

# Beyond Fitts' Law: Models for Trajectory-Based HCI Tasks

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## ABSTRACT

Trajectory-based interactions, such as navigating through nested-menus, drawing curves, and moving in 3D worlds, are becoming common tasks in modern computer interfaces. Users' performances in these tasks cannot be successfully modeled with Fitts' law as it has been applied to pointing tasks. Therefore we explore the possible existence of robust regularities in trajectory-based tasks. We used "steering through tunnels" as our experimental paradigm to represent such tasks, and found that a simple "steering law" indeed exists. The paper presents the motivation, analysis, a series of four experiments, and the applications of the steering law.

## Keywords

Fitts' law, human performance, modeling, movements, path steering, task difficulty, motor control, input techniques and devices, trajectory-based interaction

## INTRODUCTION

It has been argued that the advancement of HCI lies in "hardening" the field with quantitative, engineering-like models [14]. In reality, few theoretical, quantitative tools are available in user interface research and development. A rare exception to this is Fitts' law [6]. Extending information theory to human perceptual-motor system, Paul Fitts found a formal relationship that models speed/accuracy tradeoffs in aimed movements. It predicts that the time  $T$  needed to point to a target of width  $W$  and at distance  $A$  is logarithmically related to the inverse of the spatial relative error  $\frac{A}{W}$ , that is:

$$T = a + b \log_2 \left( \frac{A}{W} + c \right) \quad (1)$$

where  $a$  and  $b$  are empirically determined constants, and  $c$  is 0, 0.5 or 1 (See [13] for detail). The factor  $\log_2 \left( \frac{A}{W} + c \right)$ , called the index of difficulty (ID), describes the difficulty to achieve the task: the greater ID, the more difficult the task.

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Due to its accuracy and robustness, Fitts' law has been a popular research topic. Numerous studies have been conducted to explain [5, 8], extend [12] and apply Fitts' law to various domains. The value of Fitts' law in human-computer interaction research can be readily appreciated. Taking input device research as an example, it was nearly impossible to compare device performance results from different studies until the Fitts' law model was applied [2]. Without Fitts' law, performance scores (pointing/tapping times) are only meaningful under a set of specific experimental conditions (target sizes and distances). With Fitts, these scores can be translated into a performance index (in bits/second) that is independent of those experimental details.

What Fitts' laws revealed is a somewhat intuitive tradeoff in human movement: the faster we move, the less precise our movements are, or vice versa: the more severe the constraints are, the slower we move. Paul Fitts [6] formulated such a tradeoff in three experimental tasks (bar strip tapping, disk transfer, and nail insertion) that are essentially in one paradigm: hitting a target over certain distance. In human-computer interaction, such a paradigm corresponds to a frequent elemental task: pointing/target selection.

However, it is obvious that Fitts' law addresses only one type of movement. Increasingly, computer input devices are used not only for pointing to targets but also for producing trajectories, such as in drawing, writing, and steering in 3D space (e.g. VRML worlds). Fitts' law is not an adequate model for these trajectory-based tasks. Simply by trying to write with a mouse one would realize the marked difference between a mouse and a pen (stylus). Yet formal studies in Fitts' law paradigm [11] showed little performance difference between these two types of devices. Clearly the user interface / input device studies carried out in the Fitts' law paradigm are not sufficient for today's practical needs. It has long been proposed that in addition to pointing (target acquisition), pursuit tracking, free-hand inking, tracing, and constrained motion should all be considered as testing tasks for input device evaluation [1].

Given the tremendous value and success of Fitts' law, it is

surprising that the very spirit of Fitts law, namely simple quantitative relationships between task constraint and movement speed, has not been applied to other types of tasks. Are there any other regularities in human movement that can be modeled in simple mathematical equations? If so, we would have a richer set of quantitative tools for both motor control research and for user interface evaluations. The current work is one step toward such a goal.

In order to address trajectory-based tasks, the experimental paradigm we choose to focus on is steering between boundaries (also called constrained motion in Buxton's task taxonomy [1]). A simple example of such tasks is illustrated in Figure 1, where one has to draw a line from one side of the figure to the other, passing through the "tunnel". We hypothesized that for a given amplitude (tunnel length) and variability (tunnel width), the time needed to perform this kind of operations should depend directly on the amplitude and the path width, in accordance with a formal model.

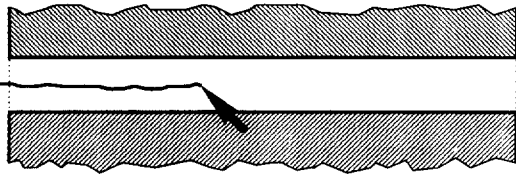


Figure 1: Self-paced movement with normal constraint

In a rather early study [7], when analyzing handwriting processes, Freeman noticed that the time needed to write a character was constant, regardless the script size, large or small. However, the characters written in larger script size were less precise (in terms of absolute accuracy) than the characters in smaller size, so that the relative accuracy (variability/amplitude) remained the same. It appears that the time to produce trajectories sets the relative speed-accuracy ratio: the larger the amplitude, the less precise the result is. This also explains why artists spend a lot of time to draw the figure contours precisely when finishing a drawing<sup>1</sup>.

Such a speed-accuracy tradeoff also seems to hold in a larger scale of movement: the faster one drives an automobile, the less precisely one can control the trajectory, such that the narrower a road, the slower one has to drive. A simple explanation for this is that, if the movement is too fast, a small deviation from the standard trajectory results in the constraints being exceeded before any feedback analysis can be completed and the movement corrected accordingly. This may be due to the fact that the time humans need to process the visual feedback information when moving has a lower bound [4, 5, 8, 16].

We took several experimental steps to derive and validate quantitative relationships between completion time and movement constraints in trajectory-based tasks. The first was a study of a "goal passing" task, in which we established a quantitative and formal model for predicting its difficulty. The result provided the theoretical basis for the second experiment, a "tunnel steering" task, as described above. We then

<sup>1</sup> Please note that precision should not be confused with smoothness

conducted two other experiments of increasing complexity. From these experiments, we derived a theoretical model that quantifies the difficulty in generalized path steering tasks.

## APPARATUS

All the experiments described below were performed on a Silicon Graphics' Impact with a 19-inch monitor (1280×1024 pixels resolution), and equipped with a Wacom UD-1825-RSB tablet (18×25 inches). With their dominant hand, subjects held and moved a stylus on the surface of the tablet, producing drawings on the computer monitor. All experiments were done in full-screen mode, with the background color set to black. The entire tablet area was mapped onto the screen, so that one centimeter on the tablet corresponded to 20 pixels on the screen.

## EXPERIMENT 1: GOAL PASSING

In this first experiment, we investigated a steering task with constraints only at the ends of the movement, as illustrated in Figure 2. We call this task the "goal passing" task: subjects were asked to pass Goal 1 and then Goal 2 as quickly as possible. The movement time between Goal 1 and Goal 2 was recorded and analyzed.

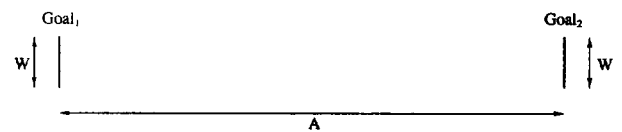


Figure 2: A goal passing task

## Procedure and design

A fully-crossed, within-subjects factorial design with repeated measures was used. Ten subjects participated in this experiment. Independent variables were the movement amplitude ( $A = 256, 512$  and  $1024$  pixels) and path width ( $W = 8, 16$  and  $32$  pixels). Subjects performed two consecutive sets of 9  $A$ - $W$  conditions. The first set was considered practice session and the second data collection session. The nine conditions were presented in a random order in each session. Subjects performed 10 trials in each condition.

At the beginning of each trial, two vertical target segments (goals) were presented on the screen, both in green color. After placing the stylus on the tablet (to the left of goal 1) and applying pressure to the tip, the subject began to draw a blue line on a screen, showing the stylus trajectory. When the cursor crossed the first goal, left to right, the line turned red, as a signal that the task had begun and the time was being recorded. When the cursor crossed the second goal, also left to right, all drawings turned yellow, signaling the end of the trial. Releasing pressure on the stylus after crossing the first goal and before crossing the second would result in an invalid trial (error). Subjects were asked to minimize errors. A beep is emitted when the condition changes.

## Results

The results shows that this goal passing task follows the same law as in Fitts' tapping task, despite the different nature of movement constraint. The scatter-plot graph (Figure 3) presenting the movement time against Fitts' ID shows a linear relationship with a high correlation between them. Quantita-

tively, the movement time  $MT$  is given by the equation:

$$MT = -1347 + 391 \log_2\left(\frac{A}{W} + 1\right) \text{ with: } r^2 = 0.987 \quad (2)$$

where  $A$  is the amplitude of movement and  $W$  is the width of the goals, i.e. the vertical variability. The error rate was 7.4% in average, with a higher rate for small widths.

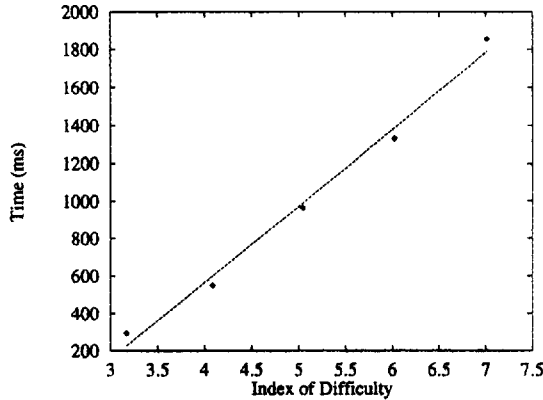


Figure 3: Scatter-plot graph of the  $MT-ID$  relationship for the goal passing task

**EXPERIMENT 2: INCREASING CONSTRAINTS**

Experiment 1 shows that a steering task with constraints on both ends (two goals) follows the same logarithmic law as Fitts' tapping task. This serves as a stepping stone towards formulating relationships between movement time and continuous constraint in steering tasks. If the time needed to pass two goals of width  $W$  over distance  $A$  follows Fitts' law, what happens if we place more "goals" on the trajectory? And what will the law become if we place infinite number of goals? Clearly, the resulting task is the straight tunnel steering task we proposed in the Introduction (Figure 1). Note that the purpose of such a recursive analysis is to formulate a hypothetical relationship for the steering task; it is not to offer an explanation with psychomotor or neuromotor understanding of the steering control process.

The recursion, illustrated by Figure 4, is defined as follows:

- The first step of the recursion is shown by Figure 4a which is the same task as in Experiment 1: two-goal passing. Experiment 1 shows that the index of difficulty to move from goal 1 to goal 2 is:

$$ID_1 = \log_2\left(\frac{A}{W} + 1\right) \quad (3)$$

- The second step of the recursion follows Step 1 by dividing the amplitude  $A$  into two identical amplitudes  $\frac{A}{2}$ , as shown in Figure 4b. Since each of the two parts is a task modeled in step 1 with amplitude  $A/2$ , it is logical to assume the index of difficulty to move from goal 1 to goal 3 via goal 2 is:

$$ID_2 = 2 \log_2\left(\frac{A}{2W} + 1\right) \quad (4)$$

- ...

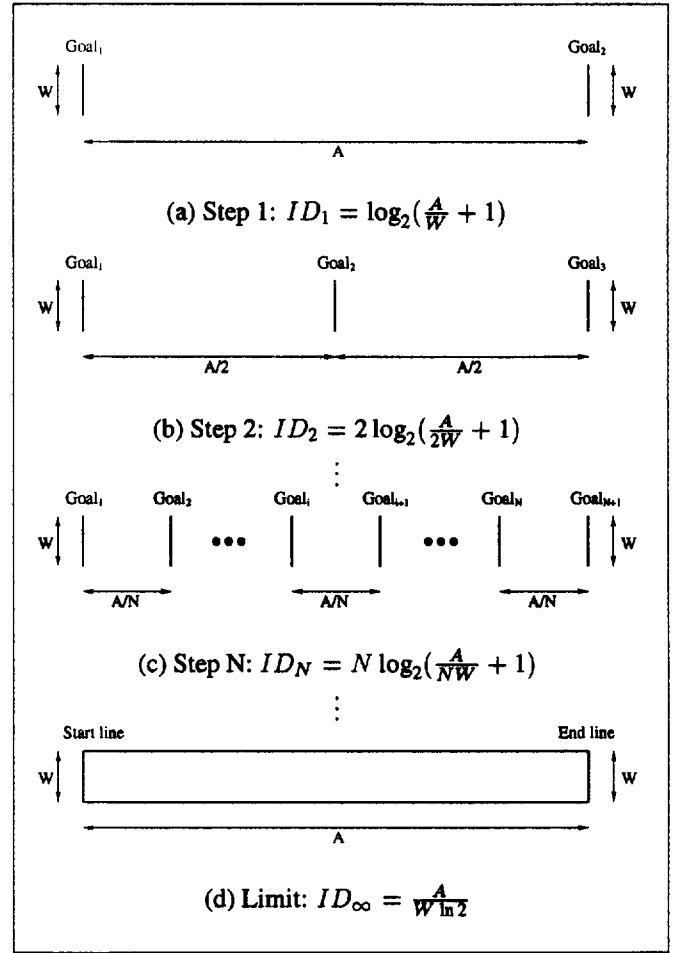


Figure 4: Defining a recursion with goal passing tasks

- The  $N^{\text{th}}$  step divides the amplitude  $A$  into  $N$  identical amplitudes  $\frac{A}{N}$ , as shown in Figure 4c. The difficulty to move from goal 1 to goal  $N+1$  via goals 2, 3, ...,  $N$  is:

$$ID_N = N \log_2\left(\frac{A}{NW} + 1\right) \quad (5)$$

This recursion is interesting because of the increasing constraint it imposes onto movements: the bigger  $N$  is, the more careful the subject has to be in order to pass through all goals. If  $N$  tends to infinity, the task becomes a "tunnel traveling" task. The tunnel has length of  $A$  and width of  $W$ . (Figure 4d). It is also possible to determine the index of difficulty for the limit task by determining the limit of the index of difficulty recursion  $ID_N$ . Indeed, using a first order Taylor series expansion of  $\log_2(1 + x)$ , we obtain:

$$ID_\infty = \lim_{N \rightarrow \infty} ID_N = \frac{A}{W \ln 2} \quad (6)$$

Therefore, such an analysis predicts that the difficulty to achieve this tunnel traveling task is not related to the logarithm of  $\frac{A}{W}$  but to  $\frac{A}{W}$ . This leads to equation 7:

$$MT = a + b \frac{A}{W} \quad (7)$$

where  $a$  and  $b$  are empirically determined constants. In the following,  $ID_\infty$  is defined as  $\frac{A}{W}$  instead of  $\frac{A}{W \ln 2}$  for simplicity.

In order to verify these assumptions, we ran an experiment corresponding to Figure 4d.

**Procedure and design**

Thirteen subjects participated in this experiment. The design of the experiment was the same as the previous one: fully-crossed, within-subjects factorial design with repeated measures. Four movement amplitudes ( $A = 250, 500, 750,$  and  $1000$  pixels) and eight path widths ( $W = 20, 30, 40, 50, 60, 70, 80,$  and  $90$  pixels) were tested in a random order. Similarly to experiment 1, this experiment included a warm-up session and the data collection session. Each combination of amplitude and width was tested with 5 trials.

At the beginning of each trial, only the rectangle, as presented by Figure 4d, was presented on the screen, in green color. Pressing on the stylus tip resulted in a blue line being drawn. The line color then turned red when the cursor crossed the left side of the rectangle, and both the rectangle and the line turned yellow when the task ended, as the stylus crosses the right side of the rectangle. A beep was also emitted when changing conditions. The crossing of the left and right sides of the rectangle was taken into account only if proceeded from left to right. Crossing the "sideways" of the path results in the cancelation of the trial and an error being recorded.

**Results**

The hypothesized model was successful in describing the difficulty of the task. Indeed, we found a strong correlation between the hypothesized model and the data collected (Figure 5). The regression analyzes on successfully completed trials, performed on all 13 subjects, gave:

$$MT = -188 + 78 \times ID \text{ with: } r^2 = 0.968 \quad (8)$$

The error rate increases significantly when the task becomes very difficult; the average error rate is 6.4%.

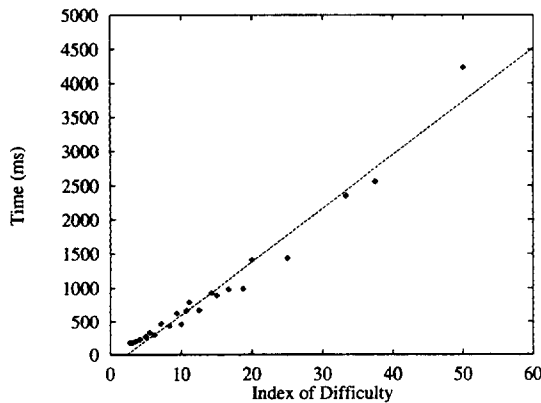


Figure 5: Scatter-plot of the MT-ID relationship. The relation fitted was  $MT = a + b \times ID$  where  $ID = \frac{A}{W}$

Note that, although subjects were asked to minimize errors in this experiment, the error rates are considerably higher than

those typically found in Fitts' law studies<sup>2</sup>. Steering through a very narrow tunnel without going out of the boundaries at any point of the trial is much more difficult than tapping on small targets. Modeling error rate as a function of task difficulty should be conducted in future studies.

**EXPERIMENT 3: NARROWING TUNNEL**

In this experiment we wanted to test if our method could be applied to linear trajectories but with a non-constant path width. The simplest configuration that satisfies these properties is a narrowing tunnel, shown on Figure 6. Subjects were asked to draw a line through the tunnel as quickly as possible.

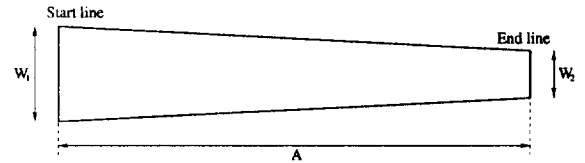


Figure 6: Narrowing tunnel

Such a task can also be decomposed into a set of elemental goal passing tasks, for which we can calculate the index of difficulty. But this method and the resulting expression of the index of difficulty (an infinite sum) is somewhat complicated compared to the simplicity of the tunnel shape. We thus applied a new, simpler method to compute the index of difficulty for this task.

The new approach considers the narrowing tunnel steering task as a sum of elemental linear steering tasks described in experiment 2. Figure 7 shows such a decomposition.

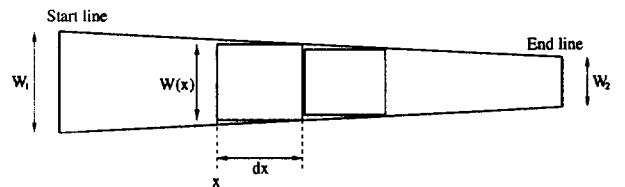


Figure 7: Decomposition of the narrowing tunnel

Let us consider an elementary path of this decomposition, situated at abscissa  $x$  and of length  $dx$ . The index of difficulty for steering this elementary path, noted  $dID_x$ , is, according to Experiment 2,  $\frac{dx}{W(x)}$ , where  $W(x)$  is the width of the path at  $x$ . To obtain the ID of the entire path, we just have to sum all  $dID_x$  along the path, that gives:

$$ID_\infty = \int_0^A \frac{dx}{W(x)} = \int_0^A \frac{dx}{W_1 + \frac{x}{A}(W_2 - W_1)} \quad (9)$$

so that the index of difficulty for the narrowing tunnel is:

$$ID_{NT} = \frac{A}{W_2 - W_1} \ln \frac{W_2}{W_1} \quad (10)$$

Moreover, it is possible to prove that decomposing a steering task into elementary steering tasks or into elementary goal passing tasks are equivalent methods, resulting in the same IDs. One can thus choose the most convenient method, depending on the shape of the path.

<sup>2</sup> This is also true for all the other experiments discussed in this paper.

**Procedure and design**

Ten subjects participated in this experiment. The design and procedure of the experiment was the same as for experiment 2. Parameters were set as follows:  $W_1 = 20, 30, 40, 50$ ;  $W_2 = 8$ ;  $A = 250, 500, 750, 1000$ .

**Results**

As shown in Figure 8, the completion time of the successful trials and index of difficulty for the narrowing tunnel steering task once again forms a linear relationship as follows:

$$MT = -532 + 93 \times ID \text{ with: } r^2 = 0.978 \quad (11)$$

Due to the high constraint on the right end of the tunnel, high error rate occurred in all conditions. The average error rate is close to 18%.

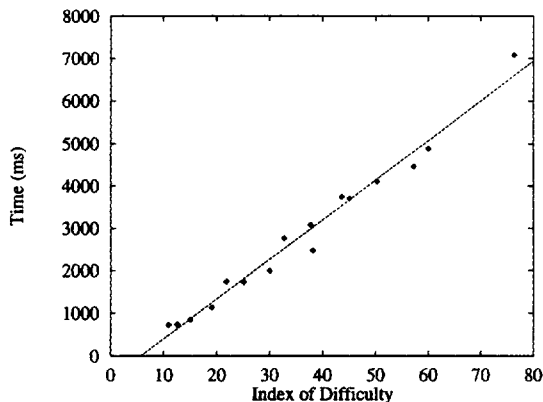


Figure 8: Scatter-plot of the MT-ID relationship for the narrowing tunnel task

**A GENERIC APPROACH: DEFINING A GLOBAL LAW**

The narrowing tunnel study brought the new concept of integrating the inverse of the path width along the trajectory. We believe that this approach is generic, that is to say that it is possible to propose an extension of this method to complex paths such as the one shown in Figure 9.

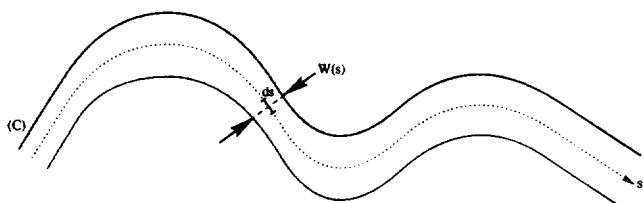


Figure 9: Integrating along a curve

To establish a generic formula, we introduced the curvilinear abscissa as the integration variable: if  $C$  is a curved path, we define the index of difficulty for steering through this path as the sum along the curve of the elementary indexes of difficulty. We thus obtain the generic expression of  $ID_C$ :

$$ID_C = \int_C \frac{ds}{W(s)} \quad (12)$$

Our hypothesis was then that the time to steer through  $C$  is linearly related to  $ID_C$ , that is:

$$T_C = a + b \int_C \frac{ds}{W(s)} \quad (13)$$

where  $a$  and  $b$  are constants. This formula is a generalization of the cases presented earlier, which can be deduced from it. As an example, let us consider the horizontal steering task corresponding to experiment 2. In this case,  $W(s)$  is constant and equal to  $W$ , so that equation 13 gives:

$$T_C = a + b \frac{1}{W} \int_C ds = a + b \frac{A}{W} \quad (14)$$

which is equation 7 found in experiment 2.

**EXPERIMENT 4: SPIRAL TUNNEL**

In order to test our method for complex paths, we studied a new configuration, the spiral tunnel, such as that shown in Figure 10. Subjects were asked to draw a line from the center to the end of the spiral.

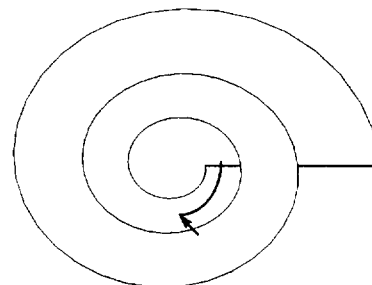


Figure 10: An instance of spiral

We defined a set of spirals  $(S_{n,w})_{n \in \mathcal{N}, w > 0}$  by varying two parameters:  $w$  is the parameter influencing the increase of the width of the spiral;  $n$  stands for the number of "turns" of the spiral. Figure 10 shows an example of such a spiral,  $S_{2,15}$ .

The equation of  $S_{n,w}$  in polar coordinates is:

$$r = (\theta + w)^3 \text{ with: } \theta \in [2\pi, 2\pi(n + 1)] \quad (15)$$

This set of spirals has been chosen to guarantee that the width of the path will vary significantly.

Our goal here is to predict the difficulty for steering these spirals. To apply the previous method, we must determine both the curvilinear abscissa function of  $\theta$  and the width of the path for any  $\theta$ .

A good approximation for the width of the path for a given angle  $\theta$  is:

$$W(\theta) = (\theta + 2\pi + w)^3 - (\theta + w)^3 \quad (16)$$

and it can be proven that:

$$ds = \sqrt{(\theta + w)^6 + 9(\theta + w)^4} d\theta \quad (17)$$

We can then apply Equation 12 and make a summation of elementary  $ID_s$ , and obtain:

$$ID_{S_{n,w}} = \int_{2\pi}^{2\pi(n+1)} \frac{\sqrt{(\theta + w)^6 + 9(\theta + w)^4}}{(\theta + 2\pi + w)^3 - (\theta + w)^3} d\theta \quad (18)$$

**Procedure and design**

A fully-crossed, within-subjects factorial design with repeated measures was used. Eleven subjects participated in this experiment. Factors were the spiral "turn" number ( $n = 1, 2, 3, 4$ ) and width factor ( $w = 10, 15, 20, 25$ ). Subjects performed twice the set of 16  $n-w$  conditions, the first time being a practice session and the second the real experiment. Conditions were presented in random order. Subjects performed 10 trial under each condition.

The procedure was similar to the previous experiment. At the beginning of each trial, a spiral, as illustrated in Figure 10, was presented on the screen. The task starts when the cursor crosses the inner small segment, and ends as the stylus crosses the outer long segment, after completing the spiral steering. Crossing the spiral boundary results in the trial being canceled.

**Results**

The experiment confirmed that the prediction of the difficulty of steering tasks is also valid for this more complex task. As shown in Figure 11, the time to steer through the spiral path is linearly related to the index of difficulty defined in Equation 18. The fitted equation is:

$$MT = 115 + 169 \times ID_{S_{n,w}} \text{ with: } r^2 = 0.971 \quad (19)$$

The average error rate for this task is 13.7%.

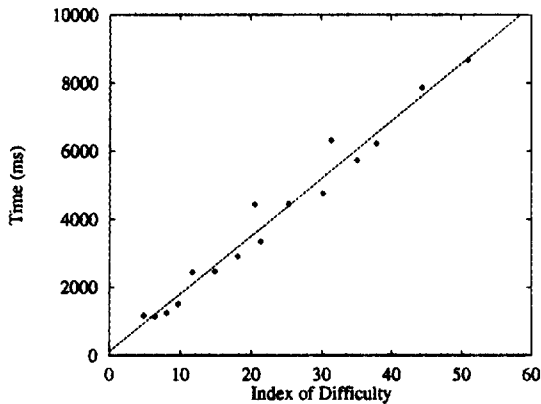


Figure 11: Spiral steering

**DERIVING A LOCAL LAW**

It has been shown that Equation 13 is a "global" law that predicts the total time to perform a steering task. A corresponding local law that models instantaneous speed can be expressed as follows:

$$v(s) = \frac{W(s)}{\tau} \quad (20)$$

where  $v(s)$  is the velocity of the limb at the point of curvilinear abscissa  $s$ ,  $W(s)$  is the width of the path at the same point and  $\tau$  is an empirically determined time constant. This local law predicts that the instantaneous speed of steering movement at any point is proportional to the variability permitted, i.e., the width of the path at this point.

The justification of this relationship between velocity and path width comes from the calculation of the time  $T_C$  needed

to steer through a path  $C$ . Indeed, along the path, the velocity  $v$  is defined as  $v = \frac{ds}{dt}$ , so that  $dt = \frac{ds}{v}$  and, considering the local law above:

$$T_C = \int_C \frac{ds}{v(s)} = \tau \int_C \frac{ds}{W(s)} \quad (21)$$

This latter expression of  $T_C$  is very close to equation 13. Indeed, the intercepts observed with real data of experiment 2 ( $-188ms$ ), experiment 3 ( $-532ms$ ), and experiment 4 ( $115ms$ ) are relatively small compared to the total trial times. It probably came from any random variation of subject performance. Ideally, the intercept should be null, but equation 13 includes it to take these variations into account.

In order to check the validity of equation 20, we used the data from previous experiments and plotted speed versus path width to check the linear relationship.

For experiment 2, for each of the eight widths of this experiment, we calculated the average speed of steering. Figure 12 represents the resulting scatter plot. The graph, built from about 120000 move events (events received from the X server), shows the linear relationship between the path width and the stylus speed. Excluding the last two points (justification in discussion section), we found that:

$$v = -6.4 \times 10^{-2} + 2.0 \times 10^{-2}W \text{ with: } r^2 = 0.986 \quad (22)$$

The small intercept can be neglected, which is coherent with equation 20. We can then derive that  $\tau \approx 50ms$ .

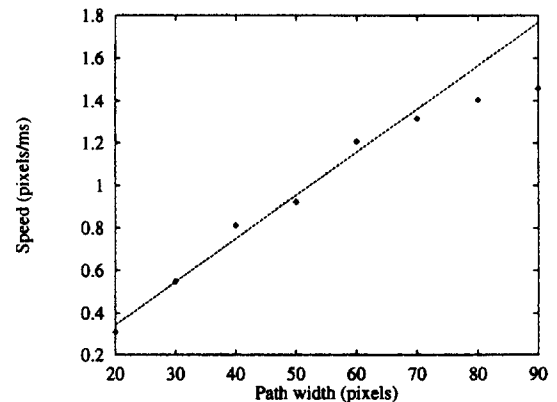


Figure 12: Speed vs. path width for experiment 2

For experiment 3 and 4, as the width is not constant, we can directly extract the average speed for any given path width. Figures 13 and 14 present respectively the scatter-plot of speed vs. path width in the cases of narrowing and spiral tunnel steering (respectively based on about 150000 and 200000 move events). These graphs also show a linear relationship between path width and hand speed. For the narrowing tunnel, considering only path widths that are less than 35 pixels, we found that:

$$v = 1.8 \times 10^{-2} + 1.4 \times 10^{-2}W \text{ with: } r^2 = 0.994 \quad (23)$$

In the case of the narrowing tunnel,  $\tau$  is thus close to 70ms.

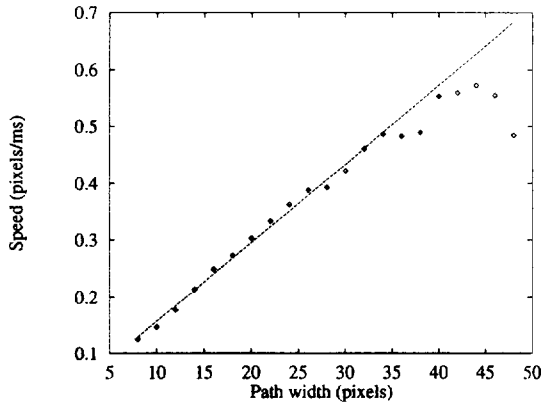


Figure 13: Speed vs. path width for experiment 3

For spiral steering, considering only path widths that are less than 80 pixels, we found:

$$v = -7.6 \times 10^{-3} + 8.9 \times 10^{-3}W \text{ with: } r^2 = 0.997 \quad (24)$$

from which we can deduce that  $\tau$  is close to 110ms.

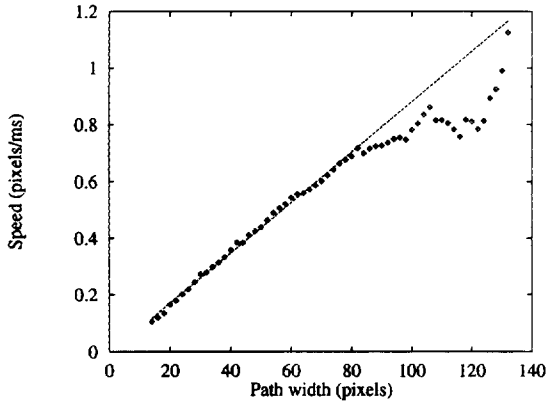


Figure 14: Speed vs. path width for experiment 4

**DISCUSSION**

Due to space limitation, we have to leave out many more detailed variations of the laws we proposed and verified. It should be pointed out, however, that there are various limitations to these simple laws.

First, due to human body limitations (speed, acceleration), there are upper bound limits to the path width that can be correctly modeled by these simple laws. Exceeding these limits leads to the saturation of the laws described above. These limitations are the reason why we had to remove the greatest widths when analyzing linear relationships between speed and path width for the local law.

Second, the local law can be modified to take path curvature into account. Indeed, our local law could be compared to the law introduced by Viviani *et al.* [15], who argued that tangential velocity  $v$  and radius of curvature  $\rho$  are proportional in unconstrained movements<sup>3</sup>. We hypothesize that a more

<sup>3</sup> Viviani *et al.* showed that their law was still valid for constrained movement, but their definition of constraints is different from ours; their constraint is purely mechanical and consists of moving a pen along the border of an object. Thus, their law is not directly applicable in our case.

general steering law should be:

$$v(s) = k\rho(s)W(s) \quad (25)$$

Finally, the starting position clearly influences the difficulty of a steering task. For instance, the performance likely depends, in Experiment 1, 2, and 3, on whether steering is performed from left to right or from right to left, and in experiment 4, on both the centripetal / centrifugal and clockwise / counter clockwise directions of steering. Steering is then probably related to handedness.

**DESIGN IMPLICATIONS**

**Modeling interaction time when using menus**

When interacting with current GUIs, one often implicitly performs various path steering tasks. One example is menu selection, such as the one shown in Figure 15. Each step in menu selection is a linear path steering task, similar to the one in Experiment 2.

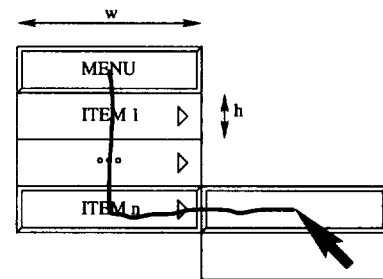


Figure 15: Interacting with menus

Selecting an item in a hierarchical menu involves two (or more) linear path steering tasks: a vertical steering to select a parent item, followed by a horizontal steering to select a sub-item. Applying the results from experiment 2, we can model the time to select a sub-menu as the sum of the vertical and horizontal steering tasks. If  $T_n$  stands for the time needed to select the  $n^{th}$  sub-menu (Figure 15), we obtain<sup>4</sup>:

$$T_n = \underbrace{a + b \frac{nh}{w}}_{\text{Vertical}} + \underbrace{a + b \frac{w}{h}}_{\text{Horizontal}} \quad (26)$$

$$= 2a + b\left(\frac{n}{x} + x\right) \text{ with: } x = \frac{w}{h} \quad (27)$$

From this equation, we can deduce that  $T_n$  is minimal when  $x = \sqrt{n}$ , that is  $w = \sqrt{n} \times h$ . Therefore, assuming that  $n$  is, on average, half the number of items in the menu, the greater the number of items is, the greater the quotient  $\frac{w}{h}$  should be.

This study may also be used as a means to compare designs, such as modeling the difference between linear hierarchic menus and hierarchic pie menus [9], for example. More generally, this is a step in the modeling of marking-based interaction and the evaluation of marking interfaces.

<sup>4</sup> We assume here that horizontal steering and vertical steering are driven by the same law. A further study is planned to prove this assumption. Moreover, the coefficient involved in these two laws are likely to be different, but of the same order of magnitude. The calculation performed here are considered as approximations.

### Performance Evaluation

By analogy to IP in Fitts' law,  $b$  in equation 13 and  $\tau$  in equation 20, can be used as indexes for performance comparisons. Device [2, 11] and limb [3, 10] comparisons have been done with Fitts' Index of Performance in pointing tasks.  $b$  and  $\tau$  in the steering laws will allow us to quantify performance in trajectory-based tasks, as a function of different devices, a function of different body limbs, or a function of any design parameter changes such as control gain and transfer function. By applying the steering law, we plan to study performance differences among various input devices such as mouse, stylus, isometric joystick, and trackball.

### CONCLUSION

Fitts' law is one of the very few robust and quantitative laws that can be applied to human-computer interaction research and design. A great number of studies have been conducted to verify and apply Fitts' law. We carried the spirit of Fitts' law a step forward and explored the possible existence of other robust regularities in movement tasks. In this study, we first demonstrated that the logarithmic relationship between movement time and tangential width of target in a tapping task also exists between movement time and normal width of the target in a "goal passing" task. A thought experiment of placing infinite numbers of goals along a movement trajectory lead us to hypothesize that there is a simple linear relationship between movement time and the "tunnel" width in steering tasks. We then confirmed such a relationship in three types of "tunnels": straight, narrowing, and spiral, all with correlations greater than 0.96. We then generalize the relationships in both integral and local forms. The integral form states that the steering time is linearly related to the index of difficulty, which is defined as the integral of the inverse of the width along the path; the local form states that the speed of movement is linearly related to the normal constraint.

The regularities presented in this study may enrich the small repertoire of quantitative tools in HCI research and design. Device comparison and menu design are just two of the many potential HCI applications.

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