BEST LINEAR UNBIASED ESTIMATOR ALGORITHM FOR RECEIVED SIGNAL STRENGTH BASED LOCALIZATION

Lanxin Lin and H. C. So

Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China
phone: + (852) 3442 7780, fax: + (852) 3442 0401, email: lxlinhk@gmail.com

ABSTRACT

Locating an unknown-position source using measurements from an array of spatially separated sensors with low complexity is quite necessary in many applications. In this paper, a linear least squares (LLS) method, which is a best linear unbiased estimator, is proposed to estimate the unknown-position source location based on the received signal strength (RSS) measurements. It is proved that the performance of our proposed method is identical to that of an existing LLS technique but the former is more computationally efficient. A relaxation method is also introduced to extend the LLS methods for RSS-based positioning with unknown path-loss factor. Furthermore, numerical examples are included to evaluate the performance of proposed algorithm by comparing with the existing LLS approach and their theoretical position variances as well as Cramér-Rao lower bound.

1. INTRODUCTION

Currently, source localization is a hot research topic in many fields such as radar, sonar [1], telecommunications [2], mobile communications [3] and wireless sensor networks [4]. Since the cost and power consumption are very important factors in many applications, estimating the location of the unknown-position source using low-cost hardware becomes more and more interesting nowadays [5]. Time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA) and received signal strength (RSS) are commonly used measurements for positioning. Clocks with the same time control schemes are needed for the TOA or TDOA measurements, while angle measuring capability is necessary for the AOA scheme, however, the RSS can be measured by each receiver during normal data communication without additional hardware or energy requirements [6]. Since some low complexity mobile receivers do not have precision instruments for accurate time or bearing measurements, the RSS information is relatively inexpensive and simple to obtain.

Many RSS-based positioning methods have been proposed, such as the maximum-likelihood (ML) [7, 8], semidefinite relaxation (SDR) [9, 10], and linear least squares (LLS) methods [11, 12]. To the best of our knowledge, the maximum-likelihood (ML) [7, 8] algorithm is hard to implement in practice because its cost function is highly nonlinear and contains multiple local minima and maxima. Additionally, the SDR method, which is a suboptimal algorithm [10], is with relatively high computational complexity. In this work, our aim is to develop a computationally efficient method for RSS-based positioning. In fact, a suboptimal but computationally efficient linear approach, which is to reorganize the nonlinear equations constructed from the noisy RSS information to linear, via subtracting the square of the reference range to remove the nonlinear terms, has been proposed [11, 12]. In this work, we contribute to positioning algorithm development and analysis given the RSS measurements by exploiting another linearization approach which transforms the nonlinear equations to linear via the introduction of a range variable. It is a best linear unbiased estimator (BLUE) whose performance is the same as the LLS methods proposed in [11, 12], but is more computationally efficient. Furthermore, these two BLUE algorithms are extended to RSS-based localization with unknown path-loss factor.

The rest of paper is organized as follows. Section 2 is devoted to derive the covariance matrix of the error due to shadow fading, and then a BLUE-LLS estimator is proposed by introducing an extra variable. The existing BLUE-LLS estimator [11, 12] is also reviewed. In Section 3, the performance of the proposed BLUE-LLS algorithm and the existing one is analyzed. Simulation results are presented in Section 4 to evaluate the localization accuracy of the proposed BLUE-LLS scheme by comparing with the existing one method and Cramér-Rao lower bound (CRLB). Finally, the conclusions are drawn in Section 5.

2. ALGORITHM DEVELOPMENT

Consider an array of $N \geq 3$ receivers in a two-dimensional (2-D) space. Note that extension to three-dimensional space is straightforward. Let $\mathbf{x} = [x\ y]^T$ be the source position to be determined and $\mathbf{x}_i = [x_i\ y_i]^T$, $i = 1, 2, \cdots, N$ be the known coordinates of the ith receiver. The distance between the source and the ith receiver, denoted by d_i, is

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2},\ i = 1, 2, \cdots, N \qquad (1)$$

Squaring both sides of (1) yields

$$d_i^2 = -2x_i x - 2y_i y + x^2 + y^2 + k_i,\ i = 1, 2, \cdots, N \qquad (2)$$

where $k_i = x_i^2 + y_i^2$. In order to construct a linear estimator, it is common to introduce an extra variable $R = x^2 + y^2$ which is considered to be independent of x and y, and then a linear estimator of $[x\ y\ R]^T$ can be built according to (2) if the estimates of d_i^2 are available. By subtracting the square of the reference range to remove the nonlinear terms x^2 and y^2, another linear estimator can be also constructed. Selecting d_1^2 as the reference, we have

$$d_i^2 - d_1^2 = -2(x_i - x_1)x - 2(y_i - y_1)y + k_i - k_1, \quad i = 2, 3, \cdots, N \qquad (3)$$

In doing so, a linear estimator of $[x\ y]^T$ can be built according to (3) if the estimates of $(d_i^2 - d_1^2)$ are obtained [11, 12].

2.1 RSS-based Localization Model

For RSS-based positioning systems, the primary source of error is multipath fading and shadowing. Averaging the signal strength can help to smooth out the effects of multipath fading, and there will still be the variability due to shadow fading which makes it hard to find a BLUE.

The most popular channel model for RSS-based localization is the lognormal shadowing path loss model [11] which foresees a linear relation between the average received power in dB and the range in logarithmic scale, plus an additional random term to account for fading effects. After collecting sufficient measurements, the average received power from ith receiver can be given as:

$$P_i = P_0 - 10\alpha \log_{10} \frac{d_i}{d_0} + n_i \quad i = 1, 2, \cdots, N \tag{4}$$

where d_0 is the reference distance, which is typically taken equal to 1 meter without loss of generality; P_i and P_0, all are known in dB, are the average power of the source received from the ith receiver and the average received power at reference distance d_0, respectively. The α is the path-loss factor and can vary from 1 to 5 depending on the propagation environment [8]. The errors $\{n_i\}$ are zero-mean uncorrelated Gaussian processes with known variances $\{\sigma_i^2\}$.

In order to construct a linear model, the estimates of d_i^2 are needed. We first express (4) as:

$$e^{\frac{-2r_i}{\alpha}} = d_i^2 e^{-\frac{2m_i}{\alpha}}, \; i = 1, 2, \cdots, N \tag{5}$$

where $r_i = 0.1\ln(10)(P_i - P_0) - \alpha \ln(d_0)$ and $m_i = 0.1\ln(10)n_i$. Note that the variance of m_i, denoted by λ_i^2, has a value of $\lambda_i^2 = 0.01(\ln(10))^2\sigma_i^2$. It is worth noting that the noise component $e^{-2m_i/\alpha}$ is now multiplicate which is the major challenge in obtaining a BLUE estimator to solve this localization problem.

2.2 Development of a BLUE-LLS Algorithm

The matrix form of (2) is

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} \tag{6}$$

where

$$\mathbf{A} = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ -2x_2 & -2y_2 & 1 \\ \vdots & \vdots & \vdots \\ -2x_N & -2y_N & 1 \end{bmatrix} \tag{7}$$

$$\boldsymbol{\theta} = [x \; y \; R]^T \tag{8}$$

and

$$\mathbf{b} = \begin{bmatrix} d_1^2 - k_1 \\ d_2^2 - k_2 \\ \vdots \\ d_N^2 - k_N \end{bmatrix} \tag{9}$$

Since $\{d_i^2\}$ are not available in practice, unbiased estimates of $\{d_i^2\}$ from $\{r_i\}$ of (5) are needed. It is well known that if q is a Gaussian variable with mean μ and variance σ^2, the mean and variance of e^q, denoted by $\mathbb{E}\{e^q\}$ and $\mathrm{var}(e^q)$, are $e^{\mu+\sigma^2/2}$ and $(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$, respectively. Applying the transformation results, the mean and variance of $e^{-\frac{2r_i}{\alpha}}$ are calculated as:

$$\mathbb{E}\left\{e^{-\frac{2r_i}{\alpha}}\right\} = d_i^2 e^{\frac{2\lambda_i^2}{\alpha^2}} \tag{10}$$

and

$$\mathrm{var}\left(e^{-\frac{2r_i}{\alpha}}\right) = d_i^4 e^{\frac{4\lambda_i^2}{\alpha^2}} \left(e^{\frac{4\lambda_i^2}{\alpha^2}} - 1\right) \tag{11}$$

Based on (10), the unbiased estimates of d_i^2 are:

$$\widehat{d_i^2} = e^{-\frac{2r_i}{\alpha} - \frac{2\lambda_i^2}{\alpha^2}} \tag{12}$$

Hence, we can use the approximation form of $\mathbf{b}$, denoted by $\widehat{\mathbf{b}}$:

$$\widehat{\mathbf{b}} = \begin{bmatrix} e^{-\frac{2r_1}{\alpha} - \frac{2\lambda_1^2}{\alpha^2}} - k_1 \\ e^{-\frac{2r_2}{\alpha} - \frac{2\lambda_2^2}{\alpha^2}} - k_2 \\ \vdots \\ e^{-\frac{2r_N}{\alpha} - \frac{2\lambda_N^2}{\alpha^2}} - k_N \end{bmatrix} \tag{13}$$

to replace $\mathbf{b}$. It is obvious that $\mathbb{E}\{\widehat{\mathbf{b}}\} = \mathbf{A}\boldsymbol{\theta}$ which corresponds to the linear unbiased data model.

Additionally, with the use of (11) and (12), the variance of $\widehat{d_i^2}$ is computed as:

$$\mathrm{var}(\widehat{d_i^2}) = \mathrm{var}\left(e^{-\frac{2r_i}{\alpha}}\right)\left(e^{-\frac{2\lambda_i^2}{\alpha^2}}\right)^2 = d_i^4 \left(e^{\frac{4\lambda_i^2}{\alpha^2}} - 1\right) \tag{14}$$

According to (14), the noise covariance for $\widehat{\mathbf{b}}$, denoted by $\mathbf{C}_{\widehat{\mathbf{b}}}$, is a diagonal matrix of the form:

$$\mathbf{C}_{\widehat{\mathbf{b}}} = \mathrm{diag}\Big(d_1^4\left(e^{4\lambda_1^2/\alpha^2} - 1\right), d_2^4\left(e^{4\lambda_2^2/\alpha^2} - 1\right), \cdots, d_N^4\left(e^{4\lambda_N^2/\alpha^2} - 1\right)\Big) \tag{15}$$

Employing the inverse of $\mathbf{C}_{\widehat{\mathbf{b}}}$ as the weighting matrix, the BLUE estimate of $\boldsymbol{\theta}$, denoted by $\widehat{\boldsymbol{\theta}}$, is obtained by finding the minimum of the following cost function:

$$J(\widetilde{\boldsymbol{\theta}}) = \left(\mathbf{A}\widetilde{\boldsymbol{\theta}} - \widehat{\mathbf{b}}\right)^T \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} \left(\mathbf{A}\widetilde{\boldsymbol{\theta}} - \widehat{\mathbf{b}}\right) \tag{16}$$

where $\widetilde{\boldsymbol{\theta}}$ is the variable for $\boldsymbol{\theta}$. The solution for (16) is [13]:

$$\widehat{\boldsymbol{\theta}} = \left(\mathbf{A}^T \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} \widehat{\mathbf{b}} \tag{17}$$

As $\{d_i^2\}$ are not available, the practical form of $\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}$ is given as:

$$\mathbf{C}_{\widehat{\mathbf{b}}}^{-1} = \mathrm{diag}\left(\frac{e^{\frac{4r_1}{\alpha}}}{1 - e^{-\frac{4\lambda_1^2}{\alpha^2}}}, \frac{e^{\frac{4r_2}{\alpha}}}{1 - e^{-\frac{4\lambda_2^2}{\alpha^2}}}, \cdots, \frac{e^{\frac{4r_N}{\alpha}}}{1 - e^{-\frac{4\lambda_N^2}{\alpha^2}}}\right) \tag{18}$$

which is obtained by substituting $\{d_i^2\}$ with $\{\widehat{d_i^2}\}$ in (15). The BLUE position estimate, denoted by $\widehat{\mathbf{x}}_1$, is simply extracted from the first and second entries of $\widehat{\theta}$, that is,

$$\widehat{\mathbf{x}}_1 = [[\widehat{\theta}]_1 \; [\widehat{\theta}]_2]^T \tag{19}$$

According to BLUE, the covariance matrix for $\widehat{\theta}$, denoted by $\mathbf{C}_{\widehat{\theta}}$, is [13]:

$$\mathbf{C}_{\widehat{\theta}} = \left(\mathbf{A}^T \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} \mathbf{A}\right)^{-1} \tag{20}$$

The variances for the estimates of x and y are thus given by the $(1,1)$ and $(2,2)$ entries of $\mathbf{C}_{\widehat{\theta}}$, respectively, that is, the mean square position error (MSPE) for $\widehat{\mathbf{x}}_1$, denoted by MSPE$(\widehat{\mathbf{x}}_1)$, is

$$\text{MSPE}(\widehat{\mathbf{x}}_1) = [\mathbf{C}_{\widehat{\theta}}]_{1,1} + [\mathbf{C}_{\widehat{\theta}}]_{2,2}. \tag{21}$$

As the relationship of $R = x^2 + y^2$ is not exploited in (17), the BLUE positioning accuracy cannot attain the CRLB.

2.3 Comparison with Existing BLUE-LLS Algorithm

The existing BLUE-LLS algorithm [11, 12] is based on (3) where the estimates of $(d_i^2 - d_1^2)$ are exploited. The matrix form of (3) is

$$\mathbf{G}\mathbf{x} = \mathbf{h} \tag{22}$$

where

$$\mathbf{G} = -2 \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ \vdots & \vdots \\ x_N - x_1 & y_N - y_1 \end{bmatrix} \tag{23}$$

and

$$\mathbf{h} = \begin{bmatrix} d_2^2 - d_1^2 - k_2 + k_1 \\ d_3^2 - d_1^2 - k_3 + k_1 \\ \vdots \\ d_N^2 - d_1^2 - k_N + k_1 \end{bmatrix} \tag{24}$$

Similar with the proposed BLUE-LLS algorithm, the unbiased approximation of $\mathbf{h}$, denoted by $\widehat{\mathbf{h}}$, is:

$$\widehat{\mathbf{h}} = \begin{bmatrix} e^{-\frac{2r_2}{\alpha} - \frac{2\lambda_2^2}{\alpha^2}} - e^{-\frac{2r_1}{\alpha} - \frac{2\lambda_1^2}{\alpha^2}} - k_2 + k_1 \\ e^{-\frac{2r_3}{\alpha} - \frac{2\lambda_3^2}{\alpha^2}} - e^{-\frac{2r_1}{\alpha} - \frac{2\lambda_1^2}{\alpha^2}} - k_3 + k_1 \\ \vdots \\ e^{-\frac{2r_N}{\alpha} - \frac{2\lambda_N^2}{\alpha^2}} - e^{-\frac{2r_1}{\alpha} - \frac{2\lambda_1^2}{\alpha^2}} - k_N + k_1 \end{bmatrix} \tag{25}$$

and the covariance matrix of $\widehat{\mathbf{h}}$, denoted by $\mathbf{C}_{\widehat{\mathbf{h}}}$, is

$$\mathbf{C}_{\widehat{\mathbf{h}}} = \text{diag}\left(d_2^4\left(e^{\frac{4\lambda_2^2}{\alpha^2}} - 1\right), d_3^4\left(e^{\frac{4\lambda_3^2}{\alpha^2}} - 1\right), \cdots, d_N^4\left(e^{\frac{4\lambda_N^2}{\alpha^2}} - 1\right)\right) + d_1^4\left(e^{\frac{4\lambda_1^2}{\alpha^2}} - 1\right)\mathbf{1}_{N-1}\mathbf{1}_{N-1}^T \tag{26}$$

where $\mathbf{1}_N$ denotes the $N \times 1$ vector with all elements 1. Similarly, the practical form of $\mathbf{C}_{\widehat{\mathbf{h}}}$ is constructed by using $e^{-2r_i/\alpha - 2\lambda_i^2/\alpha^2}$ to approximate d_i^2 according to (12).

The estimated $\mathbf{x}$ based on (22), denoted by $\widehat{\mathbf{x}}_2$, is

$$\widehat{\mathbf{x}}_2 = \left(\mathbf{G}^T \mathbf{C}_{\widehat{\mathbf{h}}}^{-1} \mathbf{G}\right)^{-1} \mathbf{G}^T \mathbf{C}_{\widehat{\mathbf{h}}}^{-1} \widehat{\mathbf{h}} \tag{27}$$

Furthermore, the covariance matrix for $\widehat{\mathbf{x}}_2$, denoted by $\mathbf{C}_{\widehat{\mathbf{x}}_2}$, is

$$\mathbf{C}_{\widehat{\mathbf{x}}_2} = \left(\mathbf{G}^T \mathbf{C}_{\widehat{\mathbf{h}}}^{-1} \mathbf{G}\right)^{-1} \tag{28}$$

Thus, the MSPE for $\widehat{\mathbf{x}}_2$, denoted by MSPE$(\widehat{\mathbf{x}}_2)$, is

$$\text{MSPE}(\widehat{\mathbf{x}}_2) = \text{Tr}(\mathbf{C}_{\widehat{\mathbf{x}}_2}). \tag{29}$$

where Tr is the trace operator.

2.4 Extension to Unknown Path-loss Factor

When α is not known *a priori*, we propose to start with an estimate $\hat{\alpha} \in [1,5]$ to compute $\widehat{\mathbf{x}}_1$ of (19) or $\widehat{\mathbf{x}}_2$ of (27). We then employ the position estimate to construct the distance estimates $\{\hat{d}_i\}$ according to (1). A more accurate estimate of α is then obtained using weighted least squares (WLS) as:

$$\hat{\alpha} = \arg\min_{\tilde{\alpha}} \sum_{i=1}^{N} \frac{\left[r_i + \tilde{\alpha}\ln(\hat{d}_i)\right]^2}{\lambda_i^2} = -\frac{\sum_{i=1}^{N} r_i \ln(\hat{d}_i)/\lambda_i^2}{\sum_{i=1}^{N}\left[\ln(\hat{d}_i)\right]^2/\lambda_i^2} \tag{30}$$

We repeat the updates of the position estimate and (30) in an alternate manner for a few iterations until parameter convergence.

3. PERFORMANCE ANALYSIS

The relationships between the proposed BLUE-LLS method and the existing one are discussed in this section.

3.1 Performance Estimation

In order to analyze the relationship between (20) and (28), we first define two matrix [14], namely, $\mathbf{L}^T = [\mathbf{I}_2 \; \mathbf{0}_{2\times1}]$ and $\mathbf{P} = [-\mathbf{1}_{N-1} \; \mathbf{I}_{N-1}]$, here $\mathbf{I}_2$ is the 2×2 identical matrix and $\mathbf{0}_{2\times1}$ is the 2×1 zero matrix. We have:

$$\begin{aligned} \mathbf{G} &= \mathbf{PAL} \\ \mathbf{C}_{\widehat{\mathbf{h}}} &= \mathbf{P}\mathbf{C}_{\widehat{\mathbf{b}}}\mathbf{P}^T \end{aligned} \tag{31}$$

Thus, the covariance matrix for $\widehat{\mathbf{x}}_2$, which is given in (28) of the existing BLUE algorithm, can be expressed as

$$\mathbf{C}_{\widehat{\mathbf{x}}_2} = \left[\mathbf{L}^T\mathbf{A}^T\mathbf{P}^T(\mathbf{P}\mathbf{C}_{\widehat{\mathbf{h}}}\mathbf{P}^T)^{-1}\mathbf{PAL}\right]^{-1} \tag{32}$$

Furthermore, the covariance matrix for the estimates of x and y in the proposed BLUE-LLS algorithm can be expressed as $\mathbf{L}^T(\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{A})^{-1}\mathbf{L}$, which corresponds to the upper left 2×2 sub-matrix of $(\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{A})^{-1}$

$$(\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{A})^{-1} = \begin{bmatrix} \mathbf{L}^T\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{AL} & \mathbf{L}^T\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{1}_N \\ \mathbf{1}_N^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{AL} & \mathbf{1}_N^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{1}_N \end{bmatrix}^{-1} \tag{33}$$

With the use of the partitioned inversion formula and $\mathbf{1}_N^T \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} \mathbf{1}_N = \mathrm{Tr}(\mathbf{C}_{\widehat{\mathbf{b}}}^{-1})$, the upper left 2×2 sub-matrix of $\mathbf{C}_{\widehat{\theta}}$, $\mathbf{L}^T(\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{A})^{-1}\mathbf{L}$, can be computed as

$$\mathbf{L}^T(\mathbf{A}^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{A})^{-1}\mathbf{L} = \left[\mathbf{L}^T\mathbf{A}^T\left(\mathbf{C}_{\widehat{\mathbf{b}}}^{-1} - \frac{\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{1}_N\mathbf{1}_N^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}}{\mathrm{Tr}(\mathbf{C}_{\widehat{\mathbf{b}}}^{-1})}\right)\mathbf{A}\mathbf{L}\right]^{-1} \quad (34)$$

Using the property of $\mathbf{P}\mathbf{1}_N = \mathbf{0}_{(N-1)\times 1}$, we construct an idempotent matrix $\mathbf{S}\in\mathbb{R}^{N\times N}$, which has the form of

$$\mathbf{S} = \frac{\mathbf{C}_{\widehat{\mathbf{b}}}^{-1/2}\mathbf{1}_N\mathbf{1}_N^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1/2}}{\mathrm{Tr}(\mathbf{C}_{\widehat{\mathbf{b}}}^{-1})} + \mathbf{C}_{\widehat{\mathbf{b}}}^{1/2}\mathbf{P}^T(\mathbf{P}\mathbf{C}_{\widehat{\mathbf{b}}}\mathbf{P}^T)^{-1}\mathbf{P}\mathbf{C}_{\widehat{\mathbf{b}}}^{1/2} \quad (35)$$

Since $\mathrm{rank}(\mathbf{S}) = \mathrm{Tr}(\mathbf{S}) = N$, employing the full rank property of $\mathbf{S}$ as well as idempotent property of $\mathbf{S}(\mathbf{I}_N - \mathbf{S}) = \mathbf{0}_{N\times N}$ yield

$$\mathbf{S} = \mathbf{I}_N \quad (36)$$

Pre-multiplying and post-multiplying both sides of (35) by $\mathbf{C}_{\widehat{\mathbf{b}}}^{-1/2}$ with the use of (36), we obtain

$$\mathbf{P}^T(\mathbf{P}\mathbf{C}_{\widehat{\mathbf{h}}}\mathbf{P}^T)^{-1}\mathbf{P} = \mathbf{C}_{\widehat{\mathbf{b}}}^{-1} - \frac{\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}\mathbf{1}_N\mathbf{1}_N^T\mathbf{C}_{\widehat{\mathbf{b}}}^{-1}}{\mathrm{Tr}(\mathbf{C}_{\widehat{\mathbf{b}}}^{-1})} \quad (37)$$

From (32), (34) and (37), it is obvious that the estimation performance of our proposed BLUE-LLS algorithm is the same as that of the existing one.

3.2 Computational Complexity

Since the covariance matrix of approximation error in our proposed method $\mathbf{C}_{\widehat{\mathbf{b}}}$ is a diagonal matrix, the weighting matrix can be easily calculated with $O(N)$ multiplications, while the inverse of $\mathbf{C}_{\widehat{\mathbf{h}}}$ needs $O(N^2)$ multiplications [14]. Therefore, the complexity of our proposed method is $O(N)$ while that of the existing algorithm is $O(N^2)$, which means that our proposed method is more computationally efficient.

4. NUMERICAL EXAMPLES

MATLAB simulations are carried out to evaluate the performance of the proposed BLUE-LLS algorithm by comparing with the existing one [11, 12] as well as CRLB [15]. We consider a 2-D geometry of 8 receivers with known coordinates at $(0,0)$, $(0,100)$, $(100,0)$, $(100,100)$, $(25,25)$, $(75,25)$, $(75,75)$ and $(25,75)$ in a 100×100 area. The errors $\{n_i\}$ are zero-mean white Gaussian processes with identical variances of $\sigma_i^2 = \sigma^2$. All results are averages of 10000 independent runs.

Figure 1 plots the MSPEs of the proposed BLUE-LLS and existing one versus $\sigma^2 \in [0.01, 10]$ at known path-loss factor $\alpha = 3$. The theoretical variances of the position estimates of the LLS estimators, given by (21) or (29) are included as well as CRLB. The source is at $\mathbf{x} = [30\ 80]^T$. It is seen that the MSPEs of the proposed and existing methods agree with (21) and (29) when the disturbance is sufficiently small, namely, $\sigma^2 < 1$, and are around 1.2 times of the CRLB which indicate their suboptimality. It is worthy to point out that the MSPEs of these two methods are not numerically identical. Figure 2 shows their MSPEs versus $\alpha \in [1,5]$ at $\sigma^2 = 0.1$. We again see the suboptimality of the proposed and existing schemes. The localization accuracy increases with α, which is also indicated by the CRLB. Moreover, the equivalence between the proposed and existing algorithms is demonstrated. The first test is repeated by considering the source position is uniformly chosen within the square bounded by the last four receivers for each trial and the result is shown in Figures 3. We see that the findings are similar to those of Figure 1. Finally, The third test is repeated with unknown α and we start with $\hat{\alpha} = 5$ while the actual path-loss factor is $\alpha = 3$. The results of the proposed relaxation scheme for both algorithms with 5 iterations are plotted in Figure 4. It is seen that MSPE gaps between both estimators and CRLB are very small.

5. CONCLUSIONS

A best linear unbiased estimator approach for received signal strength (RSS)-based localization is proposed in this paper. Both performance analysis and simulation results show that our proposed method and the existing one have identical performance while the former is more computationally efficient, that is, the former is in order of the number of receivers N while the latter is in order N^2. Furthermore, a relaxation method is introduced to extend the LLS approach to solve the RSS-based positioning problem with unknown path-loss factor.

REFERENCES

[1] G. Carter, Ed., *Coherence and Time Delay Estimation: An Applied Tutorial for Research, Development, Test, and Evaluation Engineers*. New York: IEEE, 1993.

[2] Y. Huang and J.Benesty, Eds., *Audio Signal Processing for Next-Generation Multimedia Communication Systems*. Kluwer Academic Publishers, 2004.

[3] J. J. Caffery, *Wireless Location in CDMA Cellular Radio Systems*. Kluwer Academic Publishers, 2000.

[4] M. Ilyas and I. Mahgoub, *Handbook of Sensor Networks: Compact Wireless and Wired Sensing Systems*. London, U.K.: CRC Press, 2005.

[5] F. Hu and X. J. Cao, *Wireless Sensor Networks*. Boca Raton, FL : Auerbach Publications, 2010.

[6] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, R. L. Moses, and N. S. Correal, "Locating the nodes cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54–69, Jul. 2005.

[7] N. Patwari, A. Hero, M. Perkins, N. Correal, and R. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 51, pp. 2137–2148, Aug. 2003.

[8] A. Coluccia and F. Ricciato, "On ML estimation for automatic RSS-based indoor localization," in *Proc. IEEE International Symposium on Wireless Pervasive Computing*, Modena, Italy, May 2010, pp. 495–502.

[9] C. Meng, Z. Ding, and S. Dasgupta, "A semidefinite programming approach to source localization in wire-

less sensor networks," *IEEE Signal Processing Letters*, vol. 15, pp. 253–256, 2008.

[10] R. W. Ouyang, A. K. S. Wong, and C. T. Lea, "Received signal strength-based wireless localization via semidefinite programming: Noncooperative and cooperative schemes," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 3, pp. 1307–1318, Mar. 2010.

[11] P. Tarrio, A. M. Bernardos, J. A. Besada, and J. R. Casar, "A new positioning technique for RSS-Based localization based on a weighted least squares estimator," in *Proc. IEEE International Symposium on Wireless Communication Systems*, Reykjavik, Iceland, Oct. 2008, pp. 633–637.

[12] S. D. Chitte, S. Dasgupta, and Z. Ding, "Distance estimation from received signal strength under log-normal shadowing: bias and variance," *IEEE Signal Processing Letters*, vol. 16, no. 3, pp. 216–218, Mar. 2009.

[13] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[14] F. K. W. Chan, H. C. So, J. Zheng, and K. W. K. Lui, "Best linear unbiased estimator approach for time-of-arrival based localization," *IET - Signal Processing*, vol. 2, no. 2, pp. 156–162, Jun. 2008.

[15] S. D. Chitte, S. Dasgupta, and Z. Ding, "Source localization from received signal strength under log-normal shadowing: bias and variance," in *Proc. IEEE International Congress on Image and Signal Processing*, Tianjin, China, Oct. 2009, pp. 17–19.

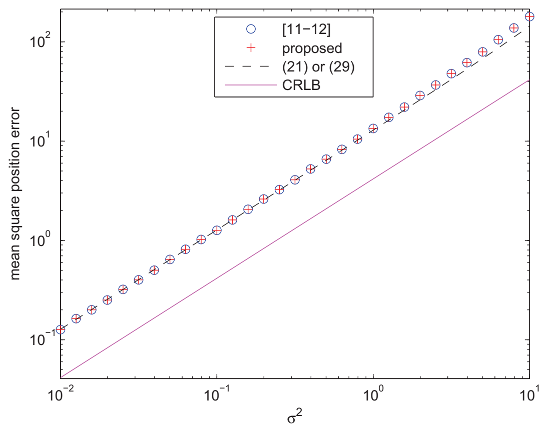

Figure 1: Mean square position error versus σ^2 at $\mathbf{x} = [30\ 80]^T$ with $\alpha = 3$

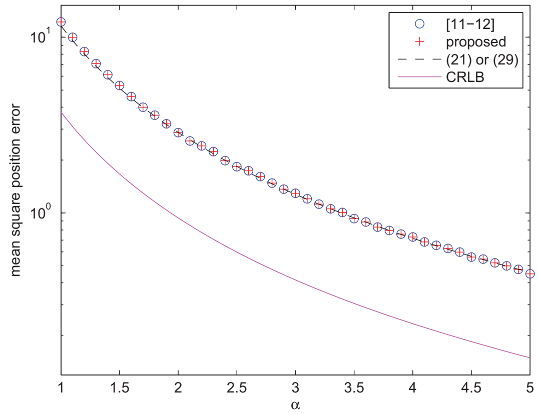

Figure 2: Mean square position error versus α at $\mathbf{x} = [30\ 80]^T$ with $\sigma^2 = 0.1$

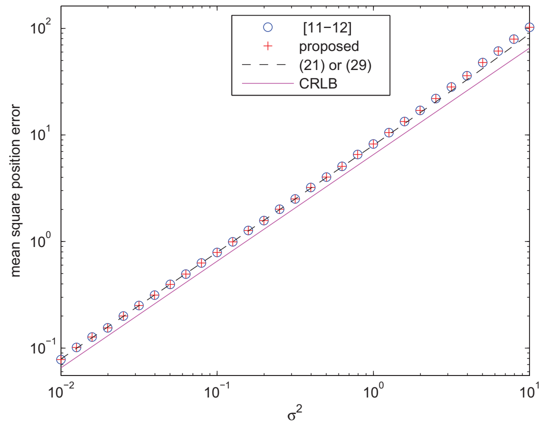

Figure 3: Mean square position error versus σ^2 with random $\mathbf{x}$ and $\alpha = 3$

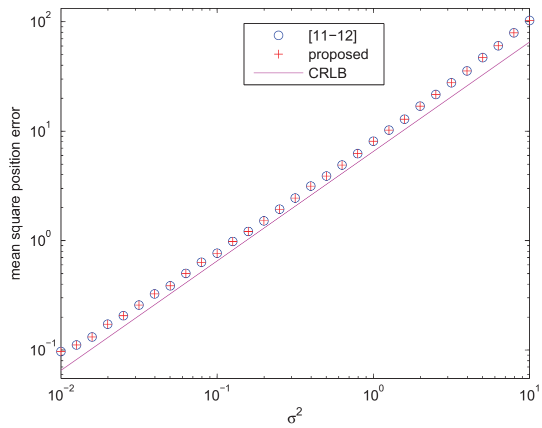

Figure 4: Mean square position error versus σ^2 with random $\mathbf{x}$ and unknown α