BET_EF: a probabilistic tool for long- and short-term eruption forecasting

Warner Marzocchi, Laura Sandri, Jacopo Selva

March 30, 2007

Istituto Nazionale di Geofisica e Vulcanologia
Sezione di Bologna
via D. Creti 12, 40128 Bologna, Italy

Corresponding author: Warner Marzocchi, e-mail: marzocchi@bo.ingv.it

Submitted to: *Bull. Volcanol.*

Abstract

The main purpose of this paper is to introduce a Bayesian event tree model for eruption forecasting (BET_EF). The model represents a flexible tool to provide probabilities of any specific event at which we are interested in, by merging all the relevant available information, such as theoretical models, a priori beliefs, monitoring measures, and any kind of past data. BET_EF is based on a Bayesian procedure and it relies on the fuzzy approach to manage monitoring data. The method deals with short- and long-term forecasting, therefore it can be useful in many practical aspects, as land use planning, and during volcanic emergencies. Finally, we provide the description of a free software package that provides a graphically supported computation of short- to long-term eruption forecasting, and a tutorial application to the recent MESIMEX exercise at Vesuvius.

Keywords: Eruption forecasting, long- and short-term volcanic hazard, Bayesian Inference, Event Tree, fuzzy sets, software, MESIMEX.

1 Introduction

One of the major goals of modern volcanology is to set up a sound risk-based decision making in land use planning and emergency management. One of the basic scientific ingredients to achieve these goals is a reliable and quantitative long- and short-term eruption forecasting (EF hereinafter).

Despite some recent researches on short-term forecasting (from hours to few days) are based on a deterministic approach (e.g., Voight and Cornelius, 1991; Kilburn, 2003; see also Hill et al., 2001), the presence of complex and different precursory patterns for distinct eruptions, as well as the exigency to consider the possibility that a precursory pattern not necessarily leads to an eruption, suggest that a probabilistic approach could be more efficient in EF (e.g., Sparks, 2003). At this purpose, it is worth remarking that the probabilistic approach is not incompatible with the deterministic approach, because the former can include deterministic rules as limit cases, i.e., when the probability tends to one. In other words, the probabilistic approach is certainly more general, and it has also the merit to be applicable at different time scales. For instance, during a quiet period of the volcano, EF is estimated by accounting for the past activity of the volcano (long-term EF; see, e.g., Marzocchi and Zaccarelli, 2006; Jaquet et al., 2006). Conversely, during an unrest, the method allows mid- to short-term EF to be estimated by considering different patterns of pre-eruptive phenomena (e.g., Newhall and Hoblitt, 2002; Aspinall and Woo, 1994; Aspinall et al., 2003; and Marzocchi et al., 2004).

The concept of short/long-term EF deserves further explanations. The terms "short" and "long" are referred to the expected characteristic time in which the process shows significant variations; in brief, during an unrest the time variations occur on time scales much shorter than the changes expected during a quiet phase of the volcano. On the other hand, these terms are not linked to the forecasting time window (for instance, we can use a forecasting time window of one day, both for short- and long-term EF). The distinction between these two time scales, besides to reflect a difference in the physical state of the volcano (quiescence and unrest), is also important in a practical perspective; in fact, for example, the long-term EF is a primary component of long-term (years to decades) volcanic hazard assessment that allows different kinds of hazards (volcanic, seismic, industrial, floods, etc.) in the same area to be compared; this comparison is very useful for cost/benefit analysis of risk mitigation actions, and for appropriate land-use planning and location of settlements. In contrast, monitoring on mid- to short-time scales assists with actions for immediate vulnerability (and risk) reduction, for instance through evacuation of people from danger areas (Fournier d'Albe, 1979).

As a general thought, we can say that a realistic EF is usually entangled by the scarce number of data and the relatively poor knowledge of the physical pre-eruptive processes. This makes any EF hypothesis/model hardly testable also in a backward analysis, overall for explosive volcanoes. On the other hand, the extreme risk posed by many volcanoes pushes us to be pragmatic and attempt to solve the problem from an "engineering" point of view: by this, we mean that the devastating potential of volcanoes close to urbanized areas forces the scientific community to address the issue as precisely as possible. This is best done by treating scientific uncertainty in a fully structured manner and, in this respect, Bayesian statistics is a suitable framework for producing an eruption forecasting (and volcanic hazard/risk assessments) in a rational, probabilistic form (e.g., UNESCO, 1972; Gelman et al., 1995). In order to illustrate the general philosophy of the approach, we quote Toffler (1990) that said "it is better to have a general and incomplete model, subject to revision and correction, than to have no model at all". We add that the model has to be necessarily "accurate", i.e., without significant biases, because a biased estimation would be useless in practice. On the other hand, the model has to be as "precise" as possible (i.e., the relative error has to be as small as possible), but "precision" has not to be achieved reducing "accuracy". In other words, the model may have a low "precision" that would reflect our scarce knowledge of some physical processes involved.

Here we address the EF issue by implementing a general quantitative model for volcanic hazard assessment based on the Bayesian Event Tree (BET hereinafter). BET represents a development of the method proposed by Marzocchi et al. (2004) based on the event tree (Newhall and Hoblitt, 2002) scheme. Specifically, BET follows the philosophy of approach described by Marzocchi et al. (2004), and it proposes some significant novelties like the introduction of the fuzzy approach, the inclusion of a node for the vent location, and an improvement of the statistics formalism. It also contains few minor conceptual changes and implementations. Finally, we put forward a scheme of a software package (BET_EF: Bayesian Event Tree for Eruption Forecasting) to calculate the probability of eruption for a generic volcano. It is worth noting that our procedure has some overlapping with the Bayesian Belief Network (BBN) adopted by Aspinall et al. (2003), and, in general, they share the same philosophy. As a matter of fact, both methods deal with multiple parameters monitoring, and with uncertainties. The main difference is that the Bayesian approach of BET allows aleatory and epistemic uncertainties to be directly accounted for in a structured and explicit fashion.

A detailed discussion on the approach adopted here can be found in Marzocchi et al. (2004; 2006a), Newhall and Hoblitt (2002), and Gelman et al. (1995). Here, we report the main features of BET, that can be summarized in four general points:

- BET is a probabilistic model that merges all kinds of volcanological information, coming from theoretical/empirical models, geological and historical data, and monitoring observations, to obtain probability of any relevant volcanic event. Such probabilities represent an homogeneous and quantitative synthesis of the present knowledge about the volcano.

- BET has the most important characteristic for a model to be "scientific", that is, it gives the possibility to "falsify" the results provided (Popper, 1959); this important feature gives also an opportunity to make scientifically testable any scientific belief/hypothesis.

- In general, BET does not rule out any possibility, but it shapes the probability distribution of the event considered around the most likely outcome accounting for all the information reported above. This is accomplished by dealing with aleatory and epistemic uncertainties in a proper way (see Woo, 1999; Marzocchi et al., 2004).

- BET estimates short- and long-term EF, depending on the present state of the volcano, providing a useful tool in several contexts: i) to compare different types of risks, ii) to carry out cost/benefit analysis of risk mitigation actions, iii) to indicate appropriate land-use planning and location of settlements, and iv) to suggest immediate risk reduction actions, such as the evacuation of people from danger areas (Fournier d'Albe, 1979).

In order to make the paper readable for a vast audience, we report almost all the technical details in two Appendixes that are published as Electronic Supplementary Material. The paper contains only the general features and philosophy of the method, the description of a free software package (BET_EF) that provides a graphically supported computation of short- to long-term eruption forecasting, and finally a tutorial application to the recent MESIMEX exercise at Vesuvius.

2 The Bayesian Event Tree scheme for the Eruption Forecasting

In a nutshell, BET is a probabilistic model to calculate the probability of any possible volcano-related event, by merging all of the information available, such as theoretical models, a priori beliefs, monitoring observations, and every kind of past data. BET is based on the concept of event tree. The event tree is a branching graph representation of events in which individual branches are alternative steps from a general prior event, state, or condition, and which evolve into increasingly specific subsequent events. Eventually the branches terminate in final outcomes representing specific hazardous phenomena that may turn out in the future. In this way, an event tree attempts to graphically display all relevant possible outcomes of volcanic unrest in progressively higher levels of detail. The points on the graph where new branches are created are referred to as *nodes* (Newhall and Hoblitt, 2002; Marzocchi et al., 2004; 2006a).

Since this definition is mainly driven by the practical utility of the event tree, the branches at each node point to the whole set of different possible events, regardless of their probabilistic features. In other words, the events at each node need not be mutually exclusive. Here, since we are interested in EF, we consider only the first nodes of a generic event tree (see upper part of figure 1), where the events at each node are mutually exclusive and exhaustive. In this case, definitions are constructed so that no sequence of events can proceed along more than a single branch of the event tree. This property makes the event tree comparable to the event trees usually reported in statistical literature (Smith, 1988).

For each possible path we have the following nodes:

- Node 1: there is an unrest, or not, in the time interval $(t_0, t_0 + \tau]$, where t_0 is the present time, and τ is the time window considered.

- Node 2: the unrest is due to magma, or to other causes (e.g., hydrothermal, tectonics, etc...), given unrest is detected.
- Node 3: the magma will reach the surface (i.e., it will erupt), or not, in the time interval $(t_0, t_0 + \tau]$, provided that the unrest has a magmatic origin.
- Node 4: the eruption will occur in a specific location, provided that there is an eruption.
- Node 5: the eruption will be of a certain "size/type" (e.g., VEI), provided that there is an eruption in a certain location.

Hereinafter we will refer to "node k" to indicate one of the possible state, event, or condition of the kth step of the event tree. At each one of these nodes we attribute a probability function. Let us define θ^E as the probability of the conditional event E (note that each event reported above is conditioned to the occurrence of other events at previous nodes); therefore, for each one of the five nodes we define $[\theta_1^{(\text{unrest})}]$, $[\theta_2^{(\text{magma})}]$, $[\theta_3^{(\text{erup})}]$, $[\theta_4^{(\text{loc})}]$, $[\theta_5^{(\text{size})}]$, where the square brackets stand for a generic "probability density function (pdf)". In other words, BET considers the conditional probability at each node as a random variable, therefore it estimates each probability through a pdf, not as a single value. As described in the following, the use of these pdfs (characteristic of the Bayesian inference) allows BET to estimate aleatory and epistemic uncertainties. Since the first three nodes have only two possible states that are mutually exclusive and exhaustive (for instance, unrest or not), we set, for the sake of simplicity, $[\theta_1^{(\text{unrest})}] = [\theta_1]$, $[\theta_2^{(\text{magma})}] = [\theta_2]$, and $[\theta_3^{(\text{erup})}] = [\theta_3]$.

Given all the pdfs at each node, BET combines them in order to obtain the absolute probability of each event at which we are interested in. For instance, the pdf of the probability to have an eruption of type k in a the time interval $(t_0, t_0 + \tau]$ at the j-th vent location, i.e, $[\Phi_1]$, is

$$[\Phi_1] = [\theta_1][\theta_2][\theta_3][\theta_4^{(\text{j})}][\theta_5^{(k)}] \tag{1}$$

In other words, $[\Phi_1]$ is a quantitative measure of EF. For a visualization of the procedure, see the upper two blocks of figure 1. The probability to have the same eruption at any location, i.e, $[\Phi_2]$, is

$$[\Phi_2] = \sum_{j=1}^{M} [\theta_1][\theta_2][\theta_3][\theta_4^{(\text{j})}][\theta_5^{(k)}] \tag{2}$$

where M is the number of possible vent location considered. Note that we assume that the distribution of possible vents is a set of completely mutually exclusive events. The functional form of $[\Phi]$ is not determined analytically, but through a Monte Carlo simulation. In practice, we sample 1000 times each pdf, and we perform the calculation by using each sample. Therefore, we obtain 1000 values of $[\Phi]$ that are used to determine the functional form numerically. In this way, we propagate in a proper way both aleatory and epistemic uncertainties at all nodes, and we estimate best guess (i.e., the average) and errors (the standard deviation) of the absolute probability of any possible event.

To summarize, BET provides quantitative estimations of EF through the evaluation of the pdfs of the five nodes, by accounting for any kind of available information. In Appendixes A and B we report all the technical details for estimating the pdf at each node. In figure 1, we report a scheme that describes the main logical steps of BET, and we indicate the parts of this paper where a detailed description of each step is reported.

3 A Bayesian Event Tree algorithm for Eruption Forecasting (BET_EF)

In this section, we describe the main features of BET_EF, a Windows software package that is available for free under request to the authors. The package gives the opportunity of computing short- to long-term probabilities for eruption forecasting with a graphical support (see figure 2). The user may directly check the flexibility of the event tree scheme and select the event at which is interested in, as well as visualize probabilities and relative aleatory and epistemic uncertainties, computed from the user defined information. With BET_EF is also provided an .HTML manual (BET_MANUAL) explaining step by step how to use BET_EF.

In practice, BET_EF implements the method described above, where the forecasting time interval is chosen by the user. Most of the volcanological information is provided by the user by means of the selection of monitoring parameters and relative thresholds, theoretical beliefs, models, and expert opinion, and past data. BET_EF allows the user to merge all of the provided information following the BET approach described above and in the Appendixes. To do that, altogether with BET_EF, it is also provided another package, called BET_UPGRADE, which permits to upload and/or upgrade all of the information, and then to run BET_EF for any given volcano.

BET_EF allows the user to easily compute and visualize:

- the Event Tree with all nodes and branches. The branch and/or path in which the user is interested can be directly selected in a graphical way
- either absolute or conditional probability distributions of the selected path/node
- the probability density functions (pdfs) and the cumulative distribution function (cdfs), as well as the parameters that describe the distribution, i.e., the averages, medians, and 10th and 90th percentiles, of the selected path/branch
- the conditional probability map for vent locations

Monitoring measures may be directly input on BET_EF, so that it may be used either in real-time during unrest episodes (e.g., useful for crisis management), or for long-term estimates during quite periods (e.g., useful for land use planning). The code also allows snapshots of results in "bitmap" (.bmp) format. Maps can be saved both in "bitmap" format and in "Google Earth" format (.kml). More details of the software package can be found in the manual.

4 A tutorial example of the application of BET_EF: the MESIMEX exercise

MESIMEX (Major Emergency Simulation Exercise) is a simulation carried out between the 17th and the 23rd of October 2006 by the Regione Campania (administrative institution of the region that includes the Neapolitan area) and the Dipartimento Nazionale della Protezione Civile (Italian Civil Protection), with the goal of improving the coordination among national institutions, the organization of civil protection operative actions, and the preparedness of the civil society in case of eruption at Vesuvius.

The core of the scientific part of the exercise was the definition of a realistic pre-eruptive scenario for Vesuvius, simulated by a pool of experts. The activity included several typical phenomena accompanying volcanic crises, such as seismic activity, deformations, gravity changes, etc... The second part of MESIMEX was devoted to exercise the evacuation. The simulated activity of the volcano was spread to the scientific community through a series of bulletins (one or two per day) distributed through a mailing list as well as a dedicated blog on Internet.

Here, we present the application of BET_EF during the MESIMEX exercise, as a tutorial example of the application of the code. We anticipate that the good performance of the method is not a "proof" that the technique will work well in a future crisis at Vesuvius, but only that it represents well and quantitatively the average opinion of experts about what can happen before an eruption.

In this application, we set $\tau = 1$ month; the choice of a 1 month of time interval is a practical one, approximating the longest lead times of precursors (see discussion Marzocchi et al., 2004). For node 5, we consider three sizes: VEI 3, VEI 4, and VEI 5+ (see Marzocchi et al., 2004). All of the volcanological input of BET_EF and the parameters and thresholds used for each node are reported in Tables 1-4. Tables 2-4 contain also a synthesis of the main content of the MESIMEX bulletins. The thresholds and parameters were defined before MESIMEX (see also Marzocchi et al., 2004; Marzocchi et al., 2006b). At each bulletin, we loaded the monitoring simulated measures (see input values in Tables 2-4) in BET_EF, and we calculated the probability of unrest, of presence of magma, and of eruption. To avoid any possible overfitting or any feedback that could lead to a bias on the results, we did not interact neither with the pool of experts simulating the pre-eruptive activity, nor with the other research groups operatively involved in MESIMEX. Our probability estimates have been exclusively sent by e-mail to the director of the INGV Osservatorio Vesuviano that coordinated all the scientific work.

The conditional probability of sizes given an eruption does not change through time (i.e., it does not depend on monitoring measures; see Appendix B and figure 3A). On the opposite, the probability of nodes 1, 2, 3, and 4 depends on the monitoring measures in input so that they change at each bulletin. In figure 3B, the time evolution of the absolute probabilities of unrest, presence of magma, and eruption are reported. The probability of unrest (node 1) increases quickly and it is found close to 1 after the second bulletin (18 Oct., 07:00). The probability of magmatic unrest (node 2) and of eruption (node

3) increases more slowly; only after the fourth bulletin (19 Oct., 18:00), the average probability of eruption has a major step and becomes about 62%. The evacuation was called by Civil Protection Department on 21 Oct., when the probability of eruption per month was about 83%.

Finally, we give a look at the probability of vent opening (see, i.e., bottom part of figures 2 and figure 3C). BET_EF allows using monitoring measures also to forecast the position of the vent (node 4). The conditional probability of vent opening has been upgraded through time, using the localization of earthquakes (both VT and LF). The best guess values are reported in the right table of the bottom panel in figure 2, for all locations, on 20 Oct (Bulletin 5). The most likely location is always the central vent (loc #1), but there is a non-negligible probability also at locations #3 and #4 (SE and NE sectors respectively), that reaches about 0.19 for the second bulletin. This effect is mainly due to 2 swarms of events located in the border between these two lateral sectors.

5 Final Remarks on BET

In this section, we report some central features of BET, trying to provide answers to possible "frequently asked questions".

What is BET? BET is a tool to calculate and to visualize probabilities related to eruption forecasting/hazard assessment. It is based on a fully probabilistic Bayesian scheme. It also introduces a fuzzy approach to manage monitoring measurements. BET is a scientific tool because it provides probabilities that can be used to test any hypothesis/models contained in BET.

What is not BET? BET is not a black box; all quantitative rules and assumptions adopted are described in detail in this paper. BET is not a magic box; the reliability of the results is strongly related to the reliability of the volcanological information provided by the user.

What is the practical usefulness of BET? BET "dynamically" assesses long-term (useful for land use planning, and for comparing the volcanic hazard with other different kinds of hazard), and short-term (useful during emergency to help managing short-term actions aimed to reduce risk) eruption forecasting.

What is BET input? BET input consists of all of the available information such as models, state of the volcano, geologic/volcanologic/historic data, present and past monitoring observations, expert opinion, and theoretical beliefs.

Which is the reliability of BET output? BET takes properly and explicitly into account the epistemic (data- or knowledge-limited) and aleatory (stochastic) uncertainties. This guarantees reliable outputs, given reliable input information.

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BET_EF input

Node	Prior distribution Models/beliefs	Likelihood Past data	Monitoring parameters	Past monitored events
NODE 1	No info uniform dist.	$y_1 = 0$ $n_1 = 384$ [months]	7 see Tab. 2	
NODE 2	No info uniform dist.	No data	5 see Tab. 3	0
NODE 3	No info uniform dist.	No data	8 see Tab. 4	0
NODE 4: 5 locs according to topography see Fig. 2	Central volcano $\Theta_4^{(1)\{\overline{\mathcal{M}}\}} = 0.99$ $\Theta_4^{(2)\{\overline{\mathcal{M}}\}} = 0.0025$ $\Theta_4^{(3)\{\overline{\mathcal{M}}\}} = 0.0025$ $\Theta_4^{(4)\{\overline{\mathcal{M}}\}} = 0.0025$ $\Theta_4^{(5)\{\overline{\mathcal{M}}\}} = 0.0025$ $\Lambda_4^{\{\overline{\mathcal{M}}\}} = 50$	13 eruptions $y_4^1 = 13$ $y_4^2 = 0$ $y_4^3 = 0$ $y_4^4 = 0$ $y_4^5 = 0$		
NODE 5: 3 groups (VEI 3, VEI 4, VEI 5+)	Power-Law dist. see Marzocchi et al., 2004 $\Theta_5^{(1)\{\overline{\mathcal{M}}\}} = 0.83$ $\Theta_5^{(2)\{\overline{\mathcal{M}}\}} = 0.14$ $\Theta_5^{(3)\{\overline{\mathcal{M}}\}} = 0.03$ $\Lambda_5^{\{\overline{\mathcal{M}}\}} = 1$	7 eruptions $y_5^1 = 4$ $y_5^2 = 2$ $y_5^3 = 1$		

Table 1: Summary of the volcanological information in input of BET_EF for the MESIMEX application; more details on the text, and on Marzocchi et al. (2004, 2006a). Next tables contain information about monitoring parameters and thresholds.

Node 1

Parameter	Order Rel., Thresholds & Units	Bulletin 1 Oct 17 9 am	Bulletin 2 Oct 18 7 am	Bulletin 3 Oct 19 9 am	Bulletin 4 Oct 19 6 PM	Bulletin 5 Oct 20 3 PM	Bulletin 6 Oct 21 5 PM
n_e monthly number of seismic events with $M_d \geq 1.9$ at OVO station	$>$ 23 ; 150 month^{-1}	10	38	61	104	183	258
M_d monthly largest duration magnitude of seismic events at OVO station	$>$ 3.4 ; 4.3 month^{-1} month^{-1}	2.8	4.2	4.2	4.2	4.2	4.2
n_{LF} monthly number of LF events deeper than 1 Km	$>$ 1 ; 3 month^{-1}	0	0	2	26	61	131
Π_{SO_2} presence of significant SO_2 (1=yes)	$=$ 1	0	0	1	1	1	1
Φ_{CO_2} daily CO_2 emission rate	$>$ 5 ; 30 Kg m^{-2} d^{-1}	10	20	20	30	300	400
$\dot{\varepsilon}$ strain rate (inflation)	$>$ 0 ; 0 d^{-1}	0	0	0	5 10^{-5}	5 10^{-5}	2 10^{-4}
T temperature of the fumaroles in the crater	$>$ 98 ; 105 C	95	95	100	110	110	110

Table 2: Monitoring parameters (column 1), order relationship, lower and upper thresholds and relative units (column 2), and measured values as in MESIMEX real time bulletins (columns 3-8), relative to node 1 for Vesuvius.

Node 2

Parameter	Order Rel., Thresholds & Units	Bulletin 1 Oct 17 9 am	Bulletin 2 Oct 18 7 am	Bulletin 3 Oct 19 9 am	Bulletin 4 Oct 19 6 PM	Bulletin 5 Oct 20 3 PM	Bulletin 6 Oct 21 5 PM
Π_{SO_2} (as in Table 2)	$=$ 1	0	0	1	1	1	1
$\dot{\varepsilon}$ (as in Table 2)	$>$ 10^{-6} ; 10^{-5} d^{-1}	0	0	0	$5\ 10^{-5}$	$5\ 10^{-5}$	$2\ 10^{-4}$
$\overline{\nu}$ average spectral frequency of earthquakes	$<$ 2.5 ; 3.5 Hz	4	4	4	3.6	3.5	2.5
ξ_e ratio between average and dispersion of earthquake depths	$<$ 0.3 ; 0.4	0.4	0.4	0.4	0.4	0.4	0.3
T (as in Table 2)	$>$ 98 ; 105 C	95	95	100	110	110	110

Table 3: Monitoring parameters (column 1), order relationship, lower and upper thresholds and relative units (column 2), and measured values as in MESIMEX real time bulletins (columns 3-8), relative to node 2 for Vesuvius.

Node 3

Parameter	Order Rel., Thresholds & Units	Bulletin 1 Oct 17 9 am	Bulletin 2 Oct 18 7 am	Bulletin 3 Oct 19 9 am	Bulletin 4 Oct 19 6 PM	Bulletin 5 Oct 20 3 PM	Bulletin 6 Oct 21 5 PM
PE presence of phreatic explosions (1=yes)	$=$ 1	0	0	0	0	0	0
$\dot{\overline{\nu}}$ rate of change of $\overline{\nu}$ (see Table 3)	$<$ 0 ; 0 Hz d^{-1}	0	0	0	-0.4	-0.5	-1.0
ξ_e (as in Table 3)	$<$ 0.3 ; 0.4	0.4	0.4	0.4	0.4	0.4	0.3
$\ddot{E}$ acceleration of seismic energy release	$>$ 0 ; 0 J^2 d^{-2}	0	0	0	0	0	0
$\ddot{\varepsilon}$ acceleration of strain (inflation)	$>$ 0 ; 0 d^{-2}	0	0	0	0	0	0
ε cumulative strain (inflation) since beginning of unrest	$>$ 10^{-5} ; 10^{-4}	0	0	0	5 10^{-5}	10^{-4}	3 10^{-4}
$\dot{\rho}$ change of the ratios HCl/SO_2 and/or HF/SO_2 (1=yes)	$=$ 1	0	0	0	0	0	0
REV sudden reversal of at least one of the above parameters (1=yes)	$=$ 1	0	0	0	0	0	0

Table 4: Monitoring parameters (column 1), order relationship, lower and upper thresholds and relative units (column 2), and measured values as in MESIMEX real time bulletins (columns 3-8), relative to node 3 for Vesuvius.

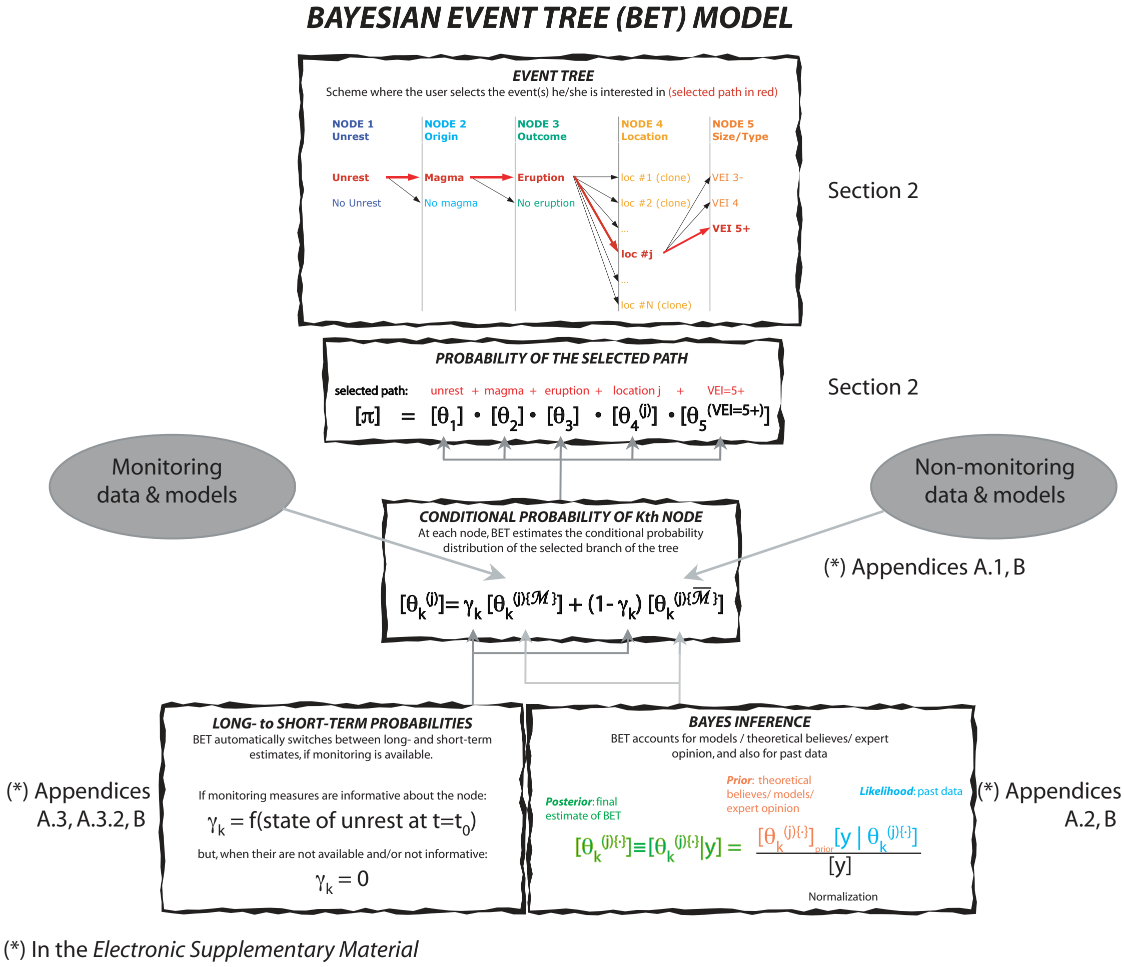

Figure 1: General scheme of BET, with references to the sections of the paper where a detailed description of each box is reported. From the top, it is reported i) the selection of a path within the event tree; ii) the computation of the probability of the path; iii) the computation of each conditional probability from all of the monitoring and non-monitoring information, iv) the computation of the weight of the monitoring part, and v) the Bayesian inference core of BET.

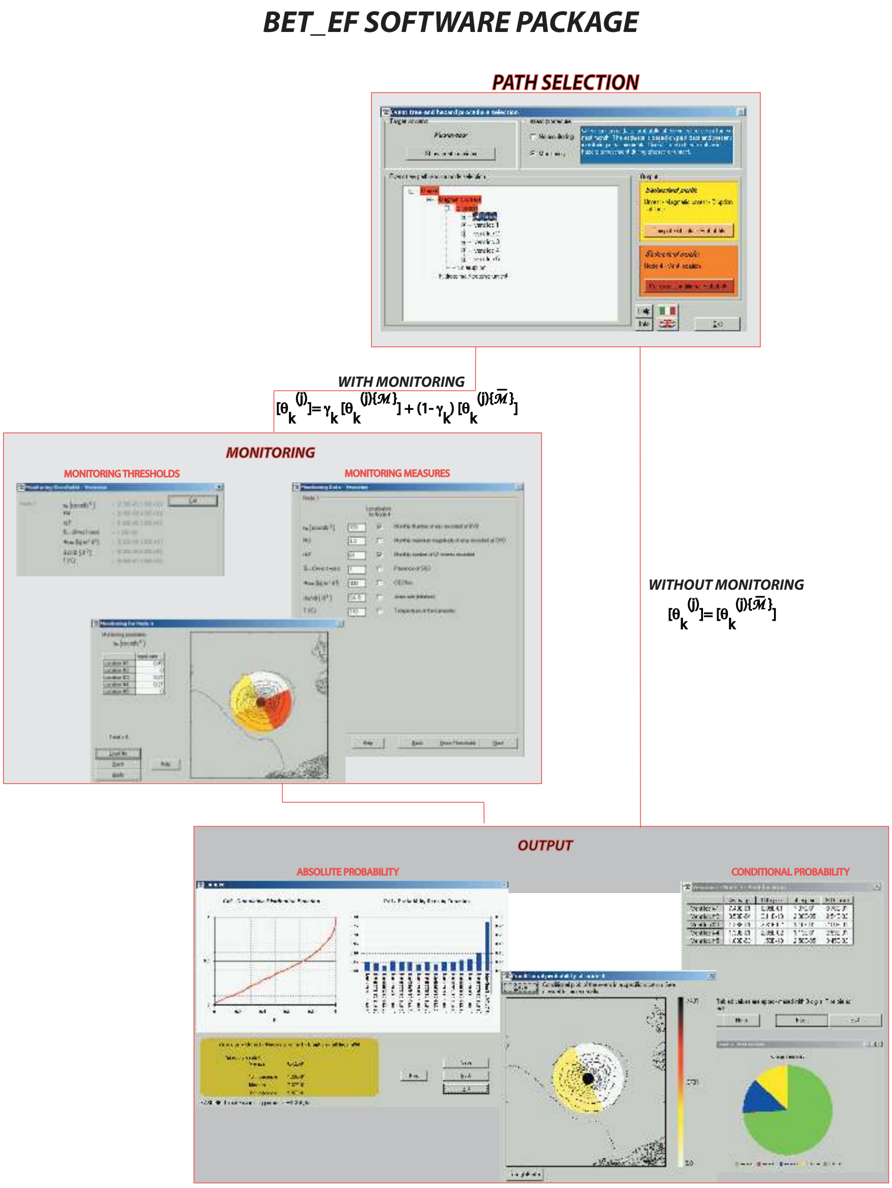

Figure 2: The scheme of the BET_EF software package. The user chooses the hazard procedure (with or without monitoring) and the path within the event tree. In the case of monitoring, the user inputs monitoring measures (node 1 to 3) and their localization (node 4). In output, BET_EF gives posterior pdfs of either conditional or absolute probability (depending on the choice of the user), uncertainties, and a map of vent opening conditional probability. The output displayed is relative to the Bulletin 5 of the MESIMEX exercise.

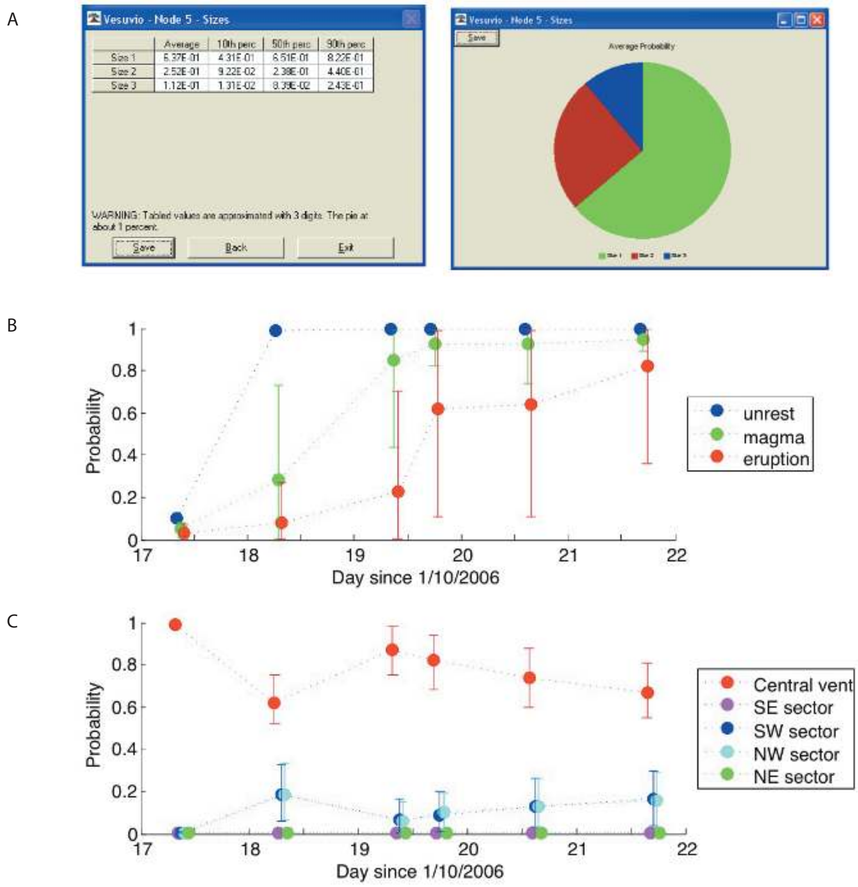

Figure 3: Probability estimations during MESIMEX. A - Conditional probabilities to have a VEI 3, 4, and 5+ eruption, given an eruption will occur (node 5). B - Time evolution of the probability of unrest (blue), magmatic unrest (green), and eruption (red). C - Conditional probability of vent opening (node 4) for different locations. Dots represent the averages, and bars are the intervals between 10th and 90th percentiles.