

BERM BREAKWATER TRUNK EXPOSED TO OBLIQUE WAVES

A. Alikhani¹, G. R. Tomasicchio² and J. Juhl³

ABSTRACT

Comprehensive three-dimensional model tests with a berm breakwater roundhead and the adjacent trunk section were carried out at the Danish Hydraulic Institute (DHI) and at the Hydraulic & Coastal Engineering Laboratory of Aalborg University (AAU). This paper describes the influence of wave obliquity on the profile shape, on the initiation of longshore transport and on the longshore transport rate at the trunk section both during and after the profile reshaping, whereas the roundhead stability is described in Juhl et al (1996). Furthermore, tests were made for studying the influence of storm duration and of short crested waves. Equations for calculation of profile development and longshore transport rate under oblique wave attack are introduced.

keywords: berm breakwaters, reshaping, threshold values, longshore transport.

1. INTRODUCTION

A berm breakwater exposed to head-on waves can hardly be destroyed unless it is overtopped, whereas for oblique waves the stones can move along the breakwater. Burcharth and Frigaard (1987) made the first systematic study of oblique wave attack on reshaping breakwaters by testing with angles of wave attack of 15° and 30°. As guideline they recommend $H_o < 4.5$ for a trunk exposed to steep oblique waves, $H_o < 3.5$ for a trunk exposed to long oblique waves and $H_o < 3$ for a roundhead, where

$$H_o = \frac{H_s}{\Delta D_{n50}} \quad (1)$$

is the stability number, Δ is the relative density, D_{n50} is the equivalent cube side length and H_s is the significant wave height. Dependency on wave obliqueness was not described. Burcharth and Frigaard (1988) mentioned that H_o is insufficient to describe the phenomena as it among other things does not contain the effect of wave length and the effect of the duration of the sea storm.

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They concluded that the erosion in oblique waves has a non-linear dependency of the sea state.

Van der Meer (1988) introduced the product of dimensionless wave height and wave period parameters for the design of a reshaping breakwater:

$$H_o T_{op} = \frac{H_s}{\Delta D_{n50}} T_p \sqrt{g/D_{n50}} \quad (2)$$

where T_p is the peak period and g is acceleration due to gravity.

Vrijling and Swart (1991) used data from tests of Burcharth and Frigaard (1988) and data from Delft Hydraulics to fit a formula for the transport of stones along the trunk of a breakwater under oblique wave attack. The formula gives the along-structure transport measured as the number of stones per wave, S :

$$S = 4.8 * 10^{-5} (H_o T_{op} - 100)^2 \quad (3)$$

The effect of obliquity of the incident waves is not included in Eq. (3). In average the onset of transport corresponds to $H_o T_{op} = 100$. van der Meer and Veldman (1992) presented Eq. (3) in a slightly different form. They concluded that the longshore transport for larger angles of wave attack such as 50° is much smaller than for 15° and 30° ; their formula is:

$$S = 5.0 * 10^{-5} (H_o T_{op} - 105)^2 \quad (4)$$

The influence of angle of wave attack for shingle beaches was studied by Van Hijum and Pilarczyk (1982) who proposed Eq. (5) for the along-structure volume transport rate under different angles of wave attack. The formula is only valid for shingle beaches, $H_o = 12 - 27$, and can thus not be applied for berm breakwaters.

$$\frac{S_V}{g D_{90}^2 T_s} = 7.12 * 10^{-4} \frac{H_{so} \sqrt{\cos \psi}}{D_{90}} \left(\frac{H_{so} \sqrt{\cos \psi}}{D_{90}} - 8.3 \right) \frac{\sin \psi}{\tanh kh} \quad (5)$$

where ψ is the incident wave angle, H_{so} is the deep water significant wave height, T_s is the 15% excess value of wave period, D_{90} is the sieve diameter for which 90% of the stones (by weight) are smaller, k is the wave number at the structure toe, h is the water depth at the structure toe, g is the acceleration of gravity and S_V is the bulk volume transport rate. With regard to the geometry of a reshaped breakwater Van Hijum and Pilarczyk (1982) concluded that profile parameters should be reduced by $\sqrt{\cos \psi}$. However, van der Meer (1988) re-analysed the data and came to a reduction factor of $\cos \psi$. The analysis was done only for 30° of wave attack and for finer material ($12 < H_o < 27$).

In the Shore Protection Manual (CERC, 1984) a longshore transport formula is given by the following relationship which is valid for H_o larger than 50 (i.e. for sand beaches):

$$S_V = 0.12 * 10^{-4} \pi H_s^2 c_{op} \sin 2\psi \quad (6)$$

where c_{op} is the wave celerity associated to the peak wave period. The formula includes the effect of the wave angle, but does not consider the grain size.

Tomasicchio et al. (1994) and Lamberti and Tomasicchio (1996) on the basis of flume tests, proposed a conceptual model relating longshore transport due to oblique wave attack to stone mobility. The transport model assumes that stones move during up-rush and down-rush in the direction of incident and reflected waves and that movement statistics is affected by obliquity through a modified stability number:

$$N_s^{**} = \frac{H_k}{C_k \Delta D_{n50}} \left(\frac{s_{mo}}{s_{mk}} \right)^{-1/5} (\cos \psi)^{2/5} \quad (7)$$

where H_k is a characteristic wave height for the phenomena under study. For stone movements at a berm breakwater $H_k = H_{1/50}$ is assumed; C_k is the ratio between the characteristic and significant wave height and has a value 1.55 if a Rayleighian wave height distribution is valid. The second factor in Eq. (7) is such that $N_s^{**} = H_o$ for $\psi = 0$ if $s_{mo} = s_{mk}$, where s_{mo} is the wave steepness in deep water based on the mean wave period and s_{mk} was assumed to be 0.03. Tomasicchio et al. (1994) defined the longshore transport, S , as the number of stones moved per wave to be related to the displacement length, l_d , a standard measure for damage, N_{od} , and to the incident wave angle, ψ . The damage index N_{od} was found to

$$N_{od} = 2.05 N_s^{**} (N_s^{**} - 2.0)^{2.2} \quad (8)$$

and for $N_s^{**} > 2.0$, S is given by:

$$\frac{S}{\sin \psi} = \frac{l_d}{D_{n50}} \frac{N_{od}}{1000} \quad (9)$$

In the following, the analysis of the effect of incident wave angle will be presented but also some other important parameters will be discussed.

2. EXPERIMENTS AND TEST PROGRAM AT DHI

2-1. Model set-up

An experimental investigation has been carried out at DHI in a 23 x 30 m wave basin in order to study the stability of a berm breakwater roundhead and the adjacent trunk section under the exposure of oblique waves. Fig. 1 shows a plan of the model basin including the various positions of the two 5.5 m wide wave generators capable of generating irregular long crested waves. Fig. 2 shows the initial profile at the trunk section, and the roundhead was made by rotating the profile around the centreline.

2-2. Stone characteristics

The berm breakwater was constructed of two stone classes, i.e. one for the core and the scour protection and one for the berm, the crest and the rear side protection. The core material had a nominal diameter of $D_{n50} = 0.010m$. A relative wide stone gradation was used for the berm, $\frac{D_{n85}}{D_{n15}} = 1.8$, with an equivalent cube length of $D_{n50} = 0.023m$. The density of the stone material was $\rho_s = 2.68t/m^3$.

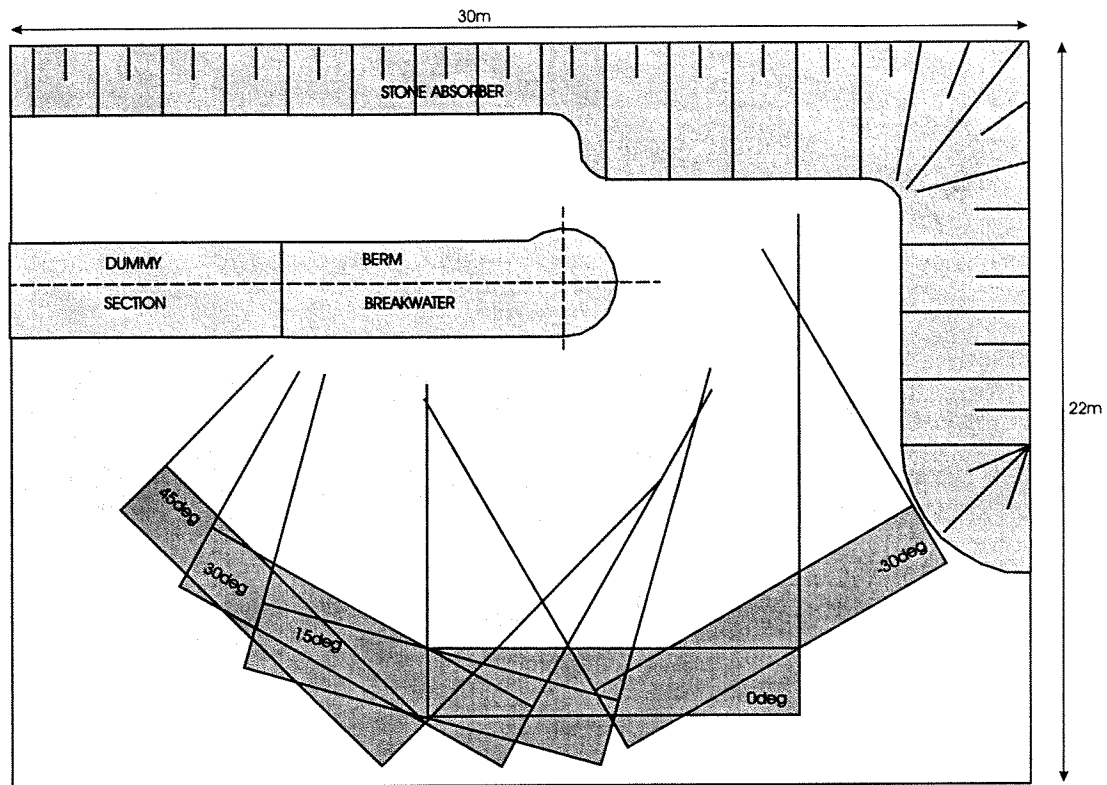


Figure 1: Plan view of the experimental set-up at DHI showing different positions of the wave paddle.

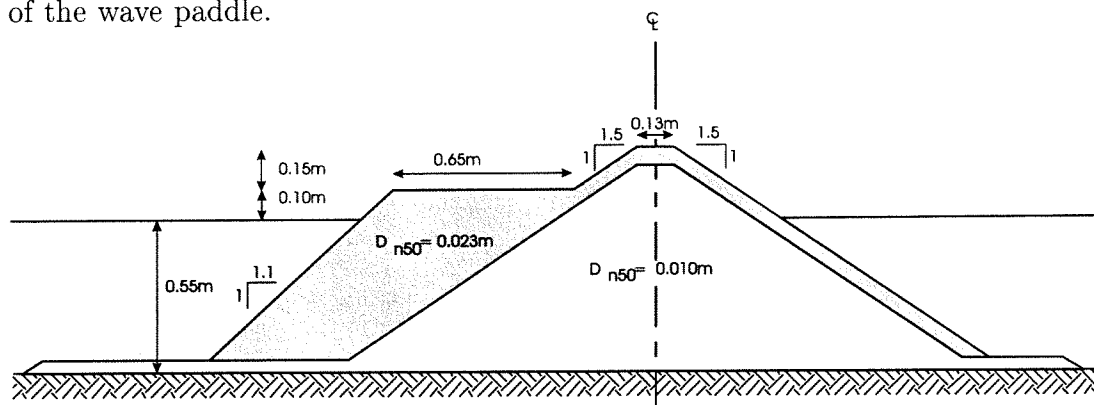


Figure 2: Initial profile for tests at DHI.

2-3. Test program

A total of six test series with irregular long crested waves were carried out in order to consider five angles of incident waves with a wave steepness of 0.05 (-30° , 0° , 15° , 30° , and 45° ; 0° is perpendicular to the trunk, see Fig. 1) and two wave steepnesses for the angle of incidence of -30° ($s_m = 0.03$ and 0.05). Each test series consisted of five tests ($H_0 = 2.0, 2.5, 3.0, 3.5$, and 4.0) with a duration corresponding to 2,000 waves for initial reshaping of the berm breakwater followed by four tests with a duration of 1,000 waves for studying the stone movements on the reshaped profile ($H_0 = 2.5, 3.0, 3.5$, and 4.0). All waves were generated on the basis of a Pierson Moskowitz spectrum.

2-4. Measurements

The waves were measured at 14 positions in the wave basin by the use of resistance type wave gauges. The wave conditions were checked by five reference wave gauges placed in a way to minimise the effect of reflection from the berm breakwater. Spectral analysis and zero-crossing analysis were carried out.

A total of 38 profiles along the 8.5 m long breakwater were measured after its construction (initial profile) and after each test run. The profiling was made with a laser running on a beam across the breakwater trunk. The horizontal position of the laser along the beam was measured by another laser, whereas the location along the breakwater axis was fixed manually. The profiles were measured for each 0.5 m along the trunk and for each 0.1 m at the roundhead. Assessment of the stone movements was made based on visual observations, photos taken after each test run and video recordings. Observations were made both during the reshaping process involving a large number of stone movements and after the reshaping. In order to facilitate observations of the threshold conditions for long-shore transport and the longshore transport rate, all the armour stones in a one meter wide section of the trunk were painted (immersed in paint) before the reshaping phase and the surface was painted again before the stone movement tests.

3. EXPERIMENTS AND TEST PROGRAM AT AAU

3-1. Model set-up

A model test program for studying the effect of directionality of the waves and the effect of the number of waves on the behaviour of a berm breakwater trunk section exposed to oblique waves has been carried out in a 8.5 x 15.7 m directional wave basin at Aalborg Hydraulic & Coastal Engineering Laboratory. Fig. 3 shows the layout of the model basin and the cross section of the structure. The waves were generated by 9 paddles each 0.9 m wide.

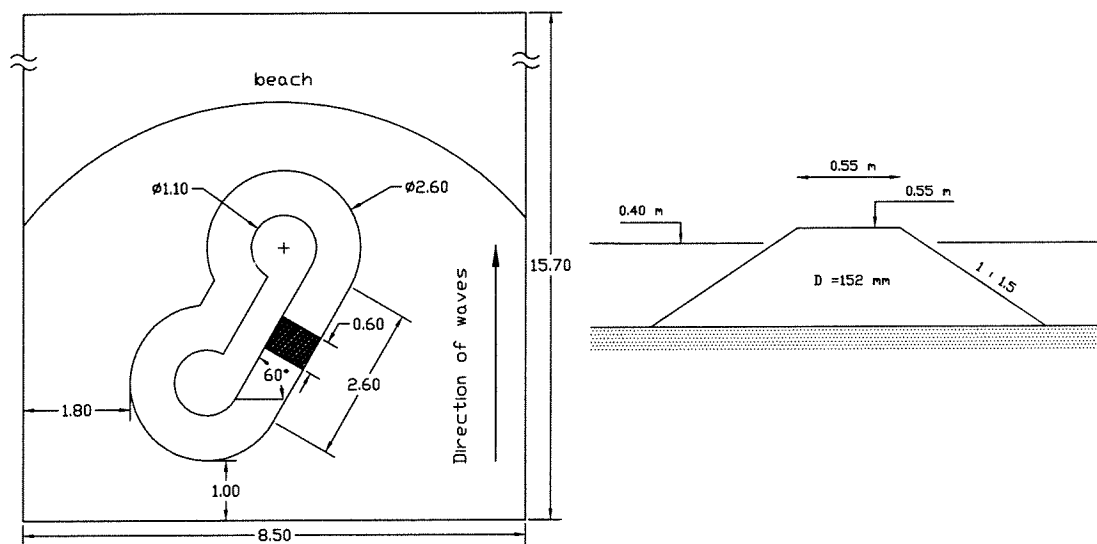


Figure 3: Lay-out of model basin and breakwater cross-section used for the directional wave tests at Aalborg University.

3-2. Stone characteristics

The berm breakwater was constructed with only one stone class having a nominal diameter of $D_{n50} = 0.0152m$ and $\frac{D_{n85}}{D_{n15}} = 1.44$. The density of the stone material was measured to $\rho_s = 2.70t/m^3$.

3-3. Test program

A total of 20 tests with irregular waves were carried out. The reshaping phase considered 5 wave attacks ($H_o = 2.2, 2.7, 3.3, 3.8,$ and 4.1) with long crested oblique waves. The tests were made with an angle of incidence of 60° with respect to the breakwater and a duration of 2,000 waves. The reshaping phase was followed by 4 tests with long crested oblique waves (60°) for studying the stone movements on the reshaped profile ($H_o = 2.9, 3.2, 3.5,$ and 4.0); each test with a duration corresponding to 1,000 waves.

After these stages, long duration tests with a total duration corresponding to 11,000 waves were carried out (11 tests with $H_o = 4.0$). Two of the tests were made with directional waves having an angle of incidence of 60° and a cosine spreading function of $s = 10$. All waves were generated on the basis of a JON-SWAP type spectrum with peakedness parameter $\gamma = 3.3$, width parameters $\sigma = 0.10$ for $f \leq f_p$ and $\sigma = 0.50$ for $f > f_p$.

3-4. Measurements

Prior to the initiation of the model tests, the directional waves were calibrated with a very gentle stone absorber in order to minimise wave reflection. The waves were measured at 6 positions in the wave basin by the use of resistance type wave gauges. The wave conditions have been checked by five reference wave gauges placed in positions with minimum effect of reflection from the berm breakwater. For the long crested waves the incident wave height was calculated using the method of Funke and Mansard (for separation of incident and reflected waves) and for the directional waves the calibrated target waves were used.

A total of 3 manual profile measurement were made: the initial profile, the profile after reshaping of the trunk and the profile after the long duration test with $H_o = 4.0$. Assessment of the stone movement was made based on visual observations and photos taken after each test. Observations were made both during the reshaping process involving a large number of stone movements and on the reshaped profile. In order to facilitate observations of the threshold conditions for the longshore transport and of the longshore transport rate, all the berm stones in a 0.6 m wide section of the trunk were painted (immersed in paint).

4. DISCUSSION

Three effects will be discussed in the following:

1. effect of wave obliquity on threshold of movement;
2. effect of wave obliquity on profile shape and sorting of the stones;
3. effect of wave obliquity on longshore transport.

4-1. Threshold of stone movement

For smaller angles of incidence the unstable stones will first start rocking and/or rolling and will not move in the longshore direction as long as the longshore

component of the energy flux is not big enough to move them. Table 1 shows the longshore transport (stones per wave) on the reshaped profile as a function of $H_o T_{op}$ for different angles of wave attack.

$H_o T_{op}$	15°	30°	45°
40	0.000	0.000	0.000
55	0.000	0.000	0.000
80	0.002	0.003	0.005
100	0.008	0.014	0.015
131	0.021	0.05	0.07
161	0.2	-	-

Table 1: Longshore transport after reshaping (tests made at DHI).

In general, the largest transport distance occur for $\psi = 45^\circ$ with a decreasing tendency for smaller and larger angles of wave attack. Now the threshold values are described as:

$$H_o T_{op} > \frac{50}{\sqrt{\sin 2\psi}} \quad (10)$$

during the reshaping phase, and

$$H_o T_{op} > \frac{75}{\sqrt{\sin 2\psi}} \quad (11)$$

after the reshaping phase

4-2. Sorting and profile shape

With regard to the tests made at DHI, Fig. 4 shows the as built structure stone distribution as well as the stone distribution around the water level and in the lower steep part of the reshaped breakwater after exposure to head-on waves with $H_o = 4.0$. w_{50} around the water level is found to be half the w_{50} in the lower

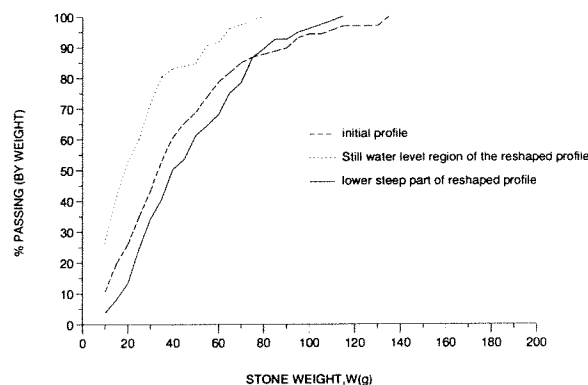


Figure 4: Stone distribution after exposure to 2,000 head-on waves with $H_o = 4.0$ (DHI tests).

steep part, where w_{50} is defined as the stone weight for which 50% of the stone are smaller. However, for more narrow stone gradations the differences will be less pronounced showing that the stone distribution might be an important parameter. The result of the tests at AAU with larger angle of wave attack and narrower stone gradation indicate less sorting of the stones. The major part of the resorting happens in the first 1000 waves. Due to the less sorting of stones for larger ψ and narrower stone gradation, a modification should be applied to the D_{n50} value. This modification might be dependent on the incident wave angle as well as on the value of $\frac{D_{85}}{D_{15}}$. From Fig. 5 it is seen that in long duration tests the lower part of the profile is not changing but the upper part is developing as a function of duration. The reason for this continued recession is lack of nourishment from the upstream of the breakwater. In the test of Van Hijum and

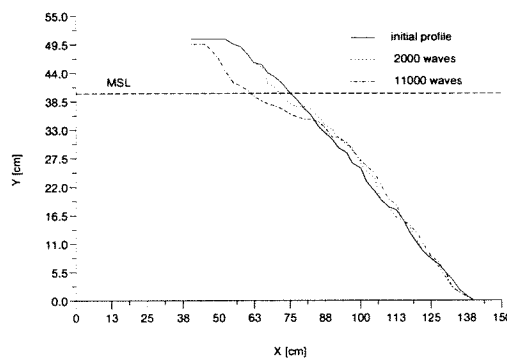


Figure 5: AAU tests, initial and reshaped profile at trunk section ($\psi = 60^\circ$).

Pilarczyk (1982) some of the structural parameters are neither fitting to $\sqrt{\cos \psi}$ nor to $\cos \psi$ as re-analysed by van der Meer (1988), e.g for the crest width the reduction factor is even bigger than unity. The fact is that the reduction factor for the upper part of the profile under oblique waves is more dependent on the storm duration, while for head-on waves this is not the case. Therefore, due to the longer duration sea state the recession of the berm under oblique waves can exceed the value for head-on waves for fixed equivalent wave parameters. Thus, a reduction factor for the upper part of the profile is meaningless under a long duration sea storm.

Equations were developed to calculate the reshaped profile under head-on and oblique wave attack. For reshaped profiles there is always a point, below SWL, which indicates the change of the curvature of the profile, see Fig. 6. This step point, h_s , can be described by

$$h_s = \frac{H_o T_{op} D_{n50} N^{0.07} \sqrt{\cos \psi}}{40} \quad (12)$$

where N , is the number of waves. From this point, depending on the water depth, wave parameters, stone weight, incident wave angle, initial shape of the profile, and repose angle of the stones, two power functions can represent the reshaped

profile namely upper part and lower part. The lower part can be described as:

$$Y = -hm_1\left(\frac{X}{h}\right)^{1.7} + h - h_s \quad (13)$$

where X and Y are the length and height of the reshaped profile counted from the center line of the breakwater, h is the water depth at the toe of the breakwater and coefficient m_1 is given by

$$m_1 = \frac{1.7e^{(-0.013H_oT_{op})}}{\cos \psi} \quad (14)$$

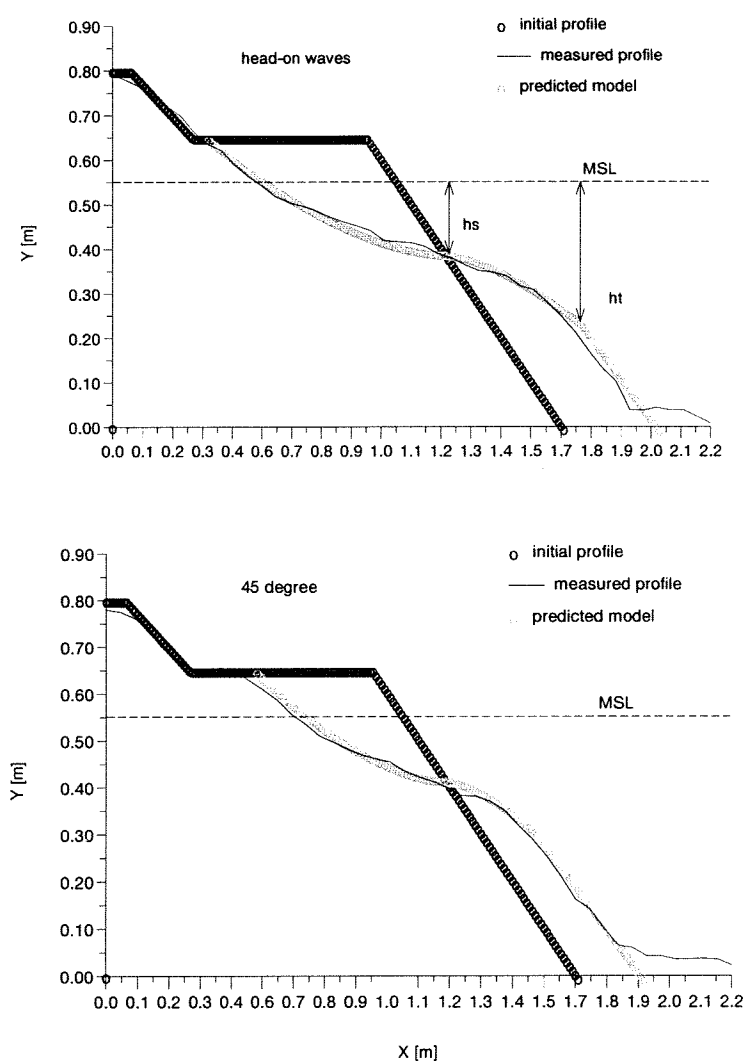


Figure 6: Initial, predicted and measured reshaped profiles for $H_oT_{op} = 161$.

Tests result at DHI show that the part of the profile below the transition point h_t follows the repose angle of the stones under water, where h_t is given by

$$h_t = H_s \cos \psi + h_s \quad (15)$$

The upper part can be described as:

$$Y = hm_2\left(\frac{X}{h}\right)^{1.7} + h - h_s \quad (16)$$

where

$$m_2 = \frac{1.7e^{(-0.01H_oT_{op})}}{\cos \psi} \quad (17)$$

The upper part above still water line is more depending on the initial shape of the profile. Tests result at DHI show that the part of the profile above the berm is following the initial profile shape.

A computer program "RESHAPED" is developed for calculating the profiles, see examples in Fig. 6. The profiles are drawn from an arbitrary point say from the center line of the breakwater and then by the use of an iterative procedure shifted along still-water line until the mass balance is fulfilled. In the case, where enough nourishment is not supplied from upstream, additional erosion due to longshore transport must be considered. For this purpose the longshore transport rate described in Eq. 18 can be used. RESHAPED is calibrated for a breakwater with core material and a relatively wide stone gradation. Comparison was made with some of the tests at AAU with a structure without core material and narrower gradation, showing more resistance of the breakwater compared to the tests at DHI. Calculations of the profile developments for the tests at DHI were also made using BREAKWAT, see van der Meer (1992), showing an overestimation of the structure response.

4-3. Longshore transport

Eq. 4 seems to be adaptable for berm breakwaters, but no influence of the wave angle is included, i.e the formula does not consider any difference between the effect of head-on waves and of oblique waves on the longshore transport.

One of the most important parameters influencing longshore transport is the longshore component of the incident wave energy. Therefore it is rational to relate the longshore transport to this parameter and consequently Eq. 4 is modified as:

$$S = 0.8 * 10^{-6} \sqrt{\cos \psi} (H_o T_{op} \sqrt{\sin 2\psi} - 75)^2 \quad (18)$$

The formula is calibrated to give the maximum longshore transport for $\psi = 45^\circ$ and zero transport for head-on waves and waves propagating parallel to the breakwater axis. As the test conditions at laboratories were different (e.g water depth, nearshore slope, stones characteristics, initial profiles, wave generation and calibration procedures, and analysis of incident and reflected waves), each set of laboratory data will be discussed separately. Eq. 18 is shown in Fig. 7 together with data from DHI tests after reshaping. The test results and Eq. 18 show comparable relationships of the wave conditions and angle of wave attack. Fig. 8 shows Eq. 18 together with longshore transport data from the tests by Burcharth and Frigaard (1987). The tests for studying the influence of the wave angle were all made with a breakwater constructed with a profile as will develop after exposure to head-on waves, as they assumed the profile to be almost independent of the wave angle. Three points may concern here:

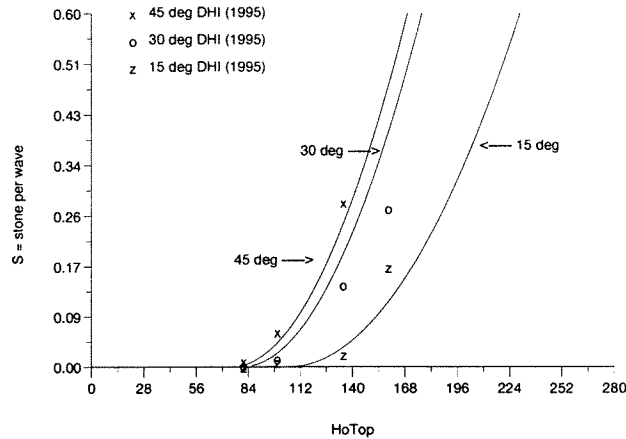


Figure 7: Longshore transport, in form of stone per wave, S , as function of $H_o T_{op}$.

- 1) The structure was not compacted during a reshaping process and redistribution of the stones did not take place. By constructing a breakwater in this manner, the additional stability due to the natural armouring that occurs as a result of stone motion induced by wave attack will not develop. This can have more influence on the tests with 15° than on the tests with 30° .
- 2) Results have shown that the profile reshaping is larger for head-on waves than for oblique waves. Running tests with oblique waves on a more reshaped profile will result in more energy dissipation and thus reduced longshore transport, which could be the reason for the small difference in longshore transport rate between 15° and 30° .
- 3) In some of the tests with increasing wave parameters the longshore transport was not increased; which can be due to the breakwater becoming more consolidated during exposure to more and more waves.

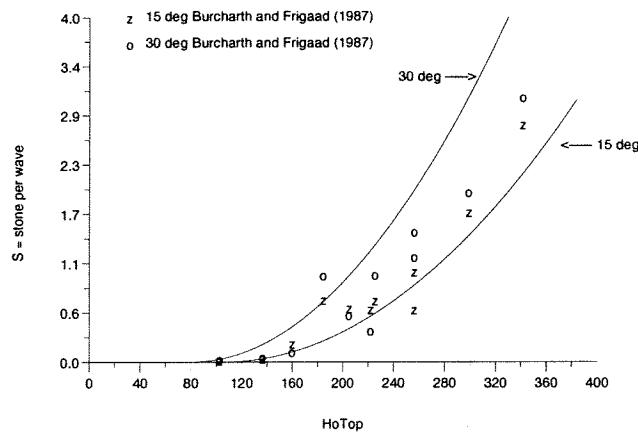


Figure 8: Longshore transport for data of Burcharth and Frigaard (1987).

Fig. 9 shows the longshore transport after reshaping based on data from van

der Meer and Veldman (1992). This figure confirms that other important pa-

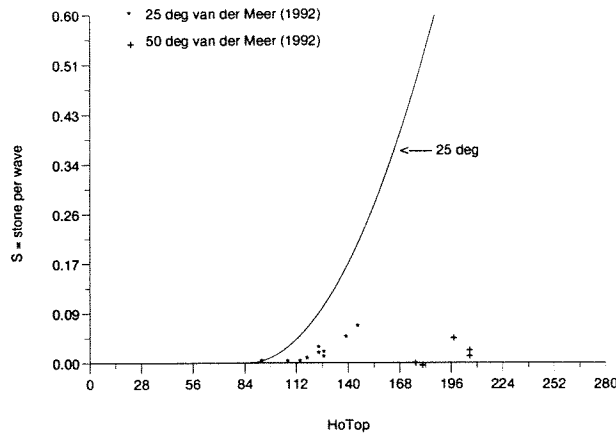


Figure 9: Longshore transport for data of van der Meer and Veldman (1992).

rameters are the directionality of waves and the number of waves. Test results from AAU for directional waves have shown that the longshore transport is much lower than for long crested waves. For example for $s = 10$, where s is a parameter that controls the angular distribution in the cosine power spreading function, the longshore transport was $1/4$ of the case for long crested waves for $H_o = 4.0$. More tests with different s values are necessary in order to formulate the directionality effects.

Fig. 10 shows a plot of the data of the tests of van der Meer and Veldman (1992), where the directionality has been taken into account applying a reduction factor of 4. For an angle of wave attack of 25° the data fit very well, but for 50° there must be some other reason which is not explainable.

In order to show the effect of the storm duration, long duration tests were carried out with a total of 11,000 waves. The number of moved stones was counted after each 1,000 waves. The longshore transport rate was decreasing with increasing number of waves, see results presented in Fig. 11. Experimental data fit in with an exponential function:

$$S = \frac{ae^{-bN}}{1000} \quad (19)$$

For tests at AAU with $\psi = 60^\circ$ and $H_o = 4.0$, a was found to be equal to 350.

5. CONCLUSIONS

Three-dimensional experiments both at DHI and Aalborg University shown that the threshold value of the stone movement, the shape of the profile and the rate of longshore transport are dependent on both the angle of wave attack and the wave energy.

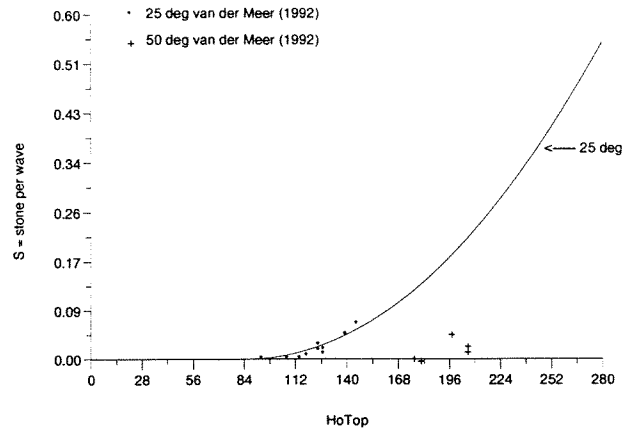


Figure 10: Longshore transport for data of van der Meer and Veldman (1992), with a reduction factor of 4 for directional waves.

Laboratory investigations shown that due to the wave induced sorting of the stones, D_{n50} does not remain constant during the reshaping process at certain locations on the cross section. The sorting is a function of the hydrodynamic vector in front of the structure and will change as a function of the incident wave energy and the wave direction.

An equation for calculation of the longshore transport rate on a reshaped berm breakwater is established, see Eq. 18. Criteria for initiation of stone movements both during and after reshaping are given, see Eq 10. and 11. Moreover, laboratory investigations from Aalborg University shown that directional waves reduce the longshore transport rates (a specific test with a wave angle of 60° showed a reduction of 4) and that the longshore transport is reduced by an exponential function when increasing the number of waves.

The presented longshore transport formula can not necessarily fit all the data because it does not consider the effects of the stone gradation, the shape of the stones, the permeability, the armour layer thickness, the spreading function and the number of waves considered for each test at different laboratories. Further analysis are required to study the influence of especially stone gradation and directional waves.

The profile of a reshaped profile can be described by two equations, one for the part above a defined step point and one below this point, see Eq. 13 and 16. It was found that the angle of wave attack can be included by a reduction factor of $\cos \psi$.

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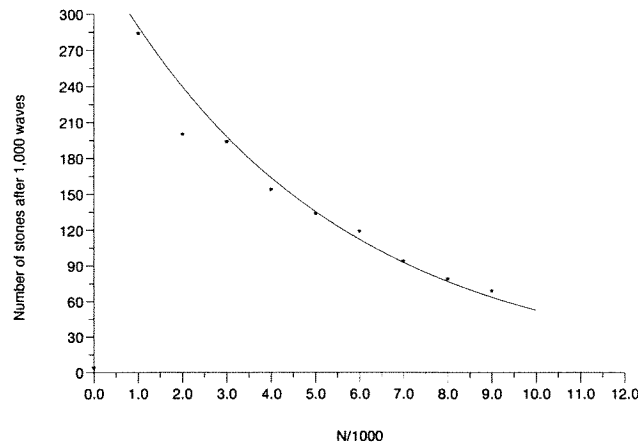


Figure 11: Long duration tests at Aalborg University $H_o = 4.0$, $\psi = 60^\circ$.

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