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Better Product Quality May Lead to Lower Product Price

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Abstract: This article analyzes the conditions under which better product quality implies higher or lower product price. In an optimal control framework, I make the following assumptions: The firm sets the dynamic pricing and product innovation policies; product innovation raises quality, which drives production cost, and consumers are sensitive to price and quality. I derive a rule of price-quality relationship that stresses the influence of quality on price through the effects of cost (positive), sales (negative), and markup (positive). This article shows that, while maximizing profit and despite a quality and cost increases, the firm may decrease product prices because of the possibility of generating more sales as a result of combining better quality with lower price. This sales effect solves the puzzle of a negative price-quality relationship. More generally, the sales effect mitigates the ability of price to convey information about quality.

Keywords: price-quality relationship, dynamic pricing, product quality, product innovation, quality-based cost, optimal control

1 Introduction

In modern manufacturing industries, such as electronic chips, automotive, or aircraft components, firms simultaneously set pricing and product innovation policies. Innovation (here intended as product innovation) enhances quality (here understood as product quality). For example, a more powerful computer, a faster car, and a more stable airplane represent quality enhancements due to innovation. Better quality increases consumer interest (and willingness to pay) as well as firm's (unit production) cost. Better quality therefore should imply a higher price. As such, numerous theoretical studies confirm a positive price-quality relationship (Scitovsky 1944; Mussa and Rosen 1978; Stiglitz 1987;

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Tapiero, Ritchken, and Reisman 1987; Gavious and Lowengart 2012), and actually analyze the ability of higher price to signal better quality (Spence 1975; Wolinsky 1983; Milgrom and Roberts 1986; Bagwell and Riordan 1991; Judd and Riordan 1994; Ellingsen 1997; Acharyya 1998; Janssen and Roy 2010). Yet a puzzle arises from empirical studies showing that the price-quality relationship may not only be null but also negative (Gardner 1971; Gerstner 1985; Monroe and Dodds 1988; Jin and Kato 2006), and consequently that better quality may not necessarily be inferred from higher price (Zhou, Su, and Bao 2002; Völckner and Hofmann 2007; Yan and Sengupta 2011).Anecdotal evidence in e-commerce suggests that the price of a textbook or a mobile application is independent from its quality as rated by consumers. Theoretical studies, although extensive, do not provide a rationale for the empirically negative relationships between price and quality. This article bridges this gap and offers a theoretical foundation for both positive and negative price-quality relationships, thereby solving the puzzle of a negative relationship.

In this article, I focus the question of the impact of quality on price, formulating the conditions under which better quality leads to higher or lower price, namely, when the price-quality relationship is positive or negative. To make this relationship explicit, I develop an optimal control model based on the following assumptions: The firm chooses its pricing and innovation policies, innovation drives quality and the cost is based on quality, which is known by consumers who are sensitive to price and quality. Consumers' preferences and the firm's organization are linked to demand and supply dynamics, which in turn are linked to pricing and innovation policies. Hence, dynamic pricing and innovation literature inform this research.

Dynamic pricing literature often focuses on demand function properties (Dockner et al. 2000; Rubel 2013; Xue, Tang, and Zhang 2016; Zhang, Kevin Chiang, and Liang 2014), yet most dynamic pricing literature ignores quality improvements because of innovation (Chatterjee 2009; Chenavaz et al. 2011; Den Boer 2015). Innovation literature instead models quality improvement mainly using parametric functions (Li and Rajagopalan 1998; Adner and Levinthal 2001; Vörös 2006; Saha 2007; Lambertini and Mantovani 2009), yet omits the study of pricing. Teng and Thompson (1996) and Mukhopadhyay and Kouvelis (1997) initiate the joint study of dynamic pricing and quality policies, in which quality is chosen by the firm but does not result from innovation. Chenavaz (2011, 2012) and Vörös (2013) explicitly analyze both dynamic pricing and innovation policies, but they ignore the relationship between price and quality. Sun (2014) studies the price-quality relationship in a two-period setting.

The modeling in this article is derived from two characteristics: The interest in the demand function properties following dynamic pricing literature, and quality improvement mechanisms building on innovation literature. Similar to other research that studies pricing and innovation together (Bayus 1995; Chenavaz 2012), quality explicitly results from innovation, and cost relates to quality. Unlike research that relies on numerical simulations (Tapiero, Ritchken, and Reisman 1987; Bayus 1995; Adner and Levinthal 2001; Saha 2007), the results derived analytically have a formal guarantee. The closest modeling framework to mine is that of Vörös (2013), but my work is based on the properties of functions, whereas Vörös (2013) studies parametric functions. Regarding prior literature, this article points out explicit mechanisms that explain both positive and negative price-quality relationships. Moreover, the mechanisms originate from a simple model that does not require considering quality uncertainty, firm competition, or past sales effects.

This article makes two main contributions. First, the findings contribute to extant literature by underlining the mechanisms by which quality affects price. Quality exerts three effects on price: The cost effect on the supply-side and the sales and markup effects on the demand-side. On the supply-side, greater quality brings costs up, and higher costs increase the price: Cost has a positive effect on price. On the demand-side, the sales and markup effects exert their influences in opposite directions. On the one hand, greater quality increases sales, which increase even further with lower price: Sales exert a negative effect on price. On the other hand, greater quality expands the markup, which expands even more with higher prices: Markup has a positive effect on price. In turn, the total impact of quality on price is linked to (positive) cost, (negative) sales, and (positive) quality effects. Depending on the relative weight of each effect, the price-quality relationship may be positive or negative. In this sense, the sales effect provides a theoretical explanation for the empirically negative relationships between price and quality. Further, if quality is unknown by the consumer, the sales effect challenges the possibility of price to signal quality, causing market failure.

Second, this contribution is based on different price-quality relationship rules, which depend on the properties of the demand function and the quality-based cost but are independent of innovation. For a joint price and quality demand function, the rule of price-quality relationships explains both positive and negative cases, and therefore the price may decline even if quality and cost rise (the sales effect exceeds the cost and markup effects). For a multiplicative and additive separable demand function, more specific rules of price-quality relationships predict a positive link. In the multiplicative case, the price dynamics mimic the cost dynamics. In the additive case, the price dynamics emulate the quality dynamics. At the conceptual level, these results shed new light on the relationship between price and quality. At the practical level, the relationship rules and the quantification of the effect of quality on price support clear-cut managerial implications.

2 General Model Formulation

2.1 Model Development

This article studies a monopoly in an optimal control framework. A monopoly describes the situation of a firm that launches a new product or that protects its product by patent. The planning horizon is fixed and finite with length T. The time $t \in [0, T]$ is continuous.

2.1.1 Quality

The firm invests in (product) innovation $u(t) \in \mathbb{R}^+$ to improve (product) quality $q(t) \in \mathbb{R}^+$, and quality may evolve autonomously. Innovation expense $u(t)$ and product quality $q(t)$ are control and state variables. The quality dynamics writes

$$\dot{q}(t) = K(u(t), q(t)), \text{with } q(0) = q_0, \qquad [1]$$

where $K : \mathbb{R}^{2+} \to \mathbb{R}$ is twice continuously differentiable. Integrate eq. [1] gives the capital stock (or cumulative level) of quality $q(t) = q_0 + \int_0^t K(u(s), q(s))ds$.

Hereafter, and when no confusion exists, I omit any argument for notational simplicity. Also, $\dot{z}$ denotes the time derivative of z and z_x denotes the derivative of z with respect to x; z_{xx} and z_{xy} state for the second order derivative of z with respect to x and the cross derivative of z with respect to x and y.

The marginal effect of innovation u on quality q is positive but declines as innovation rises:

$$K_u > 0, \ K_{uu} < 0. \qquad [2]$$

Quality may also develop autonomously in any direction, and $K_q \in \mathbb{R}$. The case $K_q < 0$ accounts for autonomous degradation; the case $K_q \geqslant 0$ applies when any improvement is cumulative.

2.1.2 Cost

The (unitary production) cost function $C : \mathbb{R}^+ \to \mathbb{R}^+$ is once continuously differentiable and increases with quality q. The cost reads $C = C(q)$ with

$$C_q \geqslant 0. \tag{3}$$

For example, a cost independent of quality $C_q = 0$ and a cost rising with quality $C_q > 0$ characterize the software and the hardware industries (Shy 2001). This cost function is used by Caulkins et al. (2015). A more general cost function could account for a learning effect reducing the production cost, but it entails rendering the analysis less tractable (Jørgensen and Zaccour 2012, 70).

2.1.3 Demand

The (product) price $p \in \mathbb{R}^+$ is a control variable. For heterogeneous consumers, the (current) demand function $D: \mathbb{R}^{2+} \to \mathbb{R}^+$ is twice continuously differentiable. All the demand is satisfied and there is no inventory. Thus sales (understood here as the quantity of products sold as opposed to the monetary value of this quantity) equal demand. The demand D depends on the price p and quality q.

$$D = D(p, q). \tag{4}$$

Demand falls with price and rises with quality. Moreover, demand is more difficult to increase demand by reducing the price when quality is high compared to when quality is low:

$$D_p < 0,\ D_q > 0,\ D_{pq} \leqslant 0. \tag{5}$$

These assumptions are satisfied for instance with the linear demand function $D = a_0 - a_1 p + a_2 q$ or the Cobb-Douglas demand function $D = a_0 p^{-a_1} q^{a_2}$ with the parameters $a_0, a_1, a_2 > 0$.

2.2 Model Analysis

Table 1 defines the notations used in the model analysis.

The current profit function $\pi: \mathbb{R}^{3+} \to \mathbb{R}$ is assumed twice continuously differentiable. The current profit corresponds to the revenues less innovation expenses, writing

$$\pi(p, u, q) = [p - C(q)]D(p, q) - u.$$

Here, innovation investment increases product quality, modeled as a capital stock. Similar to capital accumulation, investing in innovation represents a fixed cost u improving quality, which increases demand from which higher rents can be extracted. Distinct from capital accumulation, producing quality also implies a variable cost $C(q)$.

Table 1: Notation.

T	= fixed terminal time of the planning horizon
r	= interest rate
$p(t)$	= product price at time t (decision variable)
$u(t)$	= innovation expense at time t (decision variable)
$q(t)$	= product quality at time t (state variable)
$\dot{q}$	$= dq/dt = K(u,q)$ = quality dynamics
$\lambda(t)$	= current-value adjoint variable at time t
$C(q)$	= unit production cost
$D(p,q)$	= current demand
$\pi(p,u,q)$	$= [p - C(q)]D(p,q) - u$ = current profit
$H(p,u,q,\lambda)$	= current-value Hamiltonian

To ensure interior solutions, assuming that they exist, the current profit π is supposed strictly concave in the price p. The firm maximises the intertemporal profit (or present value of the profit stream) over the planning horizon, by simultaneously choosing the innovation and pricing policies according to the quality dynamics. For simplicity, the salvage value of quality is zero. The interest rate is $r \in \mathbb{R}$, and the objective function of the firm is

$$\max_{u(s),\ p(s) \geqslant 0,\ \forall s \in [0,T]} \int_0^T e^{-rt} \pi(p(t), u(t), q(t)) dt,$$

$$\text{subject to } \dot{q}(t) = K(u(t), q(t)), \text{with } q(0) = q_0.$$

The shadow price (or current-value adjoint variable) $\lambda(t)$ represents the marginal value of quality at time t. The current-value Hamiltonian H with the shadow price λ for quality dynamics is

$$H(p,u,q,\lambda) = [p - C(q)]D(p,q) - u + \lambda K(u,q).$$

The Hamiltonian H measures the intertemporal profit. It is the sum of the current profit $(p-c)D-u$ and the future profit λK.

The maximum principle implies the dynamics of the shadow price

$$\dot{\lambda} = r\lambda - H_q = (r - K_q)\lambda + C_q D - (p - C)D_q \text{ with } \lambda(T) = 0. \qquad [6]$$

As previously mentioned, I confine my interest to interior solutions (or solutions admitting prices above unit costs). Thus, the necessary and sufficient first-order conditions for H maximization are for all $t \in (0,T)$

$$H_u = 0 \Longrightarrow K_u = \frac{1}{\lambda}, \qquad [7a]$$

$$H_p = 0 \Longrightarrow p = C - \frac{D}{D_p}. \qquad [7b]$$

The first-order condition for innovation eq. [7a] matches the static innovation rule that Bayus (1995) and Chenavaz (2012) use. The solution is interior because of eq. [2]. In addition, the higher the shadow price of quality λ, the higher innovation u is. Considering the diminishing returns of innovation in eq. [2], the marginal impact of innovation on quality K_u is lower.

The first-order condition for price eq. [7b] corresponds to the static pricing rule of Amoroso-Robinson. This classical rule, exhibiting an interior solution because the price is above the cost ($D \geqslant 0$ and $D_p < 0$ imply $-\frac{D}{D_p} \geqslant 0$), shows that the price is linked to supply and demand characteristics through the terms C and $\frac{D}{D_p}$.

Assuming the sufficient second-order conditions for H maximization with interior solutions for all $t \in (0, T)$

$$H_{uu} < 0 \Longrightarrow \lambda K_{uu} < 0, \qquad [8a]$$

$$H_{pp} < 0 \Longrightarrow 2 - D\frac{D_{pp}}{D_p^2} > 0, \qquad [8b]$$

$$H_{uu}H_{pp} - H_{up} > 0. \qquad [8c]$$

Condition (8*a*) together with the diminishing returns of innovation eq. [2] and the transversality condition [6] imply

$$\lambda(t) > 0, \ \forall t \in [0, T), \qquad [9]$$

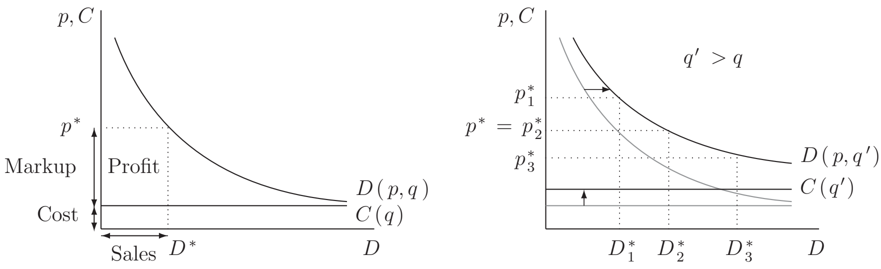
according to which better quality always raises the intertemporal profit.

Condition [8*b*] corresponds to the strict concavity of the current profit π with respect to price p. Roughly speaking, it shows that the demand function D cannot be "too" convex in the price p. As a corollary, this condition is verified if D is linear or strictly concave in p.

Condition [8*c*] is technical and not readily interpretable. This condition is satisfied because $H_{uu} < 0, H_{pp} < 0, \ \textit{and} \ H_{up} = 0$.

2.2.1 The price-quality relationship in a static setting

Figure 1 offers insights on the potential impact of an exogenous quality increase on price. First, Figure 1(a) determines the optimal price p^* for quality q, demand $D(p, q)$, and cost $C(q)$. According to the pricing rule [8], the optimal price p^* is the sum of the cost C and the markup $-\frac{D}{D_p}$. Recalling that sales equal demand,



(a) Optimal Price for a Quality Level

(b) Optimal Price after a Quality Increase

Figure 1: The effect of quality increase on price.

substituting $p = p^*$ in $D(p, q)$ yields the optimal sales $D^* = D(p^*, q)$. Assuming the elimination of innovation and its cost, the profit is measured by the markup length and sales width rectangle. The maximum profit corresponds to the maximum surface of this rectangle, which reduces with cost on the supply-side and augments with both markup and sales on the demand-side. Second, Figure 1(b) describes the effect on price of a quality raise from q to q', when the profit is maximised. Because of eq. [3], the cost moves up, and $C(q') \geqslant C(q)$. Because of eq. [5], demand shifts up and right, and $D(p, q') > D(p, q)$: Since greater quality increases the interest of new consumers, the demand shifts right (at the same price, more quantity is sold). Even if cost and demand raise, a higher quality has an ambiguous effect on price. Indeed, three optimal price candidates for profit optimality are $p_1^* > p^*$, $p_2^* = p^*$, and $p_3^* < p^*$. The supply-side effect is always clear: According to the pricing rule [8], a higher cost is passed on to the price. The demand-side effect is ambiguous: There is a trade-off between markup and sales, which move in the opposite direction along the demand function. Profit may be maximized either with greater markup and lower sales, as with p_1^*, or with lower markup and higher sales, as with p_3^*.

Built in a static setup, Figure 1 offers some insights about a possible counterintuitive phenomenon, that is a negative price-quality relationship. Intuitively, if quality improvement not only shifts the demand curve upward, but also make it flatter and thus more elastic, even if the marginal cost goes up after quality improvement, the equilibrium price may go down. Technically, submodularity of the demand function with respect to price and quality assumed in eq. [5] ($D_{pq} \leqslant 0$) is a necessary (but not sufficient) condition for a negative price-quality relationship to play out. A static setup, though offering preliminary insights, does not give the precise conditions under which the equilibrium price goes up or down following a quality increase. A dynamic setup enables to decompose the effects at play in a static setup. Thus, a dynamic setup offers a deeper analysis showing the precise conditions of change in equilibrium price after that quality increases.

2.2.2 Value of λ (t)

Define $\eta_q = D_q \frac{q}{D}$, the quality elasticity of demand, and $\eta_p = -D_p \frac{p}{D}$, the price elasticity of demand. Substitute η_q, η_p, and eq. [8] in eq. [6] implies

$$\dot{\lambda} - (r - K_q)\lambda = D\left(C_q - \frac{\eta_q}{\eta_p}\frac{p}{q}\right) \text{with } \lambda(T) = 0.$$

I abuse the notation by writing $\int K_q d\mu$ for $\int_{s-t}^{T} K_q(u(\mu), q(\mu))d\mu$. Multiply both sides of the last equation by $e^{-\left(r - \int K_q d\mu\right)t}$ gives $e^{-\left(r - \int K_q d\mu\right)t}\left[\dot{\lambda} - (r - K_q)\lambda\right]$ $= \frac{d\lambda e^{-\left(r - \int K_q d\mu\right)t}}{dt} = D\left(C_q - \frac{\eta_q}{\eta_p}\frac{p}{q}\right)$. Thus, $d\lambda e^{-\left(r - \int K_q d\mu\right)t} = e^{-\left(r - \int K_q d\mu\right)t} D\left(C_q - \frac{\eta_q}{\eta_p}\frac{p}{q}\right)dt$. Therefore, $\int_t^T d\lambda(s)e^{-\left(r - \int K_q d\mu\right)s} = \int_t^T e^{-\left(r - \int K_q d\mu\right)s} D\left(C_q - \frac{\eta_q}{\eta_p}\frac{p}{q}\right)ds$, and $\lambda(T)e^{-\left(r - \int K_q d\mu\right)T} - \lambda(t)e^{-\left(r - \int K_q d\mu\right)t} = \int_t^T e^{-\left(r - \int K_q d\mu\right)s} D\left(C_q - \frac{\eta_q}{\eta_p}\frac{p}{q}\right)ds.$

Substitute the transversality condition $\lambda(T) = 0$ gives the value of λ over time,

$$\lambda(t) = \int_t^T e^{-\left(r - \int K_q d\mu\right)(s-t)} D\left(\frac{\eta_q}{\eta_p}\frac{p}{q} - C_q\right) ds. \quad [10]$$

According to eq. [10], the shadow price of quality λ relates to the net result of the markup effect $\frac{\eta_q}{\eta_p}\frac{p}{q}$ and cost effect C_q.

The markup effect $\frac{\eta_q}{\eta_p}\frac{p}{q}$ captures the price increase that the consumer accepts paying after a rise in quality. The markup effect increases with the relative demand sensitivity to quality and to price $\frac{\eta_p}{\eta_q}$, and the quality-deflated price $\frac{p}{q}$. The markup effect has a positive effect on λ ($\frac{\eta_q}{\eta_p}\frac{p}{q} > 0$ because all terms are positive), because quality promotes willingness to pay and thus affects future profit.

The cost effect C_q captures the marginal effect of quality on cost. The cost effect has a negative effect on λ ($-C_q \leqslant 0$ because $C_q \geqslant 0$), in that better quality fosters higher cost and therefore lowers the future profit. If cost is independent of quality ($C_q = 0$), the cost effect disappears, and only the demand effect persists, as in Mukhopadhyay and Kouvelis (1997) and Chenavaz (2012). Alternatively, if cost increases with quality ($C_q > 0$), the cost effect mitigates the shadow price λ, and the innovation rule [7a] predicts lower innovation.

Equations [9] and [10] together impose

$$\frac{\eta_q}{\eta_p}\frac{p}{q} > C_q, \ \forall t \in [0, T), \tag{11}$$

namely, the markup effect $\frac{\eta_q}{\eta_p}\frac{p}{q}$ dominates the cost effect C_q at every point of time.

When quality rises, the intertemporal profit increases more from the higher markup rather than decreasing because of the higher cost: The net result of better quality on the intertemporal profit is positive. In other words, innovation generates a level of quality such that the cost of the quality increase is less than the price increase that consumers are willing to pay.

2.2.3 Variations of *u*(*t*)

Following Chenavaz (2012), eqs [6] ($\lambda(T) = 0$) and [12] ($\lambda(t) > 0, \ \forall t \in [0, T)$) imply $\exists t_1 \in [0, T) \, / \, \dot{\lambda}(t) < 0, \ \forall t \in [t_1, T)$. After time t_1, λ declines. Moreover, according to eq. [7], $\frac{d}{dt}(K_u) = \frac{d}{dt}\left(\frac{1}{\lambda}\right) = -\frac{\dot{\lambda}}{\lambda^2}$. So, $sign\, \dot{K}_u = -\, sign\, \dot{\lambda}$, and $\forall t \in [t_1, T)$, $\dot{K}_u > 0$. Assume $K_{uq} = 0$, then $\dot{K}_u = K_{uu}\dot{u}$. According to eq. [2], $K_{uu} < 0$, which implies $\text{sign}\, \dot{u} = \text{sign}\, \dot{\lambda}$. Therefore innovation falls after time t_1:

$$\exists t_1 \in [0, T) \, / \, \dot{u}(t) < 0, \ \forall t \in [t_1, T). \tag{12}$$

The *dynamic innovation rule* [12] depends solely on the first-order condition for innovation eq. [7b], and not on the first-order condition for price eq. [7a]. Innovation may increase ($\dot{u} > 0$) at the beginning of the planning horizon, from $t = 0$ to t_1. But innovation then diminishes ($\dot{u} < 0$) for the remaining planning horizon from t_1 to T, even if the firm always produces some innovation according to the innovation rule [7a]. If $t_1 = 0$, innovation declines over the entire planning horizon. The dynamic innovation rule is in line with Vörös (2006, 2013), who states that improvement activities may increase and then decrease over time, but the rule contrasts with Li and Rajagopalan (1998), who argue that such activities only reduce with time.

2.2.4 Variations of *p*(*t*)

In the static setting, eq. [7b] is a well-known pricing condition. This condition has to hold at every point in time, so that at the optimum, variations in marginal revenue must equal the corresponding variations in marginal cost. Variations in marginal revenue and cost over time in turn induce variations of price and quality, linking these dynamics. The formal link between the price and quality dynamics manifests with the differentiation of the first-order pricing condition [8] with respect to time *t*:

$$\dot{p} = C_q \dot{q} - \frac{(D_p \dot{p} + D_q \dot{q}) D_p - D(D_{pp} \dot{p} + D_{pq} \dot{q})}{D_p^2}.$$

Note $-\frac{D_p D_q}{D_p^2} = \frac{\eta_q}{\eta_p} \frac{p}{q}$ and rearrange yields

$$\dot{p}\left(2 - D\frac{D_{pp}}{D_p^2}\right) = \dot{q}\left(C_q + \frac{\eta_q}{\eta_p}\frac{p}{q} + D\frac{D_{pq}}{D_p^2}\right), \quad [13]$$

which is called the *general rule of dynamic pricing*.

The general rule of dynamic pricing eq. [13] links the dynamics of price $\dot{p}$ to the dynamics of quality $\dot{q}$ for a joint price and quality demand function $D(p,q)$. The rule [13] relies on the sole first-order condition for price eq. [8] and is independent from the first-order condition for innovation eq. [7b]. As such, the dynamic pricing rule is robust with any innovation process in eq. [1] and any innovation rule in eq. (7a). For example, introducing uncertainty or threshold effects among innovation and quality evolution in eq. [1] would have no effect on the rule [16] by itself.

Because p and q are control and state variables, the time elimination method applies (Mulligan and Sala-i Martin 1991). I assume the control p to be a once continuously differentiable function $p: \mathbb{R}^+ \to \mathbb{R}^+$ of the state q. Assuming $\dot{q} \neq 0$,[1] then $\frac{\dot{p}}{\dot{q}} = \frac{\frac{dp}{dt}}{\frac{dq}{dt}} = \frac{dp}{dq} = p_q$, and eq. [16] simplifies to

$$p_q \underbrace{\left(2 - D\frac{D_{pp}}{D_p^2}\right)}_{(+)} = \underbrace{C_q}_{(+)} + \underbrace{\frac{\eta_q}{\eta_p}\frac{p}{q}}_{(+)} + \underbrace{D\frac{D_{pq}}{D_p^2}}_{(-)}, \quad [14]$$

1 As autonomous degradation of quality is possible with $K_q < 0$, thus the case $\dot{q} = 0$ may occur. I assume $\dot{q} \neq 0$ because $\dot{q} = 0$, meaning constant quality, does not allow measuring the impact of quality on price. Note that the general rule of dynamic pricing eq. [16] holds with $\dot{q} = 0$.

which is identified as the *general rule of price-quality relationship*.

The general price-quality relationship rule [17] quantifies the effect of quality on price p_q. On the left-hand side of eq. [17], the second factor $\left(2 - D\frac{D_{pp}}{D_p^2}\right)$ is strictly positive because of the second-order condition [10]. On the right-hand side, the effect of quality on price results from three additively separable effects: The supply-side cost effect C_q, the markup effect $\frac{\eta_q}{\eta_p}\frac{p}{q}$, and the demand-side sales effect $D\frac{D_{pq}}{D_p^2}$. The cost and markup effects, which are more intuitive, have already been explained. Here, I analyze all three effects in greater depth.

– The cost effect C_q measures the marginal effect of quality on cost. The price increases with a greater cost following a quality increase, so the cost effect is positive. In practice, the cost effect is high for a manufacturing good, such as a car or computer chips, for which higher quality is expensive. By contrast, the cost effect vanishes for a digital good such as software or music, for which the marginal cost of production is zero.
– The markup effect $\frac{\eta_q}{\eta_p}\frac{p}{q}$ determines the increase in consumers' willingness to pay following an increase in quality. The markup rises with the price, and its effect is thus positive. The markup effect increases with the quality elasticity of demand η_q and the quality-deflated price $\frac{p}{q}$, but decreases with the price elasticity of demand η_p. Consequently, the markup effect is high for an upmarket product with little competition but low for a downmarket product with much competition.
– The sales effect $D\frac{D_{pq}}{D_p^2}$ quantifies the increase in sales following a price reduction together with a quality raise. Sales decrease when the price increases, so the sales effect is negative. The higher the demand D, the higher is the sales effect. Further, the larger the price sensitivity of demand when quality improves D_{pq} normalized by the square of the demand sensitivity to price D_p^2, the greater the sales effect is. Thus the sales effect is greater for a mass consumption product, such as a telephone or a television, than for a niche product.

Proposition 1: *If* $D = D(p, q)$, *the price-quality relationship is characterized by*

$D = D(p, q)$	Conditions	Results
Case 1	$C_q + \frac{\eta_q p}{\eta_p q} + D\frac{D_{pq}}{D_p^2} > 0$	$p_q > 0$
Case 2	$C_q + \frac{\eta_q p}{\eta_p q} + D\frac{D_{pq}}{D_p^2} = 0$	$p_q = 0$
Case 3	$C_q + \frac{\eta_q p}{\eta_p q} + D\frac{D_{pq}}{D_p^2} < 0$	$p_q < 0$

Proof: Immediate with the price-quality relationship rule [14]. □

For a general demand function, the price dynamics result from three effects that work in opposition: The cost effect (positive), the markup effect (positive), and the sales effect (negative). The price-quality relationship is unknown, since three alternative cases arise. Case 1, in which the cost and the markup effects exceed the sales effect, shows a positive relationship between price and quality, which follows intuition. Extending Teng and Thompson (1996)'s findings, Case 2, in which the cost and markup effects equal the sales effects, explains the lack of relationship between price and quality. Contrary to intuition, the sales effects outweigh the cost and markup effects in Case 3, stressing a negative relationship between price and quality.

Remark 1: *If $D = D(p, q)$, price may decrease, even if quality and cost both increase.*

With $D = D(p, q)$ and as a corollary to Proposition 1, if the sales effect is "large enough", the price falls even if cost and quality both rise. Following the general rule of price-quality relationship [14], the sign of p_q is undetermined, and the effect of quality on price remains ambiguous. The managerial implication is that the firm should set the pricing policy only according to the relative weight of the cost, sales, and markup effects.

Example 1: *Linear price-quality demand function.*

The linear price-quality demand function $D = a_0 - a_1 p + a_2 q + a_3 \frac{q}{p}$, with a_0, a_1, a_2, and $a_3 > 0$ verifies that price has an ambiguous effect on price. In effect, this demand specification leads to the price-quality relationship $p_q \left(2 - D\frac{a_3 q}{2p^3 (a_1 + a_3 \frac{q}{p^2})^2}\right) = C_q - \frac{a_3 D}{p^2 (a_1 + a_3 \frac{q}{p^2})^2} + \frac{a_2 + \frac{a_3}{p}}{a_1 + a_3 \frac{q}{p^2}}$. In this context, depending on the parameters, Cases 1, 2, and 3 all may occur here.

3 Subclasses of the General Formulation

The general demand function offers useful insights with the general rules of dynamic pricing and the price-quality relationship [13] and [14]. However, although general, the rules [13] and [14] may not always be easily applicable. The following specifications of the demand function offer clearer dynamic pricing and price-quality relationship rules. The gain in applicability trades off with a loss in generality.

3.1 Multiplicative Separable Demand Function

A demand function multiplicatively separable on price and quality holds remains relatively general and unconstrained. This simple and natural modeling is analytically tractable and explains the data well (Bayus 1995). In the multiplicatively separable case, demand eq. [4] becomes

$$D = h(p)l(q), \tag{15}$$

which implies $D_p = h_p < 0$, $D_q = l_q > 0$, and $D_{pq} = h_p l_q < 0$ recalling eq. [5].

The substitution of eq. [18] into eqs [16] and [17] yields the rules of dynamic pricing and price-quality relationship

$$\dot{p}\left(2 - h\frac{h_{pp}}{h_p^2}\right) = \dot{q}C_q = \dot{C}, \tag{16a}$$

$$p_q \underbrace{\left(2 - h\frac{h_{pp}}{h_p^2}\right)}_{(+)} = \underbrace{C_q}_{(+)}. \tag{16b}$$

Proposition 2: *If $D = h(p)l(q)$, the price-quality relationship is characterized by*

$D = h(p)l(q)$	Conditions	Results
Case 1	$C_q > 0$	$p_q > 0$
Case 2	$C_q = 0$	$p_q = 0$

Proof: Immediate with the price-quality relationship rule [16b]. □

For a multiplicative separable demand function, the dynamics of price stem solely from the cost effect (positive). Indeed, the markup effect (positive) and the sales effect (negative) are balanced, and they cancel each other out. In Case 1, for which the marginal effect of quality on cost strictly depends on quality ($C_q > 0$), the price-quality relationship is positive. In Case 2, for which the marginal impact of quality on cost is independent of quality ($C_q = 0$), the price-quality relationship disappears.

Remark 2: *If $D = h(p)l(q)$, the price dynamics mimic the cost dynamics.*

With $D = h(p)l(q)$, the dynamics of price result solely from the cost effect (positive). From the dynamics of price rule [16a], the sign of $\dot{p}$ is the sign of $\dot{C}$, and the dynamics of price mimics the cost dynamics. By definition in eq. [16a], the dynamics of cost are determined by the dynamics of quality. Therefore, the dynamics of price also mimic the dynamics of quality. The shape of the pricing policy follows the shape of cost. The managerial implication is straightforward: Price augments with cost, and the firm adopts a pricing policy that imitates the cost dynamics.

Example 2: *Multiplicative separable price-quality demand function.*

The multiplicative separable demand function $D = (a_0 + a_1 q)e^{-a_2 p}$ provides the price-quality relationship $p_q = C_q$. In this case, it is straightforward that Cases 1 and 2 may occur.

3.2 Additive Separable Demand Function

A demand function additively separable in price and quality is the most simple and natural modeling approach. In the additively separable case, demand from eq. [4] becomes

$$D = h(p) + l(q), \qquad [17]$$

which implies $D_p = h_p < 0$, $D_q = l_q > 0$, and $D_{pq} = 0$ recalling eq. [5].

The substitution of eq. [17] in eqs [13] and [14] provides the dynamic pricing and price-quality relationship rules

$$\dot{p}\left(2 - (h + l)\frac{h_{pp}}{h_p^2}\right) = \dot{q}\left(C_q + \frac{\eta_q}{\eta_p}\frac{p}{q}\right), \qquad [18a]$$

$$p_q \underbrace{\left(2 - (h + l)\frac{h_{pp}}{h_p^2}\right)}_{(+)} = \underbrace{C_q}_{(+)} + \underbrace{\frac{\eta_q}{\eta_p}\frac{p}{q}}_{(+)}. \qquad [18b]$$

Proposition 3: *If $D = h(p) + l(q)$, the price-quality relationship is characterized by*

$D = h(p) + l(q)$	Condition	Result
Case 1	$C_q + \frac{\eta_q p}{\eta_p q} > 0$	$p_q > 0$

Proof: Immediate with the price-quality relationship rule [18b]. □

For an additive separable demand function, the sales effect (negative) disappears. But the cost effect (positive) and the markup effect (positive) remain. Because these effects move in the same direction, Case 1 posits a strictly positive price-quality relationship.

Remark 3: *If $D = h(p) + l(q)$, price dynamics mimics quality dynamics.*

With $D = h(p) + l(q)$, the dynamics of price are carried by the cost and markup effects. According to the dynamic pricing rule [18a], the sign of $\dot{p}$ is equivalent to the sign of $\dot{q}$. The dynamics of price emulates the dynamics of quality, and the pricing policy slope imitates the quality slope. The managerial implication is simple: The price increases with quality, and the firm adopts a pricing policy that imitates the quality dynamics.

Example 3: *Additive separable price-quality demand function.*

The additive separable demand function $D = a_0 - a_1 p + a_2 q$ offers the price-quality relationship $p_q = \frac{1}{2}\left(C_q + \frac{a_2}{a_1}\right)$. Here, only Case 1 is possible.

4 Discussion

Because innovation drives quality, the rules for innovation and pricing are independent of each other. Thus the pricing rules hold for any innovation process in eq. [1], which may include uncertainty or learning, for example. If the rules of innovation and pricing by themselves show no direct link, there is still an indirect link because innovation drives quality (through the dynamic innovation rule) which affects price (via the dynamic pricing rule). As a result, the dynamics of innovation determine the dynamics of quality, which in turn affect the dynamics of price. Eliminating time from the rule of dynamic pricing provides the rule of price-quality relationship, which stresses the total effect of quality on price.

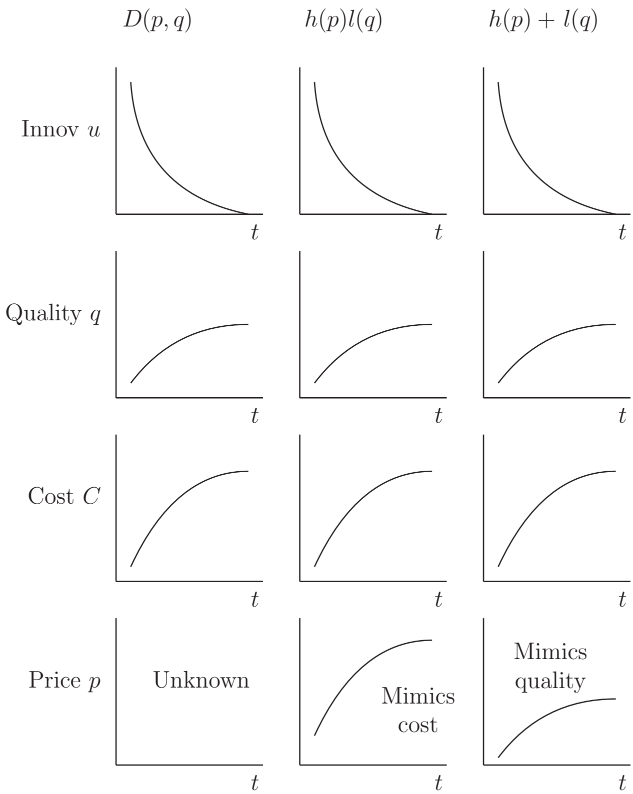


Figure 2: Innovation, quality, cost, and pricing over time.

Figure 2 shows the dynamics of innovation, quality, cost, and price over time, according to the demand function class. Considering eqs [7a] and [10], the innovation policy depends on the supply and demand characteristics. In line with eqs [7a] and [12], the firm always promotes innovation but at a decreasing level after some time. Quality and cost are therefore linked to the firm's capabilities and to consumers' preferences. The effect of quality on price results from three additively separable effects: cost (positive), sales (negative), and markup (positive) effects. These effects in turn drive the dynamics of price according to the cases of demand properties:

– In the general case $D(p, q)$, the cost, sales, and markup effects play out; Proposition 1 states that the effect of quality on price is undetermined. Therefore and contrary to intuition, if the sales effect is greater than the cost and markup effects, the price falls with a quality increase even if quality and cost rise (Remark 1): On the one hand, when quality rises, the falling price enables the firm to earn more from additional sales than lose from the additional cost and the reduced markup. On the other hand, when quality declines, the rising price grants the firm more advantages from lower costs and higher markups than disadvantages from lower sales. In this counterintuitive situation, the price-quality relationship is negative, and the sales effect undermines the ability of price to convey information on quality when quality is unknown.

- In the multiplicative case $h(p)l(q)$, the sales and markup effects cancel each other out and only the cost effect plays a role. According to Proposition 2, price increases with quality. Remark 2 notes that the price dynamics mimic the quality dynamics.
- In the additive case $h(p)+l(q)$, the sales effect vanishes. The cost and markup effects remain and come into play. Following Proposition 3, price increases with quality. Remark 3 shows that price dynamics emulate cost dynamics.

In practice, there is no reason for the demand function to be separable. Thus, the general case of demand function is a valuable framework for understanding the price-quality relationship. A positive price-quality relationship is more likely for a manufactured good (high cost effect) in a niche market (low sales effect). Examples of these goods are luxury goods, such as a watches that contain precious metals or a gastronomic restaurant. Instead, a negative relationship arises more easily for a digital good (low cost effect) in a mass market (high sales effect). Examples here are information goods, such as a textbooks or a mobile applications.

For decades, literature has identified the role of positive cost and markup effects (Scitovsky 1944; Monroe and Dodds 1988), but it has neglected the role of any negative effect, such as the sales effect. Note that the sales effect has been introduced by Chenavaz (2012), without examining the price-quality relationship. Yet in the general demand function case, the sales effect is essential in providing a theoretical explanation to the empirical puzzle of a negative price-quality relationship. The explanation of the sales effect alone originates from a simple modeling, which does not require either firm competition or unknown quality. In addition, the sales effect makes two related points with respect to prior research where quality is unknown: First, the sales effect reduces the possibility for price to act as a quality signal, as in theoretical studies (Spence 1975; Wolinsky 1983; Milgrom and Roberts 1986; Bagwell and Riordan 1991; Acharyya 1998; Janssen and Roy 2010). Second, the sales effect provides a theoretical basis for the low quality inference associated with price, as documented in empirical research (Zhou, Su, and Bao 2002; Völckner and Hofmann 2007; Yan and Sengupta 2011).

Regarding the multiplicative and additive cases of demand functions, Propositions 2 and 3 offer a formal guarantee of the intuitively positive price-quality relationship, as introduced in early contributions by Scitovsky (1944) and Monroe and Dodds (1988). The properties of each demand function directly affect the nature of the relationship between price and quality. Specifically, when there is evidence of a negative relationship, the multiplicative and additive separable modeling of demand functions, albeit convenient, are flawed.

A negative price-quality relationship is simply explained in the present framework. The framework does not require quality uncertainty (consumers cannot be misled) or the strategic behavior of firms (there is no competition). However, the framework requires "sufficiently" general demand functions for the sales effect (1) to be negative (no additive separability) and (2) not to be canceled out by the markup effect (no multiplicative separability). Further, the sales effect has to be (1) negative (negative cross-derivative of the demand function with respect to price and quality) and (2) greater than the markup and cost effects (for instance a digital good in a mass market).

To note is that a consequence of this framework simplicity is the analytical (as opposed to numerical) tractability of the model for different classes of demand functions. The cost of such tractability is the ignorance of more complex and realistic effects. Indeed, the model disregards some main managerial characteristics such as the impact of learning and diffusion effects, strategic behavior, or of a salvage value, which are emphasized in Li and Rajagopalan (1998) and Chatterjee (2009). The reason to omit here such essential elements is the focus on the sales effect in the most simplistic way. Of course, these omissions limit the scope of the conclusions that can be drawn from this model, thereby imposing an acute need for further investigations.

5 Conclusion

In this article, I define the conditions under which better quality implies a higher or lower price. Foremost, I derive a price-quality relationship rule, linked to the (positive) cost effect on the supply-side and the (negative) sales and (positive) markup effects on the demand-side. Depending on the strength of each effect, the relationship between price and quality is positive or negative. Further, I establish more specific rules for the price-quality relationship, with straightforward insights and direct managerial implications. The rules are simply derived from a stylized modeling. The counterpart of such simplicity is disregarding other effects influencing firm policies. Such simplifications call for further research using more realistic modeling.

This work sheds new light on the price-quality relationship. First, the theoretical predictions established from the general rule of price-quality relationships pertaining to the cost, sales, and markup effects provide a testable framework calling for empirical validation. Second, the sales effect, previously neglected in the literature, emerges as a fundamental element of a negative price-quality relationship. The sales effect plays a greater role in a larger market where lower price with better quality yields greater demand-expansion.

If product quality is unknown to the consumer, the sales effect mitigates the possibility of price signaling quality as analyzed in the theoretical literature; it also challenges the habit of positing a positive price-quality relationship. Further, the sale effect explains the weak inference of quality from price (especially if the quality cost is low) as documented in empirical research. Consequently, the sales effect may constitute an aggravating cause of market failure.

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