Benefits of Multiple Battery Levels for the Lifetime of Large Wireless Sensor Networks

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Abstract. In large wireless sensor networks, the few nodes close to the monitoring station are likely to prove the bottleneck in the useful lifetime of the network. We examine a strategy of equipping these nodes with a larger share of the total initial energy (battery) than the others, and generalizing this notion to the rest of the network. We solve a design problem involving optimizing the network lifetime using no more than a given number of distinct battery levels, and verify the results from the model by direct simulation.

1 Introduction

Wireless sensor networks have come to be recognized as an important research area in recent times [1]. In many practical sensor networs, the nodes are stationary and send periodic sensor readings to a monitoring station or sink. Thus the traffic is of an *egress* pattern and the routing is static. Under these conditions, it is well understood that the *first tier of nodes* (the nodes within direct radio range of the sink) will expend battery at the highest rate, because all traffic in the network is forwarded by this set of nodes. Similarly the second tier of nodes expend battery at the next highest rate, and so on. If all nodes possess the same initial battery level, the effective lifetime of the network is defined by the lifetime of the first tier. This problem is recognized in literature, and various strategies have been advanced to address it. For lack of space, we cannot discuss them comprehensively, see [2] for a full bibliography. In particular, it has been shown [3] that routing cannot alleviate this problem. Our current work is based on a strategy that has not been so far addresssed in literature, that of equipping different nodes with different levels of initial battery *i.e.* redistributing the total energy budget unequally among the nodes.

2 Design Problem

In this work we only consider a circular sensor field with the sink at the ceter, with roughly uniform sensor density. We assume that after deployment an arbi-

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trary shortest hops routing is set up by the nodes, and is recomputed whenever a node dies. With this in mind, we introduce the following notation:

N The initial number of nodes in the network.
R_M The radius of the network *i.e.*, the distance between the sink and the node farthest from it.
n The (uniform) density of node placement over the sensing area $= \pi R_M^2/N$.
R_{TX} The transmission radius of the sensor nodes.
T The number of tiers in the network, $T \approx \frac{R_M}{R_{TX}}$.
B The total energy budget.
b_u Initial energy of each node under uniform allocation, $b_i = B/N$.
P The period between the generation of two consecutive packets by a node.
e_F The energy required to forward a packet.
e_G The energy required to generate a packet.
e_I The energy expended by each node in each period P that is independent of the data traffic.
τ The traffic generated by the nodes in a unit area, in unit time.
β The amount of energy required to transmit one unit of traffic, once.

With the assumption of a uniform density, the number of nodes in tier i is easily seen to be $N_i = (2i-1)N/T^2$. We need only consider that the flows originated in all nodes outside tier i are forwarded by the N_i nodes of tier i to obtain the time at which tier i nodes will die as: $L_i = T^2 b_u N_i P/\{N\left(T^2 - i^2\right)e_F + (2i-1)N(e_G + e_I)\}$. For any i, the above is true only if no nodes in any other tier die earlier than L_i. Thus L_1, the minimum one, defines the lifetime of the network. At this time, each node in a tier $i > 1$ will only have actually consumed b_{e_i} of its energy given by:

$$\frac{b_{e_i}}{b_u} = \frac{L_1}{L_i} = \frac{\frac{T^2-i^2}{2i-1}e_F + e_G + e_I}{(T^2-1)e_F + e_G + e_I}. \tag{1}$$

Given these preliminaries, we now define our battery reallocation problem as follows: Given a total battery budget B and a maximum of k distinct battery levels, determine the optimal battery levels $b_1, \ldots, b_k$, $b_1 > b_2 > \ldots > b_k > 0$, and their assignment for each tier of nodes in a sensor network with parameters $N, R_M, R_{TX}, P, e_F, e_G$ and e_I, such that **the total lifetime L of the network is maximized**.

The problem is trivial when $k = T$, and becomes a hard one of integer optimization when T is significantly larger. We consider the asymptotic situation when the density of nodes per unit area remains the same, but the number of nodes per unit area increases sufficiently that the density can be considered to hold for any area, however small. Here, we develop the theory only considering the forwarding energy e_F and setting $e_G = e_I = 0$; in [2] we show how to take these parameters into account. Then the energy $e(r)$ expended by the nodes in a unit area around radius r in unit time is given by $2\pi r R_{TX} e(r) = \pi(R_M^2 - r^2)\tau\beta$, hence the battery power $b(r)$ expended by each individual node in that area is given by $b(r) = e(r)/n = R_M^2 - r^2)\tau\beta/(2rR_{TX}n)$. The approximation in using the continuous model arises from the assumption that the above is valid for all values of r. In reality, this is valid for the radii $r = (2m+1)R_{TX}/2$. As a check, we plot the curve $b(r)/b_u$ as given by the above definition of $b(r)$ as well as the values of b_{e_i}/b_u as given by 1 for a network of 20 tiers, in Fig 1. As expected, the values match quite well at the middle of each tier. But the former curve is continuous, this allows us to employ derivative methods to investigate the relationship of the lifetime and the battery budget.

The energy E expended in the whole network in unit time can be easily obtained by integrating $e(r)$ over the entire area as $E = (2/3)\pi R_M^3 \tau\beta/R_{TX}$, and multiplying E by a lifetime of L seconds gives the minimum amount of total energy budget that can achieve a total lifetime of L, when distributed as $b(r)$ above, such that every node exhausts its battery at the same time. Naturally, this requires continuously varying battery levels. If we are realistically constrained to using only k distinct battery levels, then the best lifetime will be obtained by approximating this battery distribution as far as possible. The nodes in an annular area will have the same battery level, and, as before, annular areas closer to the center should have higher battery levels. Let all nodes from radius $\rho_0 (= 0)$ to ρ_1 have a battery level of b_1, from ρ_1 to ρ_2 have a battery level of b_2, and so on, until all nodes from radius ρ_{k-1} to $\rho_k (= R_M)$ have a battery level of b_k. Call these the "rings" $1 \ldots k$. The problem is to determine $\rho_1 \ldots \rho_{k-1}, b_1 \ldots b_k$. We proceed by observing that the lifetime of each ring equals the lifetime of its innermost tier (we assume rings start at tier boundaries). For ease of notation, we define the variables $r_i, i = 1 \ldots k$, as $r_i = \rho_{i-1} + R_{TX}/2$, and $r_{k+1} = R_M + R_{TX}/2$. We already have $r_1 = R_{TX}/2$. For equal lifetimes of the tiers t_i, and, hence, the rings i, the battery levels b_i must be in the same ratio as $b(r_i)$. Letting L stand for the equal lifetime thus achieved, we can assert that $b_i = b(r_i)L = (R_M^2 - r^2)\tau\beta L/(2r_i R_{TX} n)$. Since b_i sums to B over all nodes, we can obtain $B = S\pi\tau\beta L/(2R_{TX})$, where $S = \sum_{i=1}^{k}(R_M^2 - r_i^2)(r_{i+1}^2 - R_{TX}r_{i+1} - r_i^2 + R_{TX}r_i)/r_i$.

S can be minimized by setting each partial derivative of S w.r.t. r_i to zero: the particular structure of S is conducive to this procedure. This is because r_i only appears in the $i-1$-th and i-th terms of the sum S. Thus we obtain

$$\frac{\partial S}{\partial r_i} = \begin{array}{l} 2R_M^2 r_i/r_{i-1} - R_M^2 R_{TX}/r_{i-1} - 2r_{i-1}r_i \\ +R_{TX}r_{i-1} - R_M^2 r_{i+1}^2/r_i^2 + R_M^2 R_{TX} r_{i+1}/r_i^2 \\ -R_M^2 - r_{i+1}^2 + R_{TX}r_{i+1} + 3r_i^2 - 2R_{TX}r_i . \end{array} \quad (2)$$

The above can be solved symbolically in MATLAB to obtain numerical solutions to r_i for any given problem instance. These values can in turn be used as outlined above to obtain the actual battery levels b_i to be used.

3 Simulation Results

Since existing network simulators such as OPNET focus on details of physical and MAC layers (which are not the focus of our work), they are unsuitable for

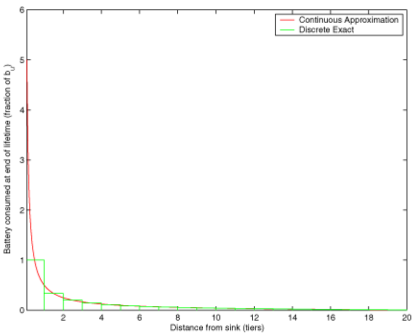

Fig. 1. Relationship of discrete battery consumption levels with continuous approximation

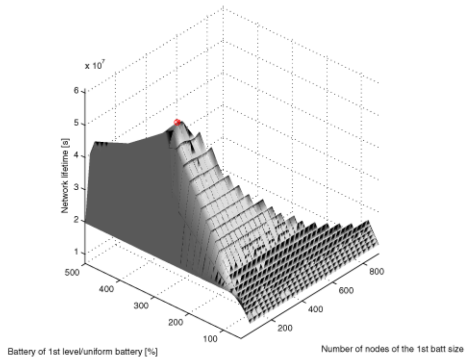

Fig. 2. Experimental validation of the best placement and best battery levels of two levels of batteries

simulation of networks of hundreds of nodes over long times. We developed a discrete even based simulation to focus on the energy consumption problem. As a base case, we considered a circular area in which 905 nodes were distributed in a square grid with the sink in the center. Each node was assumed to have power consumption characterstics and initial energy level similar to those of Berkeley motes. To assess the effect of the idealizations in Section 2 on the optimality of the battery level choices, we performed a brute-force search for the optimal values for the case when the total initial energy budget was redistributed among the nodes using two battery levels. Fig. 2 depicts the lifetime of the network as a function of the battery level of the first level of nodes and the number of nodes in the first level. The maximum lifetime is marked with the symbol ‘x’. The maximum, as predicted by theory in Section 2, is marked by the symbol ‘o’. As we can see, the location of the optimum as predicted by the continuous model matches that obtained by simulation near-perfectly.

We also simulated many deviations from the idealizations of the base case in order to verify that the approach continues to be useful under departures from the ideal. In particular, we have verified this for variations in node density, some non-uniformity in node distribution, and total number of network nodes; details can be found in [2].

In conclusion, we examined the approach of non-uniform battery allocation to the nodes of a sensor network to alleviate the problem of very early disconnection caused by the large traffic forwarding load imposed on the nodes close to the sink due to the egress nature of traffic in the network. Under a total battery budget, we demonstrated a method to approximate the optimal battery levels and number of nodes for each battery level. With this strategy, we showed that the lifetime of the network can be significantly improved, even if a small number of battery levels is used.

References

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