

# Bengali language, Romanisation and Onsager Core

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March 26, 2020

## Abstract

We continue to study Chalantika, a bengali to bengali dictionary. We romanise the bengali alphabet via wikipedia scheme. We reduce the romanised alphabet to English alphabet i.e. the alphabet which appears in an English dictionary. In the reduced alphabet scheme, we draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised( unnormalised). We observe that behind the words of the dictionary of the bengali language, in the reduced alphabet scheme, the magnetisation curve is  $BP(4, \beta H = 0.08)$ , in the Bethe-Peierls approximation of Ising model with four nearest neighbours, in presence of little external magnetic fields,  $\beta H = 0.08$ . Moreover, words of the bengali language in the reduced alphabet scheme, nearly go over to Onsager solution, on few successive normalisations.  $\beta$  is  $\frac{1}{k_B T}$  where, T is temperature, H is external magnetic field and  $k_B$  is tiny Boltzmann constant.

## I Introduction

How come a bengali boy in the author's locality in Kolkata looks like a Mising boy from upper Assam, in India? Does it mean that there is another coincidence, if not many? Do the bengali language and the Mising language coincide in one level? In the previous paper, [1], we have found that the words of the Mising language written in Roman alphabet, [2], i.e. with diacritical marks, when subjected to reduction to English alphabet, exhibits Onsager Core. The graph of  $\frac{\ln f}{\ln f_{nnnn-max}}$  vs  $\frac{\ln k}{\ln k_{lim}}$  looks like the Onsager solution.  $\frac{\ln f}{\ln f_{nnnn-max}}$  corresponds to normalisation once we decide not to use the words beginning with the first four letters in terms of ranking,[3]. k is a positive integer called rank.  $k_{lim}$  or,  $k_d$  is the limiting or, maximum rank where,  $f_{lim}$  is one. f is a number of words once set of words is arranged in strictly descending order. The two dimensional Ising model,[4], in absence of external magnetic field, is prototype of an Ising model. In case of square lattice of planar spins, one spin

interacts with four other nearest neighbour spins i.e. on an average to another one spin. Below a certain ambient temperature, denoted as  $T_c$ , the two dimensional array of spins reduces to a planar magnet with magnetic moment per site varying as a function of  $\frac{T}{T_c}$ . This function was inferred, [5], by Lars Onsager way back in 1948, [6] and thoroughly deduced thereafter by C.N.Yang[7]. This function we are referring to as Onsager solution. Moreover, systems, [8], showing behaviour like Onsager solution is rare to come across.

Studying a bengali to bengali dictionary, [9] we have found that a graphical law underlies that dictionary of the bengali language, in the article, [10]. The letters of the dictionary is of the bengali alphabet. What happens if we follow the same steps as in Mising? This paper proceeds along that. We add a new step. We write the bengali alphabet in the Roman alphabet, [11],[12] before reducing the Roman alphabet to the English alphabet i.e. to the alphabet that appears in an English dictionary. Then we do the rank analysis. We find that the graph of  $\frac{\ln f}{\ln f_{nnnnnn-max}}$  vs  $\frac{\ln k}{\ln k_{im}}$  looks almost like the Onsager solution.

We were led to the present paper as follows. We studied natural languages, [13] and have found, in the preliminary study, existence of a curve magnetisation related to two approximations of Ising model, under each language. We termed this phenomenon as graphical law. Then we looked into, [14], dictionaries of five discipline of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of graphical law behind bengali, [10], Basque, [15], Romanian, [16], five more disciplines of knowledge, [17] and Onsager core of Abor-Miri, Mising languages,[1].

The planning of this paper is as follows. We review counting of Chalantika, [9],[10]; wiki romanisation, [12], reduced alphabet scheme in the section II, the standard curves of magnetisation of Ising model in the section III. In the section IV, we describe analysis of bengali words in the reduced alphabet scheme,[1] and the Onsager solution at the core. Section V is discussion. The appendix section VI describes how to draw the curves of magnetisations used in this paper. Next section VII is acknowledgement section. The last section is bibliography.

## II Bengali

”Jato mot, tato path.”

——a bengali proverb.

We have studied Chalantika, a bengali to bengali dictionary, [9] and found that a graphical law underlies that dictionary of the bengali language, in the article, [10]. The number of words counted by us in [10], for each letter is tabulated in the table, 1.

|      |      |     |    |      |    |    |     |    |     |    |      |     |      |     |     |     |     |     |     |     |     |     |      |     |      |     |      |      |     |      |     |      |     |     |     |     |    |      |     |
|------|------|-----|----|------|----|----|-----|----|-----|----|------|-----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|-----|------|-----|------|------|-----|------|-----|------|-----|-----|-----|-----|----|------|-----|
| 1    | 2    | 3   | 4  | 5    | 6  | 7  | 8   | 9  | 10  | 11 | 12   | 13  | 14   | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24   | 25  | 26   | 27  | 28   | 29   | 30  | 31   | 32  | 33   | 34  | 35  | 36  | 37  | 38 | 39   | 40  |
| 2505 | 1307 | 177 | 35 | 1034 | 30 | 25 | 237 | 28 | 113 | 30 | 2314 | 599 | 1157 | 316 | 988 | 350 | 895 | 235 | 236 | 137 | 191 | 134 | 1078 | 102 | 1302 | 515 | 1463 | 3196 | 302 | 3170 | 791 | 1773 | 356 | 737 | 434 | 955 | 47 | 2530 | 629 |

Table 1: Bengali words: the first row represents letters of the Bengali alphabet,[9] in the serial order.

কালিগুণ

|                |                |                |                |                |                |
|----------------|----------------|----------------|----------------|----------------|----------------|
| ক<br>#<br>2595 | খ<br>#<br>1397 | গ<br>#<br>1777 | ঘ<br>#<br>35   | ঙ<br>#<br>1034 | চ<br>#<br>30   |
| ছ<br>#<br>25   | জ<br>#<br>237  | ঝ<br>#<br>28   | ঞ<br>#<br>113  | ট<br>#<br>30   | ঠ<br>#<br>988  |
| ড<br>#<br>2314 | ঢ<br>#<br>599  | ণ<br>#<br>1157 | ত<br>#<br>316  | থ<br>#<br>988  | দ<br>#<br>950  |
| ধ<br>#<br>895  | ন<br>#<br>235  | প<br>#<br>236  | ফ<br>#<br>137  | ব<br>#<br>191  | ভ<br>#<br>134  |
| ব<br>#<br>1078 | ষ<br>#<br>102  | শ<br>#<br>1392 | ষ<br>#<br>515  | ষ<br>#<br>1463 | ষ<br>#<br>3196 |
| জ<br>#<br>392  | ঝ<br>#<br>3170 | ঞ<br>#<br>791  | ট<br>#<br>1773 | ঠ<br>#<br>356  | ড<br>#<br>737  |
| ঢ<br>#<br>434  | ণ<br>#<br>955  | ত<br>#<br>47   | থ<br>#<br>2530 | দ<br>#<br>629  |                |

Figure 1: Number of words starting with various letters of the bengali alphabet

|      |      |     |      |      |      |     |      |      |     |      |      |      |      |     |     |      |     |      |     |
|------|------|-----|------|------|------|-----|------|------|-----|------|------|------|------|-----|-----|------|-----|------|-----|
| Ô    | A    | I   | Ī    | U    | Ū    | Ṛ   | E    | Oi   | O   | OU   | KÔ   | KHÔ  | GÔ   | GHÔ | CHÔ | CHHÔ | ĴÔ  | ĴHÔ  | ṬÔ  |
| 2595 | 1397 | 177 | 35   | 1034 | 30   | 25  | 237  | 28   | 113 | 30   | 2314 | 599  | 1157 | 316 | 988 | 350  | 895 | 235  | 236 |
| ṬHÔ  | ḌÔ   | DHÔ | TÔ   | THÔ  | DÔ   | DHÔ | NÔ   | PÔ   | PHÔ | BÔ   | BHÔ  | MÔ   | JÔ   | RÔ  | LÔ  | SHÔ  | ṢÔ  | SÔ   | HÔ  |
| 137  | 191  | 134 | 1078 | 102  | 1392 | 515 | 1463 | 3196 | 392 | 3170 | 791  | 1773 | 356  | 737 | 434 | 955  | 47  | 2530 | 629 |

Table 2: Bengali words: the odd rows represent letters of the Bengali alphabet,[9] in the serial order, once romanised, [12]

|      |      |      |      |     |      |     |     |      |      |     |      |      |      |      |     |      |      |      |
|------|------|------|------|-----|------|-----|-----|------|------|-----|------|------|------|------|-----|------|------|------|
| A    | B    | C    | D    | E   | G    | H   | I   | J    | K    | L   | M    | N    | O    | P    | R   | S    | T    | U    |
| 1397 | 3961 | 1338 | 2232 | 237 | 1473 | 629 | 212 | 1486 | 2913 | 434 | 1773 | 1463 | 2766 | 3588 | 762 | 3532 | 1553 | 1064 |

Table 3: Bengali words: the first row represents letters of the English alphabet in the serial order, in the reduced alphabet scheme, for the words of the bengali language([9]).

As we write explicitly in terms of letters of the bengali alphabet, the table, 1, takes the form of the figure,1. Next we take resort to Romanisation of the bengali alphabet, [11]. There are many ways of doing so. We do not know which is the royal road to Romanisation for the bengali alphabet. We take a practical approach. We choose the wikipedia romanisation,[12]. We obtain the table, 2. In the next step, we reduce the romanised alphabet in the table 2, down to the English alphabet ala [1] as follows. We combine BÔ and BHÔ into B to get the resulting number of words starting with B as three thousand nine hundred sixty one, combine CHÔ and CHHÔ into C to get the resulting number of words starting with C as one thousand three hundred thirty eight, put together ḌÔ, DHÔ, DÔ, DHÔ into D to obtain the resulting number of words starting with D as two thousand two hundred thirty two; GÔ and GHÔ into G to obtain the resulting number of words starting with G as one thousand four hundred seventy three; I and Ī into I to obtain the resulting number of words starting with I as two hundred; ĴÔ, ĴHÔ and JÔ into J to obtain the resulting number of words starting with J as one thousand four hundred eighty six; KÔ and KHÔ into K to obtain the resulting number of words starting with K as two thousand nine hundred thirteen; Ô, Oi,O and OU into O to obtain the resulting number of words starting with O as two thousand seven hundred sixty six; PÔ and PHÔ into P to obtain the resulting number of words starting with P as three thousand three hundred ninty two; Ṛ and RÔ into R to to obtain the resulting number of words starting with R as seven hundred sixty two; SHÔ, ṢÔ and SÔ into S to obtain the resulting number of words starting with S as three thousand five hundred thirty two;ṬÔ,ṬHÔ, TÔ and THÔ into T to obtain the resulting number of words starting with T as one thousand five hundred fifty three; U and Ū into U to obtain the resulting number of words starting with U as one thousand sixty four. The result is the table, 3. To visualise the content of the table,3, we plot the number of words against respective letters the figure fig.2. In the next to next section, we carry on analysis looking for onsager core

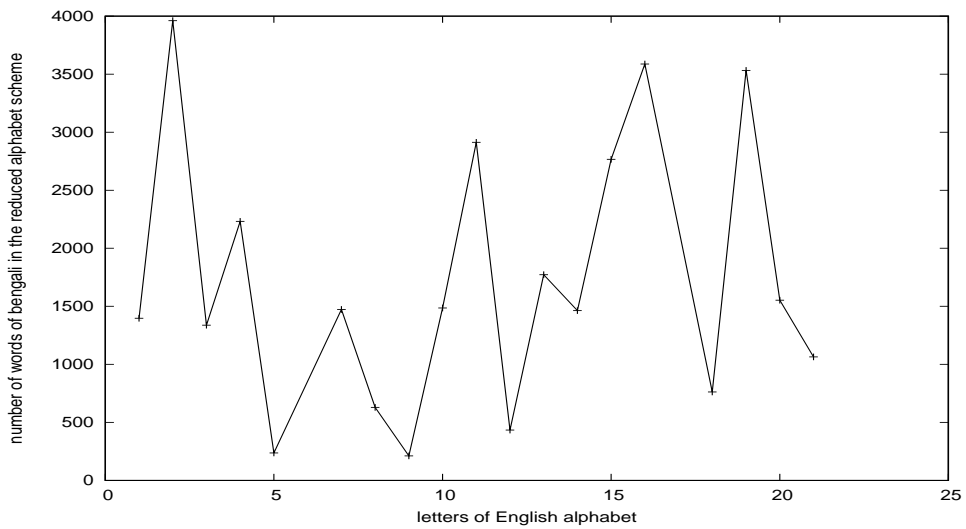


Figure 2: Vertical axis is number of words, in the table 3, of the bengali language in the reduced alphabet scheme and horizontal axis is the respective letters of the English alphabet. Letters are represented by the number in the English alphabet sequence.

as well as graphical law, basing on the table, 3. As a preparation to that, in the next section, we give an overview of the Onsager solution of the two dimensional Ising model as well as curves of magnetisation in Bethe-Peierls approximation of Ising model in presence of external magnetic field.

### III Curves of Magnetisation

The Ising Hamiltonian,[4],[18],for a lattice of spins is  $-\epsilon\sum_{n,n}\sigma_i\sigma_j-H\sum_i\sigma_i$ , where n.n refers to nearest neighbour pairs,  $\sigma_i$  is i-th spin, H is external magnetic field and  $\epsilon$  is coupling between two nearest neighbour spins.  $\sigma_i$  is binary i.e. can take values  $\pm 1$ . At a temperature T, below a certain temperature called phase transition temperature,  $T_c$ , for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [7], [18],

$$\frac{M}{M_{max}} = [1 - (\sinh \frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.3. In the Bragg-Williams and

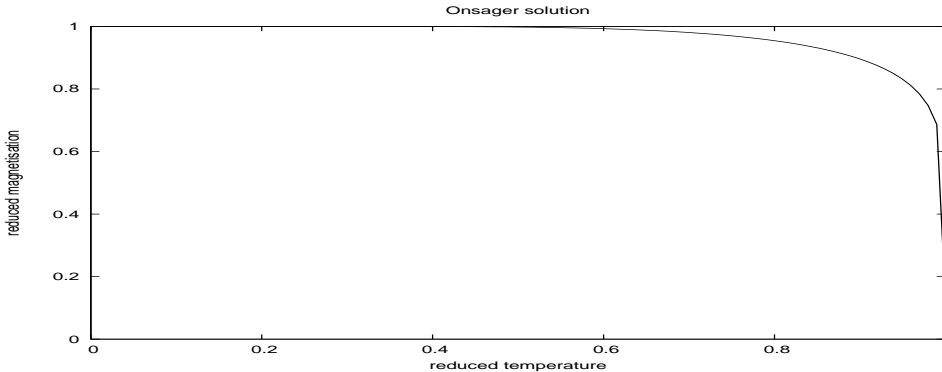


Figure 3: Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

Bethe-Peierls approximations for an Ising model in any dimension, in presence of external magnetic fields, reduced magnetisation as a function of reduced temperature, below the phase transition temperature,  $T_c$ , vary as in the figures 4-5. Related details are in the appendix.

The graphs in the figures,3-5, are used in the sections to follow as reference curves.

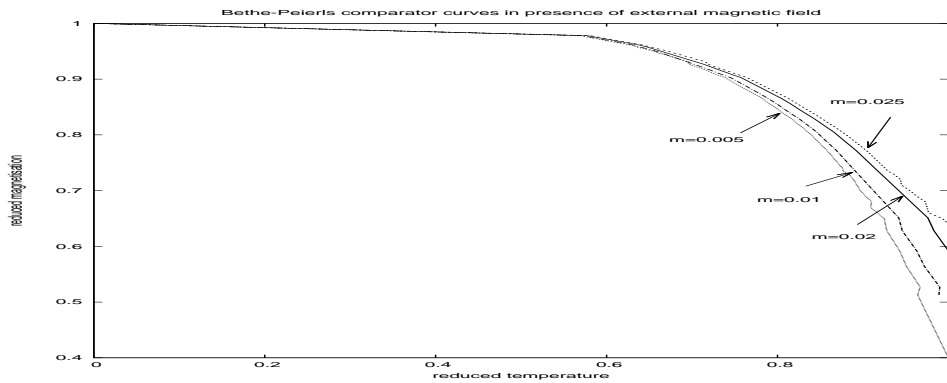


Figure 4: Reduced magnetisation vs reduced temperature curves,  $BP(4, \beta H)$ , for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

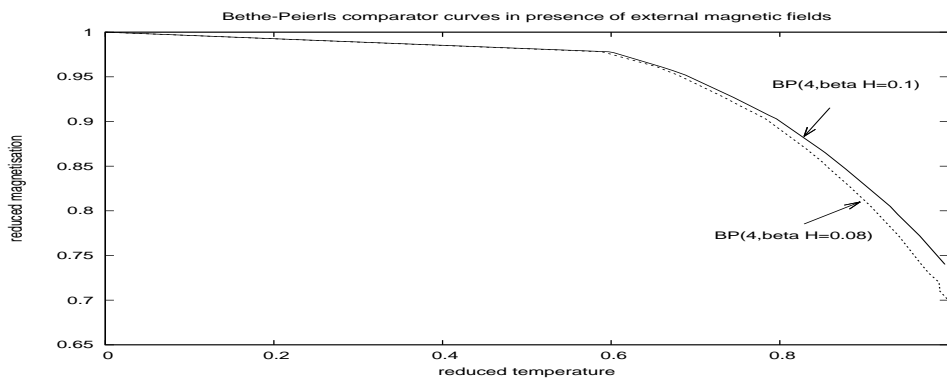


Figure 5: Reduced magnetisation vs reduced temperature curves,  $BP(4, \beta H=0.1)$  and  $BP(4, \beta H=0.08)$  in the Bethe-Peierls approximation in presence of little external magnetic field, for four nearest neighbours.



## IV Analysis in the reduced alphabet scheme

### IV.1 Method of study

For the purpose of searching for Onsager core as well as exploring graphical law, we assort the letters according to the number of words of the table, 3, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty and the limiting number of words is one. As a result both  $\frac{lnf}{lnf_{max}}$  and  $\frac{lnk}{lnk_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, 4 and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.6.

We then ignore the letter with the highest of words, tabulate in the adjoining table, 4 and redo the plot, normalising the  $lnfs$  with next-to-maximum  $lnf_{nextmax}$ , and starting from  $k = 2$  in the figure fig.7. Normalising the  $lnfs$  with next-to-next-to-maximum  $lnf_{nextnextmax}$ , we tabulate in the adjoining table, 4 and starting from  $k = 3$  we draw in the figure fig.8. Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{nextnextnextmax}$  we record in the adjoining table, 4 and plot starting from  $k = 4$  in the figure fig.9. Normalising the  $lnfs$  with next-to-next-to-next-to-next-to-maximum  $lnf_{nnnnmax}$  we record in the adjoining table, 4 and plot starting from  $k = 5$  in the figure fig.10. Normalising the  $lnfs$  with nextnextnextnextnext-maximum  $lnf_{nnnnnmax}$  we record in the adjoining table, 4 and plot starting from  $k = 6$  in the figure fig.11. Normalising the  $lnfs$  with nextnextnextnextnextnext-maximum  $lnf_{nnnnnnmax}$  we record in the adjoining table, 4 and plot starting from  $k = 7$  in the figure fig.12. Normalising the  $lnfs$  with nnnnnnn-max  $lnf_{nnnnnnnmax}$  we record in the adjoining table, 4 and plot starting from  $k = 8$  in the figure fig.13.

| k  | lnk  | lnk/lnk <sub>lim</sub> | f    | lnf   | lnf/lnf <sub>max</sub> | lnf/lnf <sub>nmax</sub> | lnf/lnf <sub>nmmax</sub> | lnf/lnf <sub>nnmax</sub> | lnf/lnf <sub>nnnmax</sub> | lnf/lnf <sub>nnnnmax</sub> | lnf/lnf <sub>nnnnnmax</sub> | lnf/lnf <sub>nnnnnnmax</sub> |
|----|------|------------------------|------|-------|------------------------|-------------------------|--------------------------|--------------------------|---------------------------|----------------------------|-----------------------------|------------------------------|
| 1  | 0    | 0                      | 3961 | 8.284 | 1                      | Blank                   | Blank                    | Blank                    | Blank                     | Blank                      | Blank                       | Blank                        |
| 2  | 0.69 | 0.230                  | 3588 | 8.185 | 0.988                  | 1                       | Blank                    | Blank                    | Blank                     | Blank                      | Blank                       | Blank                        |
| 3  | 1.10 | 0.367                  | 3532 | 8.170 | 0.986                  | 0.998                   | 1                        | Blank                    | Blank                     | Blank                      | Blank                       | Blank                        |
| 4  | 1.39 | 0.463                  | 2913 | 7.977 | 0.963                  | 0.975                   | 0.976                    | 1                        | Blank                     | Blank                      | Blank                       | Blank                        |
| 5  | 1.61 | 0.537                  | 2766 | 7.925 | 0.957                  | 0.968                   | 0.970                    | 0.993                    | 1                         | Blank                      | Blank                       | Blank                        |
| 6  | 1.79 | 0.597                  | 2232 | 7.711 | 0.931                  | 0.942                   | 0.944                    | 0.967                    | 0.973                     | 1                          | Blank                       | Blank                        |
| 7  | 1.95 | 0.650                  | 1773 | 7.480 | 0.903                  | 0.914                   | 0.916                    | 0.938                    | 0.944                     | 0.970                      | 1                           | Blank                        |
| 8  | 2.08 | 0.693                  | 1553 | 7.348 | 0.887                  | 0.898                   | 0.899                    | 0.921                    | 0.927                     | 0.953                      | 0.982                       | 1                            |
| 9  | 2.20 | 0.733                  | 1486 | 7.304 | 0.882                  | 0.892                   | 0.894                    | 0.916                    | 0.922                     | 0.947                      | 0.976                       | 0.994                        |
| 10 | 2.30 | 0.767                  | 1473 | 7.295 | 0.881                  | 0.891                   | 0.893                    | 0.915                    | 0.921                     | 0.946                      | 0.975                       | 0.993                        |
| 11 | 2.40 | 0.800                  | 1463 | 7.288 | 0.880                  | 0.890                   | 0.892                    | 0.914                    | 0.920                     | 0.945                      | 0.974                       | 0.992                        |
| 12 | 2.48 | 0.827                  | 1397 | 7.242 | 0.874                  | 0.885                   | 0.886                    | 0.908                    | 0.914                     | 0.939                      | 0.968                       | 0.986                        |
| 13 | 2.56 | 0.853                  | 1338 | 7.199 | 0.869                  | 0.880                   | 0.881                    | 0.902                    | 0.908                     | 0.934                      | 0.962                       | 0.980                        |
| 14 | 2.64 | 0.880                  | 1064 | 6.970 | 0.841                  | 0.852                   | 0.853                    | 0.874                    | 0.879                     | 0.904                      | 0.932                       | 0.949                        |
| 15 | 2.71 | 0.903                  | 762  | 6.636 | 0.801                  | 0.811                   | 0.812                    | 0.832                    | 0.837                     | 0.861                      | 0.887                       | 0.903                        |
| 16 | 2.77 | 0.923                  | 629  | 6.444 | 0.778                  | 0.787                   | 0.789                    | 0.808                    | 0.813                     | 0.836                      | 0.861                       | 0.877                        |
| 17 | 2.83 | 0.943                  | 434  | 6.073 | 0.733                  | 0.742                   | 0.743                    | 0.761                    | 0.766                     | 0.788                      | 0.812                       | 0.826                        |
| 18 | 2.89 | 0.963                  | 237  | 5.468 | 0.660                  | 0.668                   | 0.669                    | 0.685                    | 0.690                     | 0.709                      | 0.731                       | 0.744                        |
| 19 | 2.94 | 0.980                  | 212  | 5.357 | 0.647                  | 0.654                   | 0.656                    | 0.672                    | 0.676                     | 0.695                      | 0.716                       | 0.729                        |
| 20 | 3.00 | 0                      | 0    | 0     | 0                      | 0                       | 0                        | 0                        | 0                         | 0                          | 0                           | 0                            |

Table 4: Bengali words in the reduced alphabet scheme: ranking,natural logarithm, normalisations

## IV.2 Results

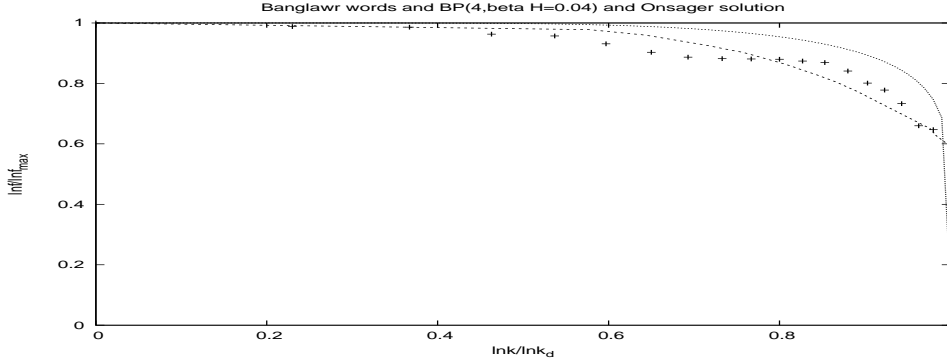


Figure 6: Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.02$  or,  $\beta H = 0.04$ . The uppermost curve is the Onsager solution.

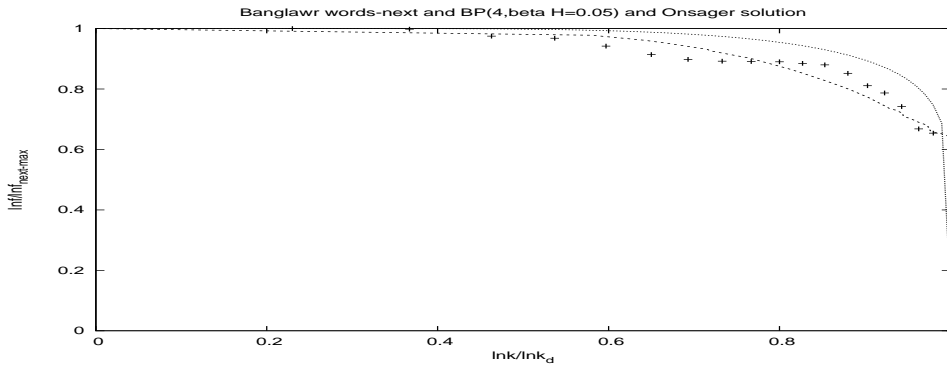


Figure 7: Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.025$  or,  $\beta H = 0.05$ . The uppermost curve is the Onsager solution.

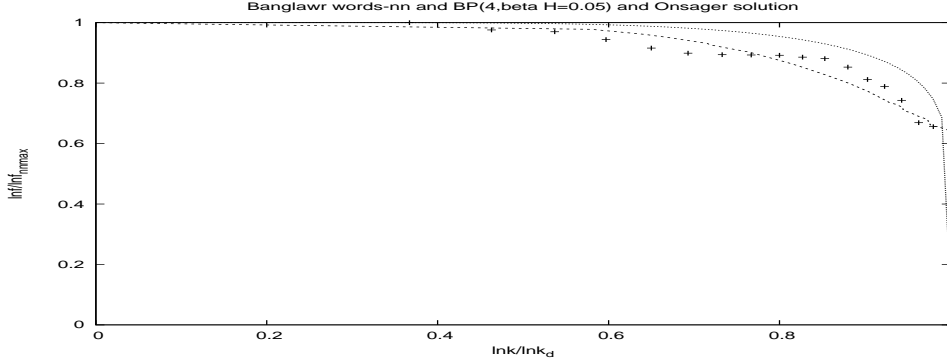


Figure 8: Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{im}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.025$  or,  $\beta H = 0.05$ . The uppermost curve is the Onsager solution

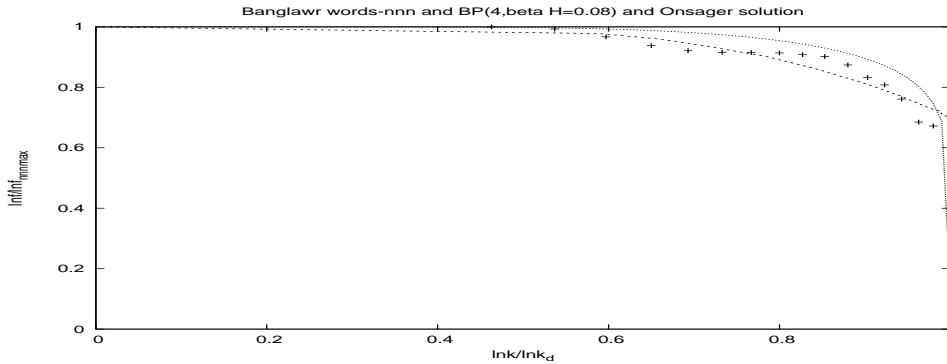


Figure 9: Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{im}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ . The uppermost curve is the Onsager solution.

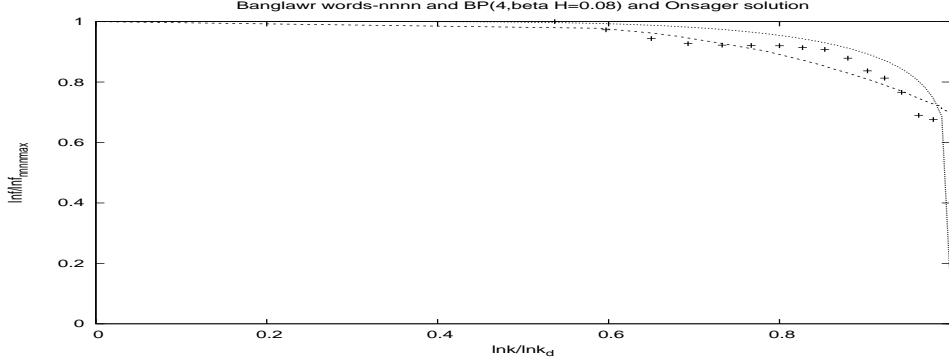


Figure 10: Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four neighbours and little magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ . The uppermost curve is the Onsager solution.

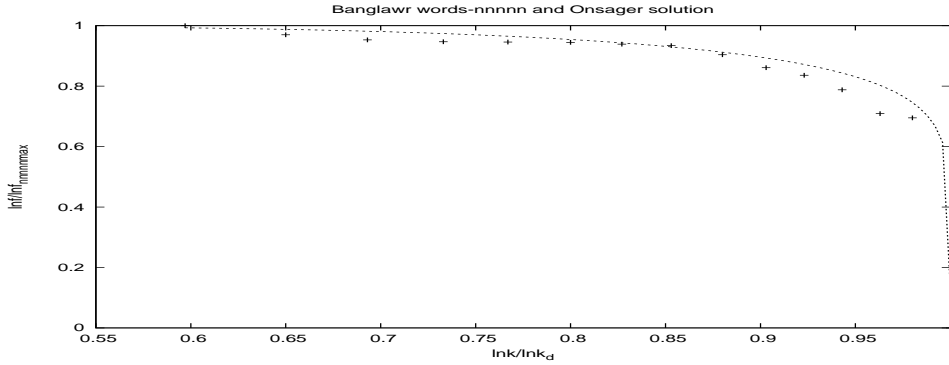


Figure 11: Vertical axis is  $\frac{\ln f}{\ln f_{nnnnn-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the Onsager solution.

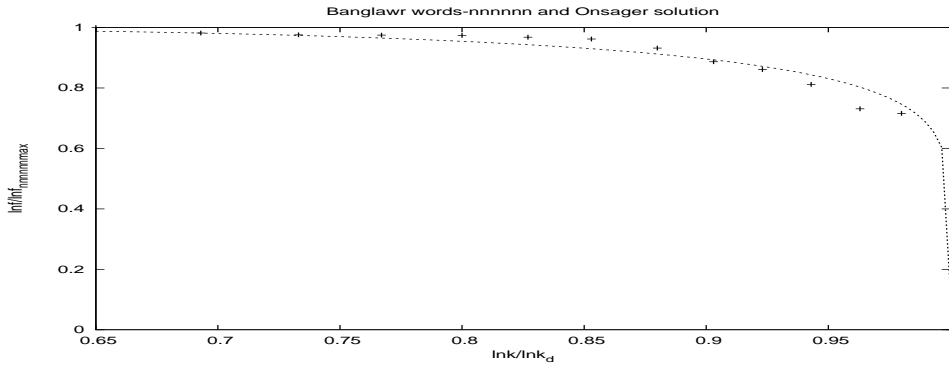


Figure 12: Vertical axis is  $\frac{\ln f}{\ln f_{nnnnnn-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the Onsager solution.

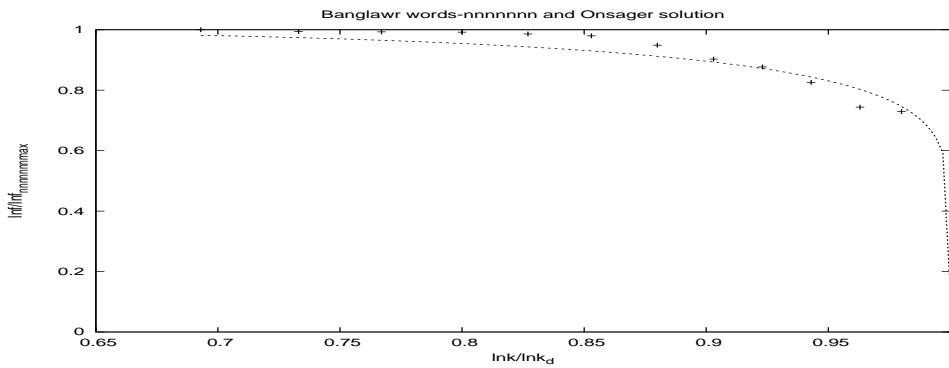


Figure 13: Vertical axis is  $\frac{\ln f}{\ln f_{nnnnnn-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the the Onsager solution.

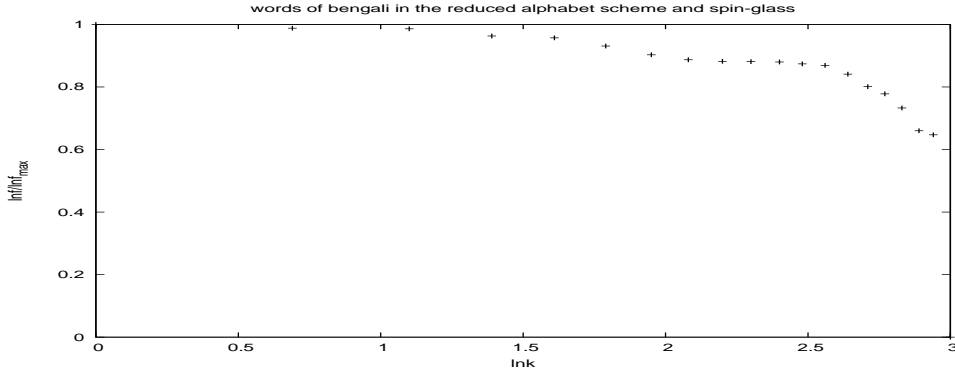


Figure 14: Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the bengali language in the reduced alphabet scheme.

### IV.3 Conclusion

From the figures (fig.6-fig.13), we observe that there is a curve of magnetisation, behind words of bengali language ([9]), in the reduced alphabet scheme. This is magnetisation curve,  $BP(4, \beta H = 0.08)$ , in the Bethe-Peierls approximation with four nearest neighbours, in presence of little magnetic field,  $\beta H = 0.08$ . Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-to-next-to-next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [19].

On the top of it, on successive higher normalisations, words of the bengali language, in the reduced alphabet scheme, almost go over to Onsager solution in the  $\frac{\ln f}{\ln f_{nnnnnn-max}}$  vs  $\frac{\ln k}{\ln k_{lim}}$  graph.

For the shake of completeness, we draw  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln f}{\ln f_{next-max}}$  against  $\ln k$  in the figures fig.(14,15) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying Bengali language words, in the reduced alphabet scheme.

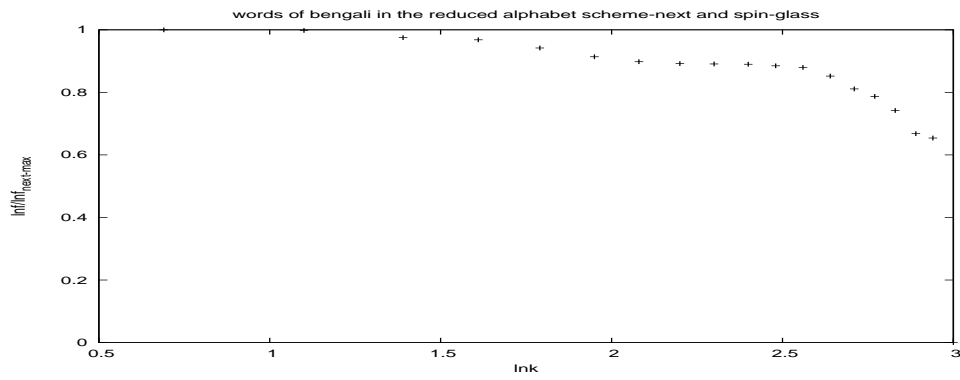


Figure 15: Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the bengali language in the reduced alphabet scheme.



In the figures 14 and 15, the points has a transition, above transition the line is almost horizontal. Hence, the words of the bengali language in the reduced alphabet scheme, can be described to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

## V Discussion

We have observed that there is a curve of magnetisation, behind words of the bengali language,[9], in the reduced alphabet scheme. This is magnetisation curve, BP(4, $\beta H = 0.08$ ), in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field,  $\beta H = 0.08$ . Moreover, words of the bengali language, [9], go over under successive normalisations nearly to the Onsager solution. i.e has Onsager core. Incidentally, this is the same conclusion we reached in [1].

It will be of interest to do the analysis counting all the words of [9], i.e. without ignoring the dialectical variation. Moreover, there is a spectrum of bengali dictionaries available. It will be more interesting to do the analysis for as many of those as possible, from the standpoint of uniqueness, as pointed out in [17].

## VI appendix:Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [18], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor}-1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (1)$$

Derivation of this formula ala [18] is given in the appendix of [17].

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For four neighbours,

$$\frac{0.693}{\frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor}-1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (2)$$

In the following, we describe datas in the table, 5, generated from the equation(2) and curves of magnetisation plotted on the basis of those datas. BP(4,  $\beta H = 0.06$ ) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.06$ . calculated from the equation(2). BP(4,  $\beta H = 0.05$ )

stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.05$ . calculated from the equation(2).  $BP(4, \beta H = 0.04)$  stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.04$ . calculated from the equation(2).  $BP(4, \beta H = 0.02)$  stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.02$ . calculated from the equation(2).  $BP(4, \beta H = 0.01)$  stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.01$ . calculated from the equation(2). The data set is used to plot fig.4 and fig.5. Empty spaces in the table, 5, mean corresponding point pairs were not used for plotting a line.

| BP( $4, \beta H = 0.1$ ) | BP( $4, \beta H = 0.08$ ) | BP( $4, \beta H = 0.06$ ) | BP( $4, \beta H = 0.05$ ) | BP( $4, \beta H = 0.04$ ) | BP( $4, \beta H = 0.02$ ) | BP( $4, \beta H = 0.01$ ) | reduced magnetisation |
|--------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|-----------------------|
| 0                        | 0                         | 0                         | 0                         | 0                         | 0                         | 0                         | 1                     |
| 0.597                    | 0.589                     | 0.583                     | 0.580                     | 0.577                     | 0.572                     | 0.569                     | 0.978                 |
| 0.603                    | 0.593                     | 0.587                     | 0.584                     | 0.581                     | 0.575                     | 0.572                     | 0.977                 |
| 0.660                    | 0.655                     | 0.647                     | 0.643                     | 0.639                     | 0.632                     | 0.628                     | 0.961                 |
| 0.673                    | 0.665                     | 0.657                     | 0.653                     | 0.649                     | 0.641                     | 0.637                     | 0.957                 |
| 0.688                    | 0.679                     | 0.671                     | 0.667                     |                           | 0.654                     | 0.650                     | 0.952                 |
|                          |                           |                           | 0.716                     |                           |                           | 0.696                     | 0.931                 |
| 0.745                    | 0.734                     | 0.723                     | 0.718                     | 0.713                     | 0.702                     | 0.697                     | 0.927                 |
| 0.766                    | 0.754                     | 0.743                     | 0.737                     | 0.731                     | 0.720                     | 0.714                     | 0.917                 |
| 0.787                    | 0.775                     | 0.762                     | 0.756                     | 0.749                     | 0.737                     | 0.731                     | 0.907                 |
| 0.796                    | 0.783                     | 0.770                     | 0.764                     | 0.757                     | 0.745                     | 0.738                     | 0.903                 |
| 0.848                    | 0.832                     | 0.816                     | 0.808                     | 0.800                     | 0.785                     | 0.778                     | 0.869                 |
| 0.854                    | 0.837                     | 0.821                     | 0.813                     | 0.805                     | 0.789                     | 0.782                     | 0.865                 |
| 0.866                    | 0.849                     | 0.832                     | 0.823                     | 0.815                     | 0.799                     | 0.791                     | 0.856                 |
| 0.878                    | 0.859                     | 0.841                     | 0.833                     | 0.824                     | 0.807                     | 0.799                     | 0.847                 |
| 0.902                    | 0.882                     | 0.863                     | 0.853                     | 0.844                     | 0.826                     | 0.817                     | 0.828                 |
| 0.931                    | 0.908                     | 0.887                     | 0.876                     | 0.866                     | 0.846                     | 0.836                     | 0.805                 |
| 0.940                    | 0.917                     | 0.895                     | 0.884                     | 0.873                     | 0.852                     | 0.842                     | 0.796                 |
| 0.966                    | 0.941                     | 0.916                     | 0.904                     | 0.892                     | 0.869                     | 0.858                     | 0.772                 |
| 0.996                    | 0.968                     | 0.940                     | 0.926                     | 0.914                     | 0.888                     | 0.876                     | 0.740                 |
| 1                        |                           |                           | 0.929                     |                           |                           | 0.877                     | 0.735                 |
|                          | 0.977                     |                           | 0.936                     |                           |                           | 0.883                     | 0.730                 |
|                          | 0.989                     |                           | 0.944                     |                           |                           | 0.889                     | 0.720                 |
|                          | 0.990                     |                           | 0.945                     |                           |                           |                           | 0.710                 |
|                          | 1.00                      |                           | 0.955                     |                           |                           | 0.897                     | 0.700                 |
|                          |                           |                           | 0.963                     |                           |                           | 0.903                     | 0.690                 |
|                          |                           |                           | 0.973                     |                           |                           | 0.910                     | 0.680                 |
|                          |                           |                           |                           |                           |                           | 0.909                     | 0.670                 |
|                          |                           |                           | 0.993                     |                           |                           | 0.925                     | 0.650                 |
|                          |                           |                           |                           | 0.976                     | 0.942                     |                           | 0.651                 |
|                          |                           |                           | 1.00                      |                           |                           |                           | 0.640                 |
|                          |                           |                           |                           | 0.983                     | 0.946                     | 0.928                     | 0.628                 |
|                          |                           |                           |                           | 1.00                      | 0.963                     | 0.943                     | 0.592                 |
|                          |                           |                           |                           |                           | 0.972                     | 0.951                     | 0.564                 |
|                          |                           |                           |                           |                           | 0.990                     | 0.967                     | 0.527                 |
|                          |                           |                           |                           |                           |                           | 0.964                     | 0.513                 |
|                          |                           |                           |                           |                           | 1.00                      |                           | 0.500                 |
|                          |                           |                           |                           |                           |                           | 1.00                      | 0.400                 |
|                          |                           |                           |                           |                           |                           |                           | 0.300                 |
|                          |                           |                           |                           |                           |                           |                           | 0.200                 |
|                          |                           |                           |                           |                           |                           |                           | 0.100                 |
|                          |                           |                           |                           |                           |                           |                           | 0                     |

Table 5: Reduced magnetisation vs reduced temperature datas in Bethe-Peierls approx. of Ising model in presence of little external magnetic fields, from right to left along a row.

## VI.1 Spin-Glass

In the case coupling between( among) the spins, not necessarily n.n, for the Ising model is( are) random, we get Spin-Glass, [20, 21, 22, 23, 24, 25, 26]. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like  $\frac{1}{T-T_c}$  upto the phase transition temperature, followed by very little increase,[20, 26], in magnetisation, as the ambient temperature continues to drop. This happens at least in the replica approach of the Spin-Glass theory, [23, 24].

## VII Acknowledgement

We have used gnuplot for plotting the figures in this paper.

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