup>Formally, let  $\hat{K}$  denote the projection of equilibrium  $K$  on  $\theta$ ; that is, consider the regression of realized investment on realized fundamentals. Since the residual  $K - \hat{K}$  is orthogonal to the projection  $\hat{K}$ , we have that  $Var(K) = Var(\hat{K}) + Var(K - \hat{K})$ . That is, aggregate volatility can be decomposed in two components:  $Var(\hat{K})$ , which represents the fundamental component, and  $Var(K - \hat{K})$ , which represents the non-fundamental component. The contribution of noise to aggregate volatility is then defined as the fraction  $Var(K - \hat{K})/Var(K)$ . In our baseline model, the residual  $K - \hat{K}$  depends on a single noise shock and the fraction  $Var(K - \hat{K})/Var(K)$  is simply an increasing transformation of  $\varphi$ . In the generalized model of Appendix B, there are multiple noise shocks driving the residual  $K - \hat{K}$ , but the results in Corollaries 1 and 2 continue to hold for the fraction  $Var(K - \hat{K})/Var(K)$ .

more to  $y$  relative to  $x$ . Next, let the concern for asset prices increase to  $\lambda_2$  for all entrepreneurs in the economy, but continue to assume that the signal  $z$  received by the financial market has a noise-to-signal ratio given by  $\varphi_1$ . As all entrepreneurs start responding more to the correlated signal  $y$ , aggregate investment starts responding relatively more to the noise  $\varepsilon$ . Formally, this argument proves that the mapping  $\Gamma$  is increasing in  $\lambda$  for any given  $\varphi$ . Next, consider what happens as the traders realize that the entrepreneurs' incentives have changed in the aforementioned manner. Because aggregate investment has become a noisier signal of the fundamentals, the traders find it optimal to respond less to it. As a result, the response of the price to  $\theta$  falls, while its response to  $\varepsilon$  could either increase or fall. This last effect, once acknowledged by the entrepreneurs, can either reinforce or dampen the initial effect of the higher  $\lambda$ . Formally,  $\Gamma(\Gamma(\varphi_1))$  could be either higher or lower than  $\Gamma(\varphi_1)$ . However, as long as the equilibrium remains unique, the fixed point of  $\Gamma$  necessarily inherits the comparative statics of  $\Gamma$ , which proves the result.

Interestingly, however, as  $\lambda$  increases enough, the two-way feedback between the real and the financial sector may get sufficiently reinforced that  $\Gamma$  may admit multiple fixed points. Different fixed points are associated with self-fulfilling prophecies regarding the quality of the signal that the real sector sends to the financial market: as the entrepreneurs respond more to the correlated signal  $y$ , they make asset prices more sensitive to noise shocks relative to fundamental shocks, which in turn justifies their stronger response to the correlated signal  $y$ .

**Proposition 5** *There is an open set  $S \subset \mathbb{R}^5$  such that, for all  $(\lambda, \pi_\theta, \pi_x, \pi_y, \pi_\omega) \in S$ , there exist multiple equilibria.*

This multiplicity originates merely from the information spillover between the real and the financial sector of the economy. It is thus distinct from the one that emerges in coordination models of crises such as Diamond and Dybvig (1983) and Obstfeld (1996). Rather, it is closer to the one in Gennotte and Leland (1990) and Barlevy and Veronesi (2003). These papers also document multiplicity results that originate in information spillovers. However, these papers abstract from real economic activity and focus on spillovers that emerge *within* the financial market, between informed and uninformed traders. In our setting, instead, the multiplicity rests on the two-way feedback between the real sector and the financial market and can manifest itself as sunspot volatility in both real investment and asset prices. Clearly, this possibility only reinforces the message of our paper: the mechanism we have identified can contribute to significant non-fundamental volatility, not only by amplifying the impact of correlated errors in information, but also by opening the door to additional volatility driven by sunspots.

## 4 Beauty contests and exuberance

In the preceding analysis, we studied the economy from the perspective of rational-expectations equilibria. We now look at the problem from a different angle, one that permits us to uncover the role that higher-order uncertainty can play in our setting. This in turn helps explain the way in which our framework provides a formalization of the notions of “beauty contests” and “exuberance.”

We do so in three steps. First, we show how our Walrasian economy can be represented as a coordination game among the entrepreneurs; this helps us highlight certain similarities to, but also differences from, previous work that has attempted to capture Keynes’ metaphor with a certain class of coordination games. Second, we explain that the role of higher-order uncertainty in our setting rests on the combination of the information spillover with the option to trade; this helps clarify that our formalization of “beauty contests” is best understood as a signaling-cum-trade game between the real and the financial sector of the economy. Finally, we discuss the various forms that “noise” and “exuberance” may take in our economy.

### 4.1 A coordination game among the entrepreneurs

Substituting condition (2) into condition (3), we can express the traders’ expectation of the fundamentals, and therefore the equilibrium price, as a linear function of aggregate investment. Replacing the resulting expression into the entrepreneurs’ optimality condition (5) leads to the following result.

**Proposition 6** *In any equilibrium, there exist scalars  $\kappa_0$ ,  $\kappa_\theta$  and  $\alpha$  such that the equilibrium investment choices solve the following fixed-point problem:*

$$k(x, y) = \mathbb{E} \left[ \kappa_0 + \kappa_\theta \bar{\theta} + \alpha K(\bar{\theta}, \bar{\varepsilon}) \mid x, y \right], \quad (7)$$

*Furthermore,  $\alpha > 0$  if and only if high investment is “good news” about profitability (i.e., conveys a positive signal about  $\theta$ ), which in turn is necessarily the case whenever the equilibrium is unique.*

This result facilitates a certain game-theoretic representation of our economy: if we fix the equilibrium response of the financial market, the equilibrium investment decisions can be represented as the Perfect Bayesian Equilibrium of a coordination game among the entrepreneurs, with best responses given by condition (7) and with the coefficient  $\alpha$  measuring the degree of strategic complementarity in this game. Importantly, the origin of the coordination motive is the informational spillover between the real and the financial sector. Although each entrepreneur *alone* is too small to have any impact on market prices, the entrepreneurs *as a group* can influence the beliefs of the traders and hence the equilibrium price. This naturally leads to a complementarity

in their investment decisions: the higher the aggregate investment, the higher the traders' expectation about the profitability of the new investment opportunity, and hence the higher the price at which each entrepreneur will be able to sell his capital. Of course, the traders recognize this, and this in turn puts constraints on the extent that the entrepreneurs can collectively “manipulate” the traders' beliefs. Nevertheless, as the above proposition establishes, a coordination motive is present among the entrepreneurs as long as the traders look at investment as a signal of the underlying fundamentals.

This game-theoretic representation, in turn, establishes a useful parallel between our model and a class of games with incomplete information and linear best responses analyzed, among others, by Morris and Shin (2002) and Angeletos and Pavan (2007, 2009). As it is known from this earlier work, strategic complementarity ( $\alpha > 0$ ) tilts the equilibrium use of information towards correlated sources of information, which in turn offers an alternative angle on what drives the result in Proposition 3 regarding the response of individual investment to the different signals. Furthermore, Morris and Shin (2002), and a growing body of research thereafter, have associated the aforementioned class of linear-quadratic games with Keynes' beauty contest metaphor. This approach puts aside the micro-foundations of what these games represent and instead focuses on the fact that these games help capture the dependence of equilibrium outcomes on higher-order beliefs. From this perspective, and if one treats the equilibrium response of the financial market as exogenous, our economy can be interpreted as a beauty contest among the entrepreneurs.

However, it is important to note that, in contrast to this earlier work, the strategic complementarity in our economy is endogenous: as already mentioned, the complementarity originates in the information spillover between the real and the financial sector, not in any direct payoff externality, production spillover, and the like.<sup>15</sup>

## 4.2 The beauty contest game between the real and the financial sector

An even more appealing—at least in our view—formalization of “beauty contests” obtains if one does not abstract from the micro-foundations and instead focuses on the endogeneity of the response of the financial market. A “beauty contest” then emerges as a game between the real and the

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<sup>15</sup>With richer payoffs and richer market interactions, the degree of strategic complementarity  $\alpha$  can be either positive or negative. For example, in Section 6.2 we consider a variant of our baseline model that introduces a competitive labor market. In this variant, higher aggregate investment raises the demand for labor, which in turn increases equilibrium wages and reduces the expected return on capital. As a result, a source of strategic substitutability ( $\alpha < 0$ ) emerges. Alternatively, Angeletos and La'O (2009a) consider a Walrasian economy in which specialization and trade of differentiated commodities introduce strategic complementarity ( $\alpha > 0$ ) even in the absence of an information spillover. However, as we show in Appendix B, the following is true within a rich class of environments that can accommodate the aforementioned effects: relative to a situation without information spillovers, the signaling role of aggregate investment necessarily tilts the entrepreneurs' best responses in the direction of more strategic complementarity (or less strategic substitutability), exactly as in the baseline model considered here. In this sense, our result that the information spillover is a source of strategic complementarity is quite robust.

financial sector of the economy, that is, between the entrepreneurs, as a group, and the traders, rather than as a game among the entrepreneurs.

To see this more clearly, note that in our model the entrepreneurs' payoffs depend only on  $\theta$ , the exogenous return they receive if they hold on their capital, and  $p$ , the price at which they may sell their capital in the financial market. It follows that the entrepreneurs only need to forecast the fundamentals and the behavior of the traders. The traders' behavior, in turn, is pinned down by their own beliefs about  $\theta$ . This implies that the behavior of the entrepreneurs is pinned down by their first-order beliefs about  $\theta$  and by their second-order beliefs about the traders' beliefs about  $\theta$ .

If the traders' beliefs about the fundamentals had been exogenous, this would have been the end of the story—beliefs of higher order would have not mattered. In contrast, higher-order beliefs do matter in our setting because the informational spillover between the real and the financial sector of the economy makes the traders' beliefs endogenous. Indeed, in order to interpret the signal conveyed by aggregate investment, the traders must form beliefs about the driving forces behind the entrepreneurs' actions. By the argument in the preceding paragraph, the entrepreneurs' actions are pinned down by their own first- and second-order beliefs. It follows that the traders' beliefs about the fundamentals, and hence their behavior, depend on (i) their second-order beliefs about the entrepreneurs' first-order beliefs about the fundamentals and (ii) their third-order beliefs about the entrepreneurs' second-order beliefs about their own first-order beliefs. But then, in order to forecast the behavior of the traders, the entrepreneurs must form beliefs about the traders' higher-order beliefs, and so on.

These observations make clear that higher-order uncertainty plays a role in our economy *only* because of the information spillover from the entrepreneurs to the traders. Furthermore, the presence of strategic uncertainty within the group of the entrepreneurs, while empirically appealing, is not strictly needed for our mechanism. In the variant with heterogeneous priors studied in Section 6.1, we can allow the entrepreneurs to share the same information. In these respects, our formalization of “beauty contests” is perhaps more closely related to Townsend (1984), who emphasizes the role of higher-order uncertainty in settings with endogenous learning, than to Morris and Shin (2002), who emphasize the role of strategic uncertainty in coordination games.

### 4.3 Noise and exuberance

The preceding discussion helps recognize the following. To facilitate the characterization of the equilibrium, we assumed a particular information structure that permitted us to guess and verify the fixed point directly instead of working with the infinite regression of higher-order beliefs. The role of higher-order uncertainty then manifested itself only in the amplification of the correlated errors in the entrepreneurs' information about the fundamentals. However, with more general

information structures, “noise” or “exuberance” could also originate from shocks to higher-order beliefs. Indeed, following Angeletos and La’O (2009b), one could introduce shocks that move higher-order beliefs without affecting either the fundamentals or any agent’s exogenous information about the fundamentals. These shocks would then cause fluctuations in investment and asset prices that would look a lot like sunspot fluctuations, even though they would not be triggered by correlation devices and they would not rest on equilibrium multiplicity.

Combining these observations with the results of Section 3, we conclude that the formalization of “exuberance” we propose in this paper can take any of the following three forms: (i) amplification of correlated errors in information about the fundamentals; (ii) fluctuations originating from shocks to higher-order beliefs; and (iii) sunspot fluctuations. Either of these forms captures variation in equilibrium outcomes that likely would seem “hard to reconcile with fundamentals” in the eyes of an outside observer; they are possible in our setting because, and only because, of the information spillover between the two sectors of the economy. In Section 6.1, we discuss a fourth complementary form of “exuberance” that can obtain in a variant of our model with heterogeneous priors—this variant permits us to reinterpret the correlated error  $\varepsilon$  of our baseline model as a form of “bias” in beliefs, without, however, abandoning the axiom of rationality.

Finally, whereas the entire preceding discussion focuses on *positive* aspects, we believe that a proper formalization of “exuberance,” “sentiments,” “beauty contests,” and the like should also capture the *normative* aspects of these ideas. Indeed, references to these notions typically come together with an argument—more or less explicit—that there is something “wrong” in the functioning of the economy and that the government should intervene. Keynes himself brought up his famous beauty-contest metaphor, and talked more generally about “animal spirits,” as part of an explicit attempt to make a case that the market mechanism can be inefficient, not merely to describe market behavior. In this regard, the normative results of the next section are an integral part of our formalizations of “exuberance” and “beauty contests.” This is in contrast to previous work where the normative aspect is either ignored (e.g., Allen, Morris and Shin, 2006; Bacchetta and Wincoop, 2005) or imposed in an ad hoc manner (e.g., Morris and Shin, 2002).

## 5 Efficiency and Policy

The analysis so far focused on the positive properties of the equilibrium. We now turn to its normative properties and to policy implications.

## 5.1 Constrained efficiency

In the environment considered in this paper, the government could obviously improve upon the competitive equilibrium if it could collect all the information dispersed in the economy and make it public—this would remove any asymmetry of information and would achieve the first-best allocation. In practice, it seems implausible that the government be able to perform this task.<sup>16</sup> The question we address here is whether the government can improve upon the equilibrium by manipulating the agents' incentives through taxes, regulation, and other policy interventions. We thus consider a notion of constrained efficiency that is designed to address this question, without getting into the details of specific policy instruments. Namely, we consider a planner who can dictate to the agents how to use their available information but that cannot transfer information from one agent to another. Angeletos and Pavan (2007, 2009) propose and study such a notion of constrained efficiency within a class of games with dispersed information; here, we adapt this notion to the Walrasian economy under consideration by embedding the aforementioned information constraint into otherwise standard definitions of feasible and Pareto-optimal allocations.

**Definition 2** *A feasible allocation is a collection of investment choices  $k_i$ , one for each entrepreneur, together with a collection of consumption choices  $c_{it}$ , one for each entrepreneur and for each trader in each period, that jointly satisfy the following constraints:*

(i) *resource feasibility:*

$$\begin{aligned} \int_{i \in [0,1]} c_{i1} di &\leq e - \int_{i \in [0,1/2]} \frac{1}{2} k_i^2 di, \\ \int_{i \in [0,1]} c_{i2} di &\leq e, \\ \int_{i \in [0,1]} c_{i3} di &\leq e + \int_{i \in [0,1/2]} \theta k_i di \end{aligned}$$

with  $c_{i3} = 0$  for all  $i$  such that  $s_i = 0$  (i.e., for all entrepreneurs hit by the shock).

(ii) *informational feasibility:* for each entrepreneur  $i \in [0, 1/2]$ ,  $c_{i1}$  and  $k_i$  are contingent on  $(x_i, y)$ ,  $c_{i2}$  is contingent on  $(x_i, y, s_i, K, \omega)$ , and  $c_{i3}$  is contingent on  $(x_i, y, s_i, K, \omega, \theta)$ ; for each trader  $i \in (1/2, 1]$ ,  $c_{i1}$  is non-contingent,  $c_{i2}$  is contingent on  $(K, \omega)$ , and  $c_{i3}$  is contingent on  $(K, \omega, \theta)$ .

**Definition 3** *An efficient allocation is a feasible allocation that is not Pareto dominated by any other feasible allocation.*

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<sup>16</sup>Why the government may not be able to centralize the information that is dispersed in the economy is an important and difficult question that, as emphasized by Hayek, rests at the heart of the market mechanism. Clearly, this question is beyond the scope of this paper.



Because of the linearity of preferences in consumption, efficiency leaves the distribution of consumption across periods indeterminate. Moreover, the distribution of consumption across agents will depend in general on the point chosen on the Pareto frontier. However, the efficient investment strategy is uniquely determined and is the one that maximizes the following welfare objective:

$$W = \mathbb{E} \left[ \tilde{Q} - \frac{1}{2} \int_i \tilde{k}_i^2 di \right]$$

where  $Q \equiv \int_i \theta k_i di$  measures the level of aggregate output at  $t = 3$  and where  $\frac{1}{2} \int_i \tilde{k}_i^2 di$  represents the social cost of producing this level of aggregate output. Equivalently,

$$W = \mathbb{E} \left[ \tilde{\theta} k(\tilde{x}, \tilde{y}) - \frac{1}{2} k(\tilde{x}, \tilde{y})^2 \right], \quad (8)$$

which leads to the following characterization of the efficient investment strategy.

**Proposition 7** *The efficient investment strategy is given by*

$$k(x, y) = \mathbb{E} \left[ \tilde{\theta} | x, y \right] = \delta_0 + \delta_x x + \delta_y y, \quad (9)$$

almost all  $(x, y)$ , where the coefficients  $\delta_0$ ,  $\delta_x$ , and  $\delta_y$  are the same as in Proposition 1.

The efficient investment strategy thus coincides with the equilibrium strategy in the benchmark with no information spillovers. It follows that our key positive result has a normative counterpart.

**Corollary 2 (Main normative prediction)** *In the presence of information spillovers, the contribution of noise to aggregate volatility is inefficiently high.*

As anticipated in the previous section, this result provides the normative basis of our formalization of “exuberance” and “beauty contests.” The intuition for this result is quite simple. The agents in charge of real investment decisions possess information that permits them to forecast not only the long-run profitability of their investments but also the mispricing of this profitability by other agents at subsequent stages of financial trades. The possibility of forecasting such a mispricing in turn gives rise to a “speculative return,” which is however purely private and hence not warranted from a social viewpoint. Such a private benefit tilts the way entrepreneurs respond to their sources of information away from efficiency, with negative implications for welfare.

The market friction that sustains this inefficiency is only the absence of perfectly revealing markets. By this we mean the following. In Walrasian settings, the absence of perfect information aggregation is tightly connected to missing markets: when markets are complete, all relevant information is perfectly revealed through prices, and a first-best allocation is obtained (Grossman,

1981). In this sense, our normative result necessarily rests on some missing market. However, this has nothing to do with borrowing constraints, incomplete risk-sharing and the like. Rather, the only essential market friction is the one that limits the aggregation of information: inefficiency emerges robustly as long as the financial market looks at the real sector’s activity as a signal of the underlying fundamentals.<sup>17</sup>

Finally, note that Corollary 2 presumes that the equilibrium is unique, which is the case we have focused on. When there are multiple equilibria, the result holds for any equilibrium in which aggregate investment increases with  $\theta$ . Since this property is clearly satisfied by the efficient allocation, this also means that, when there are multiple equilibria, all of them are inefficient.

## 5.2 Policy implications

While the preceding analysis suggests that there exist policies that improve upon equilibrium welfare without requiring the government to centralize the information that is dispersed in the economy, it does not spell out the details of the specific policies that permit to do so. We now show how policies aimed at reducing asset price volatility may achieve this goal. Our focus on this class of policies is motivated by two considerations. First, there is a vivid debate on whether central bankers, or governments more generally, should try to tame “exuberant” movements in asset prices. Second, such policies look a priori plausible in our setting, since the inefficiency in our model rests on how financial markets respond to the signals sent by the real sector.

Consider a proportional tax  $\tau$  on financial trades at  $t = 2$ . This tax is meant to capture more broadly a variety of policies that may introduce a “wedge” between the asset price and the underlying private valuations of the asset; this may include not only taxes on capital gains but also regulatory interventions. For simplicity, the tax is assumed to be paid by the buyers (here the traders). To capture the idea that policy intervention may be contingent on the level of asset prices, we let the tax rate  $\tau$  be contingent on  $p$ :

$$\tau = \tau(p) = \tau_0 + \tau_p p, \tag{10}$$

where  $\tau_0$  and  $\tau_p$  are scalars chosen by the government.<sup>18</sup>

The equilibrium price in the financial market now satisfies  $p = \mathbb{E}[\tilde{\theta}|K, \omega] - \tau(p)$ , which yields

$$p = \frac{1}{1 + \tau_p} \left( \mathbb{E}[\tilde{\theta}|K, \omega] - \tau_0 \right). \tag{11}$$

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<sup>17</sup>For example, we could allow the entrepreneurs to trade at  $t = 1$  securities (“futures”) whose returns are correlated with  $\theta$ ; as long the price of these securities is not perfectly revealing of  $\theta$ , the financial market at  $t = 2$  would continue to look at the equilibrium  $K$  as a signal of  $\theta$ , and our results would go through.

<sup>18</sup>The revenues collected by this tax are rebated as a lump-sum transfer. Because of linear preferences, the distribution of this lump-sum transfer is irrelevant.

If the tax is procyclical, in the sense that  $\tau_p > 0$ , its effect is to dampen the response of asset prices to the traders' expectation of  $\tilde{\theta}$ , and thereby to the information contained in aggregate investment. This dampens the price response to the noise  $\varepsilon$ , thereby also dampening the relative bias towards the correlated signal in the entrepreneurs' best responses (5). At the aggregate level, this tends to make investment less responsive to noise relatively to fundamentals. As this happens, a second, countervailing effect emerges: because entrepreneurs assign relative less weight to  $y$ , aggregate investment  $K$  becomes a more precise signal of the fundamentals  $\theta$ , making prices more responsive to  $K$  and thereby also to the noise  $\varepsilon$ . However, the first effect must always dominate—for, if that were not the case, the second effect would not emerge in the first place.

We infer that policies aimed at stabilizing asset prices can dampen the relative impact of noise on real economic activity. Furthermore, a wide range of numerical results suggest that it is always desirable to do so to some extent, namely it is optimal to set  $\tau_p \in (0, \infty)$ .<sup>19</sup> However, these policies reduce the impact of noise only by reducing the response of asset prices to *all* sources of variation in the traders' expectations of their valuation of capital. In so doing, they also reduce the response of asset prices to the fundamentals themselves. As the real sector anticipates this, the absolute response of real economic activity to fundamentals also goes down, which entails a welfare loss, since that response was already inefficiently low. It follows that this kind of policy intervention can improve welfare, but cannot possibly restore efficiency.

**Proposition 8** *Consider the policies defined by (10). As long as the equilibrium remains unique,<sup>20</sup> a higher  $\tau_p$  necessarily reduces the contribution of noise to aggregate volatility and, in so doing, can improve welfare. However, no policy in this class can implement the constrained-efficient allocation.*

The analysis above thus provides a rationale for policies aimed at reducing asset price volatility, without invoking either the presence of irrational forces among market participants or any superior wisdom on the side of the government. At the same time, it highlights an important limitation of such policies: they may tame unwanted exuberance only by also dampening the response of the economy to fundamentals.

The government, however, may do better by considering more sophisticated policy interventions. By this we mean policies that are contingent on a wider set of publicly-available signals about both the exogenous fundamentals and the endogenous level of economic activity. In particular, consider a tax on capital trades whose rate is contingent, not only on the asset price  $p$ , but also on aggregate

<sup>19</sup> While we have not been able to prove a formal result that the optimal  $\tau_p$  is positive, we have found this to be the case for an extensive search of the parameter space: we have randomly drawn 10,000 values of the parameter vector  $(\lambda, \pi_\theta, \pi_x, \pi_y, \pi_\omega, \lambda)$  from  $(0, 1) \times \mathbb{R}_+^4$ . For each such vector, we have numerically computed the value of  $\tau_p$  that maximizes welfare and we have found this to be strictly positive. At the same time, we could show that a policy of full price stabilization (i.e.,  $\tau_p \rightarrow \infty$ ) is never optimal.

<sup>20</sup>The equilibrium is necessarily unique if  $\lambda$  and  $\tau_p$  are small enough.

investment  $K$ :

$$\tau = \tau(p, K) = \tau_0 + \tau_p p + \tau_K K, \quad (12)$$

where  $\tau_0, \tau_p, \tau_K$  are scalars. By choosing  $\tau_K > 0$ , the government can dampen the signaling effect of investment on asset prices and thereby ensure that asset prices no longer respond to the entrepreneurs' noise  $\epsilon$ . At the same time, by choosing  $\tau_p < 0$ , the government can ensure that asset prices respond more strongly to all other sources of information that the traders have about the fundamentals (here summarized in the signal  $\omega$ ). In fact, conditioning the tax on the asset price accomplishes the same as conditioning the tax on the signal  $\omega$ , and thereby on the fundamentals  $\theta$ . In terms of the game-theoretic representation of Proposition 6, this means that an appropriate combination of  $\tau_K$  and  $\tau_p$  permits the government to control separately  $\alpha$ , the degree of strategic complementarity in investment decisions, and  $\kappa_\theta$ , the sensitivity of best responses to (expectations of) the fundamentals. It then follows that these contingencies permit the government to reduce the relative impact of the noise while at the same time raising the absolute impact of the fundamentals, therefore restoring full efficiency.

**Proposition 9** *Consider the policies defined by (12). These policies can control separately the response of aggregate investment to noise and fundamentals. Furthermore, there exists a policy in this class that implements the constrained efficient allocation as the unique equilibrium.*

This result highlights the distinct role that state-contingent policies can play in controlling the decentralized use of information, and thereby the response of the economy to the underlying fundamental and noise shocks, when information is dispersed. While we illustrated this insight focusing on taxes on financial trades, its applicability is broader. For example, consider a tax on eventual capital returns (or firms' profits). If this tax is non-contingent, then it can affect the incentives faced by the entrepreneurs and/or the traders only in a uniform way across all states of nature. In so doing, it can affect the average level of investment and the average level of the price, but cannot affect their response to the underlying shocks. In contrast, if this tax is contingent on certain public signals (e.g., the price  $p$  and aggregate investment  $K$  as of  $t = 2$ , or the realized aggregate output  $\theta K$  as of  $t = 3$ ), then this tax can impact incentives in a different way across different states of nature; this is because different states of nature, and different information sets, are associated with different expectations at  $t = 1$  regarding these contingencies. It follows that these contingencies can help control the response of the economy to the underlying fundamental and noise shocks, much alike the taxes on financial trades studied above.<sup>21</sup>

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<sup>21</sup>Whether such state-contingent policies are time-consistent or politically feasible is an important question, but well beyond the scope of this paper. Also, the ability of such state-contingent policies to restore *full* efficiency may well rest on special features of our model, such as the absence of risk aversion and the ability of the government to perfectly observe the signals that the real sector sends to financial markets. However, the (weaker) result that these

We conclude this section by considering policies that directly affect the information available to the market. In particular, towards capturing the role of the government in collecting various data on economic activity at either the sectorial or the macroeconomic level, consider a variant of our model where the financial market observes only a noisy statistic of  $K$ . Suppose further that the government can control the quality, or precision, of this statistic. Clearly, the higher the precision of this signal, the more the weight that the traders assign to it when estimating the fundamental. It follows that the government can use the precision of this statistic to manipulate the response of asset prices to aggregate investment—essentially in the same way as it could do it with the price-stabilization policies considered above.<sup>22</sup> We infer that the choice of the optimal precision is subject to essentially the same trade-offs as those emphasized for the aforementioned price-stabilization policies: increasing the precision of this statistic increases the response of real economic activity to the fundamentals, but also exacerbates the relative impact of noise. An intermediate quality of macroeconomic statistics may thus be optimal in our context, even when the cost of improving this quality is negligible.

## 6 Discussion and extensions

In this section we discuss various extensions that help reinforce the message of the paper. We start by providing a possible reinterpretation of our results within a variant that introduces heterogeneous priors. We then continue by enriching the “real” and the “financial” side of the economy.

### 6.1 Heterogeneous priors

Our analysis has imposed that all agents share a common prior. While standard in macroeconomics, this assumption may be hard to justify during the episodes of interest. Rather, because of the unfamiliarity of the new investment opportunities, different agents may have different priors about their likely profitability, as well as about the informativeness of available signals (i.e., different priors about the joint distribution of the fundamentals and the available signals). We now discuss how our analysis can accommodate this possibility, while at the same time maintaining the axiom of rationality and a non-paternalistic approach to policy.

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contingencies can control how agents respond to their different sources of information, and in so doing control the impact of noise and fundamental shocks, is not sensitive to the details of our model. See Angeletos and Pavan (2009) for the broader applicability of this insight, and Angeletos and La’O (2009a) and Lorenzoni (2010) for applications in canonical business-cycle models.

<sup>22</sup>In fact, it is easy to show that there is a formal equivalence between the two policies. Let the statistic of aggregate investment be  $K' = K + \varsigma$ , where  $\varsigma$  is Gaussian noise with variance  $1/\pi_\varsigma$ . For each  $\pi_\varsigma$ , there is a price elasticity  $\tau_p$  of the tax in (10) that induces the same response of the price to aggregate investment in the economy with perfect observability of  $K$ , and vice versa.

It is easy to show that our results extend to a variant of our model where the entrepreneurs and the traders have different priors about  $\theta$  and where these priors are common knowledge. This is because both the amplification result of Corollary 1 and the inefficiency result of Corollary 2 are driven merely by the presence of an information spillover between the two groups, which remains present irrespective of whether or not these groups share the same prior on  $\theta$ .

Perhaps more interestingly, a complementary form of “exuberance” may emerge in our setting if we allow for heterogeneous priors about the informativeness of available signals. In particular, consider a variant where the signal  $y$  is informative of the fundamentals *only* in the minds of the entrepreneurs; the traders, instead, believe that  $y$  is pure noise. Formally, the entrepreneurs’ prior is that  $y = \theta + \varepsilon$ , while the traders’ prior is that  $y = \varepsilon$ . These differences in prior beliefs are mutually known and the rest of the model is unchanged. Under this specification,  $y$  causes variation in the entrepreneurs’ beliefs that is considered “unjustified” from the traders’ perspective. However, from the entrepreneurs’ perspective, it is the traders’ refusal to believe that  $y$  contains information about  $\theta$  which is “unjustified.” Furthermore, these differences in opinions are mutually accepted: the agents have agreed to disagree. Finally, these differences in opinions do not involve any form of naivete or irrationality: given his prior and his information, each agent forms rational expectations about the fundamentals, the prices, and the other agents’ actions; and this fact is common knowledge.

Because the entrepreneurs believe that  $y$  is informative about  $\theta$ , they would find it optimal to react to it even if they believe that the price did not correlate with it. On the other hand, because the traders believe that  $y$  is pure noise, they would themselves *not* react to it if they could directly observe  $y$ . It follows that, in the absence of an information spillover, the price is uncorrelated with  $y$  while aggregate investment responds to  $y$  only in so far  $y$  impacts the entrepreneurs’ own beliefs about  $\theta$ . In contrast, when the traders do not directly observe  $y$  and instead look at aggregate investment as a signal of the fundamentals, they are not able to tell apart movements in investment that are driven by signals that the traders themselves consider informative about  $\theta$  (here captured by the signals  $x$ ) from the movements that are driven by what the traders believe to be an unjustified “bias” in the entrepreneurs’ beliefs (here captured by the signal  $y$ ). It follows that, in the presence of an information spillover, the price is correlated with  $y$ , which in turn reinforces the entrepreneurs’ incentive to respond to  $y$ . In other words, Corollary 1 continues to hold, except that the interpretation of “noise” is now different. From the eyes of the entrepreneurs, “noise” is correlated error in their own information, exactly as in the benchmark model, whereas from the eyes of the traders, “noise” is now synonymous to a “bias” in the entrepreneurs’ beliefs.

This extension permits us to capture the idea that many market players often appear to believe that they “know better” than the rest of the market, while at same time recognizing that

other market players may also think in a symmetric way. It also brings our paper closer to the recent literature that uses heterogeneous priors to model speculative movements in asset prices (Scheinkman and Xiong, 2003) and real investment (Panageas, 2005). Nevertheless, our analysis differentiates from this literature in one crucial respect: this literature allows for heterogeneous priors but imposes symmetric information, thus ruling out the information spillover that is at the core of our analysis.<sup>23</sup> In contrast, by combining heterogeneous priors with dispersed information, our approach helps uncover a novel positive result: how information spillovers may amplify the equilibrium impact of certain “biases” as some agents cannot tell apart whether the observed investment or asset price boom is driven by “hard information” or by the “biases” of other agents.

Furthermore, our approach has distinct normative implications. To appreciate this, recall that the concept of Pareto optimality allows for subjective probabilities. It follows that heterogeneous priors, and the speculative forces studied in the aforementioned literature, do not by themselves open the door to policy intervention. Rather, one has also to take a paternalistic stand that the priors of some agents are “wrong”—a stand that we have sought to avoid. In contrast, once heterogeneous priors are combined with the information spillovers we have highlighted, the consequent amplification of the perceived “biases” is undesirable even under the perspective of a non-paternalistic planner who evaluates each agent’s ex-ante utility using the agent’s own prior, not some other prior that he judges more appropriate. In other words, the property that agents respond to their own “biases” is not per se a symptom of inefficiency under a non-paternalistic perspective; but the property that information spillovers amplifies this response is.

## 6.2 Richer specification of the real sector

We now consider a certain variant of the real sector of our model, one that introduces a competitive labor market. The production technology uses as inputs not only capital as in the baseline model, but also labor. For tractability, this technology is assumed to be Leontief, with one unit of capital requiring  $n$  units of labor at  $t = 3$  in order to produce  $\theta$  units of the consumption good, for some  $n > 0$ . The net return to capital is thus  $r = \theta - wn$ , where  $w$  denotes the wage rate. For simplicity, labor is supplied only by the traders.<sup>24</sup> Their preferences are now given by  $u_i = c_{i1} + c_{i2} + c_{i3} - H(\ell_i)$ , where  $\ell_i$  denotes labor supply and where  $H$  is a strictly convex function representing the disutility of labor. Their intertemporal budget, on the other hand, is given by  $c_{i1} + c_{i2} + c_{i3} = 3e + (r - p)q_i + w\ell_i$ , where the last term represents labor income. It follows that we can express the payoff of a trader, up to a constant, as  $u_i = (r - p)q_i + w\ell_i - H(\ell_i)$ . The payoff of a surviving entrepreneur, on the

<sup>23</sup>In particular, the aforementioned papers allow different agents to disagree on the informativeness of different exogenous signals about the fundamentals, but assume that all the signals are commonly observed. It follows that there is nothing to be learned from observing the behavior of other agents.

<sup>24</sup>Allowing the entrepreneurs to also supply labor, or introducing a third class of agents (“workers”) whose only role in the economy is to supply labor would not change the results in any significant way.

other hand, is  $u_i = rk_i - \frac{1}{2}k_i^2$ , while that of an entrepreneur hit by a liquidity shock is  $u_i = pk_i - \frac{1}{2}k_i^2$ .

We characterize the equilibrium of this variant by backward induction. First, consider wages and employment (at  $t = 3$ ). Optimality of labor supply requires that  $H'(\ell_i) = w$  for all  $i \in [1/2, 1]$ , while labor market clearing requires that  $\int_{i \in [1/2, 1]} \ell_i = nK$  (by the Leontief assumption, the aggregate demand for labor is  $nK$ ). By implication, in equilibrium,  $\ell_i = \ell = nK$  for all  $i \in [1/2, 1]$ ,  $w = H'(nK)$ , and  $r = \theta - H'(nK)n$ . It follows that the payoff of a trader can be expressed as  $u_i = V^t(\theta, K, q_i) - pq_i$ , where  $V^t(\theta, K, q) \equiv [\theta - H'(nK)n]q + H'(nK)nK - H(nK)$ , while the payoff of a surviving entrepreneur can be expressed as  $u_i = V^e(\theta, K, k_i) - \frac{1}{2}k_i^2$ , where  $V^e(\theta, K, k) \equiv [\theta - H'(nK)n]k$ . Next, consider asset prices (at  $t = 2$ ). Optimality on the traders' side, along with the fact that  $q = \lambda K$  in equilibrium, implies that

$$p = \mathbb{E}_t[\tilde{v}^t], \quad (13)$$

where  $v^t \equiv V_q^t(\theta, K, \lambda K)$  and where  $\mathbb{E}_t[\cdot]$  denotes the traders' expectation. Finally, consider investment (at  $t = 1$ ). Substituting the price into the entrepreneurs' optimality condition gives the following equilibrium condition:

$$k_i = \mathbb{E}_i[\tilde{w}_i] + \lambda \mathbb{E}_i[\mathbb{E}_t[\tilde{v}^t] - \tilde{v}^t], \quad (14)$$

where  $w_i \equiv (1 - \lambda)V_k^e(\theta, K, k_i) + \lambda V_q^t(\theta, K, \lambda K)$  and where  $\mathbb{E}_i[\cdot]$  denotes the entrepreneur's expectation.

These results have a simple interpretation. First, the functions  $V^e$  and  $V^t$  represent indirect utilities, or the reduced-form payoffs, which emerge once equilibrium employment and wages have been solved out. The dependence of these payoffs on  $K$  reflects the pecuniary externalities associated with the labor market: higher aggregate investment raises the demand for labor, thereby raising wages and depressing the return to investment. Second, the variable  $v^t$  represents the marginal valuation of capital for a trader. Third, the variable  $w_i$  represents the marginal valuation for an entrepreneur who expects to get a price equal to  $v^t$  when he sells his capital. In finance jargon, these variables represent the “fundamental valuation” of capital, or the “fundamental return” to investment. Finally, the expression  $\mathbb{E}_t[\tilde{v}^t] - v^t$  represents the pricing error in the financial market, that is, the error that the traders make in estimating their own valuation of the asset. The last term in condition (14) then represents the entrepreneurs' forecast of such a pricing error; this term captures a “speculative” return component akin to the one in the baseline model.

As in the baseline model, the pricing error is forecastable in the eyes of the entrepreneurs only because they have private information about the sources of variation behind the signal (aggregate investment) that they collectively send to the financial market. Furthermore, this speculative



component is once again a zero-sum transfer between the traders and the entrepreneurs. These observations suggest that the present variant continues to feature essentially the same kind of amplification and inefficiency highlighted in the baseline model. We verify these intuitions in Appendix B, where we show that Corollaries 1 and 2 extend not only to the present variant, but also to more flexible specifications of the reduced-form payoffs  $V^e$  and  $V^t$  that may summarize the “inner workings” of the real sector of the economy.

Apart from illustrating the robustness of our insights to richer specifications of the real side of the economy, these results help clarify the notions of “fundamental” and “speculative” returns to investment. While these notions were tightly connected to the same exogenous random variable in the baseline model, here they are allowed to have deeper micro-foundations. Furthermore, these results indicate how an episode of “exuberance” can be associated not only with high investment and high asset prices, but also with high employment, high wages, and an all-around economic boom. A similar reasoning then implies that the signals that the real sector sends to the financial market need not be limited to aggregate investment; they should be interpreted more broadly as the information, in part public and in part private, that the real sector conveys to financial-market participants about real economic activity.

### 6.3 Richer specification of the financial market

Next, we discuss extensions that may help capture the role of the financial market as a provider of information, as opposed to a simple receiver, as in the baseline model. This possibility introduces additional feedbacks between real and financial activity, which may actually reinforce the message of our paper.

To start, suppose that traders are risk averse and, instead of an exogenous public signal about  $\theta$ , they observe private signals of the form  $\omega_i = \theta + \eta_i$ , where  $\eta_i$  is noise. This would make the model of the financial market closer to the literature on rational expectations in the tradition of Grossman and Stiglitz (1976) and Hellwig (1980). Specifically, assume that the traders’ preferences display constant absolute risk aversion and that all random variables are Gaussian. Each trader’s demand for the asset is then given by

$$q_i = \frac{\mathbb{E}_i[\tilde{\theta}] - p}{\Gamma \text{Var}_i[\tilde{\theta}]}, \quad (15)$$

where  $\mathbb{E}_i[\tilde{\theta}] \equiv \mathbb{E}[\tilde{\theta}|\omega_i, p, K]$ ,  $\text{Var}_i[\tilde{\theta}] \equiv \text{Var}[\tilde{\theta}|\omega_i, p, K]$  and where  $\Gamma$  is the coefficient of absolute risk aversion. Note that the market clearing price now serves as a signal about  $\theta$ , for it aggregates the information dispersed among the traders. However, as long as there are additional unobserved sources of variation in the demand or supply of the asset, the equilibrium price will not be perfectly

revealing. For example, suppose the noises  $\eta_i$  are correlated: the price will then reveal the average  $\omega_i$  which is a noisy signal of  $\theta$ . Finally, suppose that the entrepreneurs not hit by the liquidity shock are allowed to trade in the financial market, but their valuations are subject to an additional common shock that is not observable by the traders. Once again, this shock guarantees that the price does not perfectly reveal the fundamental to the traders.<sup>25</sup> As long as the price is not perfectly revealing, the traders will continue to use  $K$  as a signal of  $\theta$ . Therefore, the key source of our information-driven complementarity would still be present in this extension.

Furthermore, because our mechanism reduces the informativeness of aggregate investment, it also implies that the traders end up in equilibrium with less information: the conditional uncertainty faced by each trader,  $Var_i[\tilde{\theta}]$ , is higher. When this is the case, each trader will not only require a higher risk premium for holding the asset, but also react less to any private information she may have about  $\theta$ . The equilibrium price will then also do a worse job in aggregating this information, for it will be relatively more sensitive to other sources of aggregate noise. It follows that our mechanism may also reduce the informational efficiency of the financial market. On the positive side, this means that our mechanism may raise risk premia in financial markets and amplify their non-fundamental volatility. On the normative side, the increased uncertainty may also exacerbate the misallocation of the asset, which in turn would reinforce our welfare implications: the planner would like entrepreneurs to react more to the fundamentals and less to noise, not only for the reasons emphasized in the baseline model, but also because this would transmit more precise information to the financial market and thereby improve informational and allocative efficiency.

Next, suppose that we introduce a second round of real investment decisions, which takes place after the financial market closes. Now information would travel not only from the first round of investment to the financial market, but also from the latter to the second round of investment. This would capture the role of asset prices in guiding investment decisions by revealing valuable information that is dispersed in the marketplace and not directly available to corporate managers (e.g., Dow and Gorton, 1997, Subrahmanyam and Titman, 1999, Chen, Goldstein and Jiang, 2007). Importantly, our mechanism would then imply a deterioration in this functioning of financial markets. This is a direct implication of the argument made in the previous paragraph regarding the informational efficiency of asset prices.

Alternatively, suppose that we introduce a financial market before the first round of real investment. This market includes some informed traders, who may or may not be present at subsequent rounds of trading, as well as some uninformed liquidity traders, whose role is to preclude perfect information aggregation. Suppose further that the entrepreneurs observe some private signals about

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<sup>25</sup>Whether or not the price reveals  $\theta$  to the entrepreneurs is not crucial. The key is that the price does not perfectly reveal  $\theta$  to the traders, so that the traders continue to use  $K$  as a signal of  $\theta$ . For a full analysis of an extension along these lines, see Section 6.2 of the earlier version of this paper, Angeletos, Lorenzoni and Pavan (2007).

the trading positions of the informed traders; think of this assumption as a parable for the fact that investors and firm managers are often anxious to collect information about the positions taken by some key informed big players in financial markets.<sup>26</sup> Then, we could re-interpret some of the exogenous signals that the entrepreneurs receive in our model as imperfect private learning about the actions of these big players. In this case, the origin of the correlation in the entrepreneurs' signals—and thereby the initial source of “noise” or “exuberance” in our model—could well be the errors of these early traders.<sup>27</sup>

Furthermore, because these earlier traders may themselves have some private information about the sources of variation behind the signals they send to entrepreneurs and later traders, they may also be able to forecast the errors made by these subsequent agents. An information-driven complementarity similar to the one that emerges in the entrepreneurs' investment decisions may then emerge also in the traders' positions. This complementarity, in turn, is likely to be stronger the higher the degree of short-termism of the traders: the more the early traders' portfolio choices are driven by forecasts of future pricing errors, as opposed to forecasts of the fundamentals, the stronger the complementarity in their choices, much alike what happens in the case of the entrepreneurs in our model. An extension along these lines could thus not only reinforce our results, but also bring our analysis closer to Froot, Scharfstein, and Stein (1992), Allen, Morris and Shin (2006), and other papers that study the implications of short-termism in financial markets.

## 7 Conclusion

This paper examined the interaction between the real and the financial sector of a neoclassical economy with dispersed information about the profitability of a new investment opportunity. By conveying a positive signal about profitability, higher aggregate investment—or, more broadly, higher real activity—increases asset prices, which in turn raises the incentives to invest. This two-way feedback between real and financial activity makes real economic decisions sensitive to higher-order expectations and amplifies the impact of noise on equilibrium outcomes. As a result, economic agents may behave *as if* they were engaged in a Keynesian “beauty contest” and the economy may exhibit fluctuations that may appear in the eyes of an external observer *as if* they were the product

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<sup>26</sup>For the sake of this discussion, ignore the additional effects that may obtain when these big players attempt to manipulate asset prices and/or real economic activity.

<sup>27</sup>Indeed, suppose that the entrepreneurs, in addition to the market-clearing price in the first round of trading, observe two purely idiosyncratic signals, one about the fundamentals  $\theta$  and one about the position  $Q$  of the early informed traders. Let these signals be  $x_i = \theta + \xi_i$  and  $y_i = Q + \varsigma_i$  and impose that the noises  $\xi_i$  and  $\varsigma_i$  are independent across  $i$  and of any other random variable. Next, suppose that the equilibrium value of  $Q$  is a linear function of the early traders' average forecast of  $\theta$ , which in turn is a linear function of  $\theta$  itself and some noise  $\varepsilon$ :  $Q = \psi_0 + \psi_1\theta + \psi_2\varepsilon$ . The observation of the signal  $y_i$  is then informational equivalent to the observation of the signal  $x_{i2} \equiv \frac{1}{\psi_1}(y_i - \psi_0) = \theta + \frac{\psi_2}{\psi_1}\varepsilon + \frac{1}{\psi_1}\varsigma_i$ . Clearly, this is a private signal about  $\theta$ , whose error is correlated across the entrepreneurs. A mechanism similar to the one in the baseline model is therefore once again at play.

of “irrational exuberance.” Importantly, these effects are symptoms of inefficiency, are driven purely by the dispersion of information, and obtain in an otherwise conventional, neoclassical, setting.

While both irrationality and our mechanism can justify policy intervention, our approach does not rest on any presumption of superior wisdom on the government’s side. We showed how stabilization policies that are contingent *only* on publicly-available signals about the exogenous fundamentals and the endogenous economic activity can indirectly tax/subsidize the response of economic agents to different sources of information. Through a proper design of such contingencies, the government can dampen the impact of noise on equilibrium outcomes, improve welfare, and, in certain cases, even attain a certain constrained-efficiency target.

The effects analyzed in this paper are likely to be stronger during periods of intense technological or institutional change, when the information about the profitability of new investment opportunities is likely to be highly dispersed. At some level, this seems consistent with the recent experiences surrounding the internet revolution or the explosion of investment opportunities in emerging economies. Our mechanism may, however, also be relevant for ordinary cyclical fluctuations. Indeed, information regarding aggregate supply and demand conditions seems to be widely dispersed, as indicated by surveys of forecasts and by the financial markets’ anxiety preceding the release of key macroeconomic statistics. This opens the door to the possibility that effects similar to the ones documented in this paper may operate over the business cycle.

Finally, our analysis has left aside credit market frictions that make the availability of outside finance relevant for investment decisions. We have done so in order to focus on the informational spillover as the only source of amplification and inefficiency. However, the episodes of interest appear to involve important interactions between credit markets, asset prices, and investment. An extension of our model that introduces collateral constraints as in Kiyotaki and Moore (1997) may reveal additional amplification effects coming from the interaction of our mechanism with a credit multiplier. Episodes of “exuberance” may then manifest, not only in exuberant investment and asset prices, but also in exuberant credit booms.

## Appendix A: proofs

**Proof of Proposition 2.** The proof proceeds in three steps. Step 1 fills in the details of the equilibrium characterization in the main text. Step 2 analyzes the fixed point problem and proves existence of an equilibrium with  $\beta_x, \beta_y, \beta_\theta, \beta_\varepsilon, \gamma_\theta, \gamma_\varepsilon, \gamma_\eta > 0$ . Step 3 proves uniqueness of the equilibrium for  $\lambda$  small enough.

*Step 1.* First note that there exist no equilibria in which  $\beta_\theta = 0$ . Indeed, in any such equilibrium,  $K$  would convey no information to the financial market. The equilibrium price would then simply be equal to  $\mathbb{E}[\tilde{\theta}|\omega]$ . Because this is an increasing function of  $\theta$ , the entrepreneurs' best responses would then impose that they react positively to both signals, thus contradicting the assumption that  $\beta_\theta = 0$ . Next, note that there exists no equilibrium in which  $\beta_\varepsilon = 0$ . Indeed, in any such equilibrium,  $K$  would perfectly reveal  $\theta$  to the traders in which case the equilibrium price would be equal to  $\theta$ . But then again each entrepreneur would find it optimal to follow a linear strategy that responds positively to both  $x$  and  $y$ , contradicting the assumption that  $\beta_\varepsilon = 0$ . Hence, any equilibrium must satisfy  $\beta_\theta = \beta_x + \beta_y \neq 0$  and  $\beta_\varepsilon = \beta_y \neq 0$ .

From the analysis in the main text, we then have that in any equilibrium the price is given by (4) and the entrepreneurs' investment strategy is given by (5). Substituting (4) into (5), and using the facts that  $\pi_z = \pi_y/\varphi^2$ ,  $\mathbb{E}[\tilde{\theta}|x, y] = \mu + \delta_x(x - \mu) + \delta_y(y - \mu)$ ,  $\mathbb{E}[\tilde{\varepsilon}|x, y] = y - \mathbb{E}[\tilde{\theta}|x, y]$ , and  $\mathbb{E}[\tilde{\eta}|x, y] = 0$ , we have that the entrepreneurs' investment strategy is given by

$$k(x, y) = \beta_0 + \beta_x x + \beta_y y,$$

where the coefficients  $(\beta_0, \beta_x, \beta_y)$  are given by the following:

$$\beta_0 = (1 - \beta_x - \beta_y) \mu \tag{16}$$

$$\beta_x = \left( 1 - \lambda \varphi \frac{\pi_y + \varphi \pi_\theta}{\pi_y + \varphi^2 (\pi_\theta + \pi_\omega)} \right) \delta_x \tag{17}$$

$$\beta_y = \left( 1 + \lambda \varphi \frac{\pi_x + (1 - \varphi) \pi_\theta}{\pi_y + \varphi^2 (\pi_\theta + \pi_\omega)} \right) \delta_y \tag{18}$$

Any equilibrium must thus satisfy (16)-(18), along with

$$\varphi = \frac{\beta_y}{\beta_x + \beta_y}.$$

*Step 2.* To establish existence of an equilibrium in which  $\beta_x, \beta_y > 0$ , let  $b \equiv \beta_y/\beta_x$ . Dividing (18)

by (17) and noting that  $\varphi = b/(1+b)$  yields

$$b = \frac{\pi_y + \left(\frac{b}{1+b}\right)^2 (\pi_\theta + \pi_\omega) + \lambda \frac{b}{1+b} \left(\pi_x + \frac{1}{1+b} \pi_\theta\right) \delta_y}{\pi_y + \left(\frac{b}{1+b}\right)^2 (\pi_\theta + \pi_\omega) - \lambda \frac{b}{1+b} \left(\pi_y + \frac{b}{1+b} \pi_\theta\right) \delta_x}. \quad (19)$$

Using the definitions of  $\delta_x$ , and  $\delta_y$ , as in Proposition 1 one can then show that the right-hand side of (19) is equivalent to the following function of  $b$ :

$$F(b) \equiv \frac{\delta_y}{\delta_x} \left\{ 1 + \frac{\lambda(1+b)b}{[(1-\lambda)(1-\delta_x) + \Omega]b^2 + (2-\lambda)\delta_y b + \delta_y} \right\} \quad (20)$$

where  $\Omega \equiv \frac{\pi_\omega}{\pi_x + \pi_y + \pi_\theta} > 0$ . Noting that  $b$  is a monotone transformation of  $\varphi$ , we have that the mapping  $b' = F(b)$  identifies the mapping  $\varphi' = \Gamma(\varphi)$  mentioned in the main text.

It is easy to see that  $F$  is well defined and continuous over  $\mathbb{R}_+$ , with  $F(\delta_y/\delta_x) > \delta_y/\delta_x$  and  $\lim_{b \rightarrow +\infty} F(b)$  finite. It follows that  $F$  has at least one fixed point  $b > \delta_y/\delta_x$ . Given this value of  $b$ , existence of an equilibrium can be established by construction. First, the equilibrium value of  $\beta_y$  is obtained substituting  $\varphi = b/(1+b)$  into (18) and is clearly positive. Next, the equilibrium value of  $\beta_x$  is given by  $\beta_y/b$  and is also positive. Given  $\beta_x$  and  $\beta_y$ , the equilibrium value of  $\beta_0$  is given by (16). Finally, from the fact that  $\beta_\theta = \beta_x + \beta_y$  and  $\beta_\varepsilon = \beta_y$ , and from the formulas for  $\gamma_\theta$ ,  $\gamma_\varepsilon$  and  $\gamma_\eta$  in the main text, it is immediate to see that all these coefficients are also positive.

*Step 3.* To prove uniqueness, first notice that there exist no equilibria in which  $\beta_x = 0$ . This can be seen directly from (17). This in turn implies that all equilibria, irrespective of the sign of  $\beta_x$  and  $\beta_y$ , must correspond to a fixed point of the function  $F$  defined in (20).

Next, note there exists  $\lambda' > 0$  such that, for any  $\lambda \in [0, \lambda']$  the denominator in the fraction in the right-hand side of (20) is strictly positive, for any  $b \in \mathbb{R}$ . This implies that, when  $\lambda \in [0, \lambda']$ , the function  $F$  is defined and continuously differentiable over the entire real line, with

$$F'(b) = \lambda \frac{\delta_y [\delta_y - (1-\lambda)(1-\delta_x - \delta_y) - \Omega] b^2 + 2\delta_y b + \delta_y}{\delta_x \{[(1-\lambda)(1-\delta_x) + \Omega]b^2 + (2-\lambda)\delta_y b + \delta_y\}^2}$$

Moreover,

$$\lim_{b \rightarrow -\infty} F(b) = \lim_{b \rightarrow +\infty} F(b) = F_\infty \equiv \frac{\delta_y}{\delta_x} \left[ 1 + \frac{\lambda}{(1-\lambda)(1-\delta_x) + \Omega} \right] > \frac{\delta_y}{\delta_x}.$$

Thus, from now on, restrict attention to  $\lambda < \lambda'$ . We now need to consider two cases. First, suppose  $\delta_y = (1-\lambda)(1-\delta_x) + \Omega$ . The function  $F$  then has a global minimum at  $b = -1/2$ . In this case,  $F$  is bounded from below and above, respectively, by  $\underline{F} \equiv F(-1/2)$  and  $\overline{F} \equiv F_\infty$ . Second, suppose  $\delta_y \neq (1-\lambda)(1-\delta_x) + \Omega$ . Then  $F'(b)$  has two zeros, respectively at  $b = b_1$  and at  $b = b_2$ ,

where

$$b_1 \equiv \frac{-\delta_y - \sqrt{[(1-\lambda)(1-\delta_x - \delta_y) + \Omega]\delta_y}}{\delta_y - (1-\lambda)(1-\delta_x - \delta_y) - \Omega} \text{ and } b_2 \equiv \frac{-\delta_y + \sqrt{[(1-\lambda)(1-\delta_x - \delta_y) + \Omega]\delta_y}}{\delta_y - (1-\lambda)(1-\delta_x - \delta_y) - \Omega}.$$

When  $\delta_y \neq (1-\lambda)\delta_0 + \Omega$ , the function  $F$  then has a global minimum at  $\underline{F} \equiv F(b_2)$  and a global maximum at  $\overline{F} \equiv F(b_1)$ . It is easy to check that in all the cases considered both  $\underline{F}$  and  $\overline{F}$  converge to  $\delta_y/\delta_x$  as  $\lambda \rightarrow 0$ . But then  $F$  converges uniformly to  $\delta_y/\delta_x$  as  $\lambda \rightarrow 0$ . It follows that for any  $\varepsilon > 0$ , there exists a  $\hat{\lambda} \leq \lambda'$  so that, whenever  $\lambda < \hat{\lambda}$ ,  $F$  has no fixed point outside the interval  $[\delta_y/\delta_x - \varepsilon, \delta_y/\delta_x + \varepsilon]$ .

Now, with a slight abuse of notation, replace  $F(b)$  with  $F(b; \lambda)$ , to highlight the dependence of  $F$  on  $\lambda$ . Notice that  $\partial F(b; \lambda)/\partial b$  is continuous in  $b$  at  $(b; \lambda) = (\delta_y/\delta_x, 0)$  and  $\partial F(\delta_y/\delta_x; 0)/\partial b = 0$ . It follows that there exist  $\tilde{\varepsilon} > 0$  and  $\tilde{\lambda} \in (0, \hat{\lambda}]$  such that  $\partial F(b; \lambda)/\partial b < 1$  for all  $b \in [\delta_y/\delta_x - \tilde{\varepsilon}, \delta_y/\delta_x + \tilde{\varepsilon}]$  and  $\lambda \in [0, \tilde{\lambda}]$ . Combining these results with the continuity of  $F(\cdot; \lambda)$ , we have that there exist  $\bar{\varepsilon} > 0$  and  $\bar{\lambda} > 0$  such that, for all  $\lambda \in [0, \bar{\lambda}]$ , the following are true: for any  $b \notin [\delta_y/\delta_x - \bar{\varepsilon}, \delta_y/\delta_x + \bar{\varepsilon}]$ ,  $F(b; \lambda) \neq b$ ; for  $b \in [\delta_y/\delta_x - \bar{\varepsilon}, \delta_y/\delta_x + \bar{\varepsilon}]$ ,  $F$  is continuous and differentiable in  $b$ , with  $\partial F(b; \lambda)/\partial b < 1$ . It follows that, if  $\lambda \leq \bar{\lambda}$ ,  $F$  has at most one fixed point, which establishes the result. ■

**Proof of Proposition 3.** In any of the equilibria identified in Proposition 2, we have that  $\varphi \in (0, 1)$ . From conditions (17) and (18) in the proof of that proposition, it then follows that  $\beta_x < \delta_x$  and  $\beta_y > \delta_y$ . Moreover, the two inequalities imply

$$\frac{\varphi}{1-\varphi} = \frac{\beta_y}{\beta_x} > \frac{\delta_y}{\delta_x} = \frac{\pi_y}{\pi_x}. \quad (21)$$

Finally,

$$\begin{aligned} \beta_\theta &\equiv \beta_x + \beta_y \\ &= \delta_x + \delta_y + \lambda \frac{\varphi\pi_y}{\pi_\theta + \pi_x + \pi_y} \frac{\pi_x + (1-\varphi)\pi_\theta}{\pi_y + \varphi^2(\pi_\theta + \pi_\omega)} - \lambda \frac{\varphi\pi_x}{\pi_\theta + \pi_x + \pi_y} \frac{\pi_y + \varphi\pi_\theta}{\pi_y + \varphi^2(\pi_\theta + \pi_\omega)} \\ &= \delta_x + \delta_y + \lambda \frac{\varphi\pi_\theta}{\pi_\theta + \pi_x + \pi_y} \frac{(1-\varphi)\pi_y - \varphi\pi_x}{\pi_y + \varphi^2(\pi_\theta + \pi_\omega)} < \delta_x + \delta_y \end{aligned}$$

where the last inequality follows from (21). ■

**Proof of Proposition 4.** Consider the function  $F(b; \lambda)$  introduced in the proof of Proposition 2. For any  $\lambda \in [0, \bar{\lambda})$ , the function  $F(\cdot; \lambda)$  is continuously differentiable over  $\mathbb{R}$ . Take any pair  $\lambda', \lambda'' \in [0, \bar{\lambda})$  with  $\lambda'' > \lambda'$ , and let  $b'$  and  $b''$  be the unique solutions to  $F(b; \lambda) = b$ , respectively for  $\lambda = \lambda'$  and  $\lambda = \lambda''$  (existence and uniqueness of such solutions follows directly from Proposition 2). Furthermore, as shown in the proof of Proposition 2,  $F(b, \lambda') - b > 0$  for all  $b \in [0, b')$ . Simple

algebra then shows that  $\partial F(b; \lambda) / \partial \lambda \geq 0$  for any  $b \geq 0$ , with strict inequality if  $b > 0$ . It follows that  $b'' > b'$ . The result in the proposition then follows from the fact that  $\varphi \equiv b/(1+b)$  and the fact that the contribution of noise to aggregate volatility is an increasing function of  $\varphi$ . ■

**Proof of Proposition 5.** Consider the function  $F(b; \lambda, \delta_x, \delta_y, \Omega)$  introduced in the proof of Proposition 2; for convenience we are highlighting here the dependence on all parameters, with  $\Omega \equiv \frac{\pi_\omega}{\pi_x + \pi_y + \pi_\theta}$ . Take the parameters  $(\lambda, \delta_x, \delta_y, \Omega) = (.75, .2, .1, .1)$ . With these parameters the function  $F$  is defined and continuous over the entire real line and  $b_2 < b_1$ , where  $b_1$  and  $b_2$  are as defined in the proof of Proposition 2. Moreover, at the point  $b_2$ , we have that  $F(b_2; \lambda, \delta_x, \delta_y, \Omega) < b_2 < 0$ . These properties, together with the properties that  $F(0; \lambda, \delta_x, \delta_y, \Omega) > 0$  and  $\lim_{b \rightarrow -\infty} F(b; \lambda, \delta_x, \delta_y, \Omega) > 0 > -\infty$ , ensure that, in addition to a fixed point in  $(\delta_y/\delta_x, +\infty)$ ,  $F$  admits at least one fixed point in  $(-\infty, b_2)$  and one in  $(b_2, 0)$ . Furthermore, each of these three fixed point is “strict” in the sense that  $F(b) - b$  changes sign around them. Because  $F$  is continuous in  $(b; \lambda, \delta_x, \delta_y, \Omega)$  in an open neighborhood of  $(\lambda, \delta_x, \delta_y, \Omega) = (.75, .2, .1, .1)$ , there necessarily exists an open set  $B \subset (0, 1)^3 \times \mathbb{R}$  such that  $F$  admits at least three fixed points whenever  $(\lambda, \delta_x, \delta_y, \Omega) \in B$ . The result in the proposition then follows by noting that for any  $(\lambda, \delta_x, \delta_y, \Omega) \in B$ , there corresponds a unique set of parameters  $(\lambda, \pi_\theta, \pi_x, \pi_y, \pi_\omega) \in \mathbb{R}^5$ . ■

**Proof of Proposition 6.** Substituting condition (2) into condition (3) gives the equilibrium price as a linear function of aggregate investment and the traders’ exogenous signal:

$$p(\theta, \varepsilon, \eta) = \gamma_0 + \gamma_K K(\theta, \varepsilon) + \gamma_\omega \omega, \quad (22)$$

with

$$\gamma_0 \equiv \frac{\pi_\theta \mu - \pi_z \beta_0 / \beta_\theta}{\pi_\theta + \pi_\omega + \pi_z}, \quad \gamma_K \equiv \frac{\pi_z / \beta_\theta}{\pi_\theta + \pi_\omega + \pi_z}, \quad \text{and} \quad \gamma_\omega \equiv \frac{\pi_\omega}{\pi_\theta + \pi_\omega + \pi_z}.$$

Clearly,  $\gamma_K > 0$  if and only if  $\beta_\theta > 0$ , which means that high investment is a positive signal about  $\theta$ . The result then follows from substituting (22) into the entrepreneurs’ optimality condition (5) and letting  $\kappa_0 \equiv \lambda \gamma_0$ ,  $\kappa_\theta \equiv 1 - \lambda + \lambda \gamma_\omega$  and  $\alpha \equiv \lambda \gamma_K$ . ■

**Proof of Proposition 7.** This follows directly from the main text. ■

**Proof of Proposition 8.** Consider the first claim, namely that a high  $\tau_p$  reduces the relative contribution of noise. Substituting the price (11) into the entrepreneurs’ best response (5) and using (3), one obtains a system of equations for  $\beta_0$ ,  $\beta_x$  and  $\beta_y$ , as in the proof of Proposition 2. Following similar steps as in the proof of that proposition, it is possible to show that a equilibrium



is characterized by a ratio  $b = \beta_y/\beta_x$  that satisfies  $b = F(b; \Psi)$  where

$$F(b; \Psi) \equiv \frac{\delta_y}{\delta_x} \left[ 1 + \frac{b(1+b)\lambda\Psi}{((1-\lambda)(\delta_0 + \delta_y + \Omega) + \lambda\Omega\Psi)b^2 + (2(1-\lambda) + \lambda\Psi)\delta_y b + (1-\lambda + \lambda\Psi)\delta_y} \right]$$

with  $\Psi \equiv 1/(1 + \tau_p)$ . Following steps similar to those in the proof of Proposition 2, one can then easily see that (i) there always exists a solution to  $F(b; \Psi) = b$  with  $b > \delta_y/\delta_x$  and (ii) that, starting from such a solution, one can construct an equilibrium in which  $\beta_x, \beta_y > 0$ . Furthermore, for any  $b > 0$ ,  $F(b; \cdot)$  is increasing in  $\Psi$ . We thus conclude that, as long as the equilibrium is unique, the equilibrium value of  $b$  is increasing in  $\Psi$ . Equivalently, the ratio  $\varphi = \frac{\beta_\varepsilon}{\beta_\theta + \beta_\varepsilon}$ , and hence the relative contribution of noise to aggregate activity, is decreasing in  $\tau_p$ . Lastly, following steps similar to those in the proof of Proposition 2, one can also verify that the equilibrium is indeed unique when  $\lambda$  and  $\tau_p$  are small enough.

Next, the claim that a positive  $\tau_p$  may increase welfare can be established by numerical example; see footnote 19. Finally, consider the last claim, namely that there is no policy as in (10) that can implement the constrained efficient allocation as an equilibrium. Towards delivering a contradiction, suppose that the opposite is true. Comparing the entrepreneurs' equilibrium best responses (6) with the efficiency condition (9), one can immediately see that efficiency requires that the following condition holds:

$$\mathbb{E} \left[ \tilde{p} - \tilde{\theta} \mid x, y \right] = 0 \text{ for all } x, y.$$

Substituting the equilibrium price (11), this condition can be rewritten as

$$\mathbb{E} \left[ \frac{1}{1 + \tau_p} \left( \mathbb{E}[\tilde{\theta} \mid \tilde{K}, \tilde{\omega}] - \tau_0 \right) - \tilde{\theta} \mid x, y \right] = 0 \text{ for all } x, y. \quad (23)$$

By Proposition 7, the fact that the policy implements the efficient allocation in turn implies that  $K = \delta_0 + \delta_\theta \theta + \delta_\varepsilon \varepsilon$ . This implies that  $\mathbb{E}[\tilde{\theta} \mid K, \omega] = \gamma_0 + \gamma_K K + \gamma_\omega \omega$  where the coefficients  $(\gamma_0, \gamma_K, \gamma_\omega)$  are as in Section 4 with  $\beta_0 = \delta_0$ ,  $\beta_x = \delta_x$ , and  $\beta_y = \delta_y$ . Therefore, (23) can be rewritten as

$$\mathbb{E} \left[ \frac{1}{1 + \tau_p} \left[ \gamma_0 + \gamma_K \left( \delta_0 + \delta_\theta \tilde{\theta} + \delta_\varepsilon \tilde{\varepsilon} \right) + \gamma_\omega \tilde{\omega} - \tau_0 \right] - \tilde{\theta} \mid x, y \right] = 0.$$

Taking unconditional expectations, one can then see that  $\tau_0$  and  $\tau_p$  must satisfy

$$\frac{1}{1 + \tau_p} \left[ \gamma_0 + \gamma_K (\delta_0 + \delta_\theta \mu) + \gamma_\omega \mu - \tau_0 \right] - \mu = 0.$$

Subtracting side by side the last two equations, after some manipulation, one obtains that

$$\left[ \frac{1}{1 + \tau_p} (\gamma_K \delta_\theta + \gamma_\omega) - 1 \right] \mathbb{E} \left[ \tilde{\theta} - \mu \mid x, y \right] + \frac{1}{1 + \tau_p} \gamma_K \delta_\varepsilon \mathbb{E} \left[ \tilde{\varepsilon} \mid x, y \right] = 0.$$

Substituting for the terms in expectation yields

$$\begin{aligned} & \left[ \frac{1}{1 + \tau_p} (\gamma_K \delta_\theta + \gamma_\omega) - 1 \right] [\delta_x(x - \mu) + \delta_y(y - \mu)] \\ & + \frac{1}{1 + \tau_p} \gamma_K \delta_\varepsilon [y - \mu - \delta_x(x - \mu) - \delta_y(y - \mu)] = 0 \text{ for all } x, y. \end{aligned}$$

Given that this condition must hold for all  $x$  and  $y$ , it must be that

$$\begin{aligned} \frac{1}{1 + \tau_p} (\gamma_K \delta_\theta + \gamma_\omega) - 1 - \frac{1}{1 + \tau_p} \gamma_K \delta_\varepsilon &= 0, \\ \left[ \frac{1}{1 + \tau_p} (\gamma_K \delta_\theta + \gamma_\omega) - 1 \right] \delta_y + \frac{1}{1 + \tau_p} \gamma_K \delta_\varepsilon (1 - \delta_y) &= 0. \end{aligned} \tag{24}$$

Substituting the first condition into the second gives

$$\frac{1}{1 + \tau_p} \gamma_K \delta_\varepsilon = 0.$$

This last condition cannot be true given that, when investment is efficient,  $\gamma_K$  is necessarily positive, and given that  $\delta_\varepsilon > 0$  and  $\tau_p$  must be finite to ensure that (24) is satisfied. Therefore, there is a contradiction. We conclude that a simple stabilization policy as then one given in (10) cannot implement the constrained efficient allocation as a competitive equilibrium. ■

**Proof of Proposition 9.** To establish the result, it suffices to show that there exists a policy of the type given in (12) such that, under this policy, there exists a competitive equilibrium in which

$$\mathbb{E}[\tilde{p} - \tilde{\theta} | x, y] = 0 \text{ for all } x, y. \tag{25}$$

To see that this is indeed the case, note that, under any policy as in (12), the equilibrium price must satisfy  $p = \mathbb{E}[\tilde{\theta} | K, \omega] - \tau_0 - \tau_p p - \tau_K K$ . Equivalently,

$$p = \frac{1}{1 + \tau_p} \left[ \mathbb{E}[\tilde{\theta} | K, \omega] - \tau_0 - \tau_K K \right]. \tag{26}$$

Next note that if the policy  $(\tau_0, \tau_p, \tau_K)$  implements the constrained efficient allocation, then

$$\mathbb{E}[\tilde{\theta} | K, \omega] = \gamma_0 + \gamma_K K + \gamma_\omega \omega \tag{27}$$

with coefficients  $(\gamma_0, \gamma_K, \gamma_\omega)$  as in Section 4 with  $\beta_0 = \delta_0$ ,  $\beta_x = \delta_x$ , and  $\beta_y = \delta_y$ . Replacing (27) into (26), one can then easily see that the policy with coefficients

$$\tau_0 = \gamma_0, \quad \tau_K = \gamma_K, \quad \tau_p = \gamma_\omega - 1.$$

is such that  $p - \theta = \omega - \theta = \eta$ . Since the entrepreneurs possess no information on the shock  $\eta$  at time 1, we then have that  $\mathbb{E}[\bar{\eta}|x, y] = 0$  which verifies that, under the identified policy, condition (25) is satisfied. Finally, note that  $\tau_K > 0$  and  $\tau_p < 0$ ; this last result follows from the definition of  $\gamma_\omega$ , which implies that  $\gamma_\omega \in (0, 1)$ . ■

## Appendix B: a generalized model

This appendix considers a generalization of our model that helps illustrate the robustness of our key positive and normative results (as stated in Corollaries 1 and 2) and also forms the basis for some of the related claims made in the main text.

*Information structure.* Suppose that the entrepreneurs observe  $S > 1$  signals about  $\theta$ . Index these signals by  $s \in \{1, \dots, S\}$  and write them as  $x_{is} = \theta + \xi_{is}$ , where  $\xi_{is}$  is the error in the  $s$ -th signal observed by entrepreneur  $i$ . Let  $\rho_s \equiv \text{Corr}(\xi_{is}, \xi_{js})$ , for  $i \neq j$ , denote the correlation in the  $s$ -th signal. Finally, let  $\mathbb{E}_i[\cdot]$  denote the expectation conditional on the information available to entrepreneur  $i$ , i.e., given the signals  $x_i \equiv (x_{i1}, \dots, x_{iS})$ .

*Payoff structure.* For a surviving entrepreneur, the value of holding  $k$  units of capital in period 3 is  $V^e(\theta, K, k)$ . An entrepreneur's payoff is then  $V^e(\theta, K, k) - k^2/2$  if he is not hit by a liquidity shock, and  $pk - k^2/2$  otherwise. For a trader, on the other hand, the value of holding  $q$  units of capital is  $V^t(\theta, K, q)$ , so that his payoff is  $V^t(\theta, K, q) - pq$ . The functions  $V^e$  and  $V^t$  are meant to capture the reduced-form payoffs that the agents may obtain through a variety of market interactions outside the focus of our analysis. For example, in the variant considered in Section 6.2, these payoffs summarize the interaction of the agents in a competitive labor market during the production stage. To maintain tractability, we impose that these functions are linear-quadratic. To guarantee that individual decision problems are concave, we further impose that  $V_{kk}^e \leq 0$  and  $V_{qq}^t \leq 0$ . We also let  $V_{k\theta}^e > 0$  and  $V_{q\theta}^t > 0$ , so that higher  $\theta$  is interpreted as better fundamentals. Finally, we assume that

$$(1 - \lambda)V_K^e(\theta, K, K) + V_K^t(\theta, K, Q) = 0 \quad (28)$$

where  $Q = \lambda K$ . This assumption is motivated by the following considerations. The external effects featured in the reduced-form payoffs  $V^e$  and  $V^t$  are meant to capture only the pecuniary externalities that emerge in certain market interactions, like in the case of the labor market in Section 6.2. In Walrasian settings, such pecuniary externalities need not be the source of any inefficiency: they often wash out at the aggregate. We impose (28) only in order to capture this idea, and thereby to isolate the information spillover as the only source of inefficiency.

*Equilibrium and efficiency.* The following proposition provides a characterization of the equilibrium and efficient allocations for the more general model described above.

**Proposition 10** (i) For any equilibrium, there exist scalars  $\kappa_0$ ,  $\kappa_\theta$ , and  $\alpha$  (these scalars depend on the payoff structure, the information structure, and the particular equilibrium) such that investment satisfies the following:

$$k_i = \mathbb{E}_i[\kappa_0 + \kappa_\theta \tilde{\theta} + \alpha \tilde{K}]. \quad (29)$$

(ii) There exists scalars  $\kappa_0^*$ ,  $\kappa_\theta^*$ , and  $\alpha^*$  (these scalars depend only on the payoff structure) such that, in the unique constrained efficient allocation, investment satisfies the following:

$$k_i = \mathbb{E}_i[\kappa_0^* + \kappa_\theta^* \tilde{\theta} + \alpha^* \tilde{K}]. \quad (30)$$

(iii) The relative contribution of noise to aggregate volatility is higher in equilibrium than in the constrained-efficient allocation if and only if  $\alpha > \alpha^*$ .

(iv) There exists a constant  $\psi > 0$  such that, for any equilibrium,

$$\alpha = \alpha^* + \psi \lambda \frac{\partial \mathbb{E}_t[\tilde{\theta}]}{\partial K}. \quad (31)$$

By implication,  $\alpha > \alpha^*$  if and only if high investment serves as a positive signal about  $\theta$ .

(v) The equilibrium is constrained efficient if and only if there are no information spillovers.

Parts (i) and (ii) provide a certain isomorphism between the generalized economy of this appendix and the class of games studied in Angeletos and Pavan (2007, 2009). The scalar  $\alpha$  can then be interpreted as the degree of complementarity featured in any given equilibrium, while  $\alpha^*$  can be interpreted as the degree of complementarity featured in the efficiency allocation. Part (iii) points out that comparing the relative impact of noise in any equilibrium with that in the efficient allocation is synonymous to comparing  $\alpha$  with  $\alpha^*$ . Part (iv) then establishes that the only source of discrepancy between  $\alpha$  with  $\alpha^*$  is the information spillover, while part (v) points out that the equilibrium is efficient when and only when the information spillover vanishes. The combination of these results then imply that the key positive and normative results of the baseline model, as stated in Corollaries 1 and 2, continue to hold in the more general model consider here: as long as high investment is “good news” about the underlying fundamentals, the relative impact of noise on equilibrium investment is both higher than what it would have been in the absence of information spillovers (amplification) and excessive from a social perspective (inefficiency).

**Proof of Proposition 10.** *Part (i).* Clearly, the equilibrium price must satisfy

$$p = \mathbb{E}_t[V_q^t(\tilde{\theta}, K, \lambda K)]$$

where  $\mathbb{E}_t$  denotes the traders’ expectation, given their available information. By implication, the

equilibrium level of investment must satisfy

$$k_i = \mathbb{E}_i[w(\tilde{\theta}, \tilde{K}, k_i)] + \lambda \mathbb{E}_i \left[ \mathbb{E}_t[V_q^t(\tilde{\theta}, \tilde{K}, \lambda \tilde{K})] - V_q^t(\tilde{\theta}, \tilde{K}, \lambda \tilde{K}) \right], \quad (32)$$

where  $\mathbb{E}_i$  denotes the expectation of entrepreneur  $i$  and where

$$w(\theta, K, k) \equiv (1 - \lambda)V_k^e(\theta, K, k) + \lambda V_q^t(\theta, K, \lambda K).$$

Note that this condition must hold irrespective of whether there are information spillovers from the real sector to the financial market; in fact, this condition holds for any information structure.

Because the functions  $V^e$  and  $V^t$  are both linear-quadratic, the function  $w$  is itself linear:

$$w(\theta, K, k) = w_0 + w_\theta \theta + w_K K + w_k k, \quad (33)$$

where  $w_0, w_\theta, w_K, w_k$  are scalars pinned down by the payoff structure, with  $w_\theta \equiv (1 - \lambda)V_{k\theta}^e + \lambda V_{q\theta}^t > 0$  and  $w_k \equiv V_{kk}^e \leq 0$ . Next, because  $K$  is known to the traders, and because the function  $V_q^t$  is linear, we have that the traders' error in forecasting their valuations is proportional to their error in forecasting  $\theta$ :

$$\mathbb{E}_t[V_q^t(\tilde{\theta}, K, \lambda K)] - V_q^t(\theta, K, \lambda K) = V_{q\theta}^t \cdot (\mathbb{E}_t[\tilde{\theta}] - \theta) \quad (34)$$

Substituting (33) and (34) into (32), we have that, in equilibrium, the investment strategy must satisfy

$$k_i = \mathbb{E}_i \left[ \frac{w_0}{1 - w_k} + \frac{w_\theta}{1 - w_k} \tilde{\theta} + \frac{w_K}{1 - w_k} \tilde{K} + \lambda \psi (\mathbb{E}_t[\tilde{\theta}] - \tilde{\theta}) \right] \quad (35)$$

where

$$\psi \equiv \frac{V_{q\theta}^t}{1 - w_k} > 0.$$

Next, note that, in any equilibrium, the entrepreneurs' investment strategy is given by

$$k_i = \beta_0 + \sum_{s=1}^S \beta_s x_{i,s}$$

for some scalars  $\beta_0, \beta_1, \dots, \beta_S$ . By implication, aggregate investment is given by

$$K = \beta_0 + \beta_\theta (\theta + \varepsilon)$$

where  $\beta_\theta \equiv \sum_{s=1}^S \beta_s$  and where  $\varepsilon \equiv \sum_{s=1}^S \frac{\beta_s}{\beta_\theta} \varepsilon_s$  is a weighted average of the correlated errors in the entrepreneurs' signals. It follows that, in the eyes of the traders,  $K$  is a Gaussian signal of  $\theta$ , which

in turn implies that their forecast of  $\theta$  can be written as follows:

$$\mathbb{E}_t[\tilde{\theta}] \equiv \mathbb{E}[\tilde{\theta}|\omega, K] = \gamma_0 + \gamma_\omega\omega + \gamma_K K \quad (36)$$

where  $\omega$  is the exogenous information of the traders and where  $\gamma_0, \gamma_\omega, \gamma_K$  are scalars, with  $\gamma_K > 0$  if and only if  $\beta_\theta > 0$ . Next, note that, since the entrepreneurs have no information about the error in the traders' exogenous signal  $\omega$ , their forecast of  $\tilde{\omega}$  coincides with their forecast of  $\tilde{\theta}$ :  $\mathbb{E}_i[\tilde{\omega}] = \mathbb{E}_i[\tilde{\theta}]$ . Substituting (36) into (35) and using  $\mathbb{E}_i[\tilde{\omega}] = \mathbb{E}_i[\tilde{\theta}]$ , we conclude that the investment strategy must indeed satisfy condition (29), with the scalars  $\kappa_0, \kappa_\theta$  and  $\alpha$  defined as follows:

$$\kappa_0 \equiv \frac{w_0}{1-w_k} + \lambda\psi\gamma_0, \quad \kappa_\theta \equiv \frac{w_\theta}{1-w_k} + \lambda\psi(\gamma_\omega - 1), \quad \text{and} \quad \alpha \equiv \frac{w_K}{1-w_k} + \lambda\psi\gamma_K \quad (37)$$

*Part (ii).* Consider the constrained efficient allocation. First, note that, irrespective of the information structure and irrespective of the investment strategy at  $t = 1$ , the planner always finds it optimal to allocate the supply of capital  $\lambda K$  at  $t = 2$  uniformly across the traders:  $q_i = \lambda K$  for all  $i \in (1/2, 1]$ . This is a direct implication of the concavity of payoffs with respect to  $q$ . It follows that the welfare objective is given by

$$W = \mathbb{E} \left[ -\frac{1}{2}\tilde{k}^2 + (1-\lambda)V^e(\tilde{\theta}, \tilde{K}, \tilde{k}) + V^t(\tilde{\theta}, \tilde{K}, \lambda\tilde{K}) \right]$$

Clearly, this is the same as welfare in a variant economy where there is only one class of agents, say the entrepreneurs, whose payoffs are given by

$$U = -\frac{1}{2}k^2 + (1-\lambda)V^e(\theta, K, k) + V^t(\theta, K, \lambda K)$$

This variant economy is nested in the class of economies studied in Angeletos and Pavan (2007, 2009). Following similar steps as in the proof of Proposition 2 in Angeletos and Pavan (2007), it is easy to check that the constrained efficient allocation is pinned down by the following condition:

$$\mathbb{E}_i \left[ U_k(\tilde{\theta}, \tilde{K}, k_i) + U_K(\tilde{\theta}, \tilde{K}, \tilde{K}) \right] = 0$$

Using the definition of  $U$  in our setting, the above can be rewritten as follows:

$$\mathbb{E}_i \left[ -k_i + (1-\lambda)V_k^e(\tilde{\theta}, \tilde{K}, k_i) + (1-\lambda)V_K^e(\tilde{\theta}, \tilde{K}, \tilde{K}) + V_K^t(\tilde{\theta}, \tilde{K}, \lambda\tilde{K}) + \lambda V_q^e(\tilde{\theta}, \tilde{K}, \lambda\tilde{K}) \right] = 0$$

or, equivalently,

$$k_i = \mathbb{E}_i \left[ w(\tilde{\theta}, \tilde{K}, k_i) + (1-\lambda)V_K^e(\tilde{\theta}, \tilde{K}, \tilde{K}) + V_K^t(\tilde{\theta}, \tilde{K}, \lambda\tilde{K}) \right]$$

Under the assumption introduced in (28), the above reduces to

$$k_i = \mathbb{E}_i[w(\tilde{\theta}, \tilde{K}, k_i)]. \quad (38)$$

Using (33), we conclude that the efficient investment must indeed satisfy (30), with the scalars  $\kappa_0^*$ ,  $\kappa_\theta^*$  and  $\alpha^*$  defined as follows:

$$\kappa_0^* \equiv \frac{w_0}{1 - w_k}, \quad \kappa_\theta^* \equiv \frac{w_\theta}{1 - w_k}, \quad \text{and} \quad \alpha^* \equiv \frac{w_K}{1 - w_k}. \quad (39)$$

Finally, existence and uniqueness of the fixed point to condition (30) follows from essentially the same arguments as in the proof of Proposition 1 of Angeletos and Pavan (2009).

*Part (iii).* The preceding parts imply that the equilibrium and efficient allocations of our economy can be understood as the equilibrium and efficient allocations of the class of games studied in Angeletos and Pavan (2009). Part (iii) then follows from Proposition 3 of that paper.

*Part (iv).* This part follows directly from the definition of  $\alpha$  in (37) and  $\alpha^*$  in (39), along with the observation that, in any given equilibrium,  $\gamma_K$  gives the slope of  $\mathbb{E}_t[\tilde{\theta}]$  with respect to  $K$  and  $\gamma_K > 0$  if and only if  $\beta_\theta > 0$ .

*Part (v).* Consider the case with no information spillovers, which is nested by letting the information  $\omega$  that the traders possess be a sufficient statistics for the entire information that the entrepreneurs collectively possess. In this case,  $\mathbb{E}_t[\tilde{\theta}|\omega, K] = \mathbb{E}_t[\tilde{\theta}|\omega]$  and  $\mathbb{E}_i[\mathbb{E}_t[\tilde{\theta}] - \tilde{\theta}] = 0$ , irrespective of the investment strategy. From (35), one can then immediately see that the equilibrium allocation coincides with the constrained efficient allocation and, by implication, is also unique. Conversely, consider the case with information spillovers. That the efficient allocation cannot be an equilibrium follows directly from part (v) along with the fact that investment is a positive signal of  $\theta$  along the efficient allocation. ■

## References

- [1] Abel, Andrew B., and Olivier J. Blanchard (1986), "The Present Value of Profits and Cyclical Movements in Investment," *Econometrica* 54, 249-273.
- [2] Allen, Franklin, Stephen Morris, and Hyun Song Shin (2006), "Beauty Contests and Iterated Expectations," *Review of Financial Studies* 19, 161-177.
- [3] Amador, Manuel, and Pierre-Olivier Weill (2008), "Learning from Private and Public observations of Others' Actions," Stanford University/UCLA mimeo.

- [4] Amador, Manuel, and Pierre-Olivier Weill (2009), "Learning from Prices: Public Communication and Welfare," Stanford University/UCLA mimeo.
- [5] Angeletos, George-Marios, and Jennifer La'O (2009a), "Noisy Business Cycles," *NBER Macroeconomics Annual 2009*.
- [6] Angeletos, George-Marios, and Jennifer La'O (2009b), "Sentiments," MIT mimeo.
- [7] Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan (2007), "Wall Street and Silicon Valley: A Delicate Interaction," NBER Working Paper 13475.
- [8] Angeletos, George-Marios, and Alessandro Pavan (2007), "Efficient Use of Information and Social Value of Information," *Econometrica* 75, 1103-1142.
- [9] Angeletos, George-Marios, and Alessandro Pavan (2009), "Policy with Dispersed Information," *Journal of the European Economic Association*, 7(1), 11-60.
- [10] Bacchetta, Philippe, and Eric van Wincoop (2005), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review* 95.
- [11] Banerjee, Abhijit (1992), "A Simple Model of Herd Behavior," *Quarterly Journal of Economics* 107, 797-817.
- [12] Barlevy, G., and P. Veronesi (2003), "Rational Panics and Stock Market Crashes," *Journal of Economic Theory* 110, 234-263.
- [13] Bernanke, Ben S., and Mark Gertler (2001), "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review* 91, 253-257.
- [14] Caballero, Ricardo, Emmanuel Farhi, and Mohamad Hammour (2006), "Speculative Growth: Hints from the U.S. Economy," *American Economic Review* 96, 1159-1192.
- [15] Cecchetti, Stephen, Hans Genberg, John Lipsky, and Sushil Wadhvani (2000), *Asset Prices and Central Bank Policy*, London, International Center for Monetary and Banking Studies.
- [16] Cespa, Giovanni, and Xavier Vives (2009), "Dynamic Trading and Asset Prices: Keynes vs. Hayek," mimeo IESE Business School and University of Salerno.
- [17] Chari, V.V., and Patrick Kehoe (2003), "Hot Money," *Journal of Political Economy* 111, 1262-1292.
- [18] Chen, Qi, Itay Goldstein, and Wei Jiang (2007), "Price Informativeness and Investment Sensitivity to Stock Price," *Review of Financial Studies* 20, 619-650.



- [19] Cutler, David M., James M. Poterba, and Lawrence H. Summers (1989), "What Moves Stock Prices?" *The Journal of Portfolio Management* 15, 4-12.
- [20] Diamond, Douglas W., and Philip H. Dybvig (1983), "Bank runs, deposit insurance, and liquidity" *Journal of Political Economy* 91, 401-19.
- [21] Dow, James, and Gary Gorton (1997), "Stock Market Efficiency and Economic Efficiency: Is There a Connection?," *Journal of Finance* 52, 1087-1129.
- [22] Dow, James, Goldstein Itai, and Alexander Guembel, (2010), "Incentives for Information Production in Markets where Prices Affect Real Investment," University of Pennsylvania/Wharton mimeo.
- [23] Dupor, William (2005), "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information," *Journal of Monetary Economics* 62, 727-747.
- [24] Froot, Kenneth, David Scharfstein, and Jeremy Stein, (1992), "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation," *The Journal of Finance*.
- [25] Gennotte, G., and H. Leland (1990), "Market Liquidity, Hedging and Crashes," *American Economic Review* 80, 999-1021.
- [26] Gilchrist, Simon, Charles Himmelberg, and Gur Huberman (2005), "Do stock price bubbles influence corporate investment?" *Journal of Monetary Economics*, 52, 805-827.
- [27] Goldstein, Itay, and Alexander Guembel (2008), "Manipulation and the Allocational Role of Prices," *Review of Economic Studies*, 75(1), 133-164.
- [28] Goldstein, Itay, Emre Ozdenoren and Kathy Yuan (2009) "Trading Frenzies and Their Impact on Real Investment," University of Pennsylvania/LSE/LBS mimeo.
- [29] Grossman, Sanford and Joseph Stiglitz (1976) "Information and Competitive Price Systems," *American Economic Review*, 66(2), 246-53.
- [30] Hellwig, Martin (1980), "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22(3), 477-498.
- [31] Hellwig, Christian (2006), "Monetary Business Cycle Models: Imperfect Information," in S. N. Durlauf and L. E. Blume (eds.), *New Palgrave Dictionary of Economics*, 2nd edition.
- [32] Hellwig, Christian, and Laura Veldkamp (2008), "Knowing What Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, forthcoming.

- [33] Keynes, John Maynard (1936), *The General Theory of Employment, Interest and Money*, Macmillan, London.
- [34] Kyle, Albert (1985), "Continuous Auctions and Insider Trading," *Econometrica*, 53(6), 1315-35.
- [35] La'O, Jennifer (2010), "*Collateral Constraints and Noisy Fluctuations*," mimeo MIT.
- [36] Loisel, Olivier, Franck Portier, and Aude Pommeret (2009), "Monetary Policy and Herd Behavior in New-Tech Investment," Banque de France/UniversitŽ de Lausanne/Toulouse School of Economics mimeo.
- [37] Lorenzoni, Guido (2010), "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information," *Review of Economic Studies* 77, 305-338.
- [38] Mackowiak, Bartosz, and Mirko Wiederholt (2009), "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, forthcoming.
- [39] Mankiw, N. Gregory, and Ricardo Reis (2009), "Imperfect Information and Aggregate Supply," paper prepared for the *Handbook of Monetary Economics*.
- [40] Myatt, David, and Christopher Wallace (2009), "Endogenous Information Acquisition in Coordination Games," w.p. Nuffield College, Oxford University.
- [41] Morris, Stephen, and Hyun Song Shin (2002), "The Social Value of Public Information," *American Economic Review* 92, 1521-1534.
- [42] Obstfeld, Maurice (1996), "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review* 40, 1037-47.
- [43] Panageas, Stavros (2005), "The Neoclassical q Theory of Investment in Speculative Markets," Wharton School mimeo.
- [44] Scheinkman, Jose, and Wei Xiong (2003), "Overconfidence and Speculative Bubbles," *Journal of Political Economy* 111, 1183-1219.
- [45] Shiller, Robert (2000), *Irrational Exuberance*, Princeton University Press, Princeton.
- [46] Subrahmanyam, Avandihar, and Sheridan Titman (1999). "The Going-Public Decision and the Development of Financial Markets," *Journal of Finance* 54, 1045-1082.
- [47] Spence, Michael (1973), "Job Market Signaling," *Quarterly Journal of Economics* 87 (3): 355-374.

- [48] Veldkamp, Laura (2006), "Media Frenzies in Markets for Financial Information," *American Economic Review* 96, 577-601.
- [49] Veldkamp, Laura (2009), "Information Choice in Macroeconomics and Finance," Manuscript.
- [50] Vives, Xavier (1997), "Learning from Others: a Welfare Analysis," *Games and Economic Behavior* 20, 177-200.
- [51] Vives, Xavier (2008), *Information and Learning in Markets*, Princeton University Press.

