BED LOAD TRANSPORT OF FINE SAND BY LAMINAR AND TURBULENT FLOW

by

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ABSTRACT

An experimental study is described in which the rate of transport of fine sand over a flat bed was measured as a function of the shear stress exerted on the bed by both laminar and turbulent flows, for a range of water temperatures. The results confirm the great sensitivity of the transport rate to changes in both shear stress and temperature over the test range. The hypothesis is propounded that in certain specified circumstances, the net rate of local bed load transport generated by an arbitrary unsteady flow, can be estimated by integration, using the measured steady laminar flow transport relationships in conjunction with the time history or probability distribution of the fluctuating bed shear stress. The concept has been tested using the measured data with encouraging results. It is suggested that this method of linking the characteristics of the near bed flow field to the local induced rates of sediment transport could be productively applied in a range of studies including the initiation and development of bed sand ripples by the action of various combinations of waves and steady currents.

1. INTRODUCTION

Over the past decade, considerable progress has been made in developing instrumentation such as the laser doppler anemometer for flow velocity measurement and also numerical mathematical modelling techniques for predicting flow behaviour. These advances coupled with significant improvements in digital data processing methods, have led to a rapidly expanding knowledge and understanding of the complex turbulence and other near bed flow charactertistics in both steady and unsteady boundary layers and separated flows. (See for example Grass(1971,1982), Kemp and Simons (1982), and Etheridge and Kemp (1979)).

As in the past, progress in sediment transport research has lagged very much behind due to the formidible theoretical and experimental difficulties. In order to fully exploit the new fluid mechanics information in the study of coastal and other sediment processes, including for example, the formation of bed ripples by

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waves and currents, it will be necessary to establish improved links between the measured or computed characteristics of the fluctuating driving flow and the corresponding response of the bed sediment.

A primary objective of the present investigation was to derive one such linking relationship for the case of bed load transport of fine grained sand.

2. BED LOAD SEDIMENT TRANSPORT

Consider an arbitrary, unsteady fluid flow inducing transport on a flat bed of non-cohesive sediment. Then, for a particular fluid viscosity, physical reasoning suggests that there will be a close correlation between the instantaneous flow velocity immediately adjacent to the bed and the simultaneous local shear stress acting on the bed and hence the induced instantaneous rate of local bed load sediment transport. Experimental evidence supporting this postulation has been previously presented and discussed by Grass (1970,1971) based on photographically recorded observations of the response of fine bed sand to the rapid fluctuations in bed shear stress generated by a turbulent boundary layer flow.

For fine grained sediment producing an hydraulically smooth boundary condition in typical water flows, the fluid forces are transferred to the bed sediment primarily by viscous shear. The hypothesis is advanced that under these conditions the instantaneous local bed load transport rate produced by a particular instantaneous value of viscous bed shear stress, τ, can be represented by the transport rate, $q_{s\ell}(\tau)$, induced by a steady laminar flow with the same bed shear stress and fluid viscosity.

In a situation where the fluctuating bed shear stress is a known function of time, t, as for example in the case of an oscillatory laminar boundary layer, then the net mean sediment transport rate, q_{st}, over a time interval T, can be calculated using the following equation:

$$q_{st} = \frac{1}{T}\int_{o}^{T} q_{s\ell}(t)dt \qquad (1)$$

If however, the fluctuating bed shear stress is random and unpredictable as in the case of turbulent boundary layer flows induced by a random wave spectrum, then the following alternative equation can be used to estimate the net be load transport rate:

$$q_{st} = \int_{\tau_{min}}^{\tau_{max}} q_{s\ell}(\tau)p(\tau)d\tau \qquad (2)$$

where $p(\tau)$ is the probability density function of the fluctuating bed shear stress τ.

The hypothesis expressed in equations (1) and (2) was tested in the present investigation by comparing the measured values of bed load sediment transport produced by a steady unidirectional turbulent boundary layer flow with the corresponding values calculated using equation (2) substituting $q_{s\ell}(\tau)$ values provided by corresponding measurements in a steady laminar flow. This procedure circumvents severe difficulties in attempting to measure the instantaneous fluctuations in bed load sediment transport rate directly.

3. EXPERIMENTAL TECHNIQUES

A fine 140 micron quartz sand from the Mersey Estuary was used in the present experiments as in the earlier study of initial movement of bed sand grains by Grass (1970). The bed load sediment transport rate was measured as a function of mean bed shear stress and water temperature (4^0C,17^0C,30^0C) under laminar and fluctuating turbulent flow conditions. Novel experimental apparatus and techniques were employed which have been described in detail by Ayoub (1977). These tests broke new ground in that they were designed to obtain strictly flat bed transport rates for bed shear stress values ranging up to 25 times the critical shear stress for initial sediment movement. This was achieved by measuring over short sample times prior to the formation of flow disturbing sand ripples.

The laminar flow experiments were carried out using the specially designed apparatus illustrated in Figure 1. A low Reynolds number, high bed shear stress laminar flow was produced between two closely spaced parallel plates 2 millimetres apart and 1 metre long and 0.4 metres wide. The carefully levelled flat sand bed was contained in a recessed pit 0.4 metres long and 0.25 metres wide located at the downstream end of the lower plate. Water was supplied to the test section from a constant head tank and the bed shear stress was determined by monitoring the streamwise pressure gradient. The sediment transport rate was measured by collecting and weighing sand samples over a know time interval using the sliding box arrangement shown in Figure 1. Very short and accurately measured sampling times were achieved using a sliding plate inlet valve and an automatic timing device attached to the sampling box. Sampling times were reduced with increasing bed shear stress down to periods as low as 1 second in the case of the higher bed stress values in order to avoidsignificant erosion of the sand bed surface. Repeated tests at the same bed shear stress indicated good reproducibility of the individual measured values of bed load sediment transport rate.

The turbulent flow was generated on a flat plate 1.5 metres long and 0.3 metres wide which was supported on a carriage and towed through still water as shown in figure 2. A sand bed 0.4 metres long and 0.2 metres wide was located at the downstream end of the plate.The plate boundary layer was tripped by means of a wire close to the leading edge and achieved a steady stable state very soon after the plate had been rapidly accelerated to its constant terminal velocity. Once again, the bed load sand transport rate was measured by weighing the sand collected in a specially partitioned box located under the downstream end of the sand bed.Differences in the weights of two sand samples collected from two experimental runs in which the bed plate

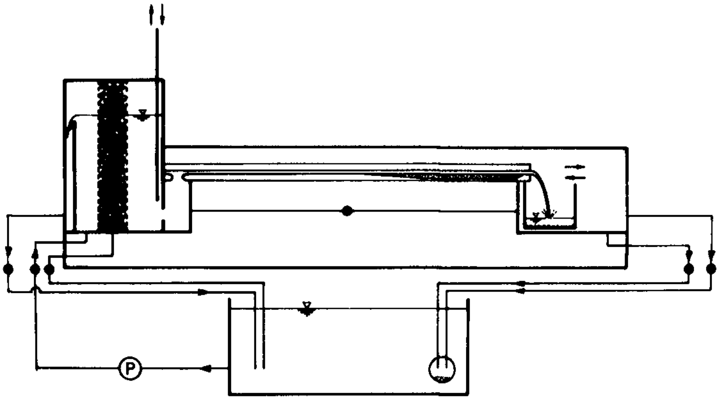

FIGURE 1. Laminar flow channel used to measure bed load sediment transport induced by a laminar boundary layer formed between closely spaced parallel plates.

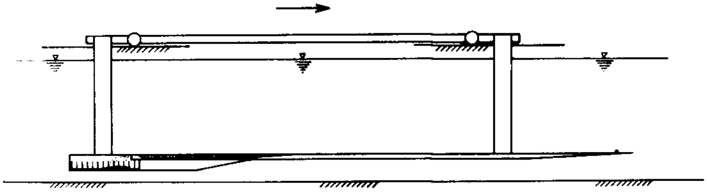

FIGURE 2. Apparatus used to measure bed load sediment transport induced by a turbulent boundary layer formed on a flat plate towed through still water.

underwent identical acceleration and deceleration phases but travelled different known intermediate distances at a common constant velocity, were used in these bed load calculations in order to eliminate end effects. Sampling times, which were simply and accurately determined from the travel distance and the known carriage velocity, were again kept very short (down to 1.5 seconds for the highest bed shear stress) in order to avoid bed erosion and also the formation of nascent sand ripples on the bed surface. Mean bed shear stresses were obtained from velocity profile measurements using a carefully calibrated hot film anemometer. This instrument was also used to record the velocity fluctuations in the viscous sublayer at the downstream end of the bed plate. The probability density functions of the fluctuating bed shear stress were calculated from these measurements. Tests were repeated four times for each of the lower values of mean bed shear stress and twice for the remainder in order to obtain closely reproducible values of the average bed load transport rates.

4. RESULTS AND DISCUSSION

The measured sediment transport rates are shown plotted in terms of the relevant non-dimensional variables in Figure 3 in which u_* is the bed shear velocity, d is the sand diameter, ρ_s and ρ are the sediment and fluid densities respectively. The slope of the fitted linear regression lines indicates that over the central experimental range the bed transport rate varies as approximately the fourth power of the bed shear stress for both the laminar and turbulent flows.

Consistent with the results obtained by Taylor and Vanoni (1972), the sediment transport rate also shows considerable sensitivity to changes in bed particle Reynolds number, R_*. Between 4^0C and 30^0C the water viscosity, ν, is halved which gives rise to a threefold increase in the bed load transport rate.

It is interesting to note that for a particular constant low value of bed transport rate, defining the initial movement stage of the bed particles, the optimized data correlation presented in Figure 3 indicates that Shields parameter, τ_{c*}, will be proportional to $R_*^{-0.64}$. This result is in good agreement with the criteria for initial motion of fine bed sand previously suggested by Grass(1970) for the same range of bed particle Reynolds numbers.

The use of equation (2) in calculating the net mean rate of bed load sediment transport from the laminar flow transport function and the probability density function of the fluctuating bed shear stress produced by a steady turbulent boundary layer flow, is illustrated in Figure 4. The particular probability distribution shown in Figure 4 and used throughout the present study, is a smoothed curve fitted to data measured in the present tests and in experiments reported by Grass (1970), Ecklemann and Reichardt (1972), and by Blinco and Simons (1974). When the various probability distributions which cover a wide range of flow Reynolds numbers and different types of steady unidirectional flow, are plotted in standard form in terms of the fluctuations in bed shear stress scaled by their standard deviation about the mean, it is found that they can be accurately summarised by a single unique curve. This takes the form of the positive skew

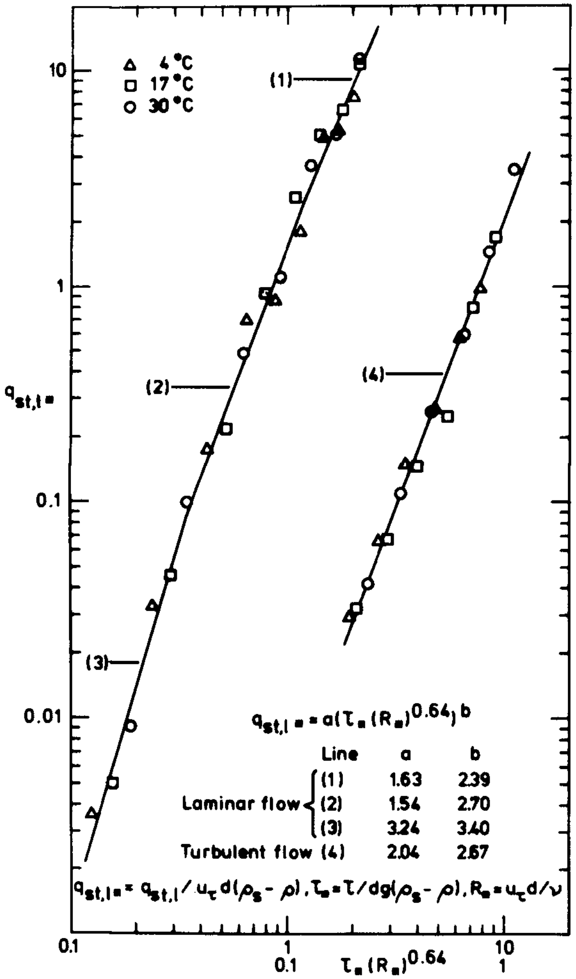

FIGURE 3. Optimized correlation of measured bed load sediment transport rates induced by laminar and turbulent flows.

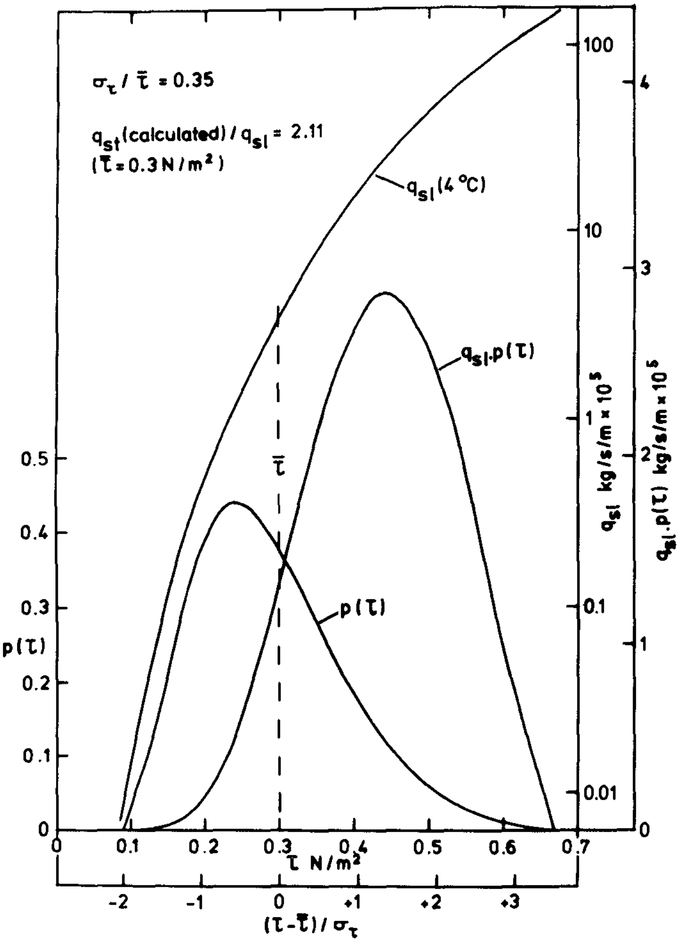

FIGURE 4. Sample illustration of the method of calculating bed load transport rate produced by a turbulent flow, q_{st}, at a particular mean bed shear stress $\bar{\tau}$ of 0.3N/m^2, from the laminar flow transport relationship, $q_{s\ell}$, and the probability distribution, p(τ), of the fluctuating bed shear stress, τ, induced by the turbulent flow.

distribution reproduced in Figure 4.

The calculation illustrated in Figure 4 is for a mean bed shear stress of $0.3N/m^2$. As can be seen, the fluctuations in instantaneous shear stress range between approximately $0.1N/m^2$ and $0.7N/m^2$. The corresponding induced rate of instantaneous bed load transport increases by four orders of magnitude over the shear stress range. For the lower instantaneous shear stresses there is negligible transport over the bed surface whilst the higher shear stress fluctuations produce very large instantaneous transport rates. This is in close accord with the evidence from direct observations of the fluctuating sand transport behaviour reported by Grass (1970).

The strong non-linearity in the sediment transport rate with increasing bed shear stress, produces maximum contributions to the next bed load transport rate at instantaneous shear stress values greatly in excess of the mean shear stress of $0.3N/m^2$, in spite ofthe rapid reduction in $p(\tau)$, as indicated by the location of the peak in the $q_{s\ell}.p(\tau)$ curve. In the particular example presented in figure 4, the calculated value of net mean sediment transport rate produced by the turbulent boundary layer flow with a mean applied bed shear of 0.3 N/m^2, is in excess of twice the corresponding laminar flow transport rate with the same bed shear stress.

A comparison between the experimental and calculated equation (2) values of the ratio between the turbulent and laminar flow transport rates is shown in figure 5. The experimental results are seen to lie satisfactorily in a zone bounded by the values calculated using bed shear stress standard deviation to mean bed shear stress ratios, $\sigma/\bar{\tau}$ of 0.25 and 0.35. These $\sigma/\bar{\tau}$, magnitudes correspond to the range of values measured in the present tests with both fixed and mobile beds and also to the measurements reported by Mitchell and Hanratty (1966) and by Eckelmann (1974).

The relatively rapid decreasing trend in the magnitude of the transport ratio from approximately 2 to 1.3 over the bed shear stress range from $0.3N/m^2$ to $0.9N/m^2$, the approximate observed the threshold shear stress for intial sediment suspension, suggests a possible decrease in the effective $\sigma/\bar{\tau}$ ratio with increaing flow Reynolds number. This is consistent with trends in data obtained in the present investigation and by Laufer (1951) and by Blinco and Simons (1974).

The rapid increase in the transport ratio at the lower end of the bed shear stress range indicated in Figure 5 is due to the fact that as the shear stress approaches the critical value for intial sand grain movement under laminar flow conditions, the laminar flow transport rate tends to zero. However, the corresponding transport rate, at thesame mean bed shear stress, under turbulent flow conditions remains finite and the transport ratio therefore increases asymptotically to infinity.

The detailed physics and charactistics of the two phase flow in the mobile sand layer and its interaction with the driving flow are in reality extremely complex. These effects are inevitably only partially allowed for in the proposed simple bed load transport model. Bearing

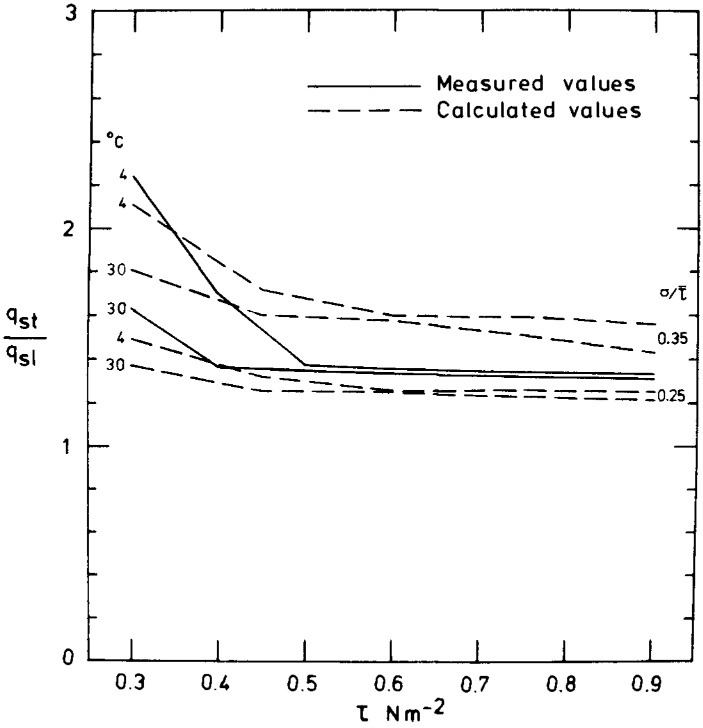

FIGURE 5. Comparison between the calculated and measured ratios of bed load transport rates, q_{st}, and $q_{s\ell}$, induced respectively by turbulent and laminar flows, as a function of mean bed shear stress.

these facts in mind, the level of agreement between the experimental and calculated results in Figure 5 is very encouraging.

5.CONCLUSIONS

a. The investigation was successful in providing completely new bed load sediment transport relationships for strictly flat bed conditions.

b. The test results confirmed the great sensitivity of the bed load sediment transport rate to changes in both bed shear stess and water temperature over the experimental range considered.

c. Under hydraulically smooth bed conditions with fine sands, the measured laminar flow transport relationship appears, on the evidence of the present results, to provide a satisfactory link function between instantaneous bed shear stress and the local simultaneously induced bed load transport rate.

d. The general validity of the proposed method for calculating local net rates of bed load transport under arbitary unsteady flow conditions is thus confirmed as are the basic concepts on which the method is based.

e. There are several areas of study where the type of approach outlined might be productively applied in both steady and unsteady flow situations. These include quantitative mathematical modelling of the instability of the sand bed interface involving the initial formation of ripples on flat beds and their subsequent development and the prediction of their equilibrium height. The relevance in this context is demonstrated by the recent work of Sleath (1982) and Ruiter (1982). The proposed relationship between the local flow characteristics and the induced bed load transport can also be applied in modelling certain types of local scour problems. Further tests will need to be carried out however in order to assess the influence of local bed slope on the laminar flow transport function.

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