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BEHAVIOUR OF BOND'S EMBEDDED OPTION WITH REGARD TO CREDIT RATING

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Abstract. In this financial engineering research, we study the behaviour of an option premium of a call/put option which is embedded in a typical fixed coupon bond with finite maturity. The contribution of the research is the conclusion about the dynamics of premium changes; represented by direction and sensitivity; with respect to the changes in credit rating and also risk-free interest rate development. The aim of the research is also to clearly demonstrate this theoretically complicated topic to the financial practitioners using a practical example. We are about to consider a 3-dimensional process where the dimensions are: time, rating development process and risk-free interest rate development. We use Standard & Poor's rating transition matrix to create rating tree and Hull-White model for modelling of risk-free interest rate development. We add embedded call/put option to the bond structure and assume the call/put option to be exercised in case of interest rates decline/rise or rating worsening/improvement. For valuation, we use the risk-neutral concept. Using a numerical solution on the 3-dimensional tree (implemented in MATLAB), we avoid problems that appear while analytical solving of partial differential equations.

Keywords: embedded call/put option, credit rating transition; more dimensional tree, Standard & Poor's rating, embedded option premium, rating development process.

JEL Classification: G1, G10, G12, G14.

Introduction

A bond is probably the most traditional and important investment instrument with regard to the volume and liquidity of the transactions. Portfolios of banks and insurance companies contain a significant percentage of this instrument. In comparison to a stock, for example, it is not a simple product, moreover in case if it contains embedded options. Issuers frequently have the right to buy back a certain amount of the debt or to repay all the instrument on certain points of time before maturity (Bermuda Style Option in this case). Sometimes there is set a certain protection period for an investor, after the issuance. In this case, we speak about the call option (callable bond) and the issuer pays for the option – the bond is cheaper for an investor. If an analogical option is in the hands of a bondholder we speak about embedded put option. This option can be of considerable value. In some cases, the issuer might even have the right to pay back a bond at any time, i.e., the bond contains an American Style Prepayment Option.

The main contribution of the research is the conclusion about the dynamics of an option premium value changes; represented by the direction and sensitivity; with respect to the changes of credit rating and also risk-free interest rate development. We are about to consider a 3-dimensional process where the dimensions are: time, rating development process and interest rates development. The research contains an example of practical usage and appropriate 3-dimensional charts. For practical valuation, we use a fixed coupon bond with finite maturity, Standard & Poor's rating transition matrix and Hull-White model for risk-free rate development. We add embedded call/put option to the bond structure and assume the call/put option to be exercised in case of interest rates decline/rise or rating worsening/improvement. All the

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presumptions have financial logic. Using a numerical solution on the 3-dimensional tree (implemented in MATLAB), we avoid problems that appear while analytical solving of partial differential equations.

When a bank or insurance company deal with the risks resulting from interest rate options (and also with other simple options), they usually calibrate an interest rate model to certain liquid instruments of the interest rate market, price the options using such model, and use the model output to manage the options' risks (Brigo and Mercurio 2006, Dozsa and Janda 2017). In the case of embedded bond options, the situation is not as simple because options embedded inside a bond are also connected to the default/credit risk of the issuer. Furthermore, issuer, in many cases, does not use his right rationally/pure logically as for example in the case of interest rate derivatives traders. This is why the techniques from simple options markets are difficult to use for the bond markets. We have to go much deeper to the model by including the debtor's credit risk. Basically, there are two types of models that can be distinguished in the literature. The first one is the structural model, describing the economic process underlying the default of a debtor explicitly and the second one is represented by reduced form approaches which model the spreads over the risk-free interest rate needed for pricing risk-free bonds (Janda and Rojcek 2014). Or in other words: while firm value models assume that the company's asset is tradeable and the company defaults when the assets are low, the reduced-form models describe default as a totally random occurrence without any concern about the assets or any other firm-specific process (Janda et al. 2013). A good overview of both types of models can be found in Aguais and Santomero (1998), Aguais and Forest (2000), Schoenbucher (2003). The pricing model used in this research models credit risk by rating transitions. This approach was introduced by Jarrow et al. (1997), where the default process was basically modelled by a Markov chain of credit ratings. Most rating systems apply statistical models that use an estimation of a default probability for each rating grade or a transition matrix for the full rating system. Further, from the empirical experience and from the history of losses that have already been observed in a bank's portfolio of loans, an estimation of recovery rates could be made. An overview of the statistical estimation of default probabilities and recovery rates can be found, by the way of example, in Engelmann and Rauhmeier (2006). For bonds without embedded options, the pricing and risk management are based on statistically and empirically estimated default probabilities and recovery rates. Based on this information all the components of a price are calculated to determine the interest rate margin which investor charges to a bond issuer. The expected credit loss of a bond investment is one of these components, and the interest margin should cover expected losses on a portfolio level. In this context, the risk is usually considered to be an uncertainty about the portfolio loss at a future point in time, and a credit risk model is used to quantify the loss distribution. Popular frameworks for such modelling are based on Gupton et al. (1997), or Wilson (1997a) and Wilson (1997b). These frameworks have been extended and upgraded in recent years by various authors. A good example of a very good and still numerically tractable approach belonging to the class of asset value models is Castagna et al. (2009). Once the loss distribution is computed, economic capital is defined by a risk measure like value-at-risk or expected shortfall. Although value-at-risk is still more popular in banking practice, the expected shortfall has the superior properties as analysed in articles by Artzner et al. (1999), Acerbi and Tasche (2002), and Tasche (2002). Once the economic capital is determined it has to be allocated to each credit exposure to measure the main drivers of credit risk in the portfolio. Reasonable approaches are explained in Kalkbrener (2005), Janda (2009, 2011), Horváth and Teplý (2013), Kalkbrener et al. (2004), and Kurth and Tasche (2003). We assume that for each bond the rating grade of a debtor is known and that either the term structure of default probabilities or a one-year transition matrix from the bank's rating system and a (possibly time-dependent) recovery rate have been estimated. There already exists a lot of literature on prepayment options in the context of mortgages and mortgage-backed securities (MBS), Kau and Keenan (1995). In these articles interest rates are driven by a term structure model and the optimality of prepayment is either derived from the interest rate level and some additional conditions like transaction costs as in Stanton (1995). Alternatively, prepayment is modelled explicitly by a prepayment process as, e.g., in Kolbe and Zagst (2008). In the latter case, the parameters of the prepayment process are determined in a calibration process from empirically observed market prices of MBS. In the mortgage literature, a default is typically resulted from an explicit modelling of the house price, as in Ciochetti et al. (2002) where debtors default when house prices fall and the drivers of default are then analysed empirically. Of course, this modelling approach is applicable to mortgages only and not to other types of loans. Furthermore, it is related to mortgages where the real estate is the only available collateral – typical for the US loan market and not for loan markets of other countries. One of the latest work on the topic of embedded option is provided by Kolman (2017) where PDE pricing is used for both firm-value and reduced-form models. Also, Kopa et al. (2017) provided new research focused on implied volatility.

Similar research of dynamics using a multifactorial approach, applied to credit default linked instruments, was done by Choro's-Tomczyk et al. (2016); Cont and Minca (2013).

1. Methodology

This research is quite generic and applicable to any type of bond. The default is modelled in a reduced form approach by explicitly modelling the credit rating of an issuer. A further advantage of this model is that it is based almost entirely on risk parameters that are already available in banks' or insurance companies' risk management systems and, therefore, hardly any model calibration is necessary. Modelling prepayment, optimality conditions are derived endogenously from the future level of interest rates and the debtor's rating at prepayment times. Other embedded options like caps and floors of loans with floating interest rates or combinations of caps with prepayment rights can be included easily into this model. Finally, if a bank decides to hedge some of its exposure to embedded options with market instruments like European swaptions the model can be used for calculating the appropriate hedge ratios. However, as the model is not complete, these hedges will not be perfect. It will be shown how techniques from credit risk modelling can be used to quantify the risk of losses when hedging embedded options in loans on the portfolio level. We finally remark that a similar approach to modelling prepayment endogenously as in this article was suggested by Aguais et al. (2000). However, they were not very detailed on model calibration nor did they work out a link between derivatives pricing and credit modelling.

We have focused so far on modelling the stochastic structure of the default event by an intensity using rating transitions, for example, Lando (1998) considers credit ratings also.

With a numerical solution on the 3-dimensional tree, using computational finance methodology (implemented in MATLAB), we avoid problems to obtain an analytical solution of partial differential equations. Nowadays, computational finance methods allow demonstrating such solutions while analytical solutions do not exist in many cases as argue Tapiero (2013), Hirsa (2016).

1.1. Example on typical bond

By the way of example, we evaluate typical coupon bond with 30 years to maturity which is callable (case 1) and putable (case 2) and taking credit rating and risk-free rate development into the consideration. Coupon of the bond is the same as the initial risk-free rate, thus the initial price must be 100% at best rating (AAA). Price of the risk-free bond (rated AAA), with the coupon, equalled current risk-free rate, is in Figure 1. Its price is 100% for each of initial interest rate (as it is mentioned above). In figure 10 a), b) we observe the deformations of the plane (in Figure 1) with regard to changes of rating.

We are about to consider a 3-dimensional process where the dimensions are:

1. dimension – time
2. dimension – risk-free interest rates development
3. dimension – rating development process

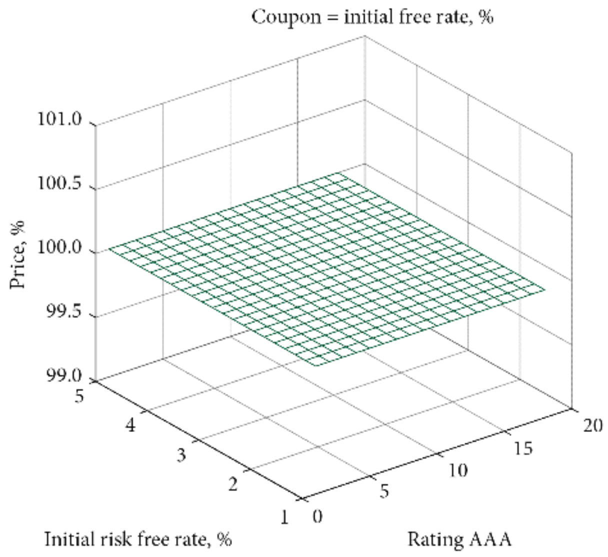

Figure 1. Price of risk-free bond with the coupon equalled current risk-free rate (source: own illustration)

1.2. 3D-Trinomial tree characteristics

We use a combination of 2 trinomial trees. The first one is the tree of credit rating development and the second one is the tree of risk-free rate development.

The tree is more intuitive than a PDE solver. The trinomial tree (Figure 2) is fully specified by the value of the risk-free rate and rating assessment at each node of the tree, where there are nine probabilities for moving to one of nine possible states in the subsequent time node at each node.

The nine probabilities sum to one. The risk-neutral probabilities and the short rate levels of the tree may be calibrated to the funding discount curve and a set of European swaptions.

This ensures that the pricing of basic market instruments is done correctly. For details on the calibration process, we refer to Brigo and Mercurio (2006).

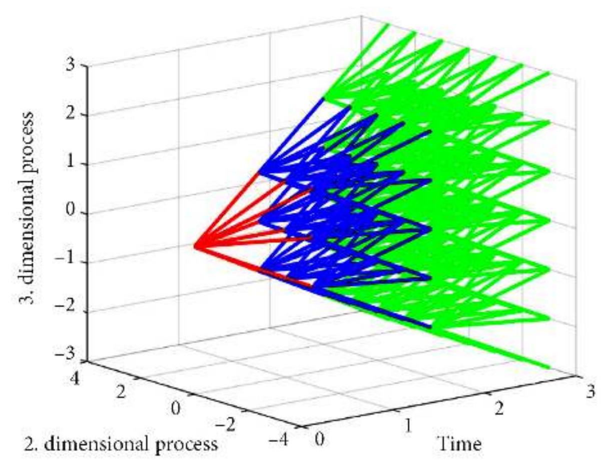

Figure 2. The trinomial tree (source: own illustration)

We note that in practical applications one would rather use a PDE solver instead of the trinomial tree because of its superior convergence properties (Randall and Tavella 2000).

The sequence of random developments which is possible to observe is for the illustration in Figure 3.

The initial price (required evaluation) is obtained using the calculations from right to left on the tree, or in other words, from the future to the purchase day (Figure 4).

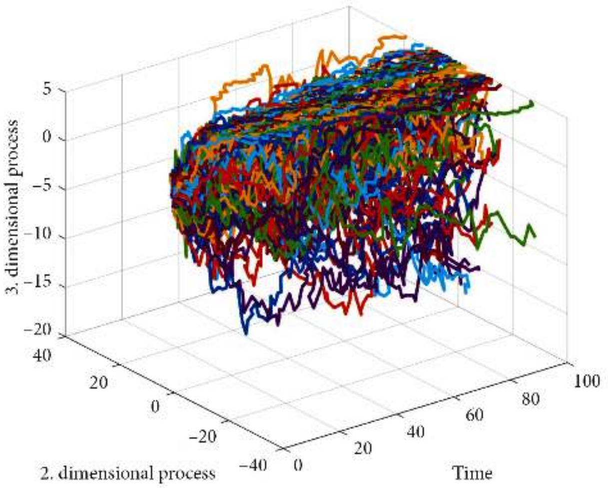

Figure 3. The sequence of random developments (simulation on the tree) (source: own illustration)

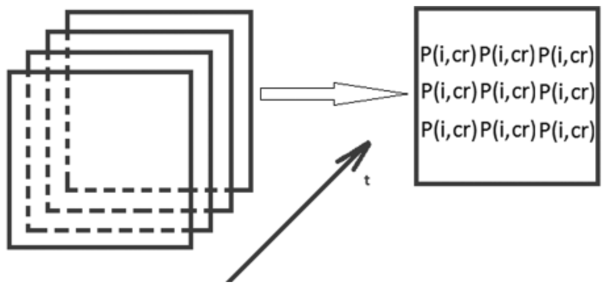

Figure 4. The principle of the calculation (source: own illustration)

The price P at each node is given by formula (1), where t is time, i is the value of risk-free rate at the point of time t, cr is current credit rating value and q denotes the probability of one way from the note.

$$P_{t,i,cr} = \sum_{s=1}^{n} q_s \frac{P_{s,t+1}}{(1+i_t)}. \quad (1)$$

As we have stated we recognize 9 ways from each node (Figure 2) means 9 prices from each node. We recognize 3 possible movements of risk-free rate and 3 possible credit rating movements.

1.3. Rating development process

We use Standard & Poor's rating transition matrix which is in figure 1 in the Appendix. We denote ratings "AAA – D" by numbers "1–18". So we recognize 18 credit rating states.

Transition matrix could be displayed as in Figure 5, and it allows the construction of rating part of the tree.

An example of a rating tree is shown in Figure 6. The thickness of the lines in the tree corresponds to the probability. Suppose we are interested in the pricing of a financial instrument that depends on the rating grade of a bond issuer at several time points T_0; T_1; … T_n.

The probability that a debtor in rating grade $cr(T_i)$ at time T_i moves to rating grade $cr(Ti + 1)$ at time $(Ti + 1)$ is given by probability according to the rating tree. There are three possible steps from each node, by the way of example: the step from BBB to BBB+ after the first year means the new rating is BBB+ or better. Simulated paths of credit rating development are in Figure 7. If the rating reaches 18 which is "D", there is not the way back.

Appropriate distribution based on Standard & Poor's rating transition matrix is in Figure 8.

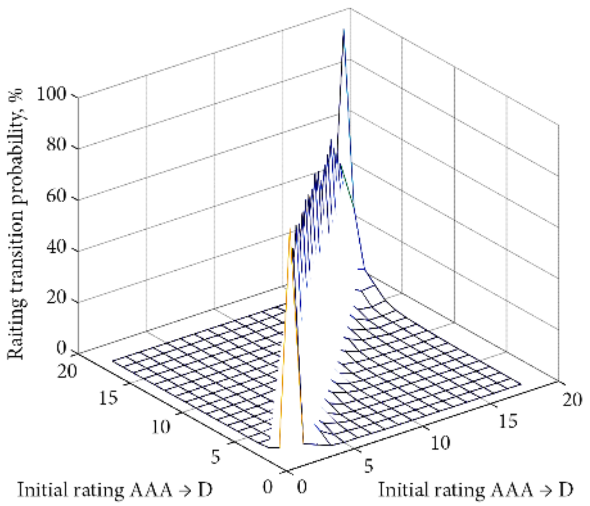

Figure 5. Rating transition chart (source: own illustration)

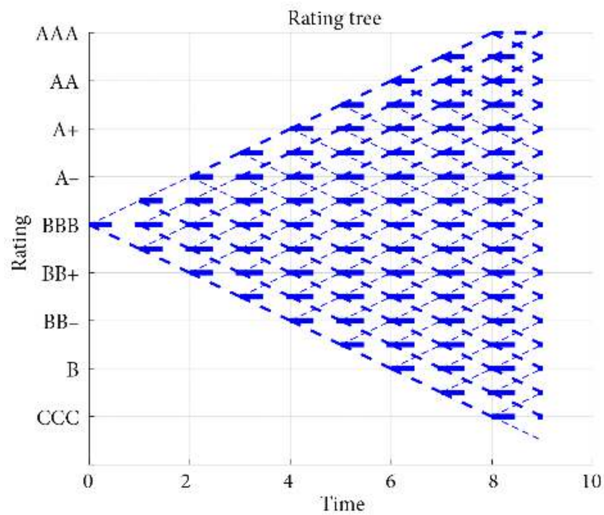

Figure 6. Rating tree (source: own illustration)

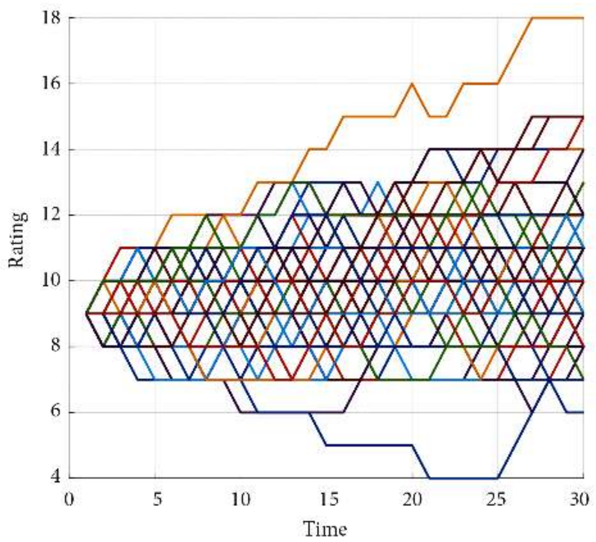

Figure 7. Simulated path of credit rating development (source: own illustration)

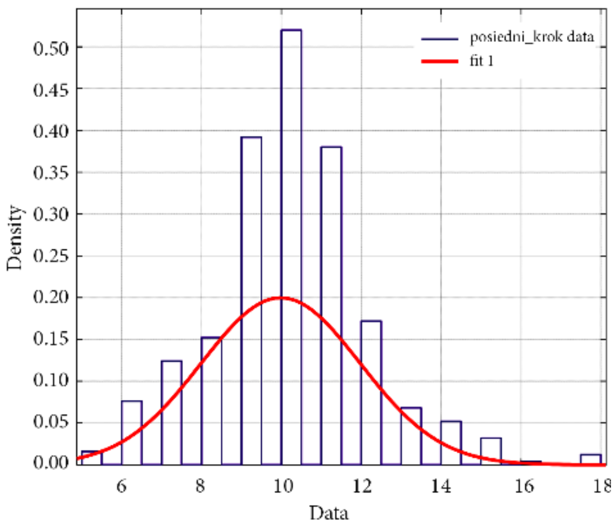

Figure 8. Credit rating distribution (source: own illustration)

1.4. Interest rate development process

For the modelling of risk-free rate development, we assume Hull-White model using parameters given from the US market. Hull-White tree is constructed using MATLAB implementation which is also providing probability values of transition between rates (Figure 9).

1.5. Options exercising presumptions

We adopt certain presumptions regarding exercising of option which are reasonable from the point of view of a bondholder and also of a bond issuer.

The call option is exercised in case the interest rate falls below the initial interest rate. The issuer has the chance to issue a new bond at lower interest costs. The similar presumption is the case of rating improvement. The issuer may issue the bond at the lower rate. Analogically the put option may be used by a bondholder in the case of the increase of interest rate (the bond price decreases below strike which is 100%) and worsening of rating which also decreases the price.

Both the call and put options are of Bermuda style and may be used at the end of each year.

1.6. Option premium calculation

The final price of the option is calculated as the difference of the bond without and with the embedded option. It is also observable from tab 1 in the Appendix. In figure 10 a) there is the recovery rate set to 0%, in figure 10 b) the recovery rate is set to the empirical value of 35%.

2. Results

Price of the bond, as an underlying asset, with respect to credit rating is in figure 10 a), b). Its value, of course, falls with worsening of credit rating. From figure 10 it is also clear that the sensitivity of the bond price with respect to credit rating changes generally increases with the rating worsening – the decreasing price/rating curve is steeper in the area of lower interest rate. This property could be well explained by higher price sensitivity in the area of lower interest rates while yield to maturity changes are caused by rating changes. For better imagination, we provide two figures.

Adequate YTM of bond with respect to credit rating is in Figure 11. YTM in Figure 11 corresponds to price chart in Figure 10. Higher values for higher risk-free rate are caused by higher coupon while the price is at 100%. It corresponds to basic bond theory.

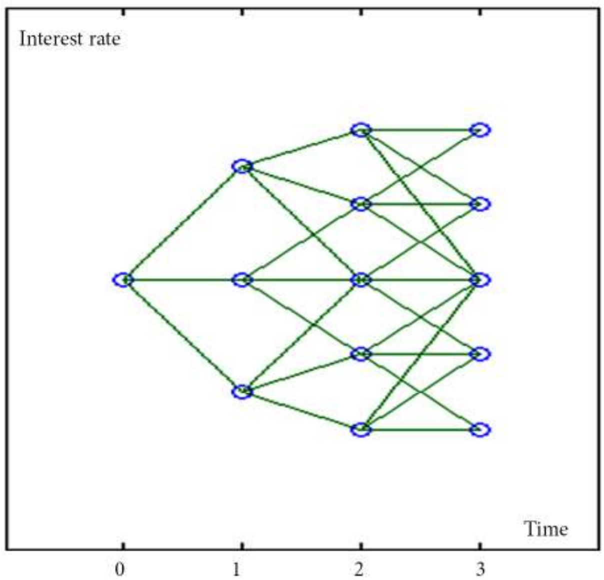

Figure 9. Hull-White trinomial tree for modelling of interest rates development (source: MathWorks)

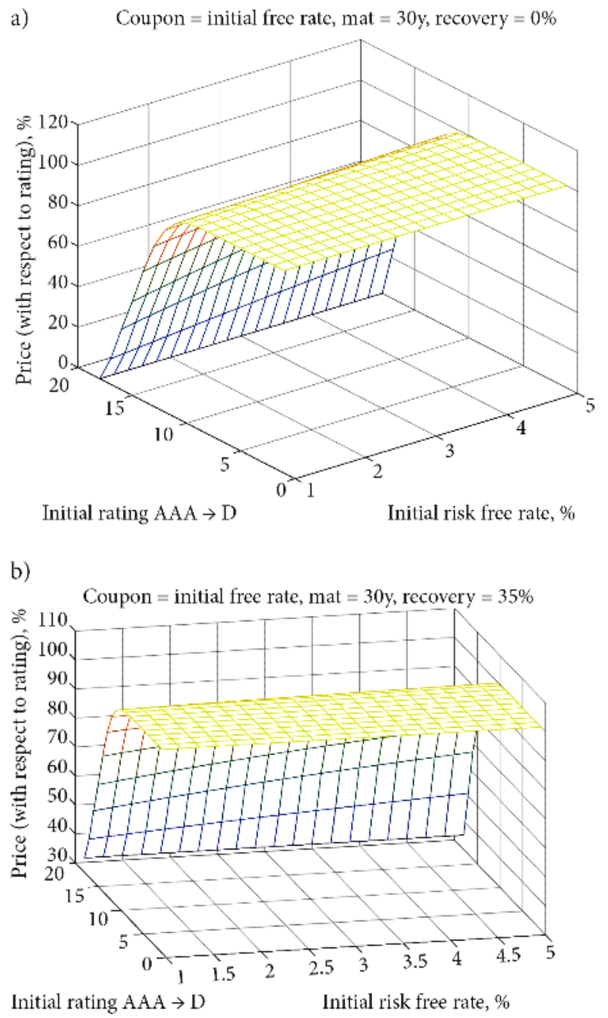

Figure 10. Price of bond with respect to credit rating, a) recovery rate = 0%, b) recovery rate = 35% (source: own illustration)

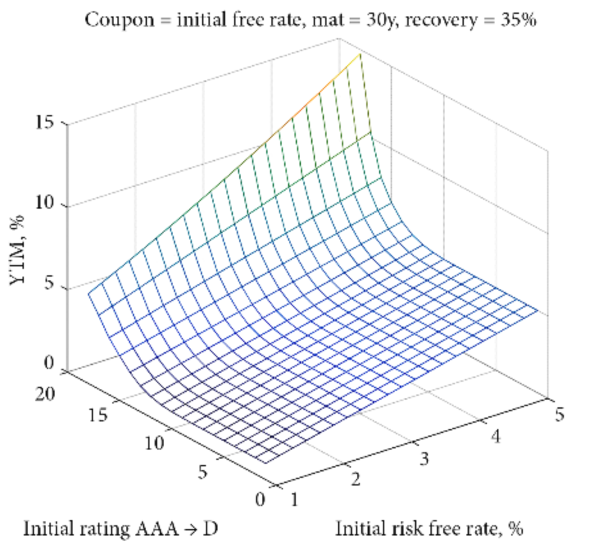

Figure 11. YTM of bond with respect to credit rating (source: own illustration)

3d chart of a price of the embedded put option is in Figure 12. The price is equalled 0 in the case of complete default and increases with rating worsening. This property could be well explained by the higher volatility of the underlying asset price in the area of worse rating (interpretation of Figure 10). The higher value of the option in the area of higher initial rate could be the reason of more factors, for example, that higher rate level in economy supports option prices or it is caused by certain disparities in the transition matrix (Figure 5).

3d chart of the embedded call option is in Figure 13. The price is equalled 0 in the case of complete default and increases with rating worsening. This property could be well explained by the higher volatility of the underlying asset price in the area of worse rating. It is the same effect as we observe

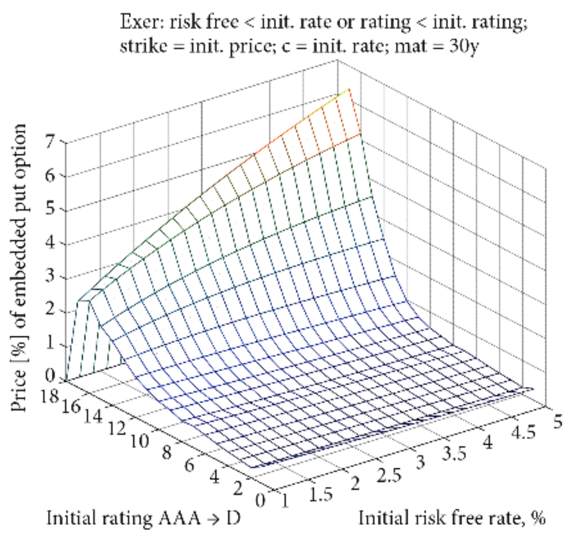

Figure 12. Price of embedded put option (source: own illustration)

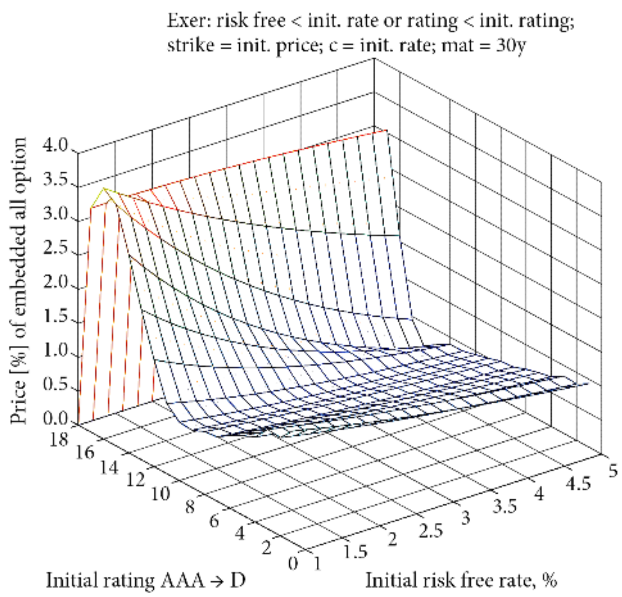

Figure 13. Price of embedded call option (source: own illustration)

in Figure 12. Certain "breakage" of the plane results from "disturbances" inside the transition matrix.

Differences of option premium values with respect to rating changes are in the Figures 14 and 15. Differences are a good measure of sensitivity and volatility. It is clear that the sensitivity of the price of the embedded option is generally higher with worsening of the credit rating, but it is not a rule. Each line in the figure is connected with the certain value of initial risk-free interest rate.

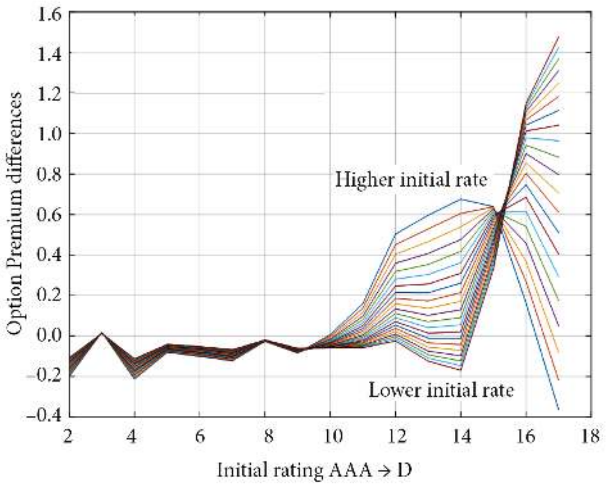

Figure 14. Call option premium sensitivity with respect to the development of credit risk (source: own illustration)

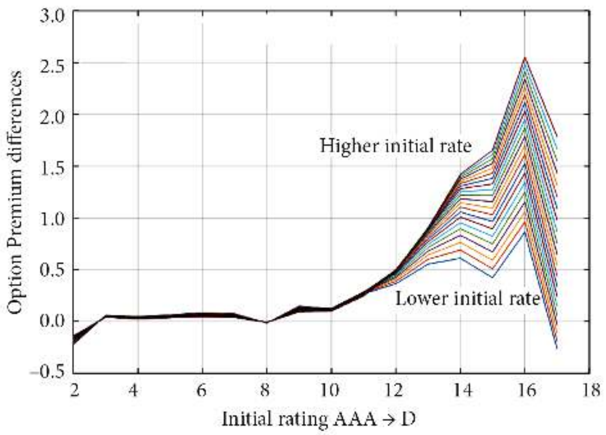

Figure 15. Put option premium sensitivity with respect to the development of credit risk (source: own illustration)

Conclusions

The main contribution of this financial engineering research are the conclusions about the dynamics of change (means the direction and value of change) of option premium of an option which is embedded in a typical fixed coupon bond with respect to the credit rating changes and risk-free rate changes. We have studied both callable and putable bonds.

From practical example, we have obtained 2 main findings in the research. Both findings we may intuitively feel, but in the research, we have demonstrated them by numerical calculation and quantification.

The first finding is that the value of option premium of embedded call/put option increases with the worsening of credit rating. It could be well explained by the higher volatility of the underlying asset price in the area of worse rating. Such property is very well observable in figure 10 a), b). This finding very well corresponds to the standard option theory, says that the price of call/put option increases with higher volatility of the underlying asset.

The second finding concludes into the assessment of the sensitivity of option premium value with respect to credit rating changes (or in other words: assessment of changes (differences) of option premium value with respect to the credit rating change). The sensitivity of option premium value with respect to credit rating changes depends on the current situation of a bond, given by credit rating and risk-free interest rate and it is demonstrated in the figures 14, 15. Based on the parameters of the rating transition matrix the sensitivity may not increase continuously; also, the surface (especially in figure 13) is not smooth because of parameters of the rating transition matrix.

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Appendix

Table 1. Prices (row) of the bond without option with respect to the credit rating and risk-free interest rate (source: own results)

Rating:	1.0	1.2	1.4	1.6	1.8	2.0
AAA	100.2	100.2	100.2	100.2	100.2	100.2
AA+	100.2	100.2	100.2	100.2	100.2	100.2
AA	100.2	100.2	100.2	100.2	100.2	100.2
AA-	100.2	100.2	100.2	100.2	100.2	100.2
A+	100.2	100.2	100.2	100.2	100.2	100.2
A	100.2	100.2	100.2	100.2	100.2	100.2
A-	100.2	100.2	100.2	100.2	100.1	100.1
BBB+	100.1	100.1	100.1	100.1	100.1	100.1
BBB	99.8	99.8	99.8	99.8	99.8	99.8
BBB-	98.7	98.8	98.9	98.9	99.0	99.0
BB+	96.0	96.2	96.3	96.5	96.6	96.8
BB	89.9	90.3	90.6	91.0	91.4	91.7
BB-	79.2	80.0	80.7	81.4	82.0	82.6
B+	64.0	65.1	66.2	67.3	68.2	69.2
B	46.4	47.7	49.0	50.2	51.4	52.6
B-	30.0	31.2	32.4	33.6	34.7	35.8
CCC	12.5	13.2	14.0	14.7	15.4	16.0
D	0.0	0.0	0.0	0.0	0.0	0.0

Global Corporate Transition Matrix (%) (1981–2010)																	
RATING	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
AAA	87.91	4.72	2.68	0.68	0.16	0.24	0.14	0.00	0.05	0.00	0.03	0.05	0.00	0.00	0.03	0.00	0.05	**0.00**
AA+	2.62	76.06	11.67	3.93	0.89	0.66	0.30	0.12	0.12	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	**0.02**
AA-	0.05	0.13	4.28	76.93	10.02	2.84	0.71	0.27	0.14	0.07	0.04	0.00	0.00	0.04	0.11	0.02	0.00	**0.04**
A+	0.00	0.11	0.58	4.46	77.42	8.80	2.57	0.71	0.40	0.09	0.09	0.12	0.01	0.09	0.04	0.01	0.00	**0.07**
A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	**0.09**
A-	0.06	0.01	0.11	0.20	0.61	6.78	75.80	7.51	2.36	0.68	0.16	0.15	0.16	0.14	0.04	0.01	0.05	**0.08**
BBB+	0.00	0.01	0.07	0.09	0.31w	1.05	6.93	73.19	8.85	2.01	0.47	0.40	0.17	0.26	0.15	0.02	0.10	**0.16**
BBB	0.01	0.01	0.06	0.04	0.17	0.48	1.23	7.04	74.22	6.30	1.62	0.83	0.37	0.31	0.17	0.04	0.09	**0.23**
BBB-	0.01	0.01	0.01	0.07	0.07	0.24	0.40	1.37	8.56	71.12	5.48	2.59	1.03	0.56	0.34	0.22	0.31	**0.38**
BB+	0.07	0.00	0.00	0.05	0.02	0.15	0.12	0.63	2.29	11.70	62.56	6.43	3.24	1.27	0.83	0.19	0.51	**0.56**
BB	0.00	0.00	0.06	0.02	0.00	0.10	0.08	0.23	0.74	2.56	8.51	64.26	7.74	2.69	1.37	0.46	0.74	**0.80**
BB-	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.13	0.30	0.48	2.06	8.23	63.76	8.43	3.06	0.97	0.91	**1.31**
B+	0.00	0.01	0.00	0.04	0.00	0.04	0.09	0.06	0.07	0.10	0.34	1.57	6.92	65.02	7.66	2.62	1.96	**2.62**
B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	**5.90**
B-	0.00	0.00	0.00	0.00	0.04	0.07	0.00	0.14	0.07	0.14	0.18	0.21	0.61	3.13	10.22	51.30	10.82	**9.15**
CCC/C	0.00	0.00	0.00	0.00	0.05	0.00	0.14	0.09	0.09	0.09	0.05	0.23	0.56	1.39	2.91	8.70	43.80	**27.43**

Figure 1. Rating transition matrix (source: Standard & Poor's)

Notations

Variables and functions

P – price at each node of the tree, given by formula (1);

t – time;

i – the value of risk-free rate at the point of time *t*;

cr – current credit rating value;

q – the probability of one way from the note.