CHAPTER 133

BED LOAD TRANSPORT DUE TO NON-LINEAR WAVE MOTION

by

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ABSTRACT

The bed load transport rate due to wave motion is measured in a wave flume. The modified stream function theory of the author (Tanaka (1988)) is applied to the formulation of the sediment transport rate in order to include the non-linearity. The proposed formula predicts well except near the surf zone where the effect of the acceleration plays an important role.

1. INTRODUCTION

Coastal sediment movement in the onshore-offshore direction has been studied by several researchers (e.g. Horikawa (1988)). Many of empirical formulas express the sand transport rate as a function of the bottom shear stress due to wave motion, which is usually estimated through the friction coefficient proposed by Jonsson (1966). His result is, in principle, only applicable to the linear wave motion. On the other hand, the non-linearity of wave motion has a considerable effect on the sediment transport in practice, especially in regard to both the direction and the rate of the net transport. Therefore, the non-linear effect of wave motion should be included in the formulation of sediment transport rate.

Up to present, however, no practical method had been available for estimation of the bottom shear stress due to non-linear wave motion, until the present author (Tanaka (1988)) made the effect of turbulence included in the stream function theory proposed by Dean (1965) for the non-linear wave motion. This modified stream function theory which enables to predict the bottom shear stress as well as the velocity profile is applied to the formulation of the bed load transport rate due to the non-linear wave motion.

The present study is restricted to the case of the

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wave motion alone, although the author's modified stream function theory can be extensively applied to the wave-current combined motion.

2. EXPERIMENTAL PROCEDURE

In the present study, two series of experiments were carried out: bed load transports over a flat bed (Experiment A) and over a sloping beach (Experiment B). The experimental setups are shown in Figs.1 (a) and (b).

The series of Experiment A were carried out in a wave flume, 16m long, 30cm wide and 50cm deep. A part of the bottom was lowered at the middle of the flume, on which the sand of mean diameter of 0.2 mm or 0.51 mm was filled to form a layer 5cm thick and 1.5m long. The experiments were carried out for 15 different wave and sediment conditions. The experimental condition was so carefully chosen as for the sand to be transported as bed load in the onshore direction. Then, the sand transported from the test bed onto the shoreward fixed bed was collected to obtain the net transport rate. The surface of the fixed bed was roughened with the pasted sand of the same size in order to make the mobility of the sand grain uniform along the wave flume.

In the series of Experiment B, a beach of slope 1:20 was built with the sand of diameter 0.78 mm at the end of the wave flume. The beach was exposed to the wave action for 10 minutes. The change of the beach profile was measured and used to calculate the sand transport rate. The direct measurement of the sand transport rate was also carried out by use of a sand trap originated by Horikawa, Sunamura and Shibayama (1977). In this case, the wave period and equivalent deep water wave height were fixed as 1.52 sec and 11.8 cm, respectively.

3. BOTTOM SHEAR STRESS DUE TO NON-LINEAR WAVE MOTION

Although the modified stream function theory can be referred in the preceding paper (Tanaka (1988)), a brief description of the theory and examples of the calculated result will be given below.

Combining Dean's stream function theory (Dean (1965)) and the time-invariant eddy viscosity model (Tanaka and Shuto (1981)), the stream function can be given by the following expression (Tanaka (1988)).

$$\psi(x,z)=\frac{L}{T}z-\frac{u_c^{*2}}{\kappa u_{cw}^{*}}z\{\ln\left(\frac{z}{z_Q}\right)-1\}+\sum_{n=1}^{N}\sinh\{nk(z-z_o)\}\{X(n)\cos(nkx)+Y(n)\sin(nkx)\}$$

$$+\,\mathrm{Real}\left[\sum_{n=1}^{N}\frac{ik\xi_n}{2c}\frac{H_1^{(1)}(\xi_n)}{H_o^{(1)}(\xi_{no})}\{X(n)-iY(n)\}\exp(inkx)\right] \tag{1}$$

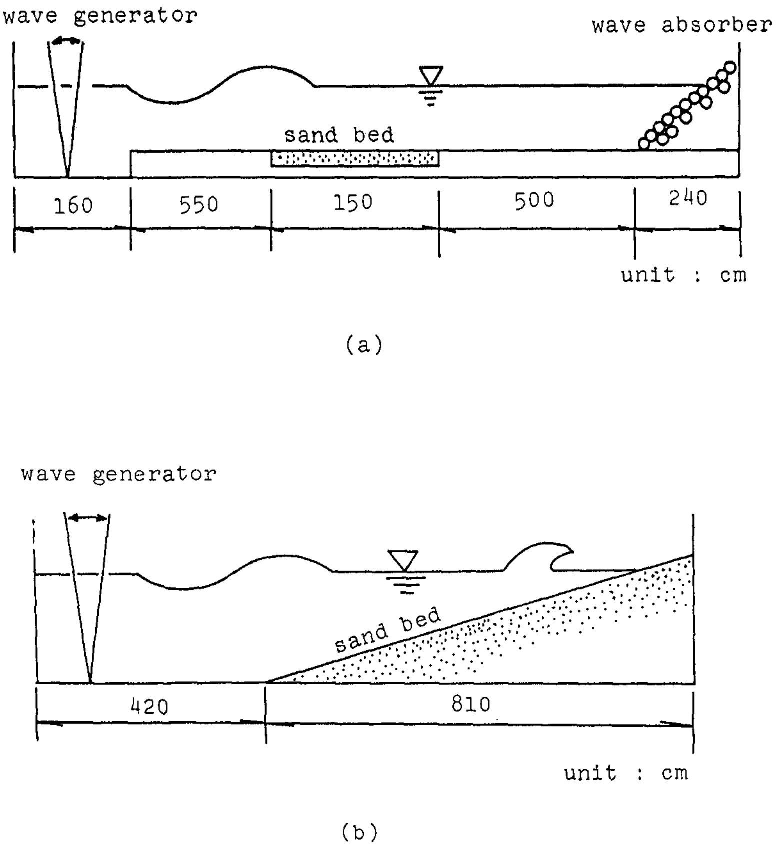

Fig. 1 Experimental setup.

where x is the horizontal coordinate taken positive in the direction of wave propagation, z the vertical axis taken positive upwards with the origin at the bottom surface, L the wave length, T the wave period, u^*_c the friction velocity due to steady current, $\hat{u}^*_{cw}$ the maximum shear velocity due to wave-current combined motion, κ the Karman constant (=0.4), z_0 the roughness length ($=k_s/30$, k_s: Nikuradse's equivalent roughness), k the wave number ($=2\pi/L$), X(n) and Y(n) the unknown coefficients, $i=\sqrt{-1}$, $\xi_n=2e^{\pi i/4}\sqrt{ncz}$, $\xi_{n0}=2e^{\pi i/4}\sqrt{ncz_0}$, $c=\sigma/(\kappa\hat{u}^*_{cw})$, and σ the angular frequency. Real denotes the real part of the function in the square bracket. In the above expression, the wave-current combined motion is described in a coordinate system traveling with the wave speed.

The unknown coefficients, X(n) and Y(n), in this equation are determined so as to minimize the difference between the measured and predicted wave profiles and to uniform the Bernoulli constant. Another unknown value, $\hat{u}^*_{cw}$, is calculated through an iteration.

The bottom shear stress can be expressed in terms of the eddy viscosity and velocity gradient as follows.

$$\tau = -\rho\kappa\hat{u}^*_{cw}z\left.\frac{\partial^2\psi}{\partial z^2}\right|_{z=z_0} \tag{2}$$

An example of the calculation is shown in Figs.2 and 3, for the case of the wave height, 8.6 cm, the wave period, 1.40 sec, and the water depth, 16.5 cm. The upper figure in Fig.2 compares the measured wave profile (open circles) with the predicted (solid line). In the lower figure, the bottom shear stress estimated with the present theory (solid line) is shown with a prediction according to Jonsson's wave friction factor (line with black circles). Jonsson's friction coefficient gives only the maximum value but not the time-variation of the shear stress, the latter of which is obtained by the square of a sine function having its maximum value at the time of the wave crest, as is usually assumed.

The bottom shear stress predicted with the modified stream function theory reflects well the non-linear wave profile. The positive part of the shear stress has larger maximum value and shorter duration than the negative part. As long as the maximum value concerns, two theories give more or less similar results. However, the details are quite different, especially at the phase of the negative shear stress. This suggests that the difference between the two predictions becomes greater in the nearshore region, where waves become more sharp-crested and more asymmetric.

The vertical distribution of the estimated horizontal velocity (solid lines) is shown in Fig.3. The prediction according to Dean's stream function theory (dotted lines) is also presented. Outside the boundary layer, the

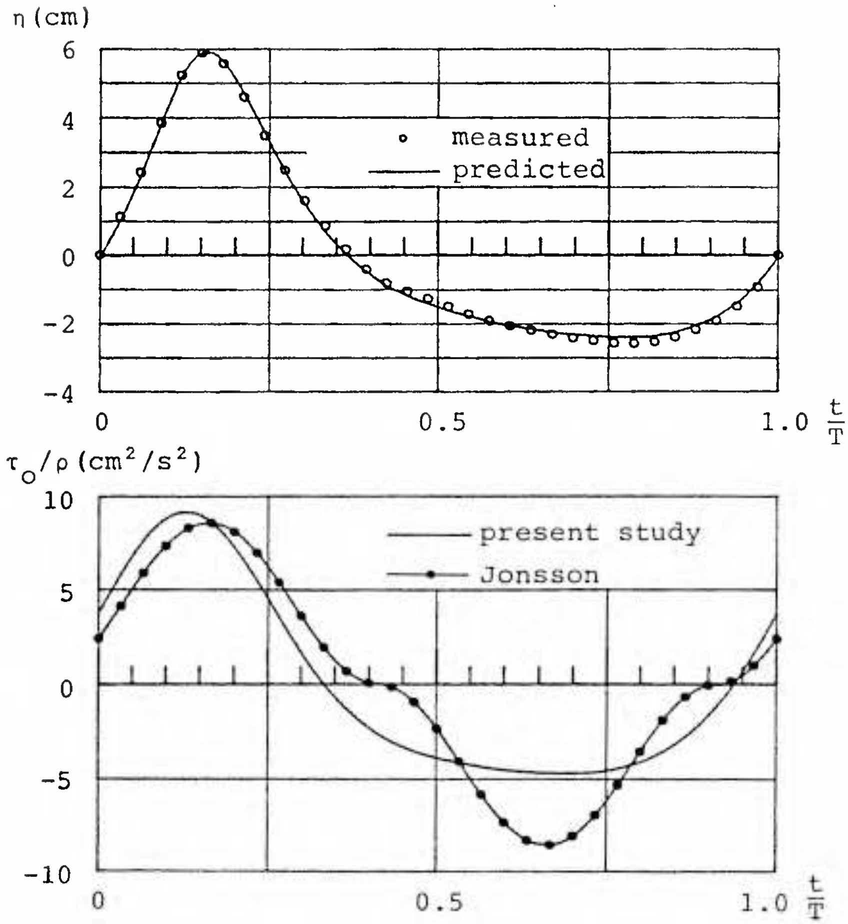

Fig.2 Time-variation of water surface level and bottom shear stress.

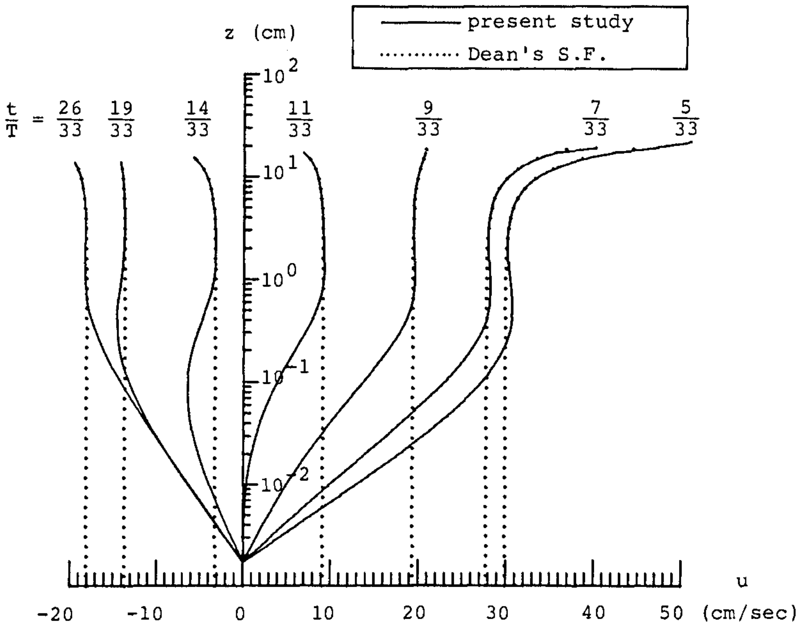

Fig.3 Vertical distribution of velocity profile.

difference between the two theories is very small. A distinct difference is found in the immediate vicinity of the bottom. This is of no wonder because Dean's method basing upon the potential theory can not satisfy the boundary condition there while the present author's theory can perfectly do.

Recent experimental measurements have revealed that Dean's stream function theory can be practically applicable to the estimation of the velocity field, except in the very vicinity of the bottom, near the surf zone over a sloping beach. Judging from this fact, it may be hopeful that the bottom shear stress obtained with the present theory can be used in the formulation of the sand transport rate not only on a flat bed but also on a sloping beach.

4.RESULTS AND DISCUSSIONS

4.1 Bedload transport over a flat bed

The instantaneous bedload transport rate is assumed to be expressed as a function of the instantaneous shear stress as follows, in an analogy to sediment movement due to steady current.

$$\frac{q_{B(t)}}{\sqrt{sgd^3}} = \alpha\, \mathrm{sign}\{\tau^*(t)\}|\tau^*(t)|^m\{|\tau^*(t)| - \tau^*_{cr}\}^n \tag{3}$$

where q_B is the instantaneous bedload transport rate, s the immersed specific weight of sand, g the gravitational acceleration, d the diameter of sand particle, $\tau^*(t)=\tau(t)/(sgd)$, and τ^*_{cr} the critical Shields number. Values of α , m and n are to be determined from experiments. The time-variation of the bottom shear stress can be calculated with the modified stream function, Eq.(1). It should be noted again that we need the whole wave profile over one wave period for the calculation of the bottom shear stress.

On the basis of Eq.(3), the net transport rate over one wave period is given by,

$$\frac{\bar{q}_B}{\sqrt{sgd^3}} = \alpha F \tag{4}$$

where

$$F = \frac{1}{T}\int_0^T \mathrm{sign}\{\tau^*(t)\}|\tau^*(t)|^m\{|\tau^*(t)| - \tau^*_{cr}\}^n dt \tag{5}$$

In the above equation, the integration should be restricted to the phase when the tractive force is greater than the critical shear stress.

The exponents, m and n, and the coefficient, α , in

Eqs.(3) and (4) will be determined so as to minimize the scattering of the experimental data.

The relationship between the measured net transport rate and the function F defined by Eq.(5) is examined for a various set of the exponents m and n. It is concluded that m=0.5 and n=1.0 are the best combination of the exponents. The result is shown in Fig.4, where the net transport rate depends linearly on the function, F, as expected in Eq.4, with the coefficient α =3.5.

In conclusion, the instantaneous transport rate of bed load can be given by

$$\frac{q_B(t)}{\sqrt{sgd^3}} = 3.5\mathrm{sign}\{\tau^*(t)\}|\tau^*(t)|^{0.5}\{|\tau^*(t)| - \tau^*_{cr}\} \tag{6}$$

or by the following expression which is convenient for comparison with the existing formulas for steady current.

$$\frac{q_B(t)}{u_*(t)d} = 3.5\mathrm{sign}\{\tau^*(t)\}\{|\tau^*(t)| - \tau^*_{cr}\} \tag{7}$$

where $u_*(t)=\sqrt{|\tau(t)|/\rho}$, and ρ is the density of the fluid.

In Fig.5, Eq.(7) is compared with the bed load formula for steady current in the alluvial channel. The dashed-and-dotted line is the result of the present study, Eq.(7). The solid line show the representative bed load formulas for the alluvial channel. The present study is close to the Sato et al. formula (1958) which is often called as the PWRI (Public Works Research Institute) formula and is widely used in Japan. Therefore, the present result will be smoothly continued to the sediment transport rate in steady current, as the excursion length of water particles becomes longer.

4.2 BEDLOAD TRANSPORT OVER A SLOPING BEACH

The bed load formula introduced in the preceding section is applied to a sloping beach case.

Beach profiles measured at t=0 min and t=10 min are shown in the upper figure of Fig.6. Since the beach profiles were not always two-dimensional but variable in the transversal direction, especially near the plunging point, values measured along three parallel longitudinal lines were averaged to determine the beach profile.

A sand bar developed near the plunging point, obviously nourished by the onshore sand transport from the seaward region. The sand transport rate is calculated from the measured beach profiles and is shown by the solid line in the lower figure. The direct measurements by use of a sand trap are also given by black circles, which show a good agreement with the solid line.

In order to apply Eq.(4) to the prediction of the bed-load transport rate on the slope, wave profiles were measured at the points where sand traps were installed.

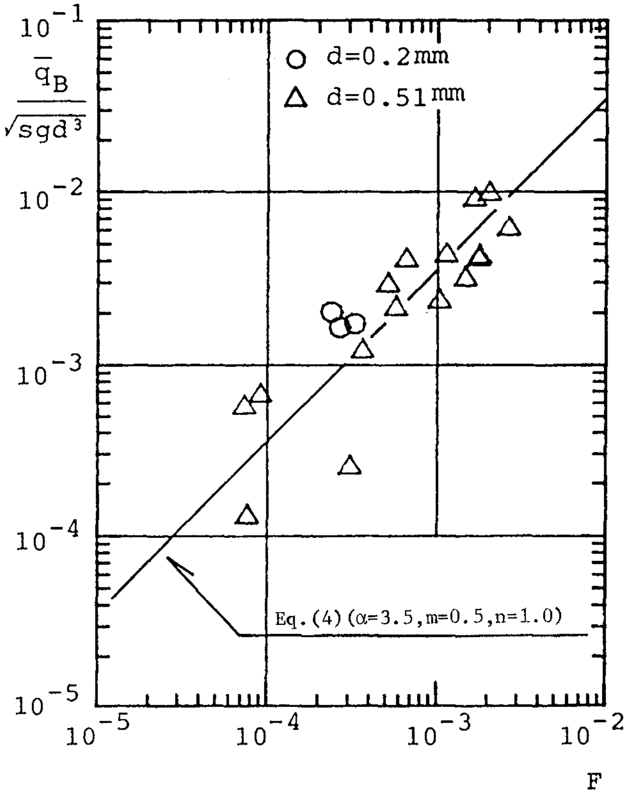

Fig.4 Bed load transport rate over a flat bed.

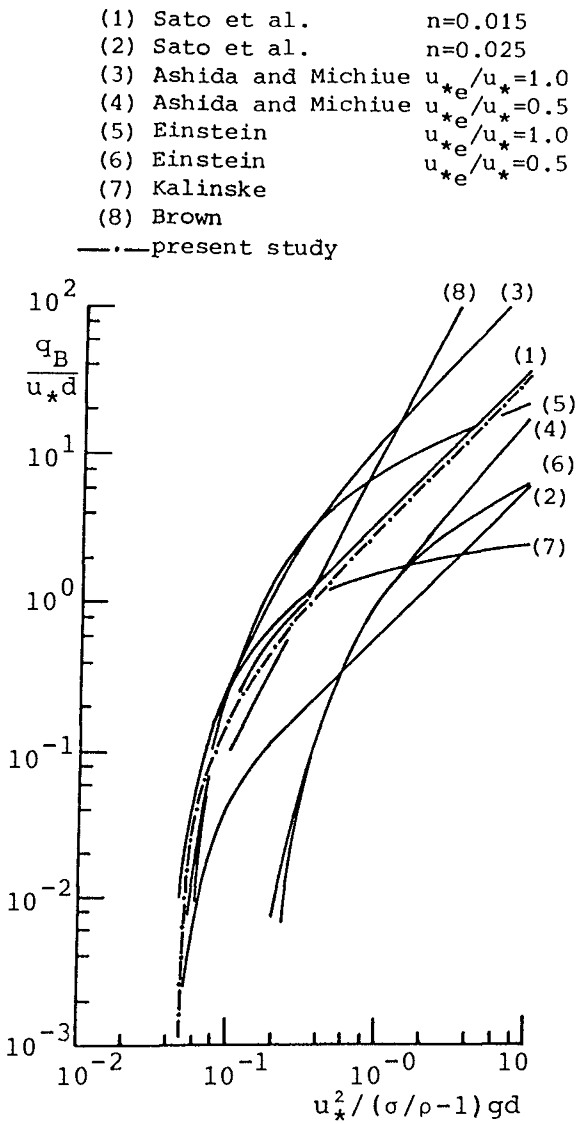

Fig.5 Comparison of bed load formulas.

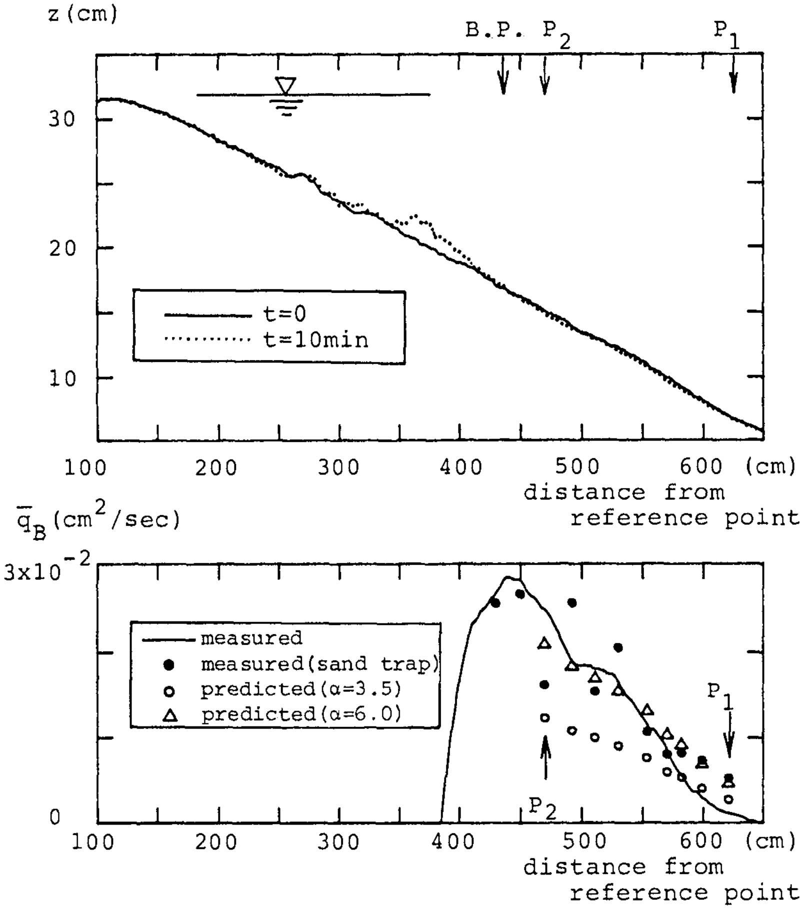

Fig.6 Change of beach profile and bed load transport rate.

Examples of the observed wave profile at P_1 and P_2 are shown in Figs.7 and 8, respectively. Open circles in the upper figures are the measured wave profiles, and solid lines the predictions. The predicted bottom shear stresses are shown in the lower figures.

At the point, P_1, in the offshore region, the wave profile is nearly symmetrical with respect to the wave crest. The time-variation of the bottom shear stress is also symmetrical. The maximum value of the positive shear stress is slightly greater than that of the negative shear stress. At the point, P_2, near the breaking point where asymmetry of the wave profile is remarkable as shown in Fig.8, the bottom shear stress, as a consequence, becomes quite asymmetrical with respect to the wave crest as well as with respect to the still water level.

The time-variation of the bottom shear stress thus obtained is substituted into Eq.(4) to predict the bed-load transport rate. The estimated values are shown by open circles in the lower figure of Fig.6.

At the measurement points in the offshore region, the measured sand transport rates show a good agreement with the predicted values. However, near the breaking point, the agreement becomes worse. If we substitute 6.0 into α, instead of 3.0, in Eq.(4), the agreement is considerably improved as shown by open triangles.

This result may suggest the importance of the acceleration in the sediment transport near the surf zone. The inertia force is usually neglected in the analysis of the motion of sand grains, compared with the drag force. Such a quasi-steady assumption can be only acceptable in the region far from the breaking point but not near the surf zone.

Therefore, it is concluded that the bed load formula established for the offshore condition needs modification to include the effect of the acceleration, if it will be extended to the more non-linear case. One possible way may be to express the coefficient α to reflect the effect of the acceleration.

5.CONCLUSION

In the previous studies on the sand movement due to wave motion, Jonsson's friction coefficient has been conveniently and widely used for the formulation of sand transport rate. This means that the non-linearity of wave motion was not adequately taken into consideration in those investigations. The present study, on the contrary, makes it possible to include the non-linear effect, by using the modified stream function theory of the author.

It is assumed that the instantaneous bed-load transport rate is expressed by Eq.(3) on the basis of the preceding studies on the sediment movement due to steady current. Exponents m and n in Eq.(3) are found to be 0.5 and 1.0, which ensure a smooth continuation from wave

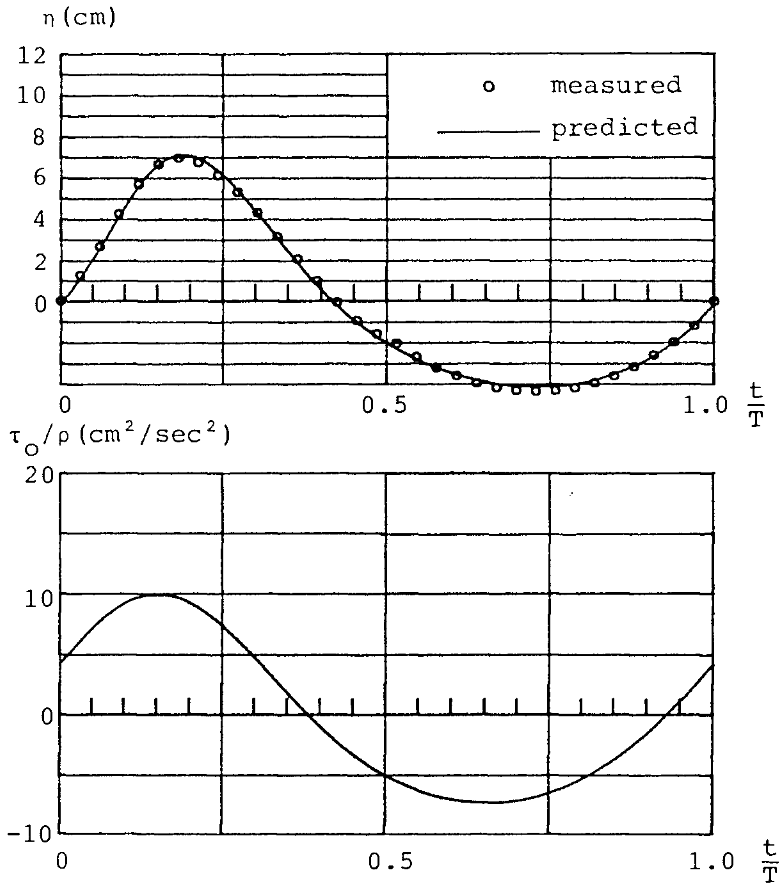

Fig.7 Time-variation of water surface level and bottom shear stress at measured point P_1.

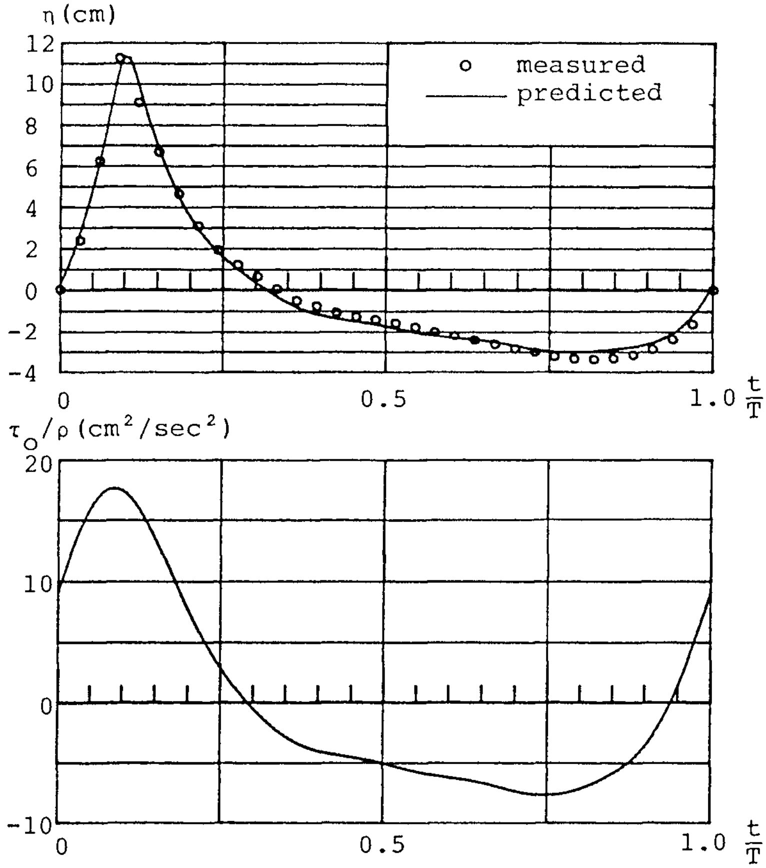

Fig.8 Time-variation of water surface level and bottom shear stress at measured point P_2.

motion to steady flow as the excursion length of water particles becomes longer. The coefficient of proportionality α is determined as 3.0 in case of a flat bottom.

The measured sediment transport rate near the surf zone is remarkably greater than the predicted. This suggests that the quasi-steady assumption can no longer be applicable in this region.

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