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BENDING OF RECTANGULAR THIN PLATES WITH FREE EDGES LAID ON TENSIONLESS WINKLER FOUNDATION

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Abstract

In this paper, the bending problem of rectangular thin plates with free edges laid on tensionless Winkler foundation has been solved by employing Fourier series with supplementary terms. By assuming proper form of series for deflection, the basic differential equation with given boundary conditions can be transformed into a set of infinite algebraic equations. Because the boundary of contact region cannot be determined in advance, these equations are weak nonlinear ones. They can be solved by using iterative procedures.

I. Introduction

The calculation of plates on elastic foundation is a common problem in engineering. Various assumptions and calculating models have been proposed in literature, in which the model based on the well-known Winkler's assumption is widely used owing to its simplicity and easiness in engineering applications. However, this model has also some weakness which becomes more conspicuous for some specific problems. For example, the soil foundation can bear compression only, but it cannot resist tension at all. Therefore calculations based on Winkler's foundation model will cause remarkable errors. In this paper, to overcome this weakness, we solve the bending problem of rectangular thin plates with free edges laid tensionless Winkler foundation by employing Fourier series with supplementary terms[2]. Results obtained are satisfactory.

II. Fundamental Differential Equation and Boundary Conditions

Fig. 1 shows a rectangular plate with side lengths a,b and thickness h laid on an elastic foundation, subjected to a transverse load $q(x,y)$.

1. Fundamental differential equation

$$D\nabla^4 w(x,y)+kH(w)w(x,y)=q(x,y) \quad (2.1)$$

in which

$$\nabla^4=\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}$$

$$H(w)=\begin{cases}1 & (w>0)\\ 0 & (w\leqslant 0)\end{cases} \quad (2.2)$$

$$D=\frac{Eh^3}{12(1-\mu^2)}$$

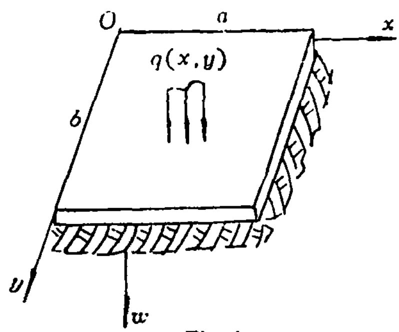

Fig. 1

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is the bending rigidity, and k is the modulus of foundation.

2. Boundary conditions

(1) On sides $x=0$ and $x=a$

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{2.3}$$

$$V_x=-D\left[\frac{\partial^3 w}{\partial x^3}+(2-\mu)\frac{\partial^3 w}{\partial x\partial y^2}\right]=0 \tag{2.4}$$

(2) On sides $y=0$ and $y=b$

$$M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{2.5}$$

$$V_y=-D\left[\frac{\partial^3 w}{\partial y^3}+(2-\mu)\frac{\partial^3 w}{\partial x^2\partial y}\right]=0 \tag{2.6}$$

(3) At all corners

$$R=-2D(1-\mu)\frac{\partial^2 w}{\partial x\partial y}=0 \tag{2.7}$$

III. Series Expansions of Deflection Function w (x, y), Heaviside Function H(w) and Loading Function q (x, y)

Assume the deflection function $w(x, y)$ is expanded into Fourier series with supplementary terms as the following form

$$\begin{aligned} w(x,y)=&\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}W_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\\ &+\sum_{m=0}^{\infty}\left\{\left[(2-\mu)\frac{b^2}{a^2}m^2\pi^2\frac{4by^3-4b^2y^2-y^4}{24b^4}+\frac{2by-y^2}{2b^2}\right]C_m\right.\\ &\left.+\left[(2-\mu)\frac{b^2}{a^2}m^2\pi^2\frac{y^4-2b^2y^2}{24b^4}+\frac{y^2}{2b^2}\right]D_m\right\}\cos\frac{m\pi x}{a}\\ &+\sum_{n=0}^{\infty}\left\{\left[(2-\mu)\frac{a^2}{b^2}n^2\pi^2\frac{4ax^3-4a^2x^2-x^4}{24a^4}+\frac{2ax-x^2}{2a^2}\right]G_n\right.\\ &\left.+\left[(2-\mu)\frac{a^2}{b^2}n^2\pi^2\frac{x^4-2a^4x^2}{24a^4}+\frac{x^2}{2a^2}\right]H_n\right\}\cos\frac{n\pi y}{b} \end{aligned} \tag{3.1}$$

This expression is a dual cosine series with two supplementary single cosine series. In this expression, W_{mn}, C_m, D_m, G_n and H_n are undetermined coefficients. It can easily be seen that expression (3.1) can be differentiated term by term at least four times and satisfies boundary conditions (2.4), (2.6) and (2.7) automatically.

Expressions for bending moments M_x and M_y can be derived:

$$M_x=D\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{m^2}{a^2}+\mu\frac{n^2}{b^2}\right)\pi^2 W_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$+D\sum_{m=0}^{\infty}\left\{\left[(2-\mu)b^2\frac{m^4\pi^4}{a^4}\,\frac{4by^3-4b^2y^2-y^4}{24b^4}+\frac{m^2\pi^2}{a^2}\left(\frac{2by-y^2}{2b^2}\right.\right.\right.$$

$$\left.\left.-\mu(2-\mu)\frac{6by-2b^2-3y^2}{6b^2}\right)+\frac{\mu}{b^2}\right]C_m+\left[(2-\mu)b^2\frac{m^4\pi^4}{a^4}\,\frac{y^4-2b^2y^2}{24b^4}\right.$$

$$\left.\left.+\frac{m^2\pi^2}{a^2}\left(\frac{y^2}{2b^2}-\mu(2-\mu)\frac{3y^2-b^2}{6b^2}\right)-\frac{\mu}{b^2}\right]D_m\right\}\cos\frac{m\pi x}{a}$$

$$+D\sum_{n=0}^{\infty}\left\{\left[\mu(2-\mu)a^2\frac{n^4\pi^4}{b^4}\,\frac{4ax^3-4a^2x^2-x^4}{24a^4}+\frac{n^2\pi^2}{b^2}\left(\mu\frac{2ax-x^2}{2a^2}\right.\right.\right.$$

$$\left.\left.-(2-\mu)\frac{6ax-2a^2-3x^2}{6a^2}\right)+\frac{1}{a^2}\right]G_n+\left[\mu(2-\mu)a^2\frac{n^4\pi^4}{b^4}\,\frac{x^4-2a^2x^2}{24a^4}\right.$$

$$\left.\left.+\frac{n^2\pi^2}{b^2}\left(\mu\frac{x^2}{2a^2}-(2-\mu)\frac{3x^2-a^2}{6a^2}\right)-\frac{1}{a^2}\right]H_n\right\}\cos\frac{n\pi y}{b} \tag{3.2}$$

$$M_y=D\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\mu\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\pi^2W_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$+D\sum_{m=0}^{\infty}\left\{\left[\mu(2-\mu)b^2\frac{m^4n^4}{a^4}\,\frac{4by^3-4b^2y^2-y^4}{24b^4}+\frac{m^2\pi^2}{a^2}\left(\mu\frac{2by-y^2}{2b^2}\right.\right.\right.$$

$$\left.\left.-(2-\mu)\frac{6by-2b^2-3y^2}{6b^2}\right)+\frac{1}{b^2}\right]C_m+\left[\mu(2-\mu)b^2\frac{m^4\pi^4}{a^4}\,\frac{y^4-2b^2y^2}{24b^4}\right.$$

$$\left.\left.+\frac{m^2\pi^2}{a^2}\left(\mu\frac{y^2}{2b^2}-(2-\mu)\frac{3y^2-b^2}{6b^2}\right)-\frac{1}{b^2}\right)D_m\right\}\cos\frac{m\pi x}{a}$$

$$+D\sum_{n=0}^{\infty}\left\{\left[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\,\frac{4ax^3-4a^2x^2-x^4}{24a^4}+\frac{n^2\pi^2}{b^2}\left(\frac{2ax-x^2}{2a^2}\right.\right.\right.$$

$$\left.\left.-\mu(2-\mu)\frac{6ax-2a^2-3x^2}{6a^2}\right)+\frac{\mu}{a^2}\right]G_n+\left[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\,\frac{x^4-2a^2x^2}{24a^4}\right.$$

$$\left.\left.+\frac{n^2\pi^2}{b^2}\left(\frac{x^2}{2a^2}-\mu(2-\mu)\frac{3x^3-a^2}{6a^2}\right)-\frac{\mu}{a^2}\right]H_n\right\}\cos\frac{n\pi y}{b} \tag{3.3}$$

Utilizing known expansions

$$\frac{4ax^3-4a^2x^2-x^4}{24a^4}=\sum_{m=0}^{\infty}\alpha_m\cos\frac{m\pi x}{a} \tag{3.4}$$

$$\frac{2ax-x^2}{2a^2}=\sum_{m=0}^{\infty}\beta_m\cos\frac{m\pi x}{a} \tag{3.5}$$

$$\frac{x^4-2a^2x^2}{24a^4}=\sum_{m=0}^{\infty}\xi_m\cos\frac{m\pi x}{a} \tag{3.6}$$

$$\frac{x^2}{2a^2}=\sum_{m=0}^{\infty}\eta_m\cos\frac{m\pi x}{a} \tag{3.7}$$

in which

$$\alpha_m=\begin{cases}-1/45 & (m=0)\\ 2m^{-4}\pi^{-4} & (m\neq 0)\end{cases} \tag{3.8}$$

$$\beta_m=\begin{cases}1/3 & (m=0)\\ -2m^{-2}\pi^{-2} & (m\neq 0)\end{cases} \tag{3.9}$$

$$\xi_m=\begin{cases}-7/360 & (m=0)\\ (-1)^{m+1}2m^{-4}\pi^{-4} & (m\neq 0)\end{cases} \tag{3.10}$$

$$\eta_m=\begin{cases}1/6 & (m=0)\\ (-1)^m 2m^{-2}\pi^{-2} & (m\neq 0)\end{cases} \tag{3.11}$$

expression (3.1) can be rewritten as

$$w(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Big\{W_{mn}+\Big[(2-\mu)\frac{b^2}{a^2}\alpha_n m^2\pi^2+\beta_n\Big]C_m +\Big[(2-\mu)\frac{b^2}{a^2}\xi_n m^2\pi^2+\eta_n\Big]D_m+\Big[(2-\mu)\frac{a^2}{b^2}\alpha_m n^2\pi^2+\beta_m\Big]G_n +\Big[(2-\mu)\frac{a^2}{b^2}\xi_m n^2\pi^2+\eta_m\Big]H_n\Big\}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{3.12}$$

Suppose there are some regions of separation $\sigma_1, \sigma_2, \cdots, \sigma_s$ in the whole plate, i.e., deflection w is negative as $(x, y)\in\sigma_i$ $(i=1,2,\cdots, s)$. Then equation (2.2) can be written as

$$H(w)=\begin{cases}0 & \text{as } ((x, y)\in\sigma_i,\ i=1, 2, \cdots, s)\\ 1 & \text{as } ((x, y)\in\sigma_i\end{cases} \tag{3.13}$$

Let the Fourier expansion of $H(w)$ be

$$H(w)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\lambda_{mn}\,a_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{3.14}$$

where

$$\lambda_{mn}=\begin{cases}1/4 & (m=0,\ n=0)\\ 1/2 & (m=0,\ n\neq 0 \text{ or } m\neq 0,\ n=0)\\ 1 & (m\neq 0,\ n\neq 0)\end{cases} \tag{3.15}$$

$$a_{mn}=\frac{4}{ab}\int_0^b\int_0^a H(w)\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}dxdy \tag{3.16}$$

Now let's expand the loading function $q(x, y)$ into dual cosine series. For concentrated forces and couples, the loading functions can be regarded as generalized functions[2]. The expansion can be expressed as

$$q(x, y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\lambda_{mn}\,q_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{3.17}$$

in which

$$q_{mn}=\frac{4}{ab}\int_0^b\int_0^a q(x,\ y)\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}dxdy \tag{3.18}$$

IV. Solution of the Fundamental Differential Equation under Given Boundary Conditions

Substituting (3.1), (3.12), (3.14) and (3.17) into equation (2.1), and utilizing the multiplication of series[1] and given expansions

$$\frac{6ax-2a^2-3x^2}{6a^2}=\sum_{m=0}^{\infty}k_m\cos\frac{m\pi x}{a} \tag{4.1}$$

$$\frac{3x^2-a^2}{6a^2}=\sum_{m=0}^{\infty}l_m\cos\frac{m\pi x}{a} \tag{4.2}$$

in which

$$k_m=\begin{cases}0 & (m=0)\\ -2m^{-2}\pi^{-2} & (m\neq 0)\end{cases} \tag{4.3}$$

$$l_m=\begin{cases}0 & (m=0)\\ (-1)^m 2m^{-2}\pi^{-2} & (m\neq 0)\end{cases} \tag{4.4}$$

and introducing the notation

$$\zeta_i=\begin{cases}1 & (i=0)\\ 0 & (i=1,\ 2,\ 3,\ \cdots)\end{cases} \tag{4.5}$$

finally, through the comparison of coefficients on both sides of equation (2.1), we obtain

$$\begin{aligned}D\Big\{&\Big(\frac{m^2}{a^2}+\frac{n^2}{b^2}\Big)^2\pi^4W_{mn}+\Big[(2-\mu)b^2\frac{m^6\pi^6}{a^6}\alpha_n+\frac{m^4\pi^4}{a^4}(\beta_n-2(2-\mu)k_n)\\
&+\mu\frac{m^2\pi^2}{a^2b^2}\zeta_n\Big]C_m+\Big[(2-\mu)b^2\frac{m^6\pi^6}{a^6}\xi_n+\frac{m^4\pi^4}{a^4}(\eta_n-2(2-\mu)l_n)\\
&-\mu\frac{m^2\pi^2}{a^2b^2}\zeta_n\Big]D_m+\Big[(2-\mu)a^2\frac{n^6\pi^6}{b^6}\alpha_m+\frac{n^4\pi^4}{b^4}(\beta_m-2(2-\mu)k_m)\\
&+\mu\frac{n^2\pi^2}{a^2b^2}\zeta_m\Big]G_n+\Big[(2-\mu)a^2\frac{n^6\pi^6}{b^6}\xi_m+\frac{n^4\pi^4}{b^4}(\eta_m-2(2-\mu)l_m)\\
&-\mu\frac{n^2\pi^2}{a^2b^2}\zeta_m\Big]H_n\Big\}+\frac{k}{4}\lambda_{mn}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\Big\{W_{pq}+\Big[(2-\mu)\frac{b^2}{a^2}\alpha_q p^2\pi^2+\beta_q\Big]C_p\\
&+\Big[(2-\mu)\frac{b^2}{a^2}\xi_q p^2\pi^2+\eta_q\Big]D_p+\Big[(2-\mu)\frac{a^2}{b^2}\alpha_p q^2\pi^2+\beta_p\Big]G_q\\
&+\Big[(2-\mu)\frac{a^2}{b^2}\xi_p q^2\pi^2+\eta_p\Big]H_q\Big\}(a_{p+m,\,q+n}+a_{p+m,\,|q-n|}\\
&+a_{|p-m|,\,q+n}+a_{|p-m|,\,|q-n|})=\lambda_{mn}q_{mn}\qquad (m=0,\ 1,\ \cdots;\ n=0,\ 1,\ \cdots)\end{aligned} \tag{4.6}$$

Then, we consider boundary conditions (2.3) and (2.5). Putting $x=0$ and $x=a$ in (3.2), $y=0$ and $y=b$ in (3.3), and considering equations (3.4), (3.5), (3.6), (3.7), (4.1), (4.2) and (4.5), we obtain

$$\sum_{m=0}^{\infty}\left(\frac{m^2}{a^2}+\mu\frac{n^2}{b^2}\right)\pi^2 W_{mn}+\sum_{m=0}^{\infty}\Big\{\Big[(2-\mu)b^2\frac{m^4\pi^4}{a^4}\alpha_n+\frac{m^2\pi^2}{a^2}(\beta_n-\mu(2-\mu)k_n)$$

$$+\frac{\mu}{b^2}\zeta_n\Big]C_m+\Big[(2-\mu)b^2\frac{m^4\pi^4}{a^4}\xi_n+\frac{m^2\pi^2}{a^2}(\eta_n-\mu(2-\mu)l_n)$$

$$-\frac{\mu}{b^2}\zeta_n\Big]D_m\Big\}+\left(\frac{2-\mu}{3}\frac{n^2\pi^2}{b^2}+\frac{1}{a^2}\right)G_n+\left(\frac{2-\mu}{6}\frac{n^2\pi^2}{b^2}-\frac{1}{a^2}\right)H_n$$

$$=0 \qquad (n=0,\ 1,\ 2,\ \cdots) \qquad (4.7)$$

$$\sum_{m=0}^{\infty}(-1)^m\left(\frac{m^2}{a^2}+\mu\frac{n^2}{b^2}\right)\pi^2 W_{mn}+\sum_{m=0}^{\infty}(-1)^m\Big\{\Big((2-\mu)b^2\frac{m^4\pi^4}{a^4}\alpha_n+\frac{m^2\pi^2}{a^2}\Big(\beta_n$$

$$-\mu(2-\mu)k_n\Big)+\frac{\mu}{b^2}\zeta_n\Big]C_m+\Big[(2-\mu)b^2\frac{m^4\pi^4}{a^4}\xi_n+\frac{m^2\pi^2}{a^2}(\eta_n-\mu(2-\mu)l_n)$$

$$-\frac{\mu}{b^2}\zeta_n\Big]D_m\Big\}+\Big[-\frac{\mu(2-\mu)}{24}a^2\frac{n^4\pi^4}{b^4}+\left(\frac{2\mu}{3}-\frac{1}{3}\right)\frac{n^2\pi^2}{b^2}+\frac{1}{a^2}\Big]G_n$$

$$+\Big[-\frac{\mu(2-\mu)}{24}a^2\frac{n^4\pi^4}{b^4}+\left(\frac{5\mu}{6}-\frac{2}{3}\right)\frac{n^2\pi^2}{b^2}-\frac{1}{a^2}\Big]H_n=0$$

$$(n=0,\ 1,\ 2,\ \cdots) \qquad (4.8)$$

$$\sum_{n=0}^{\infty}\left(\mu\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\pi^2 W_{mn}+\left(\frac{2-\mu}{3}\frac{m^2\pi^2}{a^2}+\frac{1}{b^2}\right)C_m+\Big(\frac{2-\mu}{6}\frac{m^2\pi^2}{a^2}$$

$$-\frac{1}{b^2}\Big)D_m+\sum_{n=0}^{\infty}\Big\{\Big[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\alpha_m+\frac{n^2\pi^2}{b^2}(\beta_m-\mu(2-\mu)k_m)+\frac{\mu}{a^2}\zeta_m\Big]G_n$$

$$+\Big[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\xi_m+\frac{n^2\pi^2}{b^2}(\eta_m-\mu(2-\mu)l_m)-\frac{\mu}{a^2}\zeta_m\Big]H_n\Big\}$$

$$=0 \qquad (m=0,\ 1,\ 2,\ \cdots) \qquad (4.9)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(\mu\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\pi^2 W_{mn}+\Big[-\frac{\mu(2-\mu)}{24}b^2\frac{m^4\pi^4}{a^4}+\left(\frac{2\mu}{3}-\frac{1}{3}\right)\frac{m^2\pi^2}{a^2}$$

$$+\frac{1}{b^2}\Big]C_m+\Big[-\frac{\mu(2-\mu)}{24}b^2\frac{m^4\pi^4}{a^4}+\left(\frac{5\mu}{6}-\frac{2}{3}\right)\frac{m^2\pi^2}{a^2}-\frac{1}{b^2}\Big]D_m$$

$$+\sum_{n=0}^{\infty}(-1)^n\Big\{\Big[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\alpha_m+\frac{n^2\pi^2}{b^2}(\beta_m-\mu(2-\mu)k_m)+\frac{\mu}{a^2}\zeta_m\Big]G_n$$

$$+\Big[(2-\mu)a^2\frac{n^4\pi^4}{b^4}\xi_m+\frac{n^2\pi^2}{b^2}(\eta_m-\mu(2-\mu)l_m)-\frac{\mu}{a^2}\zeta_m\Big]H_n\Big\}=0$$

$$(m=0,\ 1,\ 2,\ \cdots) \qquad (4.10)$$

Now we have five systems of equations (4.6)−(4.10). However, they are infinite equation systems. For calculation practice, we have to take a finite number of equations with a corresponding number of unknowns. Let M and N be the maximum values taken for m and n, then the total number of equations will be $(M+1)(N+1)+2(M+1)+2(N+1)$, and the number of unknowns involved is just the same. Consequently, they can be solved determinately. Nevertheless, for the tensionless Winkler foundation, $H(w)$ cannot be known in advance, thus the Fourier coefficients a_{mn} are also undetermined because they are dependent on the sign of deflection w. Therefore this problem is a nonlinear one, and cannot be solved directly. In this paper an iterative method is adopted to solve it. At first, we assume the whole plate is in contact with the foundation, then we have $H(w)\equiv 1$. By solving simultaneous equations (4.6) – (4.10), the first approximation of deflection $w_1(x,y)$ can be obtained. It is just the exact solution of a plate on classical Winkler foundation. If $w_1(x, y)>0$ holds everywhere, it will be the exact solution of the plate on tensionless Winkler foundation, too. If there is some region, in which $w_1(x, y)<0$, the solution must be modified. The region, in which $w_1(x, y)<0$ must be determined and taken as the first approximation of the actual region of separation. New modified function $H(w)$ and its Fourier coefficients a_{mn} can be determined according to the configuration of this approximated region of separation. Then we can get the second approximation of deflection $w_2(x, y)$. Repeat these procedures until the absolute value of the difference of results of two adjacent approximations is less than a preassigned small positive number. Finally, the deflection function $w(x, y)$ obtained in the last iteration will be taken as the solution of the plate on tensionless Winkler foundation under preassigned precision. Bending moments $M_x(x, y)$ and $M_y(x, y)$ can be obtained from equations (3.2) and (3.3).

V. Example

Solve the square plate on tensionless Winkler foundation shown in Fig.2. The length of each side is b. The load is a concentrated force P at the center of plate. Poisson's ratio is $\mu=0.167$ $kb^4/D=10^4$.

Solution: Along the above precedure, the first approximation $w_1(x, y)$ obtained (which is also the exact solution of plate on classical Winkler foundation) is shown in table 1.

Table 1. The first approximation of deflection $w_1\left(\frac{Pb^2}{D}\times 10^{-4}\right)$

y \ x	0	b/8	b/4	3b/8	b/2
0	−0.166	−0.233	−0.390	−0.536	−0.595
b/8	−0.233	−0.214	−0.192	−0.077	0.010
b/4	−0.390	−0.192	0.258	1.184	1.792
3b/8	−0.536	−0.077	1.184	4.143	6.560
b/2	−0.595	0.010	1.792	6.560	12.491

These results are in good coincidence with those given in [3] for plate on classical Winkler foundation.

Through five times of iteration, the solution of the plate on tensionless Winkler foundation has been obtained. The maximum deflection at the center of plate is $w_{max}=0.001531Pb^2/D$. The deflection curves of w and w_1 along $y=b/2$ are shown in Fig. 3.

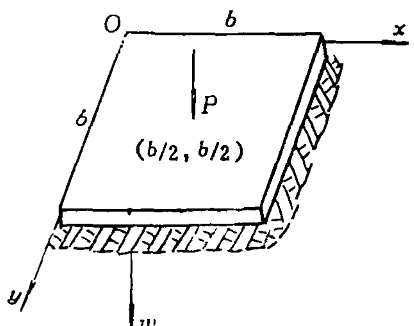

Fig. 2

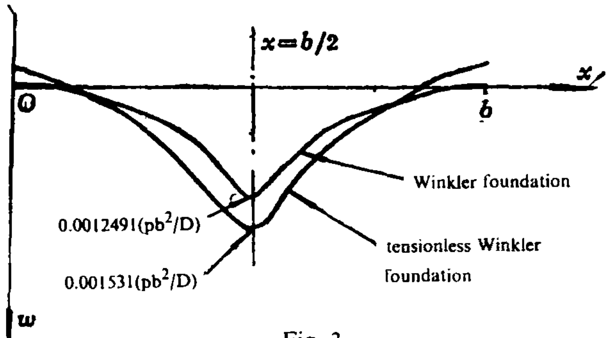

Fig. 3

VI. Concluding Remarks

It has been shown by the calculation of examples that the convergence of iteration is satisfactory. In general, a good result can be obtained through only five or six times of iteration. In addition, calculation practice shows also that the convergence of the Fourier series is rapid enough. Therefore, the method of Fourier series with supplementary terms is an efficient and generally applicable approach for solving the bending problem of rectangular plates with various boundary conditions. The authors have also extended this method to solve the Reissner plate on elastic foundation, which will be discussed later.

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