

## BERWALD TYPE INEQUALITY FOR EXTREMAL UNIVERSAL INTEGRALS BASED ON $(\alpha, m)$ -CONCAVE FUNCTION

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**Abstract.** The aim of this work is to show a Berwald type inequality for the extremal universal integrals based on  $(\alpha, m)$  concave function. Some examples are given to illustrate the validity of these inequalities.

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