

Behavior of Current Divergences under $SU_3 \times SU_3^{* \dagger}$

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We investigate the behavior under $SU_3 \times SU_3$ of the hadron energy density and the closely related question of how the divergences of the axial-vector currents and the strangeness-changing vector currents transform under $SU_3 \times SU_3$. We assume that two terms in the energy density break $SU_3 \times SU_3$ symmetry; under SU_3 one transforms as a singlet, the other as the member of an octet. The simplest possible behavior of these terms under chiral transformations is proposed: They are assigned to a single $(3, 3^*) + (3^*, 3)$ representation of $SU_3 \times SU_3$ and parity together with the current divergences. The commutators of charges and current divergences are derived in terms of a single constant c that describes the strength of the SU_3 -breaking term relative to the chiral symmetry-breaking term. The constant c is found not to be small, as suggested earlier, but instead close to the value $(-\sqrt{2})$ corresponding to an $SU_2 \times SU_2$ symmetry, realized mainly by massless pions rather than parity doubling. Some applications of the proposed commutation relations are given, mainly to the pseudoscalar mesons, and other applications are indicated.

I. INTRODUCTION

WE assume here the correctness of the $SU_3 \times SU_3$ algebra proposed for equal-time commutators of the vector and axial-vector charge operators F_i and F_i^5 for hadrons.¹ As is well known, there is some experimental evidence in confirmation of it,² and especially of the $SU_2 \times SU_2$ subalgebra. The corresponding local commutation rules proposed for the charge densities may also be correct. At infinite momentum, all these commutation relations fall into the "good-good" class³ and yield sum rules that amount to unsubtracted dispersion relations in the variable s .

We investigate in this paper the behavior under $SU_3 \times SU_3$ of the hadron energy density θ_{00} , and the closely related question of how the divergences of the axial-vector currents and strangeness-changing vector currents transform under $SU_3 \times SU_3$. The commutators involved here fall into the "good-bad" category at infinite momentum, and some of them are tractable in deriving sum rules.

We do not consider here the equal-time commutators of divergences with each other, which give "bad-bad" relations at $P_z = \infty$ and which no one has succeeded in using in a straightforward way at $P_z = \infty$ or in deriving low-energy theorems, although these commutators may be useful when acting on the vacuum state.

Our aim is to explore further the original proposal¹ (as suggested, for example, by a quark model) for commutation relations between charges and current divergences. These relations arise from an energy density θ_{00} in which the $SU_3 \times SU_3$ -violating part consists of two terms; the first breaks the chiral symmetry $SU_3 \times SU_3$ but not SU_3 itself (and corresponds in a quark model to a common quark mass), and the second breaks SU_3 (and corresponds in a quark model to a mass-splitting between isotopic singlet and doublet). The proposed behavior of these terms under $SU_3 \times SU_3$ is the simplest possible: They and all the current divergences belong to a single representation of $SU_3 \times SU_3$ and parity. This theory, because of its simplicity, contains a single universal parameter c that describes the strength of the SU_3 symmetry-breaking term relative to the chiral symmetry-breaking term and determines the commutators of charges with divergences.

If the whole $SU_3 \times SU_3$ violation were^{*}abolished, we would have a world in which all sixteen vector and axial-vector currents were conserved. We suppose that the conservation of the vector currents would be achieved through the exact degeneracy of SU_3 multiplets and the conservation of the axial-vector current through the existence and coupling of eight massless pseudoscalar mesons. The chiral symmetry violation raises the masses of the pseudoscalar mesons to finite values, and the SU_3 violation splits the SU_3 multiplets.

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¹ M. Gell-Mann, *Phys. Rev.* **125**, 1067 (1962).

² For a review of the literature, see S. L. Adler and R. F. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968) and B. Renner, *Current Algebras and Their Applications* (Pergamon Press, London, 1968).

³ R. F. Dashen and M. Gell-Mann, in *Proceedings of the Fourth Coral Gables Conference on Symmetry Principles at High Energy, 1966* (W. H. Freeman and Co., San Francisco, 1967); S. Fubini, G. Segrè, and J. D. Walecka, *Ann. Phys. (N. Y.)*, **39**, 381 (1966).

We note that SU_3 mass splittings are always of the same order as the masses of the pseudoscalar octet. This observation suggests that it would not be unreasonable for the strengths of the two symmetry-violating terms to be comparable.

Employing some simple approximate assumptions, we conclude that a consistent picture of several experimental results can be obtained and the crucial constant c estimated. This constant is not small, as originally suggested,¹ but rather is close to the value $-\sqrt{2}$. If it were exactly equal to $-\sqrt{2}$, we would have exact $SU_2 \times SU_2$ invariance and massless pions. (In the quark scheme, which we employ here only as a mnemonic, this value corresponds to a zero mass for the isotopic doublet and a finite mass for the isotopic singlet.)

Many authors have taken steps in the same direction,² and in particular we should mention the work of Glashow and Weinberg⁴ as being the most closely related to ours. Still, we find that our approach leads to results and to a formulation of the situation that, to the best of our knowledge, have not yet been reported in the literature.

In Sec. II, we discuss the commutators of charges with current divergences; in Sec. III, we estimate the constant c and give some applications.

II. TRANSFORMATION PROPERTIES OF CURRENT DIVERGENCES

The algebraic behavior of current divergences is closely related to the properties of the energy operator H under the $SU_3 \times SU_3$ algebra through the equation⁵

$$\begin{aligned} \frac{d}{dt} \int j_0(\mathbf{x}, t) d^3x &= - \int \partial_\mu j_\mu(\mathbf{x}, t) d^3x \\ &= i \left[H, \int j_0(\mathbf{x}, t) \right], \end{aligned} \quad (2.1)$$

where

$$H = \int \theta_{00}(\mathbf{x}, t) d^3x. \quad (2.2)$$

The local generalization¹ of Eq. (2.1) to

$$\partial_\mu j_\mu(\mathbf{x}, t) = -i \left[\mathcal{H}'(\mathbf{x}, t), \int j_0(\mathbf{y}, t) d^3y \right] \quad (2.3)$$

rests on the assumption that the tensor part in $\theta_{00}(x)$ commutes with the charges, and only a Lorentz scalar part $\delta_{00}\mathcal{H}'(x)$ (such as a mass term in certain Lagrangian models) breaks the symmetry. We make this our first

⁴ S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

⁵ The notation of Ref. 1 will be adopted throughout this paper, in that we use spacelike metric $g_{00} = -1$ and the pion decay constant $(1/\sqrt{2}) \langle 0 | \partial_\mu \mathcal{F}_{1\mu}^5(0) - i \partial \mathcal{F}_{2\mu}^5(0) | \pi^+ \rangle = m_\pi^2 / 2 f_\pi$. As in M. Gell-Mann [Physics **1**, 63 (1964)] f_K^{-1} and f_π^{-1} are constants that yield kaon and pion decay amplitudes when multiplied by $\sin \theta$ and $\cos \theta$, respectively.

assumption.⁶ A decomposition of $\mathcal{H}'(x)$ into terms transforming according to irreducible representations of $SU_3 \times SU_3$ specifies completely the commutators of charges and current divergences.

For simplicity, we make our second assumption: We will not admit operators of isospin or hypercharge 2 into the multiplet of the current divergences. This assumption implies the vanishing of certain commutators, for example,

$$[F_i^5 + iF_2^5, \partial_\mu \mathcal{F}_{1\mu}^5 + i \partial_\mu \mathcal{F}_{2\mu}^5] = 0, \quad (2.4)$$

$$[F_4^5 + iF_5^5, \partial_\mu \mathcal{F}_{4\mu}^5 + i \partial_\mu \mathcal{F}_{5\mu}^5] = 0. \quad (2.5)$$

These have been found consistent with experimental information.⁷ In fact, this assumption allows components of $\mathcal{H}'(x)$ only in representations $(3, 3^*) + (3^*, 3)$ and $(1, 8) + (8, 1)$ of $SU_3 \times SU_3$.

The final simplification is achieved if we make our last assumption, which is that the $(1, 8) + (8, 1)$ part of \mathcal{H}' vanishes and that the SU_3 singlet and octet parts of \mathcal{H}' belong to the same $(3, 3^*) + (3^*, 3)$ representation of $SU_3 \times SU_3$. We have arrived at the simplest theory of the behavior of \mathcal{H}' , because it introduces the least number of new operators to complete the multiplet of the current divergences. [We shall argue at the end of the article that if there is a $(1, 8) + (8, 1)$ contribution, its effects on our deductions are unlikely to be large.] We then obtain the formula

$$\mathcal{H}' = -u_0 - cu_8 \quad (2.6)$$

of Ref. 1, where the parameter c expresses the relative scale of the two parts of \mathcal{H}' and is uniquely defined by the transformation properties of the scalar and pseudoscalar nonets, u_i and v_i , in the representation $(3, 3^*) + (3^*, 3)$ of $SU_3 \times SU_3$:

$$\begin{aligned} [F_i, u_j(y)] &= i f_{ijk} u_k(y), \\ [F_i, v_j(y)] &= i f_{ijk} v_k(y), \\ [F_i^5, u_j(y)] &= -i d_{ijk} v_k(y), \\ [F_i^5, v_j(y)] &= i d_{ijk} u_k(y), \end{aligned} \quad (2.7)$$

where $i = 1 \dots 8$ and $j, k = 0 \dots 8$. We do not consider an operator F_0^5 , because we find that no such operator is "partially conserved" to anywhere near the same degree as the operators we do consider. Furthermore, such an operator is not known to play a role in the physical interactions of hadrons.

The current divergences follow from (2.3):

$$\partial_\mu \mathcal{F}_{i\mu} = c f_{ijk} u_k, \quad (2.8)$$

$$\begin{aligned} \partial_\mu \mathcal{F}_{i\mu}^5 &= -d_{i0k} v_k - c d_{i8k} v_k = -W_i(c) \delta_{ik} v_k \\ &\quad - (\sqrt{3}/2) c \delta_{i8} v_0, \end{aligned} \quad (2.9)$$

⁶ For a related discussion, see D. J. Gross and R. Jackiw, Phys. Rev. **163**, 1689 (1967).

⁷ M. A. Ahmed, Nucl. Phys. **B1**, 679 (1967); D. K. Elias, Oxford University Report, 1967 (unpublished); G. Furlan and C. Rossetti, Phys. Letters **23**, 499 (1966); G. Soliani, University of Torino Report, 1967 (unpublished).

where

$$\begin{aligned} W_i(c) &= (\sqrt{2}+c)/\sqrt{3} & \text{for } i=1, 2, 3, \\ & (\sqrt{2}-\frac{1}{2}c)/\sqrt{3} & \text{for } i=4, 5, 6, 7, \\ & (\sqrt{2}-c)/\sqrt{3} & \text{for } i=8. \end{aligned} \quad (2.10)$$

Commutation of charges with current divergences can be read off by combining Eqs. (2.7)–(2.10).

All these assumptions and conclusions are as in Ref. 1. However, it was assumed there that c was small compared to unity. If we consider the formal analogy of $-u_0 - cu_3$ with the mechanical masses of an isotopic doublet and singlet forming a fundamental triplet, then the old assumption about c amounts to saying that the doublet-singlet splitting is small compared to the average mechanical mass. If we imagine that, in the limit of conservation of all sixteen currents, the masses of the baryons (for example, the $\frac{1}{2}^+$ octet) go to zero, then this might be a reasonable idea, with c corresponding roughly to the ratio of baryon mass splitting to baryon mass. But if it is the masses of the eight pseudoscalar mesons of the 0^- octet that go to zero as we disregard $u_0 + cu_3$, then that is no longer reasonable. In fact, it seems that the real world of hadrons is not too far from a world in which we have eight massless pseudoscalar mesons, SU_3 degeneracy, and conservation of all sixteen currents. We are even closer to a world in which there are massless pions and the algebra of $SU_2 \times SU_2$ is conserved. In that limit, we would have $c = -\sqrt{2}$, since the combination $u_0 - \sqrt{2}u_3$ commutes with F_i and F_i^5 for $i=1, 2, 3$. (This combination corresponds to zero mass for the conceptual fundamental isospin doublet and a finite mass for the isospin singlet.) Since, in fact, the pion is nearly massless, we should expect that in fact the value of c is close to $(-\sqrt{2})$. We shall see that a number of experimental results can be understood with the aid of the assumed behavior of \mathcal{H}' in Eq. (2.6), a value of c something like (-1.2) , and a few approximate assumptions about the matrix elements of currents between SU_3 multiplets and about meson pole dominance.

III. APPLICATIONS

The applications given in this paper will include the estimate of c , a discussion of the approximate equality of the $\pi_{\mu 2}$ and $K_{\mu 2}$ decay constants, a demonstration of the consistency of the squared-mass formula, and some results for $K_{\mu 3}$ decays.

We shall make approximations of two kinds:

(A) Pole dominance for axial current divergences through π , K , and η mesons. We note that although the hypothesis of partially conserved axial-vector current (PCAC) for K mesons appears uncertain, there is no definite evidence known against it, and the recent estimates of K -meson Yukawa coupling constants are compatible with generalized Goldberger-Treiman rela-

tions.⁸ η -meson PCAC is not testable, but one expects it to be on a similar footing as K -meson PCAC, at least as long as $(\eta\eta')$ mixing may be neglected.

(B) Application of approximate SU_3 symmetry to vertices of certain operator octets involving multiplets with small mixing. We shall apply SU_3 only to form factors and there only at points which are far enough away from important singularities so that differences in their distance due to SU_3 violations are negligible. At small momentum transfers, form factors describing the transverse parts of vector and axial-vector currents (not the induced scalar or pseudoscalar parts) and form factors of scalar densities (u_i) may be estimated by SU_3 , while matrix elements of pseudoscalar densities (v_i) are usually too strongly distorted because of the closeness of pseudoscalar-meson poles. No such distortion by the position of singularities affects matrix elements of octet operators between the vacuum and particle octets. We shall find that the vacuum is approximately SU_3 invariant and that these matrix elements are always close to their symmetry values.

Before employing these approximations in our applications of the algebraic properties of current divergences, we give an independent example to show they are reasonable by estimating the observed SU_3 violations in the decay $Y_0^*(1405) \rightarrow \Sigma\pi$ as compared with the virtual transition $Y_0^* \rightarrow N\bar{K}$, with Y_0^* , a $(\frac{1}{2})^-$ SU_3 singlet. As discussed in (B), we apply SU_3 to the matrix elements of the transverse part of the axial-vector currents

$$\langle Y_0^* | \mathcal{F}_{\pi\mu^5} | \Sigma \rangle = ig_\pi(t) \bar{u}_Y \gamma_\mu u_\Sigma + \dots, \quad (3.1a)$$

$$\langle Y_0^* | \mathcal{F}_{K\mu^5} | N \rangle = ig_K(t) \bar{u}_Y \gamma_\mu u_N + \dots, \quad (3.1b)$$

by approximating $g_\pi(0) \approx g_K(0)$. Using generalized Goldberger-Treiman relations for the axial divergences, as discussed in (A), we find

$$g_\pi(0)(m_Y - m_\Sigma) \approx (1/2 f_\pi) g_{Y_0^* \Sigma \pi} \quad (3.2a)$$

and

$$g_K(0)(m_Y - m_N) \approx (1/2 f_K) g_{Y_0^* N \bar{K}}. \quad (3.2b)$$

Therefore,

$$g_{Y_0^* N \bar{K}} / g_{Y_0^* \Sigma \pi} \approx \frac{(m_Y - m_N) f_K}{(m_Y - m_\Sigma) f_\pi} \approx 2. \quad (3.3)$$

Experimentally, this ratio is estimated⁹ to be about 3, while exact SU_3 symmetry predicts 1. Because of the parity change ($\frac{1}{2}^- \rightarrow \frac{1}{2}^+$) the SU_3 corrections enter through the mass differences and are significantly larger than in cases considered previously¹⁰ where there is no parity change ($\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$) and the corrections enter

⁸ J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967); C. H. Chan and F. T. Meiere, *ibid.* **20**, 568 (1968); H. T. Neih, *ibid.* **20**, 1254 (1968).

⁹ C. Weil, Phys. Rev. **161**, 1682 (1967).

¹⁰ K. Raman, Phys. Rev. **149**, 1122 (1966); **152**, 1517(E) (1966); R. H. Graham, S. Pakvasa, and K. Raman, *ibid.* **163**, 1774 (1967).

through the sum of the masses. Further corrections would result from singlet-octet mixing.

We now return to our objective and start by considering the vertex of $\mathcal{H}'(0)$ between members of the pseudoscalar-meson octet $|P_i\rangle$ in the low-energy limit

$$\begin{aligned} & -\lim_{p \rightarrow 0} \langle P_i(p) | u_0 + cu_8 | P_i(p') \rangle \\ & = -2if_i \langle 0 | [F_i^5, u_0 + cu_8] | P_i(p') \rangle \\ & = 2f_i \langle 0 | \partial_\mu \mathcal{F}_{i\mu}^5 | P_i(p') \rangle = m_i^2. \end{aligned} \quad (3.4)$$

The application of low-energy limits to matrix elements involving parts of $\mathcal{H}(0)$ may appear questionable, and it certainly is misleading in some theories, but it has been our first assumption that we are not dealing with such cases in that the explicitly momentum-dependent tensor parts in $\theta_{00}(0)$ are $SU_3 \times SU_3$ symmetric and do not enter into $u_0(0) + cu_8(0)$. A free-meson theory may serve as an illustration of what we mean; there we have

$$\langle P_i(p) | \theta_{00}(0) | P_i(p') \rangle = p_0 p'_0 + \mathbf{p}\mathbf{p}' + m_i^2, \quad (3.5)$$

with $(p_0 p'_0 + \mathbf{p}\mathbf{p}')$ being contributed by the chirally symmetric kinetic tensor term (for which the low-energy limit is a bad approximation) and the symmetry-breaking mass term whose effects are not distorted in the virtual low-energy limit.

Equation (3.4) illustrates the role of the pseudoscalar-meson masses in chiral symmetry breaking and their vanishing in the symmetry limit. It gives us a way of separating the effects of u_0 from the chirally invariant part in \mathcal{H} . Only for the pseudoscalar octet, with the help of PCAC, do we know how to isolate u_0 ; for the other multiplets, this separation is yet to be achieved.

Now we consider a more general scalar vertex with pseudoscalar mesons and apply SU_3 symmetry [see (B)]. Neglecting $(\eta\eta')$ mixing, we have

$$\begin{aligned} -\langle P_i(p) | u_j | P_k(p') \rangle & = \alpha(t) \delta_{j0} \delta_{ik} + \beta(t) d_{ijk}, \\ i, k & = 1 \cdots 8; \quad j = 0 \cdots 8, \end{aligned} \quad (3.6)$$

where $\alpha(0)$ and $\beta(0)$ are related to the pseudoscalar-meson masses through the SU_3 mass formula

$$-\langle P_i(p) | u_0 + cu_8 | P_i(p) \rangle \approx m_i^2 \approx \langle m^2 \rangle_{av} + d^{i88} \Delta m^2 \quad (3.7a)$$

by

$$\alpha(0) = \langle m^2 \rangle_{av} - (\sqrt{3}/3) \Delta m^2 / c \quad \text{and} \quad \beta(0) = \Delta m^2 / c. \quad (3.7b)$$

Taking low-energy limits and neglecting the dependence of α and β on t [$\alpha(m_i^2) \approx \alpha(0) \equiv \alpha$, $\beta(m_i^2) \approx \beta(0) \equiv \beta$], we have

$$\begin{aligned} a \delta_{j0} \delta_{ik} + \beta d_{ijk} & \approx -2f_i d_{ijl} \langle 0 | v_l | P_k \rangle \\ & \approx -2f_k d_{ljk} \langle P_i | v_l | 0 \rangle. \end{aligned} \quad (3.8)$$

Using different values for (i, j, k) , we find

$$\begin{aligned} (1,4,6): \beta & \approx -2f_\pi \langle 0 | v_K | K \rangle \approx -2f_K \langle \pi | v_\pi | 0 \rangle, \\ (1,1,8): \beta/\sqrt{3} & \approx -2f_\pi \langle 0 | v_8 | \eta \rangle / \sqrt{3} - 2f_\pi \langle 0 | v_0 | \eta \rangle \sqrt{3/2} \\ & \approx -2f_\eta \langle \pi | v_\pi | 0 \rangle / \sqrt{3}, \\ (4,4,8): -\beta/2\sqrt{3} & \approx 2f_K \langle 0 | v_8 | \eta \rangle / 2\sqrt{3} \\ & \quad - 2f_K \langle 0 | v_0 | \eta \rangle \sqrt{2/3} \\ & \approx 2f_\eta \langle K | v_K | 0 \rangle / 2\sqrt{3}, \\ (8,8,8): -\beta/\sqrt{3} & \approx 2f_\eta \langle 0 | v_8 | \eta \rangle / \sqrt{3} - 2f_\eta \langle 0 | v_0 | \eta \rangle \sqrt{2/3}, \\ (1,0,1): \alpha + \beta\sqrt{2/3} & \approx -2f_\pi \langle 0 | v_\pi | \pi \rangle \sqrt{2/3}. \end{aligned} \quad (3.9)$$

From these equations, we find

$$f_\pi \approx f_K \approx f_\eta, \quad (3.10)$$

$$\langle 0 | v_i | P_i \rangle \approx -\beta/2f, \quad \text{independently of } i, \quad (3.11a)$$

$$\langle 0 | v_0 | \eta \rangle \approx 0, \quad (3.11b)$$

$$\alpha \approx 0; \quad \langle m^2 \rangle_{av} \approx (\sqrt{2/3}) \Delta m^2 / c \Rightarrow c \approx -1.25. \quad (3.12)$$

Equation (3.10) states the equality of the $\pi_{\mu 2}$ and $K_{\mu 2}$ decay constants.¹¹ Experimentally they differ by less than 25%, and we may take this as an indication of the accuracy of our estimates. Equation (3.11a) reproduces the squared meson mass formula when combined with Eqs. (2.9), (3.10), and (3.11b). This is not particularly astonishing since we began with an equivalent assumption that the pseudoscalar-meson states form an approximately pure octet, yet it appears interesting to us that the simplest ansatz gave a consistent result starting with the squared meson mass formula. The vanishing of $\langle 0 | v^0 | \eta \rangle$ corresponds to our neglecting of (η, η') mixing; if we had a significant amount of mixing we would not have the octet mass formula for pseudoscalar mesons and η dominance for $\partial_\mu \mathcal{F}_{\mu 8}^5$. In our approach we have to insist on keeping $m_{\eta'}$ still large in the limit of $SU_3 \times SU_3$ symmetry, with no conservation for a hypothetical current of the form $\mathcal{F}_{\alpha 0}^5$, because otherwise we could not explain the large splitting of singlet and octet; the term \mathcal{H}' in the energy density, as given in Eq. (2.6), is clearly not responsible for that large splitting.

The value $c \approx -1.25$ suggests the closeness of our theory to the $SU_2 \times SU_2$ symmetric limit with $m_\pi = 0$ and $c = -\sqrt{2}$. In fact, we find that in the construction of the commutators (Sec. II), if we take the limit $m_\pi = 0$, we could replace our second and third assumptions by the requirements of $SU_2 \times SU_2$ symmetry and singlet and octet behavior of $\mathcal{H}(0)$, except that some (1,8) and (8,1) would still be allowed. We see $SU_3 \times SU_3$ symmetry (with eight pseudoscalar Nambu-Goldstone bosons)

¹¹ Compare C. S. Lai, Phys. Rev. Letters **20**, 509 (1968) and R. J. Oakes, *ibid.* **20**, 513 (1968).

broken in two chains¹²:

$$\begin{array}{ccc} & SU_3 & \\ & \nearrow & \searrow \\ SU_3 \times SU_3 & & SU_2 \\ & \searrow & \nearrow \\ & SU_2 \times SU_2 & \end{array} \quad (3.13)$$

At least for the low-lying multiplets, chiral symmetry appears realized by strong coupling to massless pseudoscalar mesons, rather than by parity doubling. Fubini¹³ has recently emphasized that this is a quantitative matter in the real world; if a resonant state of opposite parity and the same spin lies closer in mass than m_π from a given state, then approximate chiral symmetry can manifest itself more in the manner of degenerate multiplets than in the Nambu-Goldstone manner.

The vacuum state should be expected to be approximately invariant under SU_3 but not $SU_3 \times SU_3$. That is consistent with Eq. (3.11), as we shall show when estimating the vacuum expectation values $\langle 0|u_0|0\rangle$ and $\langle 0|u_s|0\rangle$. The limit $p \rightarrow 0$ in Eq. (3.11) gives

$$\begin{aligned} \langle 0|v_i|P_i\rangle &= -m_i^2/(2f_i)W_i(c) \approx -\beta/2f \\ &\approx -(2f_i)(d_{i0}\langle 0|u_0|0\rangle + d_{is}\langle 0|u_s|0\rangle). \end{aligned} \quad (3.14)$$

To the accuracy of Eq. (3.14), we find $\langle 0|u_s|0\rangle \approx 0$, which indeed corresponds to an SU_3 -invariant vacuum, and $\langle 0|u_0|0\rangle \approx \frac{3}{2}\langle m^2 \rangle_{av}/(2f)^2$.

At this point, we want to argue that the effects of a possible $(1,8) + (8,1)$ admixture in \mathcal{H}' on our deductions are in fact small. We denote the scalar and pseudoscalar members of the multiplet by $g_s(0)$ and $h_s(0)$, respectively. They transform as follows:

$$\begin{aligned} [F_i, g_j] &= if_{ijk}g_k, \\ [F_i, h_j] &= if_{ijk}h_k, \\ [F_i^5, g_j] &= if_{ijk}h_k, \\ [F_i^5, h_j] &= if_{ijk}g_k. \end{aligned} \quad (3.15)$$

We note that the h_k behave oppositely under charge conjugation to the v_k . Making an ansatz similar to (3.6) for a possible contribution g_s in $\mathcal{H}'(0)$, we have

$$\begin{aligned} \langle P_i|g_j|P_k\rangle &= d_{ijk}\gamma(i) \\ &\approx -f_{ijk}\langle 0|h^k|P^k\rangle/2f_i. \end{aligned} \quad (3.16)$$

We see that, to the accuracy of low-energy limits, $\gamma(0) \approx 0$ and $\langle 0|h^k|P^k\rangle \approx 0$. Roughly speaking, we would find effects of a possible $(1,8) + (8,1)$ admixture only in the order or corrections to PCAC in our results.

This possibility should be kept in mind when trying to use the commutators of currents and divergences in

¹² Compare G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Letters **20**, 64 (1966).

¹³ S. Fubini and G. Furlan, Massachusetts Institute of Technology Report, 1968 (unpublished).

connection with multiplets other than the pseudoscalar octet. Such tests are very desirable for confirming the universality of c . An obvious possibility is the study of the so-called σ terms in the scattering of pseudoscalar mesons at low energies.¹⁴ In πN scattering, the low-energy value of the isospin-symmetric amplitude is given by the formula¹⁵

$$\begin{aligned} \langle \pi N|T|\pi N\rangle|_{q_\pi=0} &= \frac{1}{3}(\sqrt{2}+c)(4f_\pi^2) \\ &\quad \times \langle N|\sqrt{2}u_0+u_s|N\rangle. \end{aligned} \quad (3.17)$$

A precise estimate is made difficult for three reasons. The first is the occurrence of the factor $(\sqrt{2}+c)$. The second is the lack of knowledge about $\langle N|u_0|N\rangle$; it is not possible to decide *a priori* how much of the nucleon mass should be attributed to chiral symmetry violation. The third reason is that the application of PCAC to a four-point function is a delicate problem. The extreme smallness of the sum of π^+p and π^-p scattering lengths tends to suggest that in fact the quantity $\langle N|u_0|N\rangle$ is not a large fraction of the nucleon mass. In K^+p scattering, we find

$$\begin{aligned} \langle K^+p|T|K^+p\rangle|_{q_K=0} &= \frac{1}{3}(\sqrt{2}-\frac{1}{2}c)(4f_K^2) \\ &\quad \times \langle p|\sqrt{2}u_0+\frac{1}{2}\sqrt{3}u_3-\frac{1}{2}u_8|p\rangle \end{aligned} \quad (3.18)$$

and two of these difficulties are still present, with the PCAC trouble aggravated. Attempts at numerical estimates may be considered elsewhere.

Within the meson system, further applications can be made to $K_{\mu 3}$ decays. We found the low-energy theorems emerging from the commutators (2.7) compatible with and, to our accuracy, hardly distinguishable from the Callan-Treiman relation.¹⁶ The details will be given elsewhere, along with applications to hard-meson calculations of the $K_{\mu 3}$ vertex.

In conclusion, let us reemphasize that we have so far investigated the compatibility of the formula $\mathcal{H}' = -u_0 - cu_s$ (with c near $-\sqrt{2}$) only with certain data about hadrons and that further tests are desirable to establish whether a single $(3,3^*)$ and $(3^*,3)$ representation is indeed what is involved and also to decide whether there is an additional term of the form g_s belonging to $(1,8)$ and $(8,1)$.

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¹⁴ K. Kawarabayashi and W. W. Wada, Phys. Rev. **146**, 1209 (1966); K. T. Mahanthappa and Riazuddin, Nuovo Cimento **45A**, 252 (1966); R. W. Griffith (to be published), has also emphasized the connections between σ terms and the fraction of the nucleon mass arising from chiral symmetry violation.

¹⁵ We use the notation $S=1+i(2\pi)^4\delta(p'-p)T$ and covariant normalization for the states; see Footnote 5.

¹⁶ C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966).