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Betting on Death and Capital Markets in Retirement: A Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans

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Abstract

How might retirees consider deploying the retirement assets accumulated in a defined contribution pension plan? One possibility would be to purchase an immediate annuity. Another approach, called the "phased withdrawal" strategy in the literature, would have the retiree invest his funds and then withdraw some portion of the account annually. Using this second tactic, the withdrawal rate might be determined according to a *fixed benefit* level payable until the retiree dies or the funds run out, or it could be set using a *variable formula,* where the retiree withdraws funds according to a rule linked to life expectancy. Using a range of data consistent with the German experience, we evaluate several alternative designs for phased withdrawal strategies, allowing for endogenous asset allocation patterns, and also allowing the worker to make decisions both about when to retire and when to switch to an annuity. We show that one particular phased withdrawal rule is appealing since it offers relatively low expected shortfall risk, good expected payouts for the retiree during his life, and some bequest potential for the heirs. We also find that unisex mortality tables if used for annuity pricing can make women's expected shortfalls higher, expected benefits higher, and bequests lower under a phased withdrawal program. Finally, we show that delayed annuitization can be appealing since it provides higher expected benefits with lower expected shortfalls, at the cost of somewhat lower anticipated bequests.

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1. Introduction

In retirement, many people face the question of how to draw down assets that they have accumulated over their worklives. Economists often suggest that a sensible approach is to purchase a life annuity. An annuity is a financial contract between an insured person and an insurance company "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium" (*Brown et al.* 2001b: p. 1). The payments may be fixed in nominal terms (fixed annuity), or they might rise at a pre-specified fixed nominal escalation rate (graded annuity), or they could be indexed to inflation (real annuity) keeping the retiree's standard of living constant. Alternatively, they might reflect the return of a specific asset portfolio which backs the (variable) annuity, or they can depend on the insurance company's experience with mortality, investment returns, and expenses (participating annuity). As *Mitchell et al.* (1999) note, the essential attraction of a life annuity is that the individual is protected against the risk of outliving his own assets, given uncertainty about his remaining lifetime, by pooling longevity risk across a group of annuity purchasers. *Yaari* (1965) shows that risk-averse retirees without a bequest motive facing annuity markets that charge actuarially fair premiums, should annuitize 100 percent of their wealth.

Though life annuities provide invaluable longevity insurance that cannot be replicated by pure investment vehicles, they also have some disadvantages. First, the purchaser faces loss of liquidity and control over his assets, because the lump sum premium cannot be recovered after purchase of the annuity, irrespective of special needs (e.g. to cover unexpected expenditures for uninsured medical costs).[1] Second, in its simplest form, where income payments are contingent on the individual' s survival, there is no chance of leaving money for heirs, even in the case of the annui-

[1] See *Brugiavini* (1993) for a theoretical model in which the health status of the retiree is stochastic.

tant's early death. Other explanations for why individuals will be reluctant to buy annuities are the high administrative costs levied by insurance companies (*Mitchell et al.* 1999), the ability to pool longevity risk within families (*Brown/Poterba* 2000, *Kotlikoff/Spivak* 1981), and the presence of other annuitized resources from Social Security or employer-sponsored defined benefits plans (*Munnell* et al. 2002).

Recent developments in European pension systems have focused attention on alternative income withdrawal patterns for asset pools dedicated to old-age consumption. In Germany, so-called "Riester plans" offer tax inducements for voluntary saving in individual pension accounts (IPA) during the worklife, underscoring the government's interest in boosting asset accumulation in an aging population (*Börsch-Supan* et al. 2003a, b). When the age of retirement is reached, twenty percent of the accumulated assets in the IPA can be taken as a lump-sum distribution. The rest must be drawn down in the form of a lifelong annuity (offered by a commercial insurance company) or a phased withdrawal plan (typically offered by mutual fund and/or bank providers) which must partly revert into an annuity at the age of 85. In the UK, personal pensions have also grown in popularity (*Blake et al.* 2003). As in Germany, a portion of the accumulated asset can be taken as a lump sum, while with the rest, one is legally obliged to buy an annuity by the age of 75. In Canada, at age 69 retirees must either buy an annuity with their tax-sheltered savings or create a discretionary managed withdrawal plan (*Milevsky/Robinsion* 2000). In the US, no compulsory annuitization is required for 401(k) plans at retirement; instead, many workers roll over their funds as a lump sum into an Individual Retirement Account which manage themselves in old age. Though some researchers have explored aspects of the accumulation phase in these accounts (e.g. *Maurer/Schlag* 2003, *Blake* et al. 2001), thus far, relatively little attention has been devoted to the payout phase.

A key aspect of the retiree's decumulation process is the decision of how to invest these retirement plan assets and how to structure payouts during the retirement period, so as to best balance consumption flows versus bequest intentions without running out of money. An alternative strategy to buying a life annuity is associated with what has been called "self-annuitization" or *phased withdrawal* approach (c.f. *Milevsky/Robinson* 2000). At retirement, the wealth endowment is allocated across various asset categories (e.g. equity, bonds, cash) typically included in a family of mutual funds where the assets will earn uncertain rates of return. A certain amount of the invested funds can then be withdrawn periodically for consumption purposes. The particular advantage of such a phased withdrawal strategy, as compared to the life annuity, is that it offers greater liquidity, the possibility of greater consumption while alive as well as the possibility of bequeathing some of the assets in the event of early death. On the other hand, relying on income flows withdrawn directly from an IRA without any insurance provides no pooling of longevity risk. Consequently, if the retiree constantly consumes an equal amount from his account, he could outlive his assets before his uncertain date of death, particularly in the event of long-run low investment returns. An alternative withdrawal rule is to not take out some fixed *amount* per period, but rather to consume a specified *fraction* of the remaining fund wealth each period. This second strategy, in contrast to the fixed withdrawal technique, avoids the risk of outliving one's total assets, as long as the benefit-to-wealth ratio is lower than one. Nevertheless, due to stochastic investment returns, the value of the pension accounts assets change over time implying that the periodically withdrawn amount must vary in tandem – and it could be substantially lower or higher than the benefit payable under a life annuity.

To be able to evaluate the different decumulation options on a quantitative basis, it is necessary to introduce a formal risk/return framework for decisionmaking under uncertainty. The standard approach in financial economics is to maximize the expected discounted value of a (time separable) utility function for uncertain future benefits and (if necessary) for a bequest. For example, *Blake* et al. (2003) use a utility function of the constant relative risk class (CRRA), to evaluate different withdrawals plans assuming mandatory annuitization is required at age 75. *Milevsky/ Young*

(2002) use a similar objective function to determine the value of the option to defer annuitization. A shortcoming of such an approach, especially in the practical world, is that the decisionmaker rarely has explicit measures of risk preferences without knowing the shape of his utility function. As *Pye* 2000 pointed out, "neither endowment fund managers nor financial planners are using these models to help make decisions". As a result, risk-value (or risk-return) models of choice have the advantage of developing an explicit measure of risk, an explicit measure of value, and a function reflecting the trade-offs between value and risk. Clearly, individuals prefer more return to less and less risk to more, other things equal. This property allows a partial-ordering of opportunities within a risk-return dominance context, even if the exact preference weights for the risk and return tradeoff are unknown. Depending on which risk metric is selected and how the trade-off between risk and return is formulated, a risk-value model can but need not be consistent with the expected utility approach of choice (*Sarin/Weber* 1993).[2]

In this paper, we take a risk-value approach, whereby the "return" is the expected level of benefits as well as the expected possibility of bequest, and the "risk" is the possibility of not reaching a benchmark or desired level of consumption. Previous studies taking this tack focus on the *probability of consumption shortfall* as the operative risk measure.[3] Assuming that the retiree consumes a fixed real amount at specific points in time from a self-managed pension account, these authors calculate the probability of running out of money before the uncertain date of death using alternative assumption about the asset allocation, the initial consumption-to-wealth ratio, and the optimal waiting time before switching the retirement wealth into an annuity. Our work extends this literature in several directions. First, we examine the risk and return profiles of several variable self-

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[2] Perhaps the most widely used risk-return model in the area of finance is the classic mean-variance portfolio analysis elaborated by *Markowitz* (1952), which is, *inter alia,* consistent with a quadratic utility function (see *Campbell/Viceira* 2002, p. 24). A general analysis of conditions regarding the compatibility of multiparameter trade-off models of choice with the expected utility model is given in *Schneeweiß* (1967).

[3]See for instance *Milevsky/Ho/Robinson* (1997), *Milevsky/Robinson* (2000), *Milevsky* (1998, 2000), *Ameriks/Veres/Warshawsky* (2001), *Pye* (2000, 2001) and *Albrecht/Maurer* (2002).

annuitization strategies that provide payments according to predetermined benefit-to-wealth ratio. Second, we address a major shortcoming of the shortfall-probability risk measure, namely that it ignores the size of the possible loss that may be experienced. In practice, of course, both theoretical and empirical arguments suggest that investors take both the probability and the amount of a possible shortfall into consideration. Our contribution is to go beyond prior work by looking not only at the *probability of a consumption shortfall*, but also consider the *size of the shortfall* when it occurs. Third, we examine how the results change if a mandatory annuitization rule were imposed akin to those in the recent German and UK pension regulation. Fourth, we evaluate the impact of allowing the annuitization date to be endogenous, along with the asset allocation decision. We illustrate how the risk of a consumption shortfall and return profiles of fixed and variable phased withdrawal strategies compare to the life annuity, and indicate what dominant strategies might be.

The remainder of this paper is divided into four sections. The next section describes several different withdrawal strategies. To illustrate their implications, we assume conditions with respect to capital and insurance markets products and pricing found in the German annuity and capital marketplace. We adopt these so as to be informative about alternative payout options that might be contemplated under the German *Riester* plans when they reach maturity. Most results focus on an age-65 male retiree, but we also provide findings for other ages and for women. Section three reports results using a fixed asset allocation pattern, and Section four permits assets to be allocated optimally. A final section summarizes and concludes.

2. The Case of Phased Withdrawal

2.1 Withdrawal Plans with Fixed Benefits

We assume that the retiree is endowed with an initial wealth of V_0 that he can use to buy a single-premium immediate life annuity paying constant annual real benefits B at the beginning of each year, for life with no bequest. We denote this as the benchmark annuity and refer the reader to

Appendix A regarding the pricing of such an insurance product using assumption about mortality, loadings and interest rates to discount future annuity payments. If the retiree does not annuitize his wealth, he invests the retirement assets in various financial assets (e.g. equities, bonds, cash) typically represented by a family of mutual funds, and then he withdraws a certain amount at the beginning of the year for consumption purposes. Throughout the paper, we assume that benefits are taxed as ordinary income; therefore taxes will not change the desirability of voluntary annuitization or systematic withdrawal from a self-managed retirement account.[4]

Under the *fixed benefit rule,* the retiree will sell at the beginning of each year as many fund units as required to reach the same yearly benefits paid by the life annuity, until either he dies, or the retirement assets are exhausted. Formally, the benefits B_t at the beginning of each year are given by:

$$B_t = \min(B, V_t)\,, \tag{1}$$

where V_t is the value of the retirement accounts assets wealth at the beginning of year t ($t = 0, 1, \ldots$) just before the withdrawal B_t for that year is made. The retiree faces an intertemporal budget constraint that wealth next period V_{t+1} equals wealth today V_t, less what is subtracted for benefit payments B_t, times the (inflation adjusted) portfolio return R_{t+1} over the period, or zero if the fund is exhausted:

$$V_{t+1} = (V_t - B_t)\cdot(1 + R_{t+1}) = \begin{cases} (V_t - B)(1 + R_{t+1}) & V_t > B \\ 0 & V_t \le B. \end{cases} \tag{2}$$

Note that the benefit paid B_t depends on the value V_t of the retirement assets used to finance withdrawals. If these assets are risky, the benefit payouts are exposed to uncertain capital market returns. The idea of the fixed benefit rule is to replicate the income from a life annuity as long as the funds permit, while at the same time offering some bequest potential in the event of an early death.

[4] This is accurate for the German context; for more on annuity tax treatment in the US see Brown et al. (2001b).

Nevertheless, the risk of the fixed benefit rule is that adverse capital markets linked to longevity outcomes might produce a situation where V_t hits zero and therefore $B_t = B_{t+1} = \ldots = 0$, while the retiree is still alive.

2.2. Phased Withdrawal Rules with Variable Benefits

Under a variable phased withdrawal plan, the retiree receives not a fixed benefit amount per period, but rather an *ex ante* fixed fraction of the retirement assets remaining each period. This benefit-wealth ratio can be constant, increasing, or decreasing over time. Due to the stochastic nature of capital markets, the value of the retiree's fund is exposed to positive as well as negative fluctuations. Consequently, the level of benefit payments under a variable withdrawal plan also fluctuates in tandem with the accounts value. Depending on the withdrawal fraction and the realized returns of the retirement accounts assets, benefit payments could be substantially lower – or higher – than payments from a life annuity at some point during the post retirement phase. A variable phased withdrawal plan and a variable annuity have in common the fact that they pay pension benefits that vary with uncertain investment returns. Nevertheless, the former offers the possibility of bequeathing the remaining value of the retirement account in the case of the retiree's death, while the latter does not.

The path of benefits payable using a variable phased withdrawal rule can be formalized as follows. Let V_t be the value of the retirement assets at the beginning of period t ($t = 0, 1, \ldots$) before the withdrawal B_t for that year is made. At the beginning of every period t, an *ex ante* specified fraction ω_t $(0 < \omega_t \leq 1)$ is withdrawn from current wealth; hence the retiree receives a payment according to:

$$B_t = \omega_t \cdot V_t \quad . \tag{3}$$

Further let R_{t+1} denote the return of the funds over the period. Then, the intertemporal budget constraint of the retirement account is given by:

$$V_{t+1} = (V_t - B_t) \cdot (1 + R_{t+1}) = (1 - \omega_t) \cdot V_t \cdot (1 + R_{t+1}). \quad \textbf{(4)}$$

If the retiree dies at the beginning of period t+1, V_{t+1} represents the bequest potential for his heirs. Note that if the assets of the pension account are invested in risky assets (e.g. stocks and/or bonds), the returns are also uncertain, and therefore both the pension benefits B_t as well as the bequest potential V_t are random variables.

In what follows, we focus attention on three specific withdrawal rules that generate variable benefits: the fixed percentage rule, the 1/T rule, and the 1/E(T) rule. Each is discussed in turn.

"Fixed Percentage" withdrawal rule. Here a constant fraction is withdrawn each period from the remaining fund wealth, i.e. the benefit-wealth ratio is fixed over time:

$$\frac{B_t}{V_t} = \omega_t = \omega. \quad \textbf{(5)}$$

This withdrawal rule has the advantage of simplicity, requiring no information regarding the maximum possible duration of the payout phase or the retiree's characteristics (i.e. age, sex).

"1/T Rule" withdrawal rule: The idea behind this rule is to set the withdrawal fraction according to the maximum possible duration of the plan, denoted by T. One way is to set T equal to the oldest age assumed in a mortality table; another is to fix it at the retiree's life expectancy as of his retirement date (*Brown* et al., 1999). In the first case, the maximum number of payments T is given by the limiting age l of the mortality table minus the current age of the retiree x plus one:

$$T = l - x + 1. \quad \textbf{(6)}$$

The retiree gets a fraction of $1/T$ of his initial pension account as the first payment, the second payment is worth $1/(T-1)$ of the remaining assets, and so forth until the retiree either passes away or reaches the plan's limiting age l. Formally, the benefit-wealth ratio at the beginning of year t (t = 0, 1, …T-1) of this retirement plan is given according to:

$$\frac{B_t}{V_t} = \omega_t = \frac{1}{T-t}. \qquad \textbf{(7)}$$

In contrast to the fixed percentage rule discussed above, the withdrawal fraction is not constant but rather increases with age. What this means is that the longer the retiree survives, the higher the withdrawal fraction will be. For example, if l = 110 and x = 65 the first withdrawal fraction at age 65 is $\omega_0 = 1/46 = 2.17\%$, the second at age 66 is $\omega_1 = 1/45 = 2.22\%$, and at age 101 the benefit to wealth ratio is $\omega_{101} = 10\%$. The rule pay out all of the remaining wealth of the retirement account (i.e. $\omega_{110} = 100\%$) by the age of 110 – no bequest potential is left – in contrast to the fixed percentage rule.

"1/E[*T*(*x*)]" withdrawal rule: This rule, which we will call the 1/E(T) rule for short, takes into consideration the retiree's remaining life expectancy in a dynamic way. Now the withdrawal fraction is no longer determined by the maximum length of the plan, but instead by the retiree's life expectancy remaining. Let ${}_t p_x$ represents the conditional probability that an *x*-year old man will attain age x + *t*, the complete expectation of life is calculated as:

$$\mathrm{E}\left[T(x+t)\right] = \sum_{t=0}^{l-x} {}_t p_x \qquad \textbf{(8)}$$

where *l* is the maximum age according to a mortality table. Then, for an at retirement x-year old man, the benefit-to-wealth ratio in period *t* after retirement, conditional on the fact that he is still alive, is given as:

$$\frac{B_t}{V_t} = \omega_t = \frac{1}{\mathrm{E}[T(x+t)]}. \qquad \textbf{(9)}$$

The shorter his expected remaining lifetime, the higher the fraction he will withdrawal from his pension account. Therefore, the withdrawal fraction rises with the age of the retiree. Since the retiree's life expectancy is less than the maximum age of the mortality tables, the benefit-to-wealth ratio of the 1/E(T) rule exceeds that of the 1/T rule, in general. The 1/E(T) withdrawal rule is used in the US during the decumulation phase of 401(k) plans, where the tax authority seeks to ensure that retirees consume their tax-qualified pension accounts instead of leaving them as bequests for their heirs (see *Munnell et al.* 2002).

3. Risk and Reward Analysis of Phased Withdrawal Plans Conditional on Survival

3.1 Research Design

To compare the risk and value characteristics of the four phased withdrawal rules, it is useful to begin with an assessment of expected payouts conditional on retiree survival (Section 4 generalizes results with mortality-weighted risk and reward computations). For the moment, therefore, we focus only on the risk resulting from capital markets and suppress mortality. To do so, we assume a 65-year old male retiree who seeks to compare benefits under the four phased withdrawal plans given an initial asset balance. The plan assets are rebalanced annually to maintain an asset pool split evenly between stocks and bonds, consistent with recommendations by financial advisors (asset allocation is optimized in the next section).[5] We employ an annuitant mortality table provided by the German Society of Actuaries to calculate survival probabilities and expected lifetime (in the 1/E(T) case). Since this table ends at age 110, we set l = 110 for the 1/T rule. For the fixed percentage withdrawal rule, we select ω = 5.82%, since this benefit-to-wealth ratio produces an initial payout equal to the life annuity in the first year of the plan. In the case of the fixed benefit rule, we assume that the initial withdrawal continues until the retiree dies or the account is exhausted.

We next assess the risk and return patterns that emerge under these alternative phased withdrawal patterns (before taxes), compared to a fixed real annuity providing lifelong constant payouts.

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[5] *Feldstein/Ranguelova/Samwick* (2001) and *Ibbotson* (2003) assume that retirees hold their non-annuitized assets in a 60% stock, 40% bond portfolio. Here, for illustrative purposes, we use a more conservative 50-50 split, consistent with the position recommended by the President's Commission to Strengthen Social Security (see *Cogan/Mitchell*, 2003). Some financial advisers propose that investors hold equities equal to 100 minus their age; see *Canner* et al. (1997) or *Vora/McGinnes* (2000). The number 100 could (probably) be justified with the maximum age used in most population mortality tables but we note that annuitant mortality tables generally have a maximum age of 10-15 years higher.

When focusing on risks and benefits, the computations either assume that the retiree is alive, or conversely we evaluate the bequest potential if the retiree is assumed to pass away at a specific age. To do so, we specify an exogenous structure on the ex-ante probability distribution governing the financial uncertainty of future returns and estimate the parameters of such a model from independent (e.g. yearly) historical observations of real returns. With such a model in place, it is possible to look into the future and compute the expected benefit payments and different shortfall-risk measures of the four withdrawal plans in which we are interested. Implementing it relies on the assumption that the stochastic specification of the asset values in the retirement account follows Geometric Brownian motion, a standard assumption in financial economics (which can be traced back to *Bachellier*,1900). This implies that the yearly log-returns are i.i.d. and normally distributed. We also use German historical time series over the period 1967-2002 for the German Equity Index (DAX) and the German Bond Index (REXP) as proxies for stock and bond investments. The DAX represents an index portfolio of German blue-chip stocks, and the REXP represents a portfolio of German government bonds. Each of these indices is adjusted for capital gains as well as dividends and coupon payments (on a pre-tax basis). To account for potential administrative costs, we subtract the equivalent of 0.5% p.a. from the yearly portfolio return.[6] Subsequently, asset returns are adjusted for inflation by using the German Consumer Price Index.

These yearly data produce estimates (before taxes) for the real log average rate of return, the volatility and the correlation-coefficients of stocks and bonds as reported in Appendix C: Since we assume normally distributed log returns, i.e. $I_t = \ln(1 + R_t) \sim N(\mu, \sigma)$, these parameters imply a real log average rate of return on the fifty-fifty stock-bond portfolio of $\mu = 5.52$ percent with a standard deviation of $\sigma = 13.78$ %. Note that this produces an expected gross rate of return of $E(1 + R_t) = E[\exp(I_t)] = \exp[0.0552 + 0.5{*}0.1378^2] = 1.066$.

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[6] *Feldstein/Ranguelova/Samwick* (2001, p. 60) use a similar procedure to account for administration costs.

Assuming that the normality property also holds for the log portfolio returns,[7] it is straightforward to develop an analytical closed form solution for the probability distribution of future benefits of the different variable phased withdrawal rules since the intertemporal budget constraint given in equation (4) is (log)linear (see Appendix B for details). However, because the value of the retirement accounts value might hit zero, the intertemporal budget constraint in equation (2) for the fixed benefit rule is not (log)linear, and future benefits are path-dependent. Hence, even under the assumption of independent and identically distributed log portfolio returns, for the fixed benefits withdrawal plan the probability distribution of future benefits is unknown. Therefore, to obtain estimates for the different risk and return measure we use Monte-Carlo simulation to generate a large number (i.e. 100,000) of paths for the evolution of the withdrawal plan.[8]

3.2 Analysis of Expected Benefits

Figure 1 depicts the Expected Benefits profiles conditional on survival, for the four phased withdrawal rules, as compared to the annuity profile. Focusing on the fixed benefit rule shows that in the first year, mean benefits are (by construction) equal to the annuity benefit. However, in the following years, the expected payments from the plan are decreasing, reflecting the risk of running out of money. The fixed fraction rule also starts with a benefit equal to the life annuity payout, and after that, mean benefits slightly rise as the retiree ages. This is due to the fact that the pension account's expected gross rate of return is 6.66% p.a., which exceeds the constant benefit-to-wealth-ratio of 5.82% p.a. (i.e. 1.066*(1-0.0582) = 1.004 > 1).

By contrast, the 1/T rule pays a much lower expected benefit up to the age of 80, but thereafter, the expected benefit rises extremely quickly and to very high levels, reaching almost 700% of

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[7] This assumption is widely used in strategic asset allocation (e.g. *Feldstein/Ranguelova/Samwick* 2001 or *Campbell/Viceira* 2002) and can be justified by a Taylor approximation of the nonlinear function relating log-individual-asset returns to log portfolio returns. For full details see *Campbell/Viceira* 2002 p. 28-29 and *Campbell/Viceira* 2001.

[8] *Milevsky/Robinson* (2000) have developed an analytical approximation method based on moment-matching techniques and the reciprocal gamma distribution and therefore can avoid Monte Carlo-simulation.

the annuity payment late in life. This can be explained by the low withdrawal fractions of this rule during the first part of the retirement plan. Up to age 95, the benefit-to-wealth ratio is lower than the expected rate of return (i.e. 6.66%); consequently, the expected value of the pension assets grows over time. "Reserves" built up in earlier ages can be used to increase the expected benefits in later years. The 1/E(T) rule starts at a level of about 85% of the annuity payment and increases to 100% if the retiree reaches age 70. This payout approach reaches its maximum expected payment of about 150% at age of 83. After this age, the expected payments are monotonously decreasing, reaching the level of the life annuity at age 91. At ages older than 100, following the 1/E(T) rule would leave the retiree very exposed to quite low benefits, asymptotically approaching 0. Note that the withdrawal fraction under the 1/E(T) rule is higher than under the 1/T rule. Only for the first six years of the retirement plan will the benefit-to-wealth ratio be lower than the expected return earned on pension assets. If the retiree survives until age 71, his expected lifetime is about 15 years, resulting in a withdrawal fraction of 6.66% which is about the same as the expected rate of return. Beyond that age, the withdrawal fraction grows ever larger than the expected asset returns backing benefit payments. For some time (i.e. up to age 83), the increasing withdrawal fractions produce increasing expected benefits. But because less and less wealth is left in the fund, at some point (here age 83) the expected benefit amounts decrease although the withdrawal fraction increases.

3.3. Shortfall Risk Analysis

3.3.1 Shortfall Probability

In accordance with other fields of research, as well as with conventional wisdom, shortfall risk is associated with the possibility of "something bad happening", in other words, falling below a required target return. Returns below the target (losses) are considered to be undesirable or risky, while returns above the target (gains) are desirable or non-risky. In this sense, shortfall-risk-measures are called “relative” or “pure” measures of risk.[9] To analyze this risk in the case of our phased withdrawal strategies, we employ several different shortfall risk measures. We begin with the shortfall probability, defined as:

$$SP(B_t) = P(B_t < z). \quad \textbf{(10)}$$

This measures the probability that the periodic withdrawal B_t is smaller than a chosen benchmark z, which is here the payment provided by the life annuity.

Figure 2 depicts the Shortfall Probability for the fixed benefit rule, the fixed fraction rule (5.82%), the 1/T approach and the 1/E(T) rule, as compared to the annuity benefit. In the first year, all the strategies except the fixed benefit program face a high probability of shortfall, and the only reason the fixed benefit approach does not is that it is set by construction to pay the initial annuity value as long as the funds have not been exhausted. The fixed benefit program offers a Shortfall Probability close to zero at the beginning of the retirement period, but this risk metric begins to rise over time, reaching about 20% around age 85. By contrast, both the 1/T and 1/E(T) rules have very high shortfall probabilities early in the retirement period. This is because a retiree investing his assets in a mutual fund hoping to generate the same payment offered by the life annuity must withdraw about 6.50% of the fund annually. But the withdrawal fractions under the 1/T and the 1/E(T) rules are smaller early in retirement, meaning that the wealth remaining grows quickly. Consequently the shortfall probability declines over time, though the withdrawal fraction is growing. The retiree that withdraws a fixed fraction each year faces a risk profile that is remarkably high for all

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[9] The concept of shortfall risk was introduced in the area of finance by *Roy* (1952) and *Kataoka* (1963), and it was expanded and theoretically justified by *Bawa* (1975) and *Fishburn* (1977, 1982, 1984). It was widely applied to investment asset allocation by *Leibowitz/Bader/Kogelman* 1996 and used by *Leibowitz/Kraske*r 1988 and *Maurer/Schlag* 2003 among others to judge the long term risk of stocks and bonds. In addition *Libby/Fishburn* 1977; *Kahneman/Tversky* 1979; *Laughhuun/Payne/Crum* 1980 and *March/Shapira* 1987 show that in empirical business decision-making, many individuals judge the risk of an alternative relative to a reference point.

ages. In early years, the probability of receiving a benefit below the benchmark life annuity is about 50%, gradually increasing to about 60% at the end of the period.[10]

Another interesting finding has to do with the gradient of the Shortfall Probability under the 1/E(T) rule. Early in the retirement period there is a very fast decline in this risk, but if the retiree is still alive at age 83, the SP begins to rise very quickly due to the special construction of this spending rule. In contrast to the 1/T rule, expected payments at the beginning of the plan are already higher, meaning that few "reserves" are built up in the beginning of the plan. Also, the 65-year-old retiree has an expected remaining lifetime of 19 years, and his expected remaining lifetime decreases over time, especially after the age of 80. The shorter is the remaining expected lifetime, the more wealth will be withdrawn in the 1/E(T) case. As the withdrawal fractions increase, less and less wealth is left in the fund; at some point, wealth remaining is insufficient to provide high enough payments, so the shortfall probability again increases.

3.3.2 Shortfall Measures That Incorporate Severity

As *Bodie* (2001: 308) notes, a major shortcoming of the popular SP risk metric is that it "completely ignores how large the potential shortfall might be." The shortfall probability answers the question "how often" consumption falls short, but not "how bad" the loss is if it occurs, under each of the different withdrawal rules. To provide information about the potential extent of a shortfall, we next calculate the Mean Excess Loss (MEL) as an additional risk measure. Formally this risk metric is given by:

$$\mathrm{MEL}(B_t)=\mathrm{E}[\, z - B_t \mid B_t < z \,]. \tag{11}$$

[10] This results from the lognormal distribution of future benefits which become increasingly skewed to the right, the longer the retiree remains alive. Note that the expected level return (i.e. exp(0.0552 + 0.5*0.1378²) - 1 = 6.66%) of the retirement account assets is greater than the withdrawal fraction (i.e. 5.82%), but the median level return (i.e. exp(0.0552) - 1 = 5.68%) is slightly below the withdrawal fraction, so the shortfall probability rises with age.

It indicates the expected loss with respect to the benchmark, under the condition that a shortfall occurs. Therefore, given a loss, the MEL answers the question "how badly on average" does the strategy perform; it MEL can be characterized as a 'worst case' risk measure, which is highly sensitive with respect to realisations at the tail of the distribution (i.e. large-scale shortfalls). [11]

An additional shortfall risk measure that links both the probability and the extent of the conditional shortfall in an intuitive way is the Shortfall Expectation (SE):

$$\mathrm{SE}(B_t) = \mathrm{E}[\max(z - B_t, 0)] = \mathrm{MEL}(B_t) \cdot \mathrm{SP}(B_t). \qquad \textbf{(12)}$$

The shortfall expectation is the sum of losses weighted by their probabilities, and hence it is a measure of the unconditional "average loss". As equation (17) shows, the SE is simply the product of the shortfall probability and the mean level of shortfall given the occurrence of a shortfall.[12]

In Figure 3 we plot the Mean Excess Loss results for the various withdrawal strategies of interest, namely the fixed benefit rule, the fixed fraction rule (5.82%), the 1/T approach and the 1/E(T) rule. Here we compare the MEL for each tactic versus the annuity benefit. Results are similar in form: that is, in the first year, all strategies but the fixed benefit program have a positive MEL, since the fixed benefit approach pays for as long as possible an amount equal to the initial annuity. The 1/T rule has a particularly high initial MEL, at 60% of the value of the annuity payment, and this falls only to 30% some 15 years into the retirement period. Both the 1/E(T) and the fixed fraction rules have 30% MEL profiles through about age 90, but then the 1/E(T) rule confronts the retiree with a rapidly rising mean excess loss attaining close to 100% late in life. By contrast, the 1/T

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[11] The MEL is closely connected with the Tail Conditional Expectation (TCE), which is given by TCE = $\mathrm{E}(R \mid R < z)$ = z–MEL. The TCE has some favourable features, e.g. it is (in contrast to the shortfall probability) a coherent risk measure with respect to the axioms developed by *Artzner* et al. (1999).

[12] In addition, the SE is related to the price of a (derivative) financial contract which allows the annuitant to transfer the downside risk of a phased withdrawal plan into the capital market. For example, if the retiree buys a put option paying $P_t = \max(z - B_t, 0)$ at time t, then he is completely hedged against the risk that the benefit from the withdrawal plan is lower than the payment from the benchmark annuity z. Note that the future benefits are directly related to the market value of the retirement accounts assets V_t. Using standard arguments from option pricing theory, the price of such a (European) put option is given by $p_0 = \mathrm{E}_Q[\max(z - B_t, 0)]/\exp(R_f \cdot t)$, where E_Q denotes the expectation operator with respect to the risk adjusted ("martingale") probabilities and R_f is the risk free interest rate.

plan faces the retiree with a gradually declining expected loss after age 90, falling to about 30%. The results make clear that point that from a worst-case risk perspective, the fixed fraction rule, and the 1/E(T) rule, are not proper financial instruments for insurance against longevity.

The profiles for the Shortfall Expectation appear in Figure 4, and it will be recalled that these combine the Shortfall Probability and the Mean Excess Loss, all conditional on survival. This graph underscores the patterns revealed by the two previously analyzed risk measures. Now the fixed benefit rule has a very low Shortfall Expectation through about age 83, whereas the 1/T rule is initially the riskiest with a 60% SE. It takes a very long time until the SE of the 1/T rule declines to a negligible level – older than age 90 for the hypothetical individual under study. The fixed fraction and the 1/E(T) rules have a SE of less than 20% through at least age 80, but the 1/E(T) rule again traces out what is perhaps unexpected behaviour – after falling to low levels through about age 84, the risk begins to rise substantially 20 years after retirement, and it has the highest expected short-fall for the long-lived individual.

3.4. Analysis of Expected Bequests

The other side of the story behind these rules is that the retiree must in effect compare his own consumption with the potential value of the bequest that would go to the heirs if he should die. Figure 5 illustrates the expected bequest under the various formulations, conditional on death. The pattern exhibiting most stability is the fixed fraction rule, but the other three are highly divergent. For example, the 1/T rule shows an interesting path, first rising in the early retirement period when withdrawals are small. About 35 years after retirement, however, the expected bequest begins to decline very quickly – a fact that is directly attributable to the construction of this plan. The older a retiree gets, the more he or she withdraws from his account: thus five years before the plan ends, the retiree withdraws 1/5=20% of the remaining wealth. If the retiree should by chance live beyond age 110, this approach offers no continued payment or bequest potential. The 1/E(T) rule also offers only a very low bequest potential after reaching a limiting age. In contrast with the 1/T rule, however, the 1/E(T) plan offers lower expected inheritance at every age. Particularly if the retiree does not die until 20 years into retirement, the inheritance will decline dramatically.

4. Risk-Minimizing Phased Withdrawal Strategies

4.1. Optimized Withdrawal Rules in a Risk-Return Context

Thus far, our analysis has assumed that the retiree holds his pension plan assets in a fixed-weight portfolio comprised of 50% stocks and 50% bonds over a fixed investment horizon. Thus the payouts during retirement take into account only capital market uncertainty, and there was no possibility of optimization around risk/reward tradeoffs. In this section, we extend the analysis by including a consideration of mortality risk, and further we discuss two additional phased withdrawal rules that permit the retiree to optimize the design of the withdrawal patterns. In the next subsection, our analysis varies the investment weights of the associated with stock, bonds, and cash investments, to attain a "risk-minimizing" static asset allocation. The portfolio weights are therefore determined endogenously (excluding short-selling), following *Albrecht/Maurer* (2002). In the following subsection, we go on to examine the impact of mandatory shifting to annuitization at a specific age. This is currently required in tax qualified German *Riester* plans at the age of 85 and for UK income drawdown plans at the age 75. In both countries, the restriction of mandatory switching has already considerable criticism in the public debate (c.f. *Blake* et al. 2003; *Börsch-Supan/Wilke* 2003).

To evaluate how the relative ranking of the alternative withdrawal rules might change with an endogenous asset mix in the retiree's investment fund and other plan design parameters, it is useful to define the expected present value of the shortfall, called here "EPVShortfall":

$$EPVShortfall = \sum_{t=0}^{l-x} \frac{{}_t p_x SE(\mathrm{B_t})}{(1+R_f)^t} \tag{13}$$

Here, $SE(B_t) = E[\max(z - B_t, 0)]$ denotes the expected shortfall with respect to the target z, which is equal to the benefit provided by the benchmark life annuity. The possible expected shortfall in year t are weighted by the conditional probability ${}_tp_x$ that a man aged x at the beginning of the retirement phase is still alive, in the case when a shortfall occurs. All possible expected shortfalls are discounted back to the beginning of the retirement period using the risk-free interest rate R_f (i.e. assuming a flat term structure of real interest rates) and summed over the maximum length of the mortality table used. This useful summary measure of the risk associated with a phased withdrawal strategy may be interpreted as the lump sum premium that would be required for the retiree to transfer this shortfall risk to an insurer, assuming actuarially fair pricing and no additional loading.[13] Given this function, we minimize it with regard to asset allocation and other plan design parameters, to derive the asset allocation patterns most amenable to alternative withdrawal rules.

Previous studies, most notably *Milevsky* (1998, 2000) and *Albrecht/Maurer* (2002), approach the issue of optimal fixed benefits withdrawal rules by adopting the criterion of controlling the probability of a consumption shortfall in retirement. On the other hand, as we have argued, this perspective does not account for the size of the loss when it happens, which our risk measure does.

To extend the approach, we adopt two additional reward measures associated with each optimized phased withdrawal strategy, namely, the expected present value of benefits received during life (EPVBenefits) and the expected present value of bequests at death (EPVBequest). These are defined, respectively, as:

$$EPVBenefits = \sum_{t=0}^{l-x} \frac{{}_tp_x E(\mathrm{B}_t)}{(1+R_f)^t}, \text{ and} \qquad \textbf{(14)}$$

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[13] Note, if $SE(B_t)/(1+R_f)^t$ is calculated with respect to the corresponding risk adjusted ("martingale") probabilities (consistent with an arbitrage free capital market) of the underlying asset process, it is consistent with the price of an European put option which pays the difference if a shortfall happens in year t after retirement. Then the EPVShortfall is the value of a portfolio consisting of (European) put options weighted by survival probabilities ${}_tp_x$.

$$EPVBequest = \sum_{t=1}^{l-x} \frac{{}_{t-1}p_x q_{x+t} E(\mathrm{V_t})}{(1+R_f)^t} \quad \textbf{(15)}$$

Here, the EPVBenefits is similar defined as the money worth concept used by *Mitchell* et al. (1999) and reflects the expected present value of benefit payments conditional on survival. Finally, EBVBequest measures the expected present value of the inheritance that the retiree would pass on to heirs, in the event of death.

We implement these metrics instead of adopting a specific utility function for several reasons. First, these risk measures are consistent with expected utility analysis, since they are the primitives that enter into utility maximizers' objective functions.[14] Any particular functional form must embody specific tradeoffs between risk and return components, whereas our approach can remain agnostic about the specific weights attached to each (*Sarin*/*Weber* 1993). Second, risk minimization is consistent with many studies in the literature (c.f. *Albrecht/Maurer*, 2002; *Chen/Milevsky* 2003; and *Milevsky*, 1994), and it is also consistent with conventional wisdom offered by money managers and financial planners when providing advice regarding retirement income payouts (c.f. *Ameriks,* 2002*, Ameriks et al.,* 2001; *Ibbotson Associates*, 2003).

The specific optimized rules we propose are two: a "Fixed Percent Optimized" rule, and a "1/T Optimized" rule. The first (Fixed Percent Optimized) rule minimizes the expected present value of the shortfall by selecting jointly the optimal constant withdrawal fraction and the retiree's asset allocation. This contrasts from our earlier constant withdrawal rule, by endogenizing the withdrawal fraction. Compared to the non-optimized Fixed Percent rule, we expect that allowing two

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[14] For example, assume that the individual trades off expected benefit payments versus the expected shortfall vis a vis the benchmark annuity z, i.e. $\Phi(B_t)=E(B_t)-kE[\max(z-B_t, 0)]$ with risk aversion parameter $k>0$ (and ignoring bequests). This risk value model is consistent with a utility function suggested by *Fishburn* (1977) of the form $u(x)=x$ if $x\geq z$ resp. $u(x)=x-k(z-x)$ if $x<z$. As *Bawa* (1975, 1978) has shown, the mean/SE-optimization model studied here corresponds with the concept of second order stochastic dominance. To allow tradeoffs between EPVBenefit and EPVShortfall, we make the usual assumption that the individual's objective function is given by a time separable utility function of the *Fishburn* type, and that the individual's time preference is equal to the risk-free real interest rate. See also Brachinger/Weber (1997) for risk as a primitive.

parameters, the fraction consumed as well as the asset allocation, will be more successful in controlling both mortality and capital market risk. The second rule, denoted as "1/T Optimized" minimizes the EPVShortfall by selecting jointly the maximum duration of the plan conditional on survival, along with the asset allocation. We expect that the 1/T Optimized rule will permit more consumption when the probability is high that the retiree remains alive, as compared to the non-optimized 1/T rule, but it will also offer lower expected bequests.

4.2. Comparative Results: Annuity versus Phased Withdrawal Plans

Table 1 reports results for the various withdrawal rules of interest here allowing optimized asset allocation. These may be compared to results for the benchmark case of a life annuity benefit which appear in Row 1: a 65-year old male who pays €100 for an immediate real annuity will receive annual benefits of €5.82 for life (at the beginning of each year). By construction, both the EPVShortfall and EPVBequest are zero for the annuity purchase; the EPVBenefits measure is slightly below €100 due to the annuity load assumed. In Row 2 we report results for a phased withdrawal program where the Fixed Benefit is equal to the annuity at €5.82 as before; of course, the retiree may run short of funds since he is not actually annuitizing. The optimized asset allocation associated with minimizing the EPVShortfall for this Fixed Benefit withdrawal plan consists of 20% stocks and 80% bonds, and associated with this plan is an expected shortfall worth €3.58 per €100 of initial assets. As long as the retiree lives, he can expect benefits totalling €93.41 (in present value), or about 4.3% below the real annuity. Of course, on other hand, the bequest his heirs can expect is quite large, at €53.2 (or more than half the initial investment). Clearly, unless a retiree had an enormous taste for bequests, annuitization would be judged far superior to taking a fixed benefit at 5.82€ per annum until the fund is probably exhausted. Rows 3 and 4 of Table 1 displays results for two Fixed Percentage strategies. The first is determined by selecting a Fixed Percentage rule that pays out a first-year benefit equivalent to the €5.82 real lifelong annuity purchased by the 65-year old male paying €100. Given this constant benefit-wealth-ration (i.e. $\omega = 5.82\%$) we solve for the optimal asset mix minimizing the EPVShortfall.

The second strategy selects a fixed fraction that is now also optimized with regard to EPVShortfall. What is different here is that both the asset allocation and the withdrawal fraction is simultaneously optimized at the beginning of the retirement phase. These two rows indicate is that, in both cases, the risk measured by the EPVShortfall is almost four times as large as under the Fixed Benefit approach. Offsetting this could be the higher benefit stream conditional on survival and higher bequest value to the heirs. Both Fixed Percentage strategies have slightly higher equity exposures (30%) than the Fixed Benefit approach (20%). This is in contrast to the high equity exposures recommended by *Albrecht/Maurer* (2002) and *Vora/McGinnes* (2000) using a fixed benefit withdrawal approach. Of course the optimized strategy that permits a fixed percentage payout of 7% of the account annually has a lower expected shortfall and higher expected benefits than the non-optimized strategy.

Next we turn to the two 1/T rules, where again the first simply sets T to the maximum plan duration (the oldest age in the mortality table), and optimizing asset allocation with minimizing EPVshortfall. The second endogenously evaluates both, the asset allocation and the plan duration that minimizes EPVShortfall. It is interesting that the simple 1/T rule (Row 5) results in the highest equity exposure, and it is also unlikely to be preferred by many: the size of the expected shortfall is the largest of those considered (€35 of the initial €100 asset), and the expected benefits are the lowest of those examined. The only clear gainers are likely to be the heirs. We contrast this with the pattern that would result from optimizing the maximum plan duration, which the retiree could do if he had Social Security or welfare to live on in the event that his asset is extinguished and he is still alive. This would occur around age 87, according to the program computed. Row 6 indicates using the 1/T rule optimized for asset allocation and the date of running out of assets offers lower risk, higher expected than the annuity, a reasonable bequest, and the asset allocation is not too risky (15% equity, 75% bonds, and 10% cash).

Finally we examine Row 7 which refers to the 1/E(T) rule, which is consistent with the phased withdrawal scheme allowed by the US tax authority for 401(k) pension plans. This is an interesting strategy, because it offers quite low expected shortfalls, and 6% higher expected benefits as compared to the life annuity, while still affording a decent bequest potential. The asset allocation implied is rather conservative, with 20% in equity and 80% in bonds. Overall, looking across the phased withdrawal plans, there is no clearly dominant strategy, since all involve tradeoffs between risk, benefit, and bequest measures, and individual preferences may vary. Nevertheless, the 1/E(T) rule seems relatively appealing as compared to the others, as long as the retiree has only a moderate appetite for bequests.

The second panel of Table 1 reports results for a female age-65 retiree considering the same phased withdrawal patterns. To summarize results, we find that women confront lower expected shortfall risks in all cases and can anticipate higher EPVBenefits. This occurs because the lower female mortality translates into a lower initial annuity payment; i.e. her actuarially fair benefit is €5.02 per year for a €100 purchase (versus the male payout of €5.82). Consequently, variable withdrawal plans have the woman withdraw less early in life, leaving more assets in the fund to earn future capital market returns. Since the woman also is expected to live longer, she will more likely be alive to reap the fruits of the investment. We would therefore predict, and the results confirm, that the 1/E(T) rule is more attractive to women than men, since it offers rather low expected shortfalls, and 15% higher expected benefits as compared to the annuity, while still affording a decent bequest potential. It is also interesting that the asset allocation strategies for women are similar to those for men but do have slightly lower equity exposure overall.

Thus far, the analysis has assumed the retiree begins the payout phase at age 65, but it may be of interest to explore how the phased withdrawal patterns behave with alternative retirement ages. Table 2 displays the findings for a male retiring at age 60 or age 70, which can be directly compared with the top panel of Table 1. What the results show is that the phased withdrawal patterns are unambiguously more attractive for an age 60 retiree, as compared to the 65 year old. In other words, all expected shortfall risk measures are lower, expected benefit payouts to the living retirees are higher, and expected bequests are similar; the portfolios selected are slightly lighter in equities. This is because the mortality drag for the life annuity purchased by a younger person, and therefore the benchmark, is substantially lower. By contrast, the higher mortality faced by a 70-year old retiree produces a higher benchmark annuity which translates into greater EPVShortfalls, lower expected benefits, and also lower expected bequests. This is despite having 10-15% higher equity exposure. This would lead one to conclude that annuitization would be relatively more appealing to older retirees, as compared to phased withdrawal patterns.[15]

Thus far we have assumed that the annuity benchmark is computed in each case using the sex-specific mortality tables relevant to the individual making the purchase. Nevertheless, in some contexts, insurers are required to use a unisex table when selling annuities: for example, this is the case in the United States if an annuity is purchased using accruals in an employer-based defined contribution plan (*McGill* et al. 1996). Likewise in the UK, unisex tables are used to price annuities in the Personal Pension arrangements. A unisex mortality table is generated by averaging mortality probabilities for men and women at each age. Naturally, using a unisex table slightly boosts the annuity paid to a female retiree and slightly reduces the male's benefit, as compared to using sex-specific tables. Thus when German mortality tables are used to value unisex payouts (as in Appen-

[15] Similar conclusions apply to women, though the differences by retirement age are less pronounced; results are available on request.

dix A), the payout for a female from a €100 annuity purchase would rise by 7% from €5.02 to €5.37, whereas for the male it would decline by 7.7% from €5.82. Using a unisex table for annuitization would obviously change the benchmark for comparisons with phased withdrawal plans. Yet the phased withdrawal plans would still embody the sex-specific mortality patterns relevant to the individual decisionmaker.

Results using the unisex table for the annuity benchmark appear in Table 3. The annuity benefit is now equal by construction for men and women, at €5.37 annually for a €100 purchase. As a result, the female annuitant would clearly do better than she would under the sex-specific table, and the male would do worse. One surprise is that the results are less clearcut under the phased withdrawal patterns. For men, expected shortfalls under all withdrawal patterns are lower, expected benefits are lower, and bequests are higher. The pattern is the opposite for women: expected shortfalls are higher, expected benefits are higher, and bequests lower. Hence when a government requires unisex tables for annuity pricing, a woman who elected a phased withdrawal plan would be exposed to greater risk. It is interesting that this might be an unexpected and undesired result for those advocating unisex tables in pension plans.

4.3. Phased Withdrawal Plans with Mandatory Deferred Annuities

The results above suggest that some retirees might prefer to engage in a mixed strategy: that is, to undertake phased withdrawals during the early portion of the retirement period, and then to switch to an annuity thereafter. Furthermore, some researchers have suggested that such a mixed strategy would be attractive: it enhances the payout early on, in exchange for relatively low risk, and it also adds the insurance feature later in life (*Blake* et al. 2003; *Milevsky* 1998). In addition, some governments have recently required that the elderly annuitize after a phased income drawdown period. For example, in the UK, compulsory annuitization is required at age 75, and German Riester Plans require annuitization at age 85.

To examine the risks and rewards associated with phased withdrawal followed by annuitization at some later age, we revisit our calculations under each withdrawal rule, assuming annuitization is required if the individual is still alive at either age 75 or 85. Two approaches are considered. In the first case, which we call the "*switching strategy*", a retiree would follow the relevant phased withdrawal rule until reaching a mandatory switching age. In all cases, for the benchmark, we use the real annuity that the retiree could have purchased at age 65, to compare our new results with prior findings. If, at the switching point, the fund is inadequate to purchase this real annuity, the gap represents a shortfall; conversely, if the account holds more than is needed to buy the benchmark annuity, this excess can be allocated to increase the bequest or used for higher consumption. In the following, we assume that an excess (if any) is used to increase the level of the annuity starting at age 75 or 85, enhancing the EPVBenefits rather than EPVBequest measure.

For the second case, we examine an "*immediate purchase deferral strategy*". In this case, the retiree purchases an annuity on retirement, with deferred payouts beginning at age 75 (or 85). The deferred annuity benefit is set equal to the benchmark that the retiree could have received if he initiated annuity payments at age 65. It is worth noting that it is unclear what one might expect from these switching strategies, in terms of risks and rewards. Some analysts suggest that switching may be a preferred strategy, relying on the fact that the mortality drag rises with age, so annuities pay out more for a given premium, the older one is when purchasing them (*Milevsky*, 2001). On the other hand, this work focuses only on the probability of a shortfall but does not weight the size of the loss, conditional on the shortfall occurring. By delaying annuitization, the retiree can benefit from capital market returns if they are favourable, so benefit payments can be higher while he lives, or bequests higher if he dies. Yet delaying annuitization also exposes him to shortfall risk.

In Table 4 we indicate findings for a male retiring at age 65, making the decision to switch from a phased withdrawal to an annuity at either age 75 (or 85).[16] Comparing results in Panel A of Tables 1 and 4, we see that if delayed annuitization is available, this generally increases the EPVBenefits and shrinks the EPVShortfall, both of which are beneficial. The EPVBequest falls, indicating that the deferred annuitization strategy is likely to be most attractive to those seeking to secure consumption while alive, without completely stripping the heirs of some promised funds. In other words, the risk/return profile of the phased withdrawal plan that includes a delayed annuity is enhanced, as compared to no annuity, at the cost of a smaller bequest potential. Also interesting is the fact that switching to an annuity later in life (i.e. at age 85; compare panels A and B in Table 4) raises the equity share of the portfolio slightly, but greatly enhances the bond exposure. Also, buying the annuity later obviously promise more bequest potential, at the cost of higher shortfall.

Table 5 displays results for a 65-year old male purchasing a deferred annuity at the beginning of the retirement period, with annuity payouts beginning at age 75 (or 85) assuming he is alive. In contrast to the mandatory annuitization strategy, we see that the risk and return profile depends heavily on the chosen withdrawal rule. In the case of the 1/T rule combined with a deferred annuity payable from age 75, the logical strategy is to consume all remaining wealth using the phased withdrawal tactic by age 74, secure in the knowledge that one is protected against longevity risk thereafter. This pattern provides a benefit stream worth slightly more than the real annuity, and it offers low shortfall risk and low expected bequests. This is an important result since it indicates the advantage of allowing flexibility until age 75, paired with protected consumption after that age. Similar results hold if the deferred annuity were to begin at age 85, with slightly higher benefit and bequest levels at the expense of somewhat higher shortfalls. By contrast, the 1/E(T) rule combined with a deferred annuity at age 75 provides the retiree with relatively low payouts up to age 75, producing a

[16] Results for women are available on request.

high EPVShortfall, but after that age, benefits flow from both the annuity and the phased withdrawal plan which raises EPVBenefits (and higher potential bequests). Delaying the annuity payout date to age 85 instead of 75 exposes the retiree to much higher shortfall risk, along with higher possible wealth for the heirs.

5. Discussion and Conclusions

Though standard economic models imply that most people would highly value the protection against longevity risk that annuitization offers, many retirees do not purchase annuities with their disposable wealth. Certainly if older people have no desire to leave a bequest, annuities would seem to be strongly preferred. Yet there is evidence that many older people do intend to leave a bequest: for instance, *Hurd/Smith* (1999) find that more than half of the elderly expect to leave a bequest worth more than $10,000. As a result, there would seem to be great need for models to guide retirees as they examine tradeoffs between consumption versus the possibility of leaving a bequest. Of course such tradeoffs generally require the retired worker to exchange some risk for some return, in which case there is a natural role for phased withdrawal programs during the retirement period.

Taking risk and value as primitives is appealing for several reasons (*Brachinger/Weber*, 1997). First, from a descriptive perspective, a risk-value model such as ours is likely to be useful in explaining retiree preferences by understanding how they trade off expected benefits, bequests, and the risk of consumption shortfalls. Second, policymakers and regulators may benefit from evidence on the risk-return patterns of different withdrawal options in tax-favored individual retirement plans. Of course financial intermediaries offering retirement products such as banks, insurance companies, and mutual funds, can use this information to design and market products that have typical benefit, bequest, and risk features. Finally, professional financial planers may offer better information to their clients when they make retirement investment choices.

Our approach uses the concept of shortfall-risk, whereby the benefits of a life annuity serves as the benchmark, building on research by *Milevsky* (2998, 2001), *Milevesky/Robinson* (1994, 2000), *Milevesky/Ho/Robinson* (1997) and *Albrecht/Maurer* (2002). We extend this research in two directions. First, we use a risk metric which considers both the *probability of a consumption shortfall* as well as the *size of the shortfall* when it occurs. Second, we focus not only on phased withdrawal plans with fixed benefits, but also on variable benefit patterns in conjunction with a predetermined benefit-to-wealth ratio. We evaluate several alternative designs for phased withdrawal strategies, investigating withdrawal rules while allowing for endogenous asset allocation patterns, and allowing the worker to make decisions both about when to retire and when to switch to an annuity. Of course, selecting a specific withdrawal pattern requires further information on utility weights to trade consumption against bequests, but many retirees and their financial counsellors may find it difficult to articulate their utility functions in advance. For this reason we find that it is useful to explore various explicit risk and return measures for alternative withdrawal plans in a stochastic environment, allowing for randomness in both the time of death and investment returns.

We conclude the following:

- Discretionary management of accumulated assets with systematic phased withdrawals for consumption purposes offers many advantages: flexibility, bequests, and possibly higher rates of consumption than can be paid by standard life annuities. But following phased withdrawal plans also requires the retiree to devote attention to asset allocation and withdrawal rules.
- Retirees using a phased withdrawal plan who seek to minimize the risk of consuming less than the real annuity benchmark will allocate their retirement assets more in fixed income than in equities. Nonetheless, which specific mix is elected must depend on plan design, age, mortality risk, and other factors.
- A phased withdrawal strategy paying the same benefit as the annuity exposes one to the risk of outliving one's assets while still alive. A phased withdrawal plan using a fixed benefit-to-wealth ratio avoids the risk of running out of money, since benefits fluctuate in tandem with the pension fund's value. But a fixed benefit withdrawal rule affords lower risk than variable withdrawal rules, if one uses a mortality-weighted shortfall-risk measure (which includes both shortfall probability and magnitude of loss).
- Mandatory deferred annuitization with a fixed withdrawal rule can enhance expected payouts and cut expected shortfall risk but at the cost of reduced expected bequests, as com-

pared to no annuity. For a variable withdrawal plan, a simple deferred annuitization may not reduce risk: rather, it requires optimization of the benefit to wealth ratio.

- As a standalone strategy, the 1/E(T) phased withdrawal rule is appealing since it offers relatively low expected shortfall risk, good expected payouts for the retiree during his life, and some bequest potential for his heirs. But if mandatory annuities are combined with this phased withdrawal plan, the 1/E(T) rule is less attractive.
- The optimized 1/T rule and the fixed benefit rule both have appealing risk characteristics, particularly when combined with a mandatory deferred annuity.
- Unisex mortality tables have been advocated by some who believe they are "fairer" to women in annuity calculations. However, we show that if phased withdrawal plans are available as an alternative, unsex tables used for annuity pricing can make women's expected shortfalls higher, expected benefits higher, and bequests lower under a phased withdrawal program, as compared to annuitization.

These findings have general relevance for national retirement policy. For example, the 1/E(T) rule has been adopted by the US tax authority for the "default" withdrawal pattern in private defined contribution accounts (401k's), and our results show that this is a relatively appealing standard in the US context where annuitization is not mandatory. We also find that mandating annuitization after a phased withdrawal period can be quite appealing in terms of risk, so it is interesting that this approach has recently been implemented in both the UK and Germany. Some degree of mandated annuitization has also proposed for the US by the recent Commission to Strengthen Social Security in the US context (*Cogan*/*Mitchell*, 2003). Our results also imply that a government mandate requiring that unisex tables be adopted for annuity pricing (as in the UK) exposes women who elected a phased withdrawal plan to greater risk. Finally, our results have implications for the asset mix retirees will optimally want to hold: later annuitization (say, at age 85) would imply a larger fraction of the financial assets would be held in bonds.

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Table 1: Results for Risk-Minimizing Phased Withdrawal Strategies

Using Sex-specific Mortality Tables for Annuity Pricing, Allowing Optimized Asset Allocation: Male and Female Retirees

A. Results for Male (Retirement Age 65): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
1. Real Annuity €5.82	0	97.291	0			
2. Fixed Benefit = €5.82	3.579	93.408	53.191	20	80	0
3. Fixed Pct. = 5.82%	12.582	92.528	66.055	30	70	0
4. Fixed Pct. Opt ω = 7.0%	11.303	98.450	52.929	30	70	0
5. 1/T Rule Age 110	34.953	82.680	134.410	50	50	0
6. 1/T Rule Opt. Age 87	15.155	104.439	32.997	15	75	10
7. 1/E(T) Rule	8.271	103.075	39.801	20	80	0
B. Results for Female (Retirement Age 65): Benchmark Real Life Annuity €5.02 p.a./ €100						
8. Real Annuity €5.02	0	97.291	0			
9. Fixed Benefit = €5.02	1.507	95.652	54.188	15	65	20
10. Fixed Pct. = 5.02%	9.246	98.732	70.474	25	75	0
11. Fixed Pct. Opt ω= 6.1%	7.889	105.382	58.535	25	75	0
12. 1/T Rule Age 110	26.554	97.951	122.997	40	60	0
13. 1/T Rule Opt. Age 91	12.279	116.192	32.072	15	75	10
14. 1/E(T) Rule	5.688	113.469	35.482	15	85	0

Notes:
EPV Shortfall: expected present value of future benefit payments below the life annuity (shortfall)
EPV Bequest: expected present value of future bequest payments
EPV Payments: expected present value of future benefit payments

Table 2: Results for Risk-Minimizing Phased Withdrawal Strategies

Using Sex-specific Mortality Tables for Annuity Pricing, Allowing Optimized Asset Allocation: Male Retirees Only

A. Results for Male (Retirement Age 60): Benchmark Real Life Annuity €4.95 p.a/€100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €4.95	0	97.291	0			
Fixed Benefit = €4.95	1.734	95.444	57.367	15	70	15
Fixed Pct. Opt ω=6.0%	7.826	105.931	55.863	25	75	0
1/T Rule Opt Age 88	13.244	116.233	34.711	15	80	5
1/E(T) Rule	6.051	112.150	38.541	15	85	0
B. Results for Male (Retirement Age 70): Benchmark Real Life Annuity €7.03 p.a./ €100						
Real Annuity €7.03	0	97.291	0			
Fixed Benefit = €7.03	6.628	90.086	45.104	25	75	0
Fixed Pct. Opt ω=8.5%	15.450	92.839	50.585	40	60	0
1/T Rule Opt. Age 87	17.601	91.870	30.274	10	60	30
1/E(T) Rule	11.913	93.692	41.185	25	75	0

Note: See Table 1

Table 3: Results for Risk-Minimizing Phased Withdrawal Strategies

Using Unisex Mortality Tables for Annuity Pricing, Allowing Optimized Asset Allocation: Male and Female Retirees

A. Results for Male (Retirement Age 65): Benchmark Real Life Annuity €5.37 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.37	0	89.871	0			
Fixed Benefit €5.37	1.946	87.918	56.332	15	80	5
Fixed Pct. Opt ω=6.6%	8.130	94.569	54.878	25	75	0
1/T Rule Opt.Age 88	11.588	98.398	34.376	10	65	25
1/E(T) Rule	5.140	100.679	38.650	15	85	0
B. Results for Female (Retirement Age 65): Benchmark Life Annuity €5.37 p.a./ €100						
Real Annuity €5.37	0	104.206	0			
Fixed Benefit €5.37	2.776	100.986	51.264	15	85	0
Fixed Pct. Opt ω=6.4%	10.716	109.501	53.819	30	70	0
1/T Rule Opt Age 91	15.531	118.108	32.643	15	85	0
1/E(T) Rule	8.567	116.704	36.822	20	80	0

Note: See Table 1

Table 4: Results for Risk-Minimizing Phased Withdrawal Strategies Allowing Switching to Life Annuities

Using Sex-specific Tables for Annuity Pricing, Allowing Optimized Asset Allocation and Withdrawal Fraction: Male Retirees Only

A. Results for Male (Retirement Age 65 Switching Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.82	0	97.291	0			
Fixed Benefit until 75	0.934	100.321	12.590	5	25	70
Fixed Pct. Opt ω=6.8%	2.820	104.098	12.595	10	45	45
1/T Rule Opt Age 83	2.893	103.894	12.814	10	40	50
1/E(T) Rule	3.210	101.109	13.090	5	35	60
B. Results for Male (Retirement Age 65 Switching Age 85): Benchmark Life Annuity €5.82 p.a./ €100						
Real Annuity €5.82	0	97.291	0			
Fixed Benefit until 85	2.819	103.425	33.575	15	80	5
Fixed Pct. Opt ω=7.4%	7.400	108.844	32.235	25	75	0
1/T Rule Opt Age 88	9.521	108.265	35.141	20	80	0
1/E(T) Rule	5.406	104.143	31.194	15	75	10

Note: See Table 1

Table 5: Results for Risk Minimizing Phased Withdrawal Strategies with Immediate Purchase of Mandatory Deferred Life Annuities

Using Sex-specific Tables for Annuity Pricing, Allowing Optimized Asset Allocation and Withdrawal Fraction, Male Retirees Only

Strategy	EPV-Shortfall	EPV-Benefits	EPV-Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Results for Male (Retirement Age 65 annuity deferred up to Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Real Annuity €5.82	0	97.291	0			
Fixed Benefit until 75	0.55	97.426	3.606	5	15	80
Fixed Pct. Opt ω=15.3%	9.267	106.984	8.457	50	50	0
1/T-Rule Opt Age 74	1.242	99.810	3.590	5	20	75
1/E(T) Rule	21.773	121.689	31.474	85	15	0
Results for Male (Retirement Age 65 annuity deferred up to Age 85): Benchmark Life Annuity €5.82 p.a./ €100						
Real Annuity €5.82	0	97.291	0			
Fixed Benefit until 85	1.850	101.352	20.551	10	55	35
Fixed Pct. Opt ω= 8.7%	11.008	104.750	34.698	35	65	0
1/T-Rule Opt Age 84	7.074	106.822	21.387	15	85	5
1/E(T) Rule	10.624	102.280	34.094	20	80	0

Figures

Figure 1: Mean Benefit of Withdrawal Plan Conditional on Survival (50% Equities / 50% Bonds): Life Annuity Benchmark

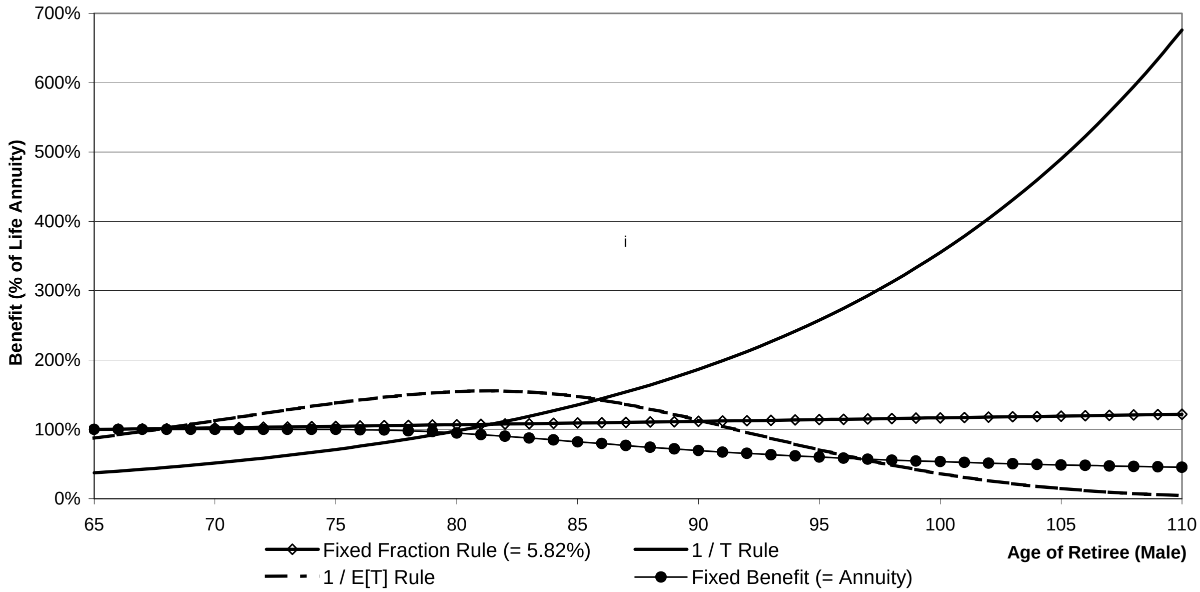


Figure 2: Shortfall Probability of Withdrawal Plan Conditional on Survival (50% Equities / 50% Bonds): Life Annuity Benchmark

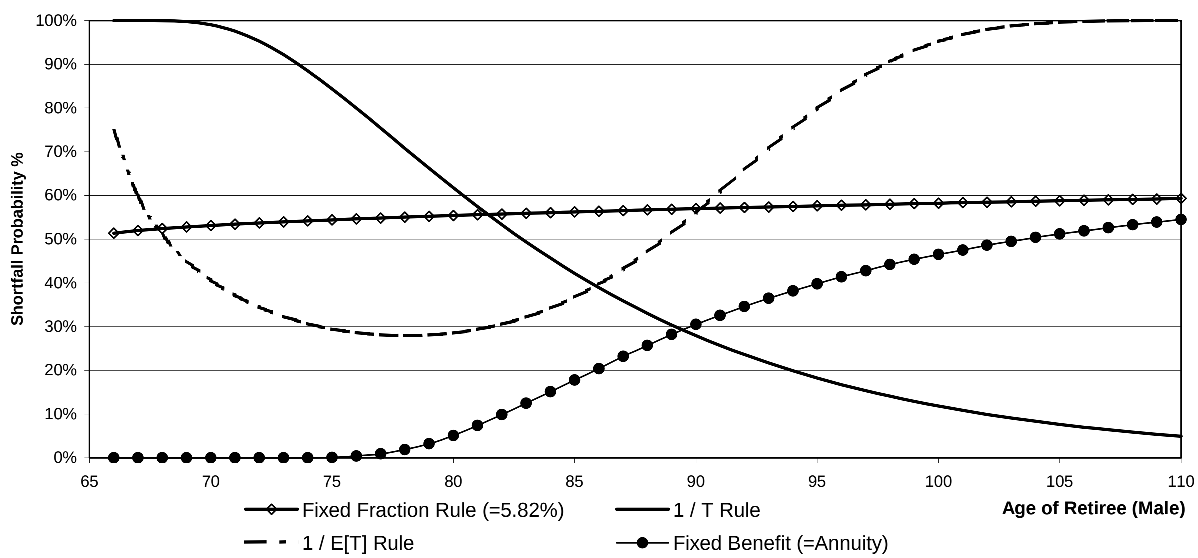


Figure 3: Mean Excess Loss of Withdrawal Plan Conditional on Survival (50% Equities / 50% Bonds): Life Annuity Benchmark

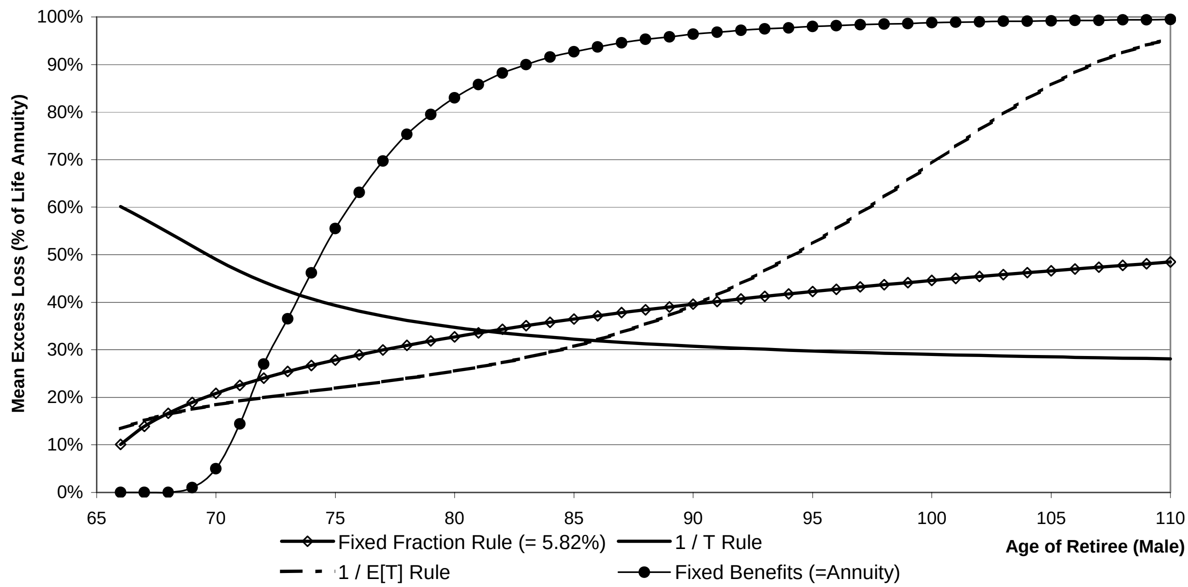


Figure 4: Expected Shortfall of Withdrawal Plan Conditional on Survival (50% Equities / 50% Bonds): Life Annuity Benchmark

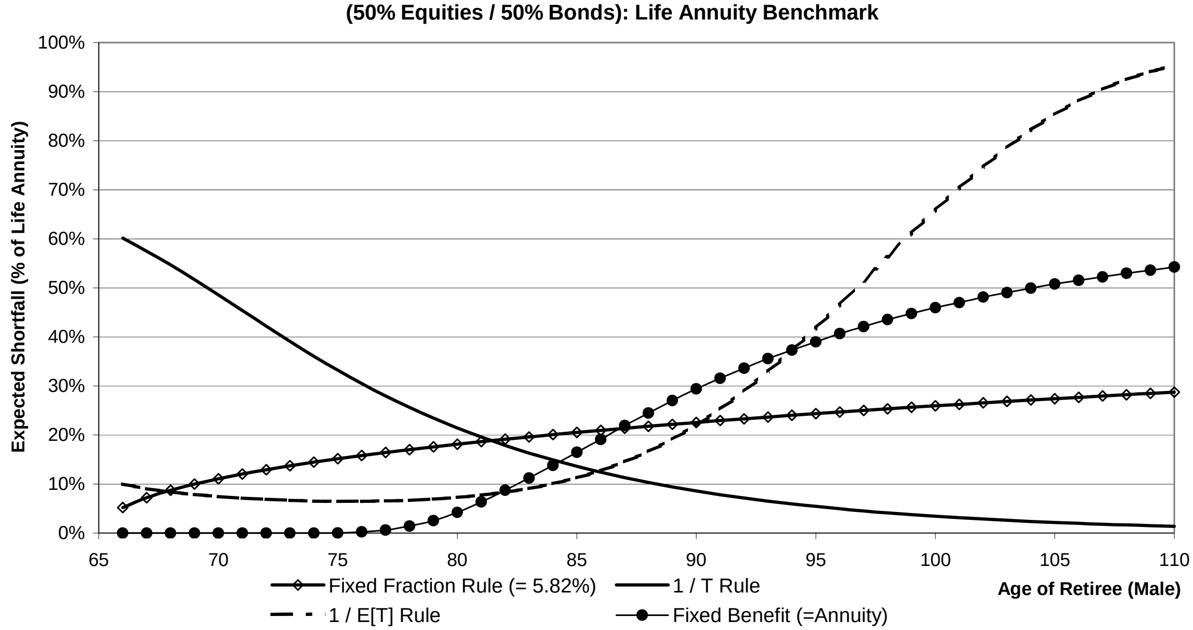
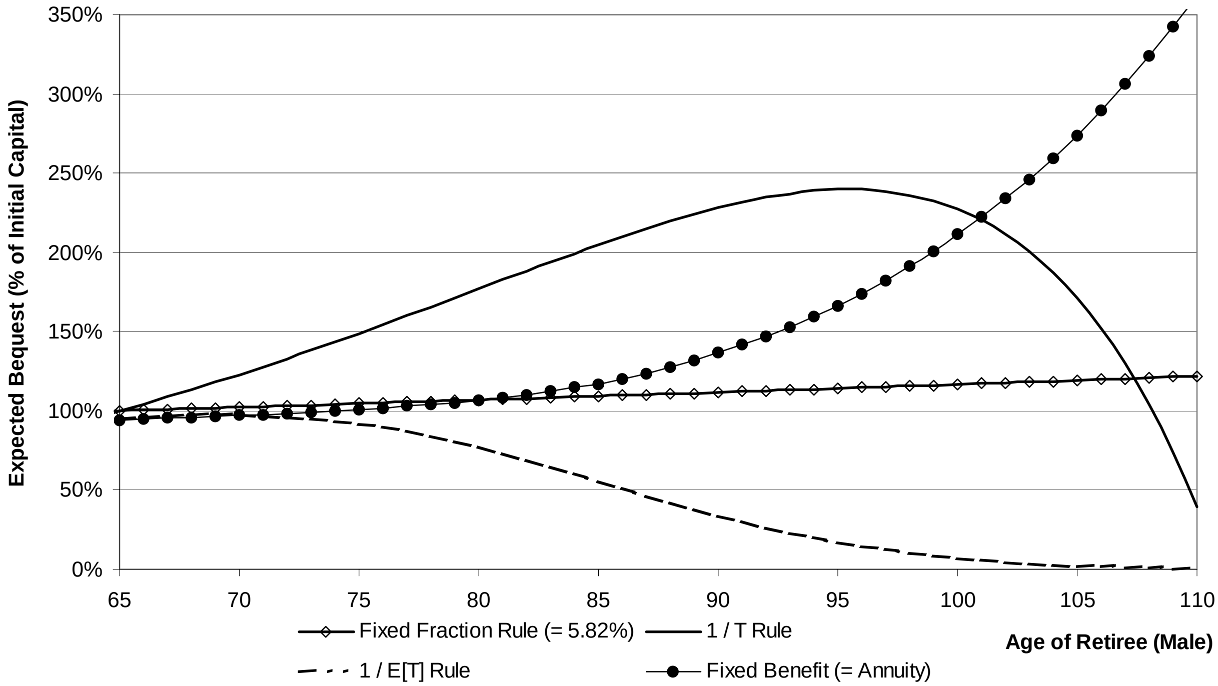


Figure 5: Mean Bequest of Withdrawal Plan Conditional on Death (50% Equities / 50% Bonds) as % of Initial Capital



Appendix A: Determining Annuity Benefits

Using the actuarial principle of equivalence, we estimate the gross single premium of the annuity by calculating the present value of expected benefits paid to the annuitant including provider expense loadings (i.e. commissions and administration fees). Formally, the annuity benefits paid in advance of each year until death are given according to the following equation

$$B = \frac{V_0}{(1+\lambda)\sum_{t=0}^{l-x} {}_t p_x \cdot (1+R_f)^{-t}}. \qquad \textbf{(A1)}$$

Here R_f denotes the (deterministic) interest rate used by the insurance company to discount future expected cash flows, ${}_tp_x$ the conditional probability that a man aged x will attain age x+t with respect to a mortality table (with maximum age l), and $1+\lambda$ is the expense loading factor. To calculate the annuity benefits for females, the ${}_tp_x$ in equation (B1) must be substituted with the sex specific survival probabilities ${}_tp_y$ for females. In the case when a unisex table is used, the survival probabilities are calculated using a specific mortality table for females.

Explicit assumptions must be made about mortality risk, the annuitant's age, the interest rate used by the insurance company to discount expected benefit payments, and the cost structure of the insurance company. Following *Albrecht* and *Maurer* (2002), we take into consideration the basic annuitant mortality table DAV 1994 R for the specification of the demographic parameters. This table is provided by the German Society of Actuaries and is widely used in the German annuities market. The table offers sex-specific mortality rates q_x (q_y) for male and female. From these sex specific mortality rates we construct the mortality rates of a unisex table as a weighted average of q_x and q_y. The interest rate used to discount expected annuity payments is set to an annual real 1.5%, consistent with the current yield of Euro-based inflation-linked bonds. Regarding the cost-structure of the insurance company, it is assumed that the total expense loading relative to the pure actuarial premium is 2.785%, i.e. $\lambda = 1.02785$.

Given these assumptions, table A1 shows the yearly benefits (adjusted for inflation and before personal income taxes) a retiree with age 60, 65 and 70 would receive per 100 EUR of premium.

Table A1: Immediate Annual Life-long Real Annuity Benefits per EUR 100 Single Premium: Total Expense Loadings 2.785%; Discount Factor 1.5%; DAV R 94 Mortality Tables

Mortality Table	**Male**	**Female**	**Unisex**
Retirement Age	**Life Annuity € p.a.**		
60	4.9480	4.3215	4.6063
65	5.8177	5.0174	5.3738
70	7.0330	5.9900	6.4421

Source: Authors' calculations

Appendix B: Determining Expected Benefits, Expected Bequest and the Risk of a Consumption Shortfall for Phased Withdrawal Plans with given Benefit-to-Wealth Ratios

Let $\omega_t = B_t/V_t$ (t = 0, 1, ..,) be a predetermined sequence of benefit-to-wealth ratios $0 \leq \omega_t \leq 1$, and define $c\omega_t = \prod_{i=0}^{t}(1-\omega_i)$. The retirement accounts assets (adjusted for inflation) used to fund the variable pension benefits B_t are assumed to follow a geometric random walk with drift. This implies that the real log returns I_t over the year are serially independent and identically normal distributed with given mean μ and volatility σ. Given an initial endowment V_0 at the beginning of the retirement phase, the market value of the retiree's account at the beginning of year t (t = 1, 2, …) just before the withdrawal B_t for that year is made:

$$V_t = (1-\omega_{t-1})V_{t-1}\exp(I_t) = c\omega_{t-1}V_0\exp\left(\sum_{i=1}^{t} I_t\right) \qquad \textbf{(B1)}$$

is distributed log-normally, i.e. $\ln(V_t) \sim N(m_t, v^2_t)$ follows a normal-distribution with mean $m_t = \ln[c\omega_{t-1} V_0] + t\mu$ and variance $v_t^2 = t\sigma^2$. Consequently, the benefit payments B_t at the beginning of each period:

$$B_t = \omega_t V_t = \omega_t c\omega_{t-1}V_0\exp\left(\sum_{i=1}^{t} I_t\right) \qquad \textbf{(B2)}$$

are also log-normally distributed, i.e. $\ln(B_t) \sim N(n_t, v^2_t)$ with parameters $n_t = \ln[\omega_t\, c\omega_{t-1}\, V_0] + t\mu$. With these formulas in hand, and additional assumptions about the expected return μ and volatility σ of the retirement accounts assets, it is possible to compute for the variable phased withdrawal rules - i.e. fixed fraction, 1/T and 1/E(T) - various risk and return measures of future benefits if the retiree is alive as well as the possible bequest in the case he dies.

The expected benefit payments E[B_t] in each period t = 0, 1, … are given by:

$$\mathrm{E}[B_t] = \exp(n_t + \tfrac{1}{2}v_t^2) = \omega_t c\omega_{t-1}V_0\exp(t\mu + \frac{1}{2}t\sigma^2) \qquad \textbf{(B3)}$$

and the expected bequest if the retiree dies in period t = 1, 2, … according to:

$$\mathrm{E}[V_t] = \exp(m_t + \tfrac{1}{2}v_t^2) = c\omega_t V_0\exp(t\mu + \frac{1}{2}t\sigma^2) \qquad \textbf{(B4)}$$

The shortfall probability that the benefits from a variable withdrawal plan is lower than a target annuity z can be calculated as:

$$\mathrm{SP}(B_t) = \Phi(q_t) \qquad \textbf{(B5)}$$

where Φ is the cumulative density function of the Standard Normal Distribution at the point $q_t = (z - m_t)/\sigma\sqrt{t}$. Using the results given in *Winkler* et al., (1972) the shortfall expectation is:

$$\mathrm{SE}(B_t) = z\cdot\Phi(q_t) - \mathrm{E}[B_t]\cdot\Phi(q_t - \sqrt{\sigma^2\cdot t})\,. \qquad \textbf{(B6)}$$

Finally, combining (B5) and (B6) the Mean Excess Loss can be computed as:

$$\mathrm{MEL}_t = \frac{\mathrm{SE}_t}{\mathrm{SP}_t} = z - \frac{\mathrm{E}[B_t]\cdot\Phi(q_t - \sqrt{\sigma^2\cdot t}}{\Phi(q_t)} \qquad \textbf{(B7)}$$

Appendix C: Summary Statistics for Real Annual Log-Returns (before tax) on Stocks and Bonds in the German Capital Market 1967-2002

Asset Class	Mean return (% p.a)	Volatility (% p.a.)	Correlations		
			Stocks	Bonds	Cash
Stocks	5.53	25.36	1		
Bonds	3.98	5.21	0.235	1	
Cash	2.84	1.69	-0.174	0.326	1

Source: Authors' calculations;