

## Beyond performance/cost tradeoffs in motion control

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# Beyond Performance/Cost Trade-Offs in Motion Control: A Multirate Feedforward Design with Application to a Dual-Stage Wafer System

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**Abstract**—Motion systems with multiple control loops often run at a single sampling rate for simplicity of implementation and controller design. The achievable performance in terms of position accuracy is determined by the data acquisition hardware, such as sensors, actuators, and analog-to-digital/digital-to-analog converters, which is typically limited due to economic cost considerations. The aim of this paper is to develop a multirate approach to go beyond this traditional performance/cost trade-off, i.e., to use different sampling rates in different control loops to optimally use hardware resources. The approach appropriately deals with the inherent time-varying behavior that is introduced by multirate sampling. A multirate feedforward control design framework is presented to optimize tracking of a dual-stage multirate system. Application of the proposed approach to an industrial dual-stage wafer system demonstrates the advantages of multirate control, both in simulations and experiments.

**Index Terms**—Multirate control, feedforward design, performance/cost trade-off, dual-stage system, wafer stage application, experiments.

## I. INTRODUCTION

Multivariable control systems, including those in motion systems, are often implemented digitally since it offers flexibility and directly connects to the digital supervisory layers. The digital implementation requires analog-to-digital and digital-to-analog conversion. For motion systems, these processes are often executed using fixed, single-rate sampling schemes [1], [2], i.e., homogeneous for all loops, since for linear time-invariant (LTI) systems it enables controller design using well-developed design approaches. In particular, it allows the use of frequency domain techniques such as Bode plots and Nyquist diagrams [3], which find application in various areas of controller design, including feedback control [3], [4], feedforward control [5], and iterative learning control [6].

Fixed, single-rate sampling is preferred from a controller design point of view, but not from a performance versus cost point of view. As an example, consider systems with multiple control loops, where only one limits the overall performance.

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The performance of a control loop can be increased by increasing the sampling frequency of that loop. For single-rate implementations this implies that if the performance of one of the loops is increased, the sampling frequency of all loops needs to be increased. Obviously, such an approach is expensive in terms of the required hardware, such as sensors, actuators, and analog-to-digital/digital-to-analog converters, since all loops are affected while only one is limiting performance.

From a performance versus cost point of view, flexible sampling is preferred over fixed sampling, see also Fig. 1. Examples of flexible sampling include multirate control [7]–[16], sparse control [17], and non-equidistant sampling [18], [19]. Indeed, a multirate approach is more natural for multiloop systems with different performance requirements, but also for systems with different time scales such as thermomechanical systems [20]. Sparse control and non-equidistant sampling are used in, e.g., systems with limited resources and optimal resource allocation [18], [21].

Flexible sampling has a large potential, but its deployment is hampered by a lack of control design techniques. This is mainly caused by the fact that flexible sampling introduces time-varying behavior [1, Section 3.3]. In particular, flexible sampling of a linear time-invariant (LTI) system yields a linear (periodically) time-varying (L(P)TV) system. Due to the time variance, the frequency domain control design techniques mentioned earlier are not (directly) applicable. Frequency domain design for linear time-varying systems is investigated in [22]–[26] and linear time-varying feedforward design is investigated in [19], [27], but at present there is no systematic control design framework available.

Although flexible sampling has the potential to go beyond the traditional performance/cost trade-off for fixed sampling, as shown in Fig. 1, at present its deployment is hampered by a lack of control design techniques for such sampling schemes. In this paper, a framework to exploit multirate feedforward controller design is presented to overcome this restriction and thereby go beyond the traditional performance/cost trade-off. Application of the framework focuses on precision motion systems. In particular, the framework is demonstrated on an experimental dual-stage system, as standard in, e.g., wafer stages [28, Chapter 9].

The main contribution of this paper is a framework to exploit multirate control for performance improvement. The following subcontributions are identified: (I) multirate controller design based on multirate system descriptions, including

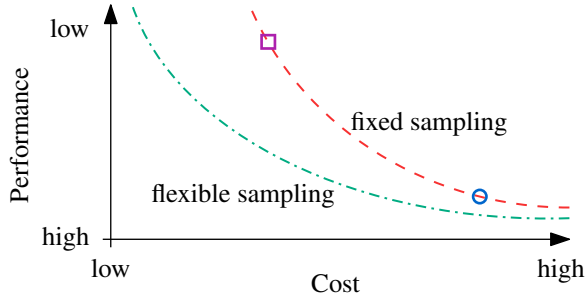


Fig. 1. A low sampling frequency is inexpensive in terms of implementation cost, but yields low performance (□). A high sampling frequency yields high performance, but is expensive (○). This performance/cost trade-off is inherent to traditional fixed sampling (---). Flexible sampling goes beyond this trade-off through use of different sampling frequencies in different control loops. Essentially, the performance/cost trade-off can be decided upon per control loop, resulting in an improved overall trade-off (-.-.).

time variance; (II) controller optimization addressing non-perfect models; (III) performance improvement by exploiting time variance; (IV) application of the design framework in simulation; and (V) experimental validation on a dual-stage system. Initial results on simulation level can be found in [29] and related work on minimizing intersample behavior in digital control systems can be found in [1], [25], [30]. The present paper contains substantial original contributions including Contribution (I), Contribution (II), and Contribution (V). Related work on wafer stage control design includes feedback control [31], [32], feedforward control [33], LPV control [34], and sparse control [17]. In the present work, previously unexplored freedom in sampling is exploited, which makes the approach complementary to other approaches.

This paper is organized as follows. In Section II, the main problem that is considered to improve the performance/cost trade-off through multirate control is presented. In Section III, the multirate control system is modeled. The multirate controller design is presented in Section IV. Furthermore, the performance is further improved by exploiting properties of time-varying systems. The controller design is applied to an experimental setup resembling a dual-stage wafer system. The experimental setup is detailed in Section V. Simulation results are presented in Section VI and experimental results are presented in Section VII. Conclusions are given in Section VIII.

## II. PROBLEM DEFINITION

In this paper, a framework is presented to enhance the performance/cost trade-off through multirate control. In this section, the main problem is presented.

### A. Application motivation: Dual-stage motion systems with large differences in performance requirements

In many motion control applications, a high positioning accuracy is required over a large range. For such systems, a single-stage design may not suffice due to the large dynamic range. To achieve high precision over a large range, a dual-stage system can be used.

A dual-stage system, as illustrated in Fig. 2, consists of two subsystems: a short stroke with a high positioning accuracy

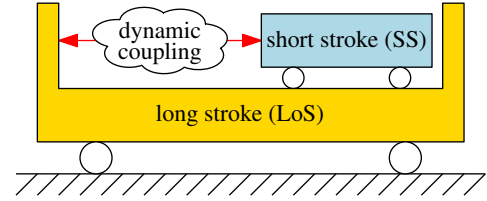


Fig. 2. Dual-stage systems consist of two subsystems: a short stroke for high precision and a long stroke to cover large ranges. The combined system provides high positioning accuracy over a large range.

(and limited range) connected to a long stroke with a large range (and limited positioning accuracy). If designed properly, the dual-stage system is able to cover a large range with high positioning accuracy. Clearly, there is a large difference between the performance requirements of the two subsystems.

An example of a dual-stage system is a wafer stage in lithography machines [28, Chapter 9]. Wafer stages require an accuracy up to nanometer level over a range of one meter ([35]; [28, Section 9.3.1]), resulting in a large dynamic range of  $\mathcal{O}(10^9)$ . Therefore, wafer stages are typically constructed as dual-stage systems. More details on the wafer stage application are presented in Section V.

### B. A performance/cost perspective on multivariable systems with large differences in performance requirements

In view of the performance/cost trade-off in Fig. 1, the different (control) requirements for the subsystems of the dual-stage design provide an excellent opportunity to exploit multirate control to go beyond performance/cost trade-offs in motion control.

The considered multirate control architecture is shown in Fig. 3 where a high sampling frequency  $f_h$  is used for the short stroke  $G_{SS,h}$  (○ in Fig. 1) and a low sampling frequency  $f_l$  is used for the long stroke  $G_{LoS,l}$  to reduce cost (□ in Fig. 1). The short-stroke system  $G_{SS,h}$  tracks reference trajectory  $\rho_{SS,h}$ . The long-stroke system  $G_{LoS,l}$  tracks the position of  $G_{SS,h}$  to ensure the short stroke is within range and reaction forces are limited. The downsampler  $\mathcal{D}_F$  facilitates the sampling rate conversion. The control design of both subsystems consists of feedback control ( $C_{FB}$ ), feedforward control ( $C_{FF}$ ), and input shaping ( $C_\psi$ ).

For design of the long-stroke controllers, the interest is in the position error between the two stages during exposure, i.e., during the scanning motion, to limit reaction forces to the short stroke. This error measured at the highest possible sampling frequency  $f_*$  is denoted  $\varepsilon_*$  and not available for real-time control, but typically available afterwards for performance evaluation. The sampling frequencies are related by

$$f_* = F_h f_h = F_l f_l, \quad f_h = F f_l, \quad (1)$$

where  $F_h \geq F_l \geq 1$ ,  $F := \frac{F_h}{F_l}$ , with  $F_h, F_l, F \in \mathbb{N}$ . In this paper, finite-time signals are considered of which the signal lengths are related as

$$N_* = F_h N_h = F_l N_l, \quad N_h = F N_l \quad (2)$$

as directly follows from (1).

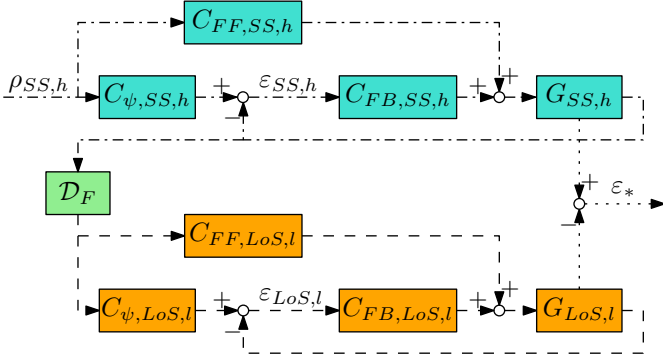


Fig. 3. Multirate control configuration for a dual-stage system. The top part relates to the short stroke (SS) at high rate  $f_h$ . The bottom part relates to the long stroke (LoS) at low rate  $f_l$ . The long stroke tracks the output position of the short stroke, where downsampler  $\mathcal{D}_F$  facilitates the sampling rate conversion. Dotted lines ( $\cdots$ ) indicate extreme high sampling rates  $f_*$ , dashdotted lines ( $-\cdots$ ) high sampling rates  $f_h$ , and dashed lines ( $- - -$ ) low sampling rates  $f_l$ . Both control loops include a feedback controller  $C_{FB}$ , a feedforward controller  $C_{FF}$ , and an input shaper  $C_\psi$ . The objective is to minimize position difference  $\varepsilon_*$  through design of  $C_{\psi,LoS,l}$  and  $C_{FF,LoS,l}$ .

**Remark 1.** The assumption of integer sampling rate factors in (1) is imposed for ease of notation, but can easily be relaxed if the factor is a rational number. The proposed approach is not applicable for irrational factors, although these can often be closely approximated with rational factors.

### C. Problem formulation: Framework for exploiting multirate sampling for enhanced control performance

In this paper, the following problem is considered.

**Main problem.** Given the multirate control configuration in Fig. 3 with sampling frequencies admitting (1), a given finite-time reference trajectory  $\rho_{SS,h} \in \mathbb{R}^{N_h}$  for  $\rho_{SS,h}$ , models  $G_{SS,*}, G_{LoS,*}$  of  $G_{SS,h}, G_{LoS,l}$  at sampling frequency  $f_*$ , and controllers  $C_{FF,SS,h}, C_{\psi,SS,h}, C_{FB,SS,h}, C_{FB,LoS,l}$ , determine

$$(C_{FF,LoS,l}, C_{\psi,LoS,l}) = \arg \min_{C_{FF,LoS,l}, C_{\psi,LoS,l}} \|\underline{\varepsilon}_*\|_2^2, \quad (3)$$

where  $\underline{\varepsilon}_* \in \mathbb{R}^{N^*}$  denotes the position error  $\varepsilon_*$  over the considered interval.

Controllers  $C_{FF,SS,h}, C_{\psi,SS,h}, C_{FB,SS,h}$  are often available from earlier control designs based on the single-rate short-stroke system only, neglecting the long-stroke system and multirate aspects. A similar reasoning holds for  $C_{FB,LoS,l}$ . It is assumed that  $C_{FB,SS,h}$  and  $C_{FB,LoS,l}$  stabilize the short-stroke and long-stroke system, respectively. Note that stability is not affected by  $C_{FF,LoS,l}, C_{\psi,LoS,l}$ .

Importantly, control objective (3) incorporates the dynamics of the short stroke for design of the long-stroke controllers  $C_{FF,LoS,l}, C_{\psi,LoS,l}$ . Moreover, it considers  $\underline{\varepsilon}_*$  rather than  $\underline{\varepsilon}_{LoS,l}$  and thereby takes intersample behavior into account, which is an important aspect in multirate control [25]. Note that (3) is posed in terms of finite-time signals, rather than infinite-time signals, since, in practice, tasks have a finite length.

The presented framework allows to recover single-rate control as a special case of multirate control by setting  $F_h = F_l$ . In Section VI and Section VII, multirate control is compared with single-rate control.

### D. Notation

Matrix variables are underlined, with  $\underline{I}_n$  the  $n \times n$  identity matrix,  $\underline{0}_{m \times n}$  the  $m \times n$  zero matrix,  $\underline{1}_n$  the  $n \times 1$  ones vector with all elements 1, and  $\underline{e}_n$  the  $n \times 1$  unit vector with the first element 1 and others 0. Vector  $\underline{\alpha} \in \mathbb{R}^N$ ,  $N \in \mathbb{N}$ , is given by  $\underline{\alpha} = [\alpha[0] \ \alpha[1] \ \dots \ \alpha[N-1]]^\top$ , with transpose  $(\cdot)^\top$  and  $\|\underline{\alpha}\|_2^2 = \underline{\alpha}^\top \underline{\alpha}$ . The Kronecker product is denoted  $\otimes$  and  $\text{diag}\{(\cdot)\}$  denotes a diagonal matrix with diagonal entries  $(\cdot)$ . The floor operator is given by  $\lfloor x \rfloor = \max\{m \in \mathbb{Z} \mid m \leq x\}$ . The discrete-time delay operator is denoted  $z^{-1}$ .

## III. MULTIRATE CONTROL SYSTEM

In this section, the model-based multirate controller design is presented, which constitutes Contribution (I). In Section III-A, the time-varying aspects of multirate systems are modeled. In Section III-B, these models are used to describe the multirate control diagram in Fig. 3. Based on these results, the multirate controller is presented in Section IV.

### A. Modeling multirate systems: Time-varying aspects

In this section, building blocks to model the multirate system in Fig. 3 are presented. The system is modeled over the finite-time length considered in the main problem in Section II-C.

Consider a causal, single-input, single-output (SISO), discrete-time, linear time-invariant (LTI) system  $H$  with Markov parameters  $h(k) \in \mathbb{R}$ ,  $k = 0, 1, \dots, N-1$ . The mapping from the finite-time input  $\underline{\alpha} \in \mathbb{R}^N$  to the finite-time output  $\underline{\beta} \in \mathbb{R}^N$  is given by  $\underline{H} \in \mathbb{R}^{N \times N}$  via

$$\underline{\beta} = \underline{H}\underline{\alpha}, \quad (4)$$

$$\begin{bmatrix} \beta[0] \\ \beta[1] \\ \beta[2] \\ \vdots \\ \beta[N-1] \end{bmatrix} = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 \\ h(1) & h(0) & 0 & \dots & 0 \\ h(2) & h(1) & h(0) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & h(N-3) & \dots & h(0) \end{bmatrix} \begin{bmatrix} \alpha[0] \\ \alpha[1] \\ \alpha[2] \\ \vdots \\ \alpha[N-1] \end{bmatrix}. \quad (5)$$

Since  $\underline{\alpha}, \underline{\beta}$  have the same sampling frequency,  $\underline{H}$  is square. Moreover, since  $H$  is causal and time-invariant,  $\underline{H}$  is lower triangular and Toeplitz, respectively [1].

The multirate system in Fig. 3 involves different sampling frequencies. The conversions between the different sampling frequencies are given as follows, see also [36, Section 4.1.1] and [25, Definition 5]. Let  $\underline{\alpha} \in \mathbb{R}^{FN}$ ,  $F, N \in \mathbb{N}$ , then the downsampling operator  $\mathcal{D}_F : \mathbb{R}^{FN} \mapsto \mathbb{R}^N$  with factor  $F$  yields  $\underline{\beta} = \mathcal{D}_F(\underline{\alpha}) \in \mathbb{R}^N$  where

$$\beta[k] = \alpha[Fk], \quad k = 0, 1, \dots, N-1. \quad (6)$$

Let  $\underline{\alpha} \in \mathbb{R}^N$ ,  $N \in \mathbb{N}$ , then the upsampling operator  $\mathcal{S}_{u,F} : \mathbb{R}^N \mapsto \mathbb{R}^{FN}$  with factor  $F \in \mathbb{N}$  yields  $\underline{\beta} = \mathcal{S}_{u,F}(\underline{\alpha}) \in \mathbb{R}^{FN}$  where

$$\beta[k] = \begin{cases} \alpha[\lfloor \frac{k}{F} \rfloor], & k = 0, F, 2F, \dots, (N-1)F, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The upsampling operator inserts zeros in between the values of the low rate signal to create a high rate signal. The interpolation is performed using a zero-order-hold interpolator. In terms of discrete-time transfer functions, the zero-order-hold interpolator with factor  $F \in \mathbb{N}$  is defined as

$$\mathcal{I}_{ZO,H,F} = \sum_{f=0}^{F-1} z^{-f}. \quad (8)$$

The zero-order-hold interpolator is used in combination with the upsampling operator for upsampling. The resulting zero-order-hold upsampler is defined by  $\mathcal{H}_F := \mathcal{I}_{ZO,H,F} \mathcal{S}_{u,F}$ , i.e., let  $\underline{\alpha} \in \mathbb{R}^N$ ,  $N \in \mathbb{N}$ , then  $\mathcal{H}_F$  with factor  $F \in \mathbb{N}$  yields  $\underline{\beta} = \mathcal{H}_F(\underline{\alpha}) \in \mathbb{R}^{FN}$  where

$$\beta[k] = \alpha[\lfloor \frac{k}{F} \rfloor], \quad k = 0, 1, \dots, (N-1)F. \quad (9)$$

The system description and controller design are based on finite-time descriptions. The finite-time description of the downsampling operator  $\mathcal{D}_F$  with factor  $F \in \mathbb{N}$  is given by

$$\underline{\mathcal{D}}_F = \underline{I}_N \otimes \underline{e}_F^\top \in \mathbb{R}^{N \times FN}, \quad (10)$$

i.e., let  $\underline{\alpha} \in \mathbb{R}^{FN}$ ,  $N \in \mathbb{N}$  and let  $\underline{\beta} \in \mathbb{R}^N$  be given by (6), then  $\underline{\beta} = \underline{\mathcal{D}}_F \underline{\alpha}$  with  $\underline{\mathcal{D}}_F$  in (10). The finite-time description of the zero-order-hold upsampling operator  $\mathcal{H}_F$  with factor  $F \in \mathbb{N}$  is given by

$$\underline{\mathcal{H}}_F = \underline{I}_N \otimes \underline{1}_F \in \mathbb{R}^{FN \times N}, \quad (11)$$

i.e., let  $\underline{\alpha} \in \mathbb{R}^N$ ,  $N \in \mathbb{N}$  and let  $\underline{\beta} \in \mathbb{R}^{FN}$  be given by (9), then  $\underline{\beta} = \underline{\mathcal{H}}_F \underline{\alpha}$  with  $\underline{\mathcal{H}}_F$  in (11). Examples of  $\underline{\mathcal{D}}_F$  and  $\underline{\mathcal{H}}_F$  are provided by Example 2.

**Example 2** (Downsampler and upsampler). Let  $F = 2$ ,  $N = 3$ , then  $\underline{\mathcal{D}}_F$  in (10) and  $\underline{\mathcal{H}}_F$  in (11) are given by

$$\underline{\mathcal{D}}_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \underline{\mathcal{H}}_F = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \quad (12)$$

Let  $\underline{\alpha} = [1 \ 2 \ 3 \ 4 \ 5 \ 6]^\top$ , then  $\underline{\beta} := \underline{\mathcal{D}}_F(\underline{\alpha}) = \underline{\mathcal{D}}_F \underline{\alpha} = [1 \ 3 \ 5]^\top$  and  $\underline{\gamma} := \underline{\mathcal{H}}_F(\underline{\beta}) = \underline{\mathcal{H}}_F \underline{\beta} = [1 \ 1 \ 3 \ 3 \ 5 \ 5]^\top$ . Note that  $\underline{\gamma} = \underline{\mathcal{H}}_F \underline{\mathcal{D}}_F \underline{\alpha} \neq \underline{\alpha}$ , since

$$\underline{\mathcal{H}}_F \underline{\mathcal{D}}_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \neq \underline{I}_6. \quad (13)$$

Example 2 shows that down-up sampling affects the signal. More generally, using the Kronecker mixed-product property

$$(\underline{A} \otimes \underline{B})(\underline{C} \otimes \underline{D}) = (\underline{AC}) \otimes (\underline{BD}), \quad (14)$$

it can be shown that

$$\underline{\mathcal{D}}_F \underline{\mathcal{H}}_F = \underline{I}_N, \quad \underline{\mathcal{H}}_F \underline{\mathcal{D}}_F = \underline{I}_N \otimes (\underline{1}_F \underline{e}_F^\top) \neq \underline{I}_{FN}. \quad (15)$$

A key observation is that up-down sampling  $\underline{\mathcal{D}}_F \underline{\mathcal{H}}_F$  has no effect on the signal, whereas down-up sampling  $\underline{\mathcal{H}}_F \underline{\mathcal{D}}_F$  does affect the signal. In fact,  $\underline{\mathcal{H}}_F \underline{\mathcal{D}}_F$  is block Toeplitz with block size  $F$ , see also Example 2, and hence the down-up sampling operation is not LTI, but linear periodically time-varying (LPTV) with period  $F$ . An important consequence

is that if an input-output operation involves any sampling rate lower than the input sampling rate, then the operation is LPTV. Indeed, this is the case for the multirate control diagram in Fig. 3, which is thus LPTV. The presented finite-time descriptions enable to exactly describe this time-varying multirate system.

In the following section, the multirate control diagram is presented, based on the finite-time descriptions, presented in this section.

**Remark 3.** A more general definition of the downsampler  $\mathcal{D}_F$  in (6) is obtained by considering  $\alpha \in \mathbb{R}^M$ ,  $\beta \in \mathbb{R}^{\lceil \frac{M}{F} \rceil}$ ,  $F, M \in \mathbb{N}$ . For ease of notation, it is assumed that  $M = FN$ .

### B. Multirate control diagram

The full control diagram of the architecture in Fig. 3 is shown in Fig. 4 and includes the modeling of systems  $G_{SS,h}$  and  $G_{LoS,l}$ . The systems are modeled through  $G_{SS,*}$  and  $G_{LoS,*}$  operating at the extremely high rate  $f_*$ , which approximate the underlying continuous-time systems  $G_{SS}$  and  $G_{LoS}$ , respectively. Here,  $\mathcal{H}_*$ ,  $\mathcal{S}_*$  are the continuous-time hold (digital-to-analog) and sampling (analog-to-digital converter). Recall that signals at rate  $f_*$  are not available for real-time feedback control. However, this approach enables evaluation of the tracking error  $\varepsilon_*$  at rate  $f_*$ .

To determine the optimal controllers, the relation between  $C_{FF,LoS,l}$ ,  $C_{\psi,LoS,l}$  and  $\varepsilon_*$  is required. The dependence of finite-time  $\varepsilon_*$  on  $\underline{\rho}_{SS,h}$ ,  $\underline{\nu}_{FF,LoS,l}$ ,  $\underline{\rho}_{\psi,LoS,l}$  is given by Lemma 4.

**Lemma 4.** Given the finite-time descriptions in Section III-A,  $\varepsilon_*$  in Fig. 4 is given by

$$\varepsilon_* = \underline{\psi}_{SS,*} - \underline{\mathcal{A}} \begin{bmatrix} \underline{\nu}_{FF,LoS,l} \\ \underline{\rho}_{\psi,LoS,l} \end{bmatrix}, \quad (16)$$

with

$$\underline{\psi}_{SS,*} = \underline{G}_{SS,*} \underline{\mathcal{H}}_{F_h} \underline{S}_{SS,h} \times (\underline{C}_{FF,SS,h} + \underline{C}_{FB,SS,h} \underline{C}_{\psi,SS,h}) \underline{\rho}_{SS,h}, \quad (17)$$

$$\underline{\mathcal{A}} = \underline{G}_{LoS,*} \underline{\mathcal{H}}_{F_l} \underline{S}_{LoS,l} [\underline{I}_{N_l} \quad \underline{C}_{FB,LoS,l}], \quad (18)$$

$$\underline{S}_{SS,h} = (\underline{I}_{N_h} + \underline{C}_{FB,SS,h} \underline{G}_{SS,h})^{-1}, \quad (19)$$

$$\underline{S}_{LoS,l} = (\underline{I}_{N_l} + \underline{C}_{FB,LoS,l} \underline{G}_{LoS,l})^{-1}. \quad (20)$$

*Proof.* See Section A.  $\square$

An important observation in Lemma 4 is that  $\underline{\mathcal{A}}$  includes sampling rate changes and hence the transfer function from  $\underline{\nu}_{FF,LoS,l}$ ,  $\underline{\rho}_{\psi,LoS,l}$  to  $\varepsilon_*$  is LPTV and cannot be described using traditional frequency domain transfer functions. In the next section, the controllers are designed.

## IV. MULTIRATE CONTROLLER DESIGN

In the previous section, the multirate system in Fig. 4 was modeled. In this section, the controllers are parameterized and the optimal controller parameters are presented, constituting Contribution (II). Furthermore, the multirate system is further improved by modifying the controller implementation and design, which constitutes Contribution (III).

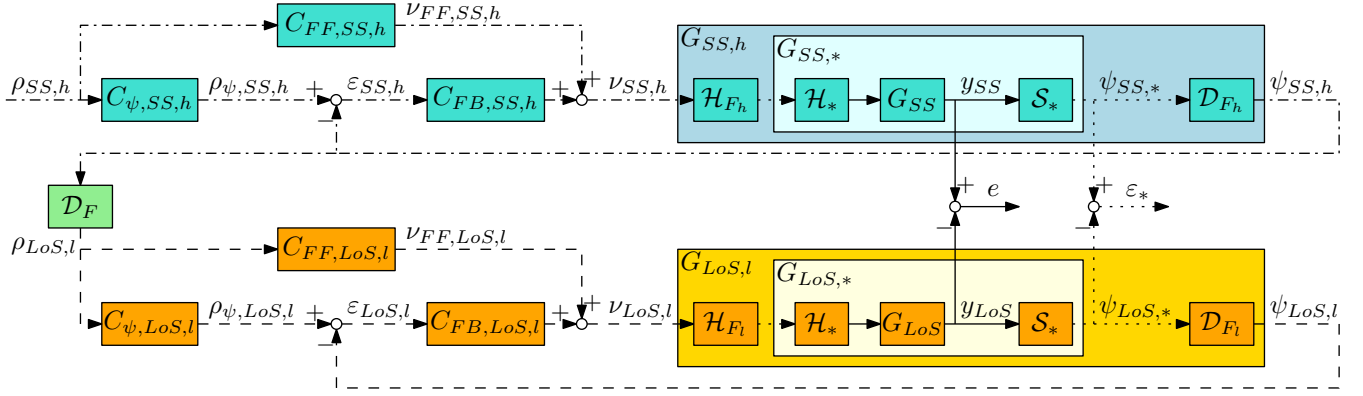


Fig. 4. Multirate control architecture where the short-stroke (SS) loop (top part) runs at high rate  $f_h$  and the long-stroke (LoS) loop (bottom part) at low rate  $f_l$ . The interconnection is provided through downsampler  $\mathcal{D}_F$ . Error  $\varepsilon_*$  is an approximation of the continuous-time signal  $e$  at extreme high rate  $f_*$ . Solid lines (—) indicate continuous-time signals, dotted lines (····) extreme high sampling rates  $f_*$ , dashdotted lines (-·-·) high sampling rates  $f_h$ , and dashed lines (- - -) low sampling rates  $f_l$ . Both subsystems ( $G$ ) are controlled through feedback ( $C_{FB}$ ), feedforward ( $C_{FF}$ ), and input shaping ( $C_\psi$ ). The objective is to minimize  $\varepsilon_*$  through design of  $C_{FF,LoS,l}$  and  $C_{\psi,LoS,l}$  such that the long stroke tracks the short stroke. In this configuration,  $C_{FF,LoS,l}$  and  $C_{\psi,LoS,l}$  are implemented at the low rate, i.e.,  $f_c = f_l$ .

### A. Controller parameterization

To address arbitrary reference trajectories, the feedforward and input shaping filters are parameterized in terms of basis functions, see, for example, [37], [38]. Basis functions decouple the parameters from the reference trajectory, allowing variations in the reference trajectories without affecting the parameters. This is in contrast to standard learning approaches [6] in which a command signal for one specific reference trajectory is learned.

Inspired by [39], controllers  $C_{FF,LoS,l}$ ,  $C_{\psi,LoS,l}$  are parameterized in terms of difference operators according to Definition 5. Note that  $C_{FF,LoS,l}(\underline{0}) = 0$  and  $C_{\psi,LoS,l}(\underline{0}) = 1$  such that if the parameters are zero, only feedback control is used.

**Definition 5.**  $C_{FF,LoS,l}$  and  $C_{\psi,LoS,l}$  in Fig. 4 are given by

$$C_{FF,LoS,l}(\underline{\theta}_{FF}) = \sum_{i=0}^{n_{FF}-1} \theta_{FF}[i] \left( \frac{f_l(z-1)}{z} \right)^{i+1}, \quad (21)$$

$$C_{\psi,LoS,l}(\underline{\theta}_\psi) = 1 + \sum_{i=0}^{n_\psi-1} \theta_\psi[i] \left( \frac{f_l(z-1)}{z} \right)^{i+1}, \quad (22)$$

with design parameters  $\underline{\theta}_{FF}$ ,  $\underline{\theta}_\psi$ .

Theorem 6 shows that  $\nu_{FF,LoS,l}$  and  $\rho_{\psi,LoS,l}$  depend affine on parameters  $\underline{\theta}_{FF}$  and  $\underline{\theta}_\psi$ , respectively.

**Theorem 6.** Given Definition 5, the finite-time descriptions of  $\nu_{FF,LoS,l}$  and  $\rho_{\psi,LoS,l}$  are given by

$$\nu_{FF,LoS,l} = \underline{C}_{FF,LoS,l} \underline{D}_F \underline{\psi}_{SS,h} = \underline{\Phi}_{FF,l} \underline{\theta}_{FF}, \quad (23)$$

$$\rho_{\psi,LoS,l} = \underline{C}_{\psi,LoS,l} \underline{D}_F \underline{\psi}_{SS,h} = \underline{D}_F \underline{\psi}_{SS,h} + \underline{\Phi}_{\psi,l} \underline{\theta}_\psi, \quad (24)$$

with

$$\underline{\Phi}_{x,l} = \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_x+1} \otimes \underline{e}_F \\ \underline{0}_{(N_h-F(n_x+1)) \times (n_x+1)} \end{bmatrix} \underline{R}_{x,l}, \quad (25)$$

$$\underline{T}_{\psi_{SS,h}} = \begin{bmatrix} \psi_{SS,h}[0] & 0 & 0 & \dots & 0 \\ \psi_{SS,h}[1] & \psi_{SS,h}[0] & 0 & \dots & 0 \\ \psi_{SS,h}[2] & \psi_{SS,h}[1] & \psi_{SS,h}[0] & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \psi_{SS,h}[N_h-1] & \psi_{SS,h}[N_h-2] & \psi_{SS,h}[N_h-3] & \dots & \psi_{SS,h}[0] \end{bmatrix}, \quad (26)$$

$$\underline{R}_{x,l} = \begin{bmatrix} \frac{1}{-1} & \frac{1}{-2} & \frac{1}{-3} & \dots & \frac{1}{-n_x} \\ 0 & 1 & 3 & \dots & * \\ 0 & 0 & -1 & \dots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (-1)^{n_x} \end{bmatrix} \text{diag}\{f_l^1, \dots, f_l^{n_x}\}, \quad (27)$$

where  $x$  refers to  $FF$  or  $\psi$ .

*Proof.* See Section B.  $\square$

Combining Theorem 6 with Lemma 4 reveals an affine dependence of  $\varepsilon_*$  on  $\underline{\theta}_{FF}$  and  $\underline{\theta}_\psi$  as made explicit in Lemma 7.

**Lemma 7.** Error  $\varepsilon_*$  is given by

$$\varepsilon_* = \underline{b} - \underline{A} \underline{\Phi} \underline{\theta}, \quad (28)$$

with

$$\underline{b} = \underline{\psi}_{SS,*} - \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{S}_{LoS,l} \underline{C}_{FB,LoS,l} \underline{D}_F \underline{\psi}_{SS,h}, \quad (29)$$

$$\underline{\Phi} = \begin{bmatrix} \underline{\Phi}_{FF,l} & 0 \\ 0 & \underline{\Phi}_{\psi,l} \end{bmatrix}, \quad (30)$$

$$\underline{\theta} = \begin{bmatrix} \underline{\theta}_{FF} \\ \underline{\theta}_\psi \end{bmatrix}. \quad (31)$$

*Proof.* See Section C.  $\square$

Lemma 7 provides the dependence of  $\varepsilon_*$  on the controller parameters  $\underline{\theta}$ . In the next section, the parameters  $\underline{\theta}$  are optimized.

### B. Controller optimization

The optimal parameters for the control objective in (3) are given by the solution of the optimization problem

$$\min_{\underline{\theta}} \|\varepsilon_*\|_2^2 \quad \text{s.t.} \quad \varepsilon_* = \underline{b} - \underline{A} \underline{\Phi} \underline{\theta}. \quad (32)$$

If  $\mathcal{A}\Phi$  is full rank, the solution to this quadratic optimization problem is given by the least-squares solution  $\underline{\theta} = \underline{\theta}_0$ , with

$$\underline{\theta}_0 = ((\mathcal{A}\Phi)^\top (\mathcal{A}\Phi))^{-1} (\mathcal{A}\Phi)^\top b. \quad (33)$$

For perfect models, solution (33) provides the optimal solution.

In practice, there are always model mismatches for which the parameters are iteratively learned through an approach that closely resembles norm-optimal iterative learning control (ILC) [6] based on the models and data of previous executions. A key observation is that the models are time-varying, which is in sharp contrast to standard learning techniques. One execution of the learning approach is referred to as a trial or task and indicated with subscript  $j = 0, 1, 2, \dots$ . The parameters  $\underline{\theta}_{j+1}$  for the next trial are determined as those minimizing the performance criterion in Definition 8 [6] based on measured data from trial  $j$ .

**Definition 8** (Performance criterion). *The performance criterion for trial  $j + 1$ ,  $j = 0, 1, 2, \dots$  is given by*

$$\begin{aligned} \mathcal{J}(\underline{\theta}_{j+1}) = & \|\underline{\varepsilon}_{j+1,*}\|_{\underline{W}_\varepsilon}^2 + \|\underline{\xi}_{j+1,l}\|_{\underline{W}_\xi}^2 \\ & + \|\underline{\xi}_{j+1,l} - \underline{\xi}_{j,l}\|_{\underline{W}_{\Delta\xi}}^2 \end{aligned} \quad (34)$$

where  $\|(\cdot)\|_W^2 = (\cdot)^\top W(\cdot)$ , with  $\underline{W}_\varepsilon \in \mathbb{R}^{N_* \times N_*}$  positive definite,  $\underline{W}_\xi, \underline{W}_{\Delta\xi} \in \mathbb{R}^{2N_l \times 2N_l}$  semi-positive definite, and

$$\underline{\varepsilon}_{j+1,*} = \underline{\varepsilon}_{j,*} - \mathcal{A}\Phi(\underline{\theta}_{j+1} - \underline{\theta}_j), \quad (35)$$

$$\underline{\xi}_{j,l} = \Phi \underline{\theta}_j. \quad (36)$$

Performance criterion (34) can be used to address several control goals. For example, for  $\underline{W}_\varepsilon = \underline{I}_{N_*}$  and  $\underline{W}_\xi = \underline{W}_{\Delta\xi} = \underline{0}_{2N_l}$ , the control goal in (3) is addressed, i.e., minimizing  $\|\underline{\varepsilon}_*\|_2^2$ . The optimal parameters for the general criterion are given by Theorem 9.

**Theorem 9** (Iterative solution). *The parameters  $\underline{\theta}_{j+1}$ ,  $j = 0, 1, 2, \dots$ , that minimize  $\mathcal{J}(\underline{\theta}_{j+1})$  in Definition 8 are given by*

$$\underline{\theta}_{j+1} = \underline{Q}\underline{\theta}_j + \underline{L}\underline{\varepsilon}_{j,*}, \quad (37)$$

with

$$\begin{aligned} \underline{Q} = & ((\mathcal{A}\Phi)^\top \underline{W}_\varepsilon (\mathcal{A}\Phi) + \Phi^\top (\underline{W}_\xi + \underline{W}_{\Delta\xi}) \Phi)^{-1} \\ & \times ((\mathcal{A}\Phi)^\top \underline{W}_\varepsilon (\mathcal{A}\Phi) + \Phi^\top \underline{W}_{\Delta\xi} \Phi), \end{aligned} \quad (38)$$

$$\begin{aligned} \underline{L} = & ((\mathcal{A}\Phi)^\top \underline{W}_\varepsilon (\mathcal{A}\Phi) + \Phi^\top (\underline{W}_\xi + \underline{W}_{\Delta\xi}) \Phi)^{-1} \\ & \times (\mathcal{A}\Phi)^\top \underline{W}_\varepsilon. \end{aligned} \quad (39)$$

Theorem 9 directly follows from substitution of (35) and (36) in (34) and equating  $\nabla \mathcal{J}(\underline{\theta}_{j+1}) = \underline{0}$ , see also [37]. Note that  $\underline{W}_\varepsilon, \underline{W}_\xi, \underline{W}_{\Delta\xi}$  should be chosen such that the inverse in (38) and (39) exists. A step-by-step procedure for the iterative algorithm is provided in Algorithm 10, where (33) provides initial parameters based on models only.

**Algorithm 10** (Iterative tuning procedure). *Calculate  $\underline{Q}, \underline{L}$  using (38), (39), set  $j = 0$  and determine  $\underline{\theta}_0$  in (33). Then, perform the following sequence of steps:*

1. Execute task  $j$  and record data  $\underline{\varepsilon}_{j,*}$ .
2. Determine  $\underline{\theta}_{j+1}$  through (37).

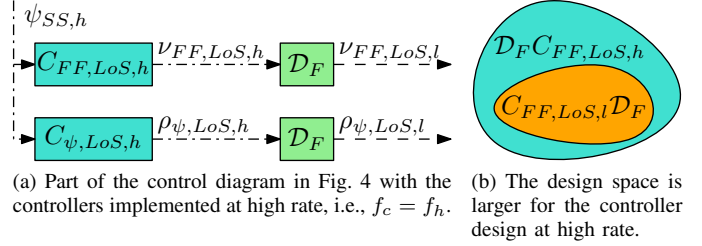


Fig. 5. Designing and implementing the controllers at high rate allows to exploit all information in  $\psi_{SS,h}$  and thereby improve performance.

3. Set  $j \rightarrow j + 1$  and repeat from step 1 until satisfactory convergence in  $\underline{\theta}_j$  or a user-defined maximum number of trials is reached.

Algorithm 10 provides the iterative tuning solution for the time-varying multirate system with controller design at the low rate. In the next section, the controllers are explicitly designed and implemented at the high rate to enhance the performance/cost trade-off in Fig. 1.

### C. Performance enhancement: High-rate control

In the previous sections, the optimal controller for the multirate system in Fig. 4 is presented. In this section, the performance of the multirate system is further improved by modifying the controller implementation and design, which constitutes Contribution (III). The results of the previous section are recovered as a special case.

In contrast to time-invariant systems, time-varying systems do generally not commute, i.e., interchanging the order affects the output. One key advantage of the proposed approach is that this property can be directly exploited to enhance the performance/cost trade-off in Fig. 1. In Fig. 4, both the feedforward controller and input shaper of the long stroke are implemented at the low rate  $f_l$ . In this section, these controllers are implemented at high rate  $f_h$  as shown in Fig. 5(a). This implementation has the potential to improve the performance since  $\psi_{SS,h}$  contains more information than  $\rho_{LoS,l} = \mathcal{D}_F \psi_{SS,h}$ . This also follows from the noble identity  $\mathcal{D}_F H(z^F) \equiv H(z) \mathcal{D}_F$ , with  $H$  a discrete-time system rational in  $z$  [36, Section 4.2]. Indeed, since the frequency response of  $C_{FF,LoS,h}$  is independent from that of  $C_{FF,LoS,l}$ , there is more design freedom as illustrated in Fig. 5(b).

The additional cost of the high-rate implementation is negligible since it only involves a different controller design in software, without effecting hardware. In particular, it uses sensor information of the short-stroke loop at high rate, which is also required for feedback control on the short stroke. The new design does not require sensor information of the long-stroke loop at a higher rate. The actuation of the long-stroke loop remains at low rate.

The parameterization of the controllers at high rate is similar to that in Definition 5 and provided by Definition 11.

**Definition 11.**  $C_{FF,LoS,h}$  and  $C_{\psi,LoS,h}$  in Fig. 5 are given by

$$C_{FF,LoS,h}(\theta_{FF}) = \sum_{i=0}^{n_{FF}-1} \theta_{FF}[i] \left( \frac{f_h(z-1)}{z} \right)^{i+1}, \quad (40)$$

$$C_{\psi,LoS,h}(\theta_{\psi}) = 1 + \sum_{i=0}^{n_{\psi}-1} \theta_{\psi}[i] \left( \frac{f_h(z-1)}{z} \right)^{i+1}. \quad (41)$$

The finite-time descriptions for this parameterization are provided in Lemma 12. Using these results, the iterative approach outlined in Algorithm 10 is directly applicable.

**Lemma 12.** Given Definition 11, the finite-time descriptions (23), (24), and (30) change to

$$\underline{\nu}_{FF,LoS,l} = \underline{\mathcal{D}}_F \underline{C}_{FF,LoS,h} \underline{\psi}_{SS,h} = \underline{\mathcal{D}}_F \underline{\Phi}_{FF,h} \underline{\theta}_{FF}, \quad (42)$$

$$\underline{\rho}_{\psi,LoS,l} = \underline{\mathcal{D}}_F \underline{C}_{\psi,LoS,h} \underline{\psi}_{SS,h} = \underline{\mathcal{D}}_F \underline{\psi}_{SS,h} + \underline{\mathcal{D}}_F \underline{\Phi}_{\psi,h} \underline{\theta}_{\psi}, \quad (43)$$

$$\underline{\Phi} = \begin{bmatrix} \underline{\mathcal{D}}_F \underline{\Phi}_{FF,h} & 0 \\ 0 & \underline{\mathcal{D}}_F \underline{\Phi}_{\psi,h} \end{bmatrix}, \quad (44)$$

with

$$\underline{\Phi}_{x,h} = \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_x+1} \\ \underline{0}_{(N_h-(n_x+1)) \times (n_x+1)} \end{bmatrix} \underline{R}_{x,h}, \quad (45)$$

$$\underline{R}_{x,h} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ -1 & -2 & -3 & \dots & -n_x \\ 0 & 1 & 3 & \dots & * \\ 0 & 0 & -1 & \dots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (-1)^{n_x} \end{bmatrix} \text{diag}\{f_h^1, \dots, f_h^{n_x}\}, \quad (46)$$

where  $x$  refers to  $FF$  or  $\psi$ .

*Proof.* See Section D.  $\square$

The controller design and implementation at high rate completes the multirate controller design. Next, the advantages of multirate control over single-rate control are demonstrated in both simulation and experiments.

## V. EXPERIMENTAL SETUP: A DUAL-STAGE WAFER STAGE SYSTEM

In the remainder of this paper, the multirate control design framework presented in the previous section is validated on a dual-stage system, both in simulations and experiments. In this section, the wafer stage system is introduced in more detail and the experimental setup of the dual-stage system is presented.

### A. Wafer stages: Key components in lithography machines

Wafer stages are key components in wafer scanners. Wafer scanners are state-of-the-art lithography machines for the automated production of integrated circuits. In Fig. 6, a schematic illustration of a wafer scanner system is depicted. Ultra-violet light from a light source ① passes through a reticle ②, which contains a blueprint of the integrated circuits to be manufactured. The reticle is clamped atop the reticle stage ③ which performs a scanning motion. The resulting image of the reticle is scaled down by a lens system ④ and projected onto the light sensitive layers of a wafer ⑤. The wafer is clamped

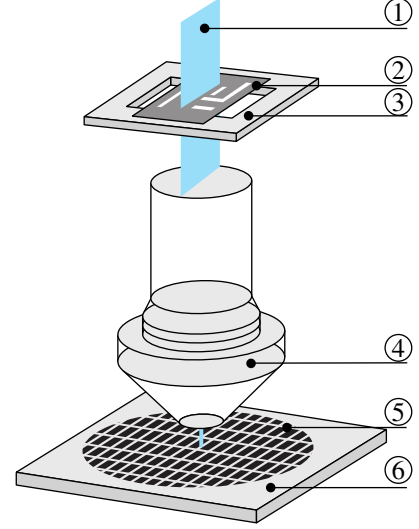


Fig. 6. Schematic illustration of a wafer scanner system, consisting of light source ①, reticle ②, reticle stage ③, lens system ④, wafer ⑤, and wafer stage ⑥.

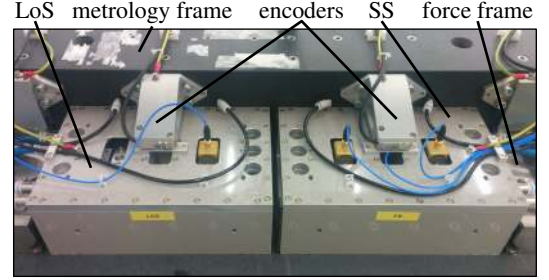


Fig. 7. Experimental setup resembling a one degree-of-freedom wafer stage. The setup consists of two air-guided stages that can translate in one horizontal direction. The positions are measured through 1 nm resolution optical encoders.

on the wafer stage ⑥ and performs a synchronized scanning motion with the reticle stage.

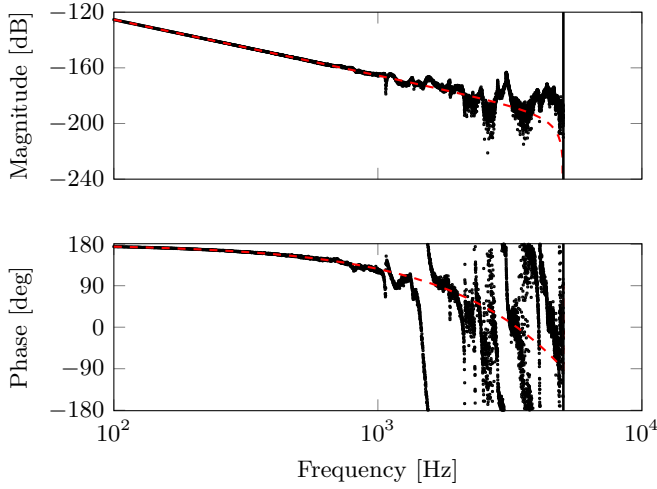
During the scanning process, the wafer stage and reticle stage track reference signals with nanometer positioning accuracy. In this paper, the focus is on control of the wafer stage, which has more stringent performance requirements than the reticle stage [40].

### B. Experimental setup

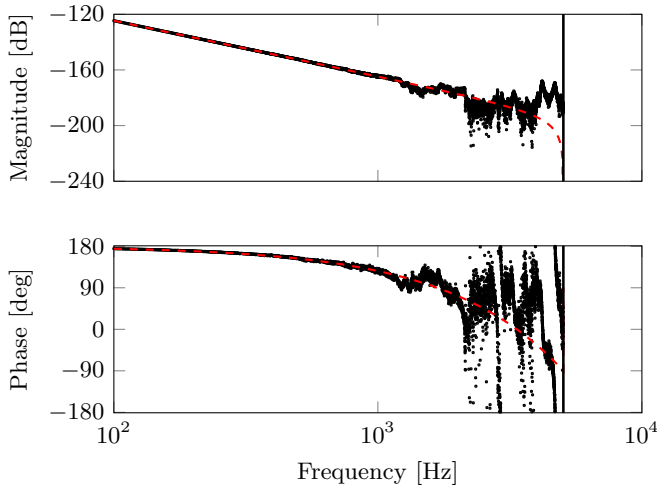
The experimental setup is shown in Fig. 7 and consists of two stages: a long stroke (LoS) and a short stroke (SS). Both stages can translate in one horizontal direction and are air guided. Each stage is actuated through a Lorentz actuator attached to the force frame. The position of each stage is measured through 1 nm resolution optical encoders attached to the metrology frame, which is separated from the force frame to reduce interaction. The total stroke is 16.0 mm.

The sampling rate of  $\varepsilon_*$  is  $f_* = 10080$  Hz. The identified frequency response functions of both stages are shown in Fig. 8. The stages are modeled as freely moving masses with





(a) Short-stroke (SS) stage.



(b) Long-stroke (LoS) stage.

Fig. 8. Measured frequency response functions (●) with sampling rate  $f_* = 10080$  Hz and the identified models (---) in (47).

one sample I/O delay:

$$G_{x,*} = z^{-1} \frac{(z+1)}{2m_x f_*^2 (z-1)^2}, \quad (47)$$

with masses  $m_{SS} = 4.70$  kg and  $m_{LoS} = 4.33$  kg. The sampling rate factors  $F_h, F_l$  are varied and provided when relevant.

Reference trajectory  $\rho_{SS,h}$  consists of a forward and backward movement with a total duration of 0.25 s and is shown in Fig. 9. The point-to-point profile is representative for the application in terms of distance, maximum acceleration, etc. A fourth-order profile is used to guarantee a smooth signal with limited high-frequency content to avoid excitation of higher-order dynamics, see also Fig. 8 and, e.g., [41].

Experiments show that the measurement noise on both  $\psi_{SS,*}$  and  $\psi_{LoS,*}$  has a variance of  $(45 \text{ nm})^2$ . This value is used during simulation to mimic experimental conditions.

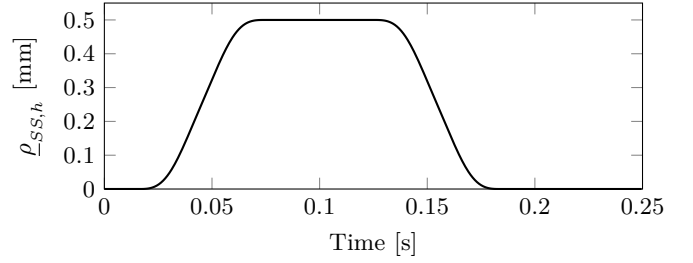


Fig. 9. Reference trajectory  $\rho_{SS,h}$  is a forward and backward movement over 0.5 mm constructed from fourth-order polynomials.

### C. Controller design

The fixed feedback controllers  $C_{FB,SS,h}$  and  $C_{FB,LoS,l}$  both consist of a lead filter, weak integrator, and second order lowpass filter based on loop-shaping techniques [5]. The controllers stabilize their respective closed-loop systems and yield a bandwidth (first 0 dB crossing of the open-loop) of 100 Hz for both loops. The feedforward controller and input shaper for the short stroke are given by

$$C_{FF,SS,h} = m_{SS} \frac{f_h^2 (z-1)^2}{z^2}, \quad (48)$$

$$C_{\psi,SS,h} = G_{SS,h} C_{FF,SS,h}. \quad (49)$$

Hence,  $C_{FF,SS,h}$  generates mass feedforward and the combination results in  $\varepsilon_{SS,h} = 0$ , if  $G_{SS,h}$  is exact.

The design of the long-stroke feedforward controller and input shaper aims to minimize  $\|\varepsilon_*\|_2^2$  by setting the weights in Definition 8 to

$$\underline{W}_\varepsilon = \underline{I}_{N_*}, \quad \underline{W}_\xi, \underline{W}_{\Delta\xi} = \underline{0}_{2N_l \times 2N_l}. \quad (50)$$

Note that these settings also facilitate fast convergence of the iterative procedure in Algorithm 10.

## VI. SIMULATION RESULTS

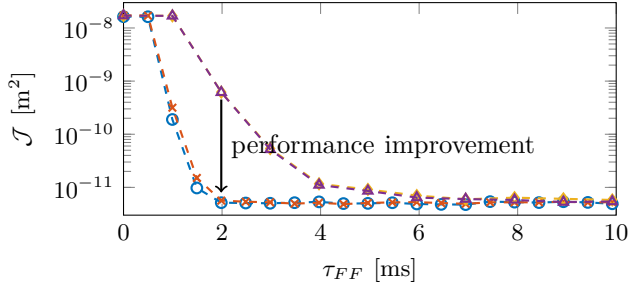
In this section, the simulation results are presented, which serve as a benchmark for the experimental results presented in Section VII. The simulations enable validation of the experimental results and constitute Contribution (IV).

### A. Comparing controllers at different rates

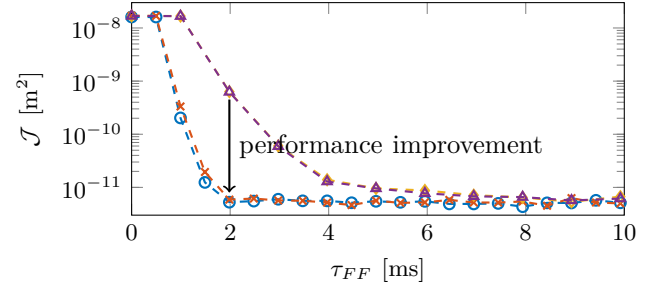
The considered control configurations are listed in Table I, see also (1). Due to the difference in sampling rate between the controller parameterization on low rate (Definition 5) and high rate (Definition 11), the number of parameters  $n_{FF}$  and  $n_\psi$  alone does not provide a fair comparison between the controllers. Therefore, the controller buffer lengths

$$\tau_{FF} := \frac{n_{FF}}{f_c}, \quad \tau_\psi := \frac{n_\psi}{f_c}, \quad (51)$$

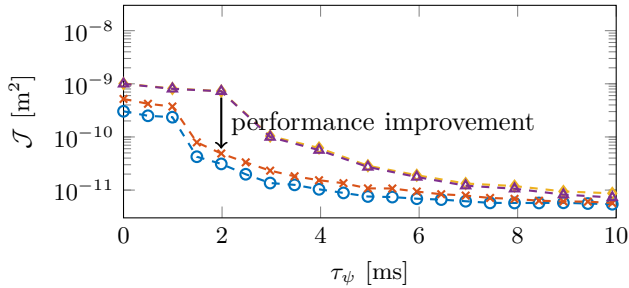
are defined, where  $f_c$  is the sampling rate of the optimized controllers, see Table I. These buffer lengths are an indication for the implementation cost of the controller.



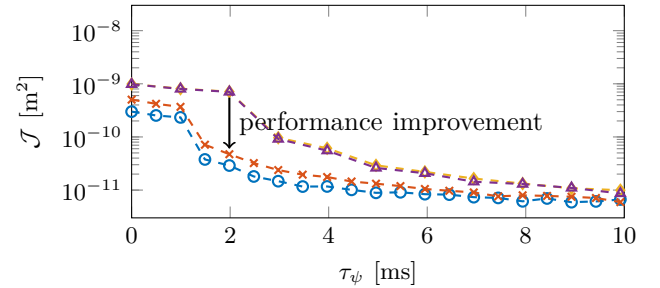
(a) Simulation results for varying  $\tau_{FF}$  show that, due to more design freedom in terms of parameters  $n_{FF}$ , the performance increases ( $\mathcal{J}$  decreases) for increasing cost (increasing buffer length  $\tau_{FF}$ ). The results shown are for fixed  $C_{\psi,LoS} = 1$  and varying  $n_{FF}$ .



(b) Experimental validation of the simulation in (a). The results are in line with the simulation results.



(c) Simulation results for varying  $\tau_{\psi}$  show that larger cost (larger buffer length  $\tau_{\psi}$ ) yields better performance (lower  $\mathcal{J}$ ). The results shown are for mass feedforward ( $n_{FF} = 2$  and  $\theta_{FF}[0] = 0$ ) and varying  $n_{\psi}$ .



(d) Experimental validation of the simulation in (c). The results are in line with the simulation results.

Fig. 10. Simulation results in (a) and (c), and experimental results in (b) and (d) for the four control configurations in Table I. As is expected, single-rate high ( $\cdot \ominus \cdot$ ) outperforms single-rate low ( $\cdot \blacktriangle \cdot$ ). The performance of multirate low ( $\cdot \blacklozenge \cdot$ ) is similar to that of single-rate low ( $\cdot \blacktriangle \cdot$ ). The performance of multirate high ( $\cdot \star \cdot$ ) is close to the performance of single-rate high ( $\cdot \ominus \cdot$ ). The results demonstrate the advantages of multirate control. Indeed, a high level of performance is achievable with multirate control for limited cost since one of the feedback control loops is evaluated at a lower rate.

TABLE I

THE FOUR DIFFERENT CONTROL CONFIGURATIONS THAT ARE EVALUATED.

Label	Symbol	$f_*$ [Hz]	$f_h$ [Hz]	$f_l$ [Hz]	$f_c$ [Hz]
Single-rate high	$\circ$	10080	2016	2016	2016
Single-rate low	$\blacktriangle$	10080	1008	1008	1008
Multirate high	$\star$	10080	2016	1008	2016
Multirate low	$\blacklozenge$	10080	2016	1008	1008

### B. Simulation setup

For comparison with the experimental results in Section VII, measurement noise is added to  $\underline{\psi}_{SS,*}$  and  $\underline{\psi}_{LoS,*}$ . The noise is modeled as zero mean, Gaussian white noise with variance  $\sigma^2 = (45 \text{ nm})^2$  based on experimental data, see also Section V-B.

In simulation, the models are exact and hence the initial parameters  $\underline{\theta}_0$  in (33) provide the optimal solution. Note that the noise introduces trial-varying disturbances, which cannot be compensated through the iterative tuning algorithm and thereby limits the achievable performance.

### C. Results

The performance/cost trade-off curves for the configurations in Table I are shown in Fig. 10(a) and Fig. 10(c). Both figures show the enhancement of the performance/cost trade-off through multirate control as illustrated in Fig. 1. In particular, both figures show I) increasing performance (decreasing  $\mathcal{J}$ )

for increasing cost (increasing  $\tau$ ); and II) excellent performance through multirate control with design at high rate.

As a direct consequence of a higher sampling rate, single-rate high outperforms single-rate low. Multirate control is a trade-off between these two and hence the performance is somewhere in between. The performance improvement of multirate low is limited compared to single-rate low. In contrast, the performance of multirate high is close to that of single-rate high. The results show that multirate control can achieve high performance with limited cost, when designed and implemented at the high rate. Indeed, the long-stroke feedback control loop remains executed at the low rate.

The results in Fig. 10(a) show the importance of adding the acceleration profile as basis function in terms of performance improvement, as is also apparent from the frequency response functions in Fig. 8 and identified models in (47). Indeed, especially for low frequencies, the stages behave as a rigid body mass. Therefore, a mass feedforward controller  $C_{FF,LoS} = \theta_{FF}[1] \frac{f_c^2(z-1)^2}{z^2}$  is used in Fig. 10(c), where parameter  $\theta_{FF}[1]$  is also optimized. Note that mass feedforward is also used for the short-stroke feedforward controller in (48).

Time-domain results for multirate high with  $n_{FF} = 2$ ,  $n_{\psi} = 0$  are shown in Fig. 11. Compared to mass feedforward, there is an additional parameter in the feedforward filter as can be observed in  $\underline{\nu}_{FF,LoS,l}$ , resulting in improved performance.

The simulation results demonstrate the potential of multirate control, especially when the controllers are designed and im-

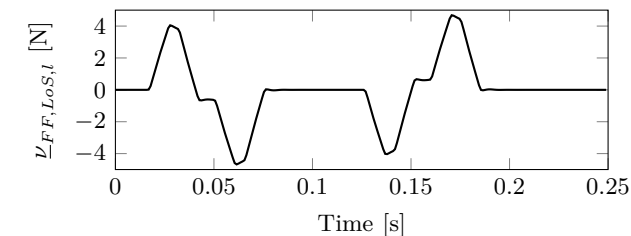
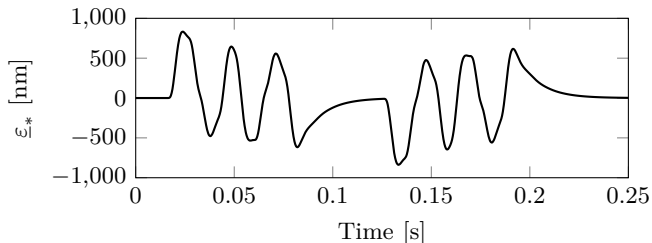
(a) Feedforward signal  $u_{FF,LoS,l}$ .(b) Error signal  $\varepsilon_*$ .

Fig. 11. Time-domain simulation results for single-rate high with  $n_{FF} = 2$ ,  $n_\psi = 0$ . The results show the importance of mass feedforward.

plemented at the high rate. Next, the results are experimentally validated.

## VII. EXPERIMENTAL RESULTS

In this section, the simulation results of the previous section are experimentally validated on the setup described in Section V. The results experimentally validate the advantages of multirate control and constitute Contribution (V).

### A. Application of iterative tuning

In contrast to simulation, the models do not exactly describe the system in experiments. Therefore, the iterative tuning procedure in Algorithm 10 is invoked to iteratively update the parameters based on measured data. The convergence of the iterative tuning algorithm is shown in Fig. 12 for the various control configurations in Table I with a fixed buffer length  $\tau_{FF} = 1$  ms ( $\tau_\psi = 0$ ).

The results in Fig. 12 show fast convergence (one trial) of the iterative algorithm as desired. Note that the deviations over the trials are caused by trial-varying disturbances for which the algorithm cannot compensate. In the remainder, five trials are used and only the results of the fifth trial are shown.

### B. Results

The experimental results for the simulations in Fig. 10(a) and Fig. 10(c) are shown in Fig. 10(b) and Fig. 10(d), respectively. The results are in line with the simulation results and the conclusions in Section VI, i.e., higher performance (lower  $\mathcal{J}$ ) for increasing number of parameters (increasing  $\tau$ ), and excellent performance for multirate control with control design at high rate (multirate high).

Time-domain signals for several parameterizations with multirate high are shown in Fig. 13. Clearly, mass feedforward only ( $n_{FF} = 2$ ,  $\theta_{FF}[0] = 0$ ,  $n_\psi = 0$ ) is restrictive

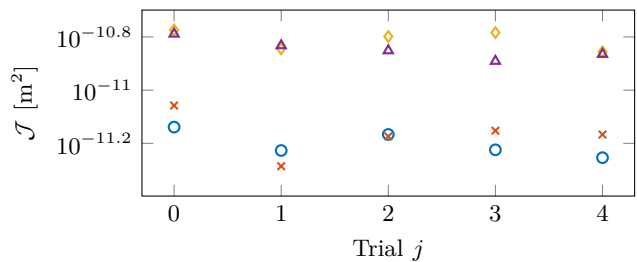


Fig. 12. Experimental results of the performance criterion over trials for  $\tau_{FF} = 1$  ms,  $\tau_\psi = 0$  with single-rate high (○), multirate high (×), multirate low (◇), and single-rate low (△). The results show that all control configurations converge in one trial up to the level of trial-varying disturbances for which the iterative tuning algorithm cannot compensate.

and achieves moderate performance. When using  $n_{FF} = 2$ ,  $n_\psi = 0$  there is more design freedom resulting in better performance. Adding design freedom in the input shaper by using  $n_{FF} = 2$ ,  $n_\psi = 4$  yields even better performance. Most design freedom is obtained by fully parameterizing the feedforward signal as in traditional learning control with  $n_{FF} = N_h$  ( $n_\psi = 0$ ) and yields the best performance. Indeed, the performance of standard learning control in which the full signal is learned is superior for repeating tasks. However, the performance deteriorates drastically when the trajectory  $\rho_{SS,h}$  is changed, see for example [37], [38], which conflicts with the requirement on reference task flexibility in Section II-C. Hence, there is a trade-off between performance and task flexibility, which can be balanced using basis functions.

### C. Summary

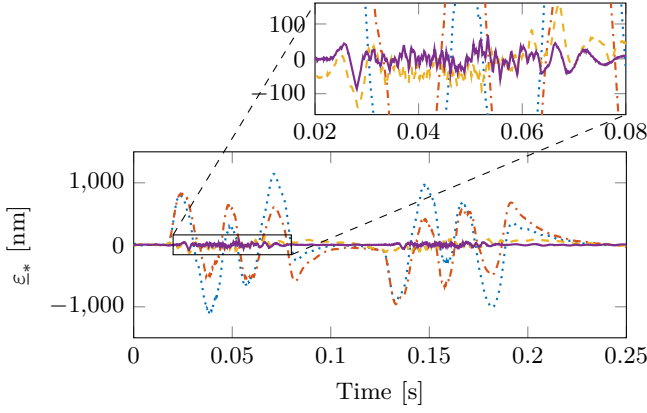
The experimental results validate the simulation results and thereby demonstrate the potential of multirate control for dual-stage systems. Both the simulations and experiments show that a multirate design approach with control design at the high rate can significantly enhance the performance compared to traditional single-rate control on the low rate. In fact, the performance is similar to that of single-rate control at the high rate, but obtained with a lower cost since one of the control loops is executed at the low rate which reduces hardware cost.

## VIII. CONCLUSION

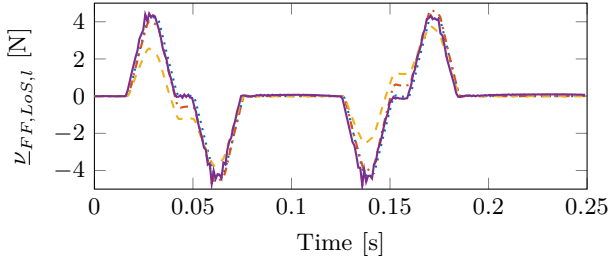
In most motion systems, all control loops are operated on a single, fixed sampling rate since this allows the use of well-known control design techniques. However, for such a design, increasing the sampling rate to increase performance is costly in terms of required hardware since all control loops are affected.

In this paper, a multirate approach is exploited to enhance the traditional performance/cost trade-off. In essence, this allows to allocate the performance and cost over the different control loops. The time variance introduced by multirate sampling complicates control design and constitutes the main challenge addressed in this paper.

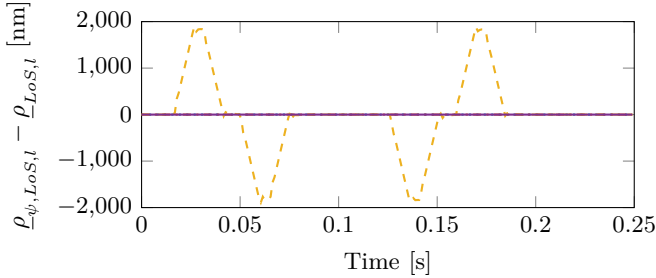
The main contribution of this paper is a control design framework for multirate systems. The framework facilitates optimal feedforward control design through iterative tuning



(a) Parameterizations with more design freedom yield a smaller error  $\varepsilon_*$  which is also apparent in the performance criterion  $\mathcal{J}$  shown in Fig. 10.



(b) The different parameterizations yield different feedforward signals  $\underline{v}_{FF,LoS,l}$ .



(c) The shaped input  $\rho_{\psi,LoS,l}$  is only different from  $\rho_{LoS,l}$  for the parameterization with  $n_{\psi} = 4$  (---) since  $n_{\psi} = 0$  for the other parameterizations.

Fig. 13. Time-domain results for multirate high for different parameterizations. In ascending order of design freedom: mass feedforward ( $n_{FF} = 2$ ,  $\theta_{FF}[0] = 0$ ,  $n_{\psi} = 0$ ) (⋯⋯);  $n_{FF} = 2$ ,  $n_{\psi} = 0$  (-.-.);  $n_{FF} = 2$ ,  $n_{\psi} = 4$  (- - -); and full learning of  $\underline{v}_{FF,LoS,h}$  ( $n_{FF} = N_h$ ,  $n_{\psi} = 0$ ) (—). More design freedom reduces the error  $\varepsilon_*$ .

control. Through simulations and experiments on a dual-stage wafer stage system, the advantages of the multirate control approach are demonstrated. In particular, it is shown that by design of multirate control on the high rate excellent performance is achieved, with limited cost. The results demonstrate the potential of flexible sampling in motion systems.

Ongoing research focuses on feedback control design for multirate systems, see [26] for preliminary results, and control design for other classes of flexible sampling.

## APPENDIX A PROOF LEMMA 4

The following identity, known as the push-through rule, is exploited:

$$(\underline{I}_m + \underline{A}\underline{B})^{-1} \underline{A} = \underline{A} (\underline{I}_n + \underline{B}\underline{A})^{-1}, \quad (52)$$

with  $\underline{A} \in \mathbb{R}^{m \times n}$ ,  $\underline{B} \in \mathbb{R}^{n \times m}$ ,  $n, m \in \mathbb{N}$ .

Using Fig. 4 and (52),  $\underline{\psi}_{SS,*}$  is expressed in  $\underline{\rho}_{SS,h}$ :

$$\begin{aligned} \underline{\psi}_{SS,*} &= \underline{G}_{SS,*} \underline{H}_{F_h} (\underline{C}_{FF,SS,h} + \underline{C}_{FB,SS,h} \underline{C}_{\psi,SS,h}) \underline{\rho}_{SS,h} \\ &\quad - \underline{G}_{SS,*} \underline{H}_{F_h} \underline{C}_{FB,SS,h} \underline{D}_{F_h} \underline{\psi}_{SS,*} \end{aligned} \quad (53)$$

$$\begin{aligned} &= (\underline{I}_{N_*} + \underline{G}_{SS,*} \underline{H}_{F_h} \underline{C}_{FB,SS,h} \underline{D}_{F_h})^{-1} \underline{G}_{SS,*} \underline{H}_{F_h} \\ &\quad \times (\underline{C}_{FF,SS,h} + \underline{C}_{FB,SS,h} \underline{C}_{\psi,SS,h}) \underline{\rho}_{SS,h} \end{aligned} \quad (54)$$

$$\begin{aligned} &= \underline{G}_{SS,*} \underline{H}_{F_h} (\underline{I}_{N_h} + \underline{C}_{FB,SS,h} \underline{D}_{F_h} \underline{G}_{SS,*} \underline{H}_{F_h})^{-1} \\ &\quad \times (\underline{C}_{FF,SS,h} + \underline{C}_{FB,SS,h} \underline{C}_{\psi,SS,h}) \underline{\rho}_{SS,h} \end{aligned} \quad (55)$$

$$\begin{aligned} &= \underline{G}_{SS,*} \underline{H}_{F_h} \underline{S}_{SS,h} \\ &\quad \times (\underline{C}_{FF,SS,h} + \underline{C}_{FB,SS,h} \underline{C}_{\psi,SS,h}) \underline{\rho}_{SS,h}, \end{aligned} \quad (56)$$

with  $\underline{S}_{SS,h}$  in (19). Using Fig. 4 and (52),  $\underline{\psi}_{LoS,*}$  is expressed in  $\underline{\rho}_{\psi,LoS,l}$ ,  $\underline{v}_{FF,LoS,l}$ :

$$\underline{\psi}_{LoS,*} = \underline{G}_{LoS,*} \underline{H}_{F_l} (\underline{v}_{FF,LoS,l} + \underline{C}_{FB,LoS,l} \underline{\rho}_{\psi,LoS,l}) \quad (57)$$

$$\begin{aligned} &\quad - \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{C}_{FB,LoS,l} \underline{D}_{F_l} \underline{\psi}_{LoS,*} \\ &= (\underline{I}_{N_*} + \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{C}_{FB,LoS,l} \underline{D}_{F_l})^{-1} \underline{G}_{LoS,*} \underline{H}_{F_l} \\ &\quad \times (\underline{v}_{FF,LoS,l} + \underline{C}_{FB,LoS,l} \underline{\rho}_{\psi,LoS,l}) \end{aligned} \quad (58)$$

$$\begin{aligned} &= \underline{G}_{LoS,*} \underline{H}_{F_l} (\underline{I}_{N_l} + \underline{C}_{FB,LoS,l} \underline{D}_{F_l} \underline{G}_{LoS,*} \underline{H}_{F_l})^{-1} \\ &\quad \times (\underline{v}_{FF,LoS,l} + \underline{C}_{FB,LoS,l} \underline{\rho}_{\psi,LoS,l}) \end{aligned} \quad (59)$$

$$\begin{aligned} &= \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{S}_{LoS,l} \\ &\quad \times \begin{bmatrix} \underline{I}_{N_l} & \underline{C}_{FB,LoS,l} \end{bmatrix} \begin{bmatrix} \underline{v}_{FF,LoS,l} \\ \underline{\rho}_{\psi,LoS,l} \end{bmatrix} \end{aligned} \quad (60)$$

$$= \underline{A} \begin{bmatrix} \underline{v}_{FF,LoS,l} \\ \underline{\rho}_{\psi,LoS,l} \end{bmatrix}, \quad (61)$$

with  $\underline{S}_{LoS,l}$  in (20) and  $\underline{A}$  in (18). The result follows from  $\varepsilon_* = \underline{\psi}_{SS,*} - \underline{\psi}_{LoS,*}$ .

## APPENDIX B PROOF THEOREM 6

It is shown that for the parameterization

$$C_l(\theta) = \sum_{i=0}^{n-1} \theta[i] \left( \frac{f_l(z-1)}{z} \right)^{i+1} \quad (62)$$

it holds

$$\xi_l = \underline{C}_l \underline{D}_F \underline{\rho}_{SS,h} = \underline{\Phi}_l \theta. \quad (63)$$

Relations (23) and (24) directly follow from this result.

Parameterization (62) can equivalently be written as a finite impulse response (FIR) structure of order  $n_\alpha = n + 1$ :

$$C_l(\underline{\theta}) = \sum_{i=0}^{n-1} \theta[i] \left( \frac{f_l(z-1)}{z} \right)^{i+1} = \sum_{i=0}^{n_\alpha-1} \alpha[i] z^{-i}. \quad (64)$$

By equating coefficients, it directly follows that the relation between the parameters is given by  $\underline{\alpha} = \underline{R}_l \underline{\theta}$  with  $\underline{R}_l \in \mathbb{R}^{n_\alpha \times n}$  as in (27). Note that  $\underline{R}_l$  is the product of a truncated transposed (lower triangular Cholesky factor of the) Pascal matrix of order  $n_\alpha$ , with a diagonal scaling matrix depending on  $f_l$ .

The finite-time description of  $C_l$  in terms of  $\underline{\alpha}$  is given by

$$\underline{C}_l = \begin{bmatrix} \alpha[0] & 0 & 0 & \dots \\ \alpha[1] & \alpha[0] & 0 & \dots \\ \vdots & \alpha[1] & \alpha[0] & \dots \\ \alpha[n_\alpha-1] & \vdots & \alpha[1] & \ddots \\ 0 & \alpha[n_\alpha-1] & \vdots & \ddots \\ 0 & 0 & \alpha[n_\alpha-1] & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \quad (65)$$

Using the Kronecker mixed-product property rule (14) the order of  $\underline{C}_l$  and  $\underline{D}_F$ , see (10), is interchanged:

$$\underline{C}_l \underline{D}_F = (\underline{C}_l \otimes 1) (\underline{I}_{N_l} \otimes \underline{e}_F^\top) \quad (66)$$

$$= (\underline{C}_l \underline{I}_{N_l}) \otimes (1 \underline{e}_F^\top) \quad (67)$$

$$= (\underline{I}_{N_l} \underline{C}_l) \otimes ((\underline{e}_F^\top \underline{e}_F) \underline{e}_F^\top) \quad (68)$$

$$= (\underline{I}_{N_l} \otimes \underline{e}_F^\top) (\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)) \quad (69)$$

$$= \underline{D}_F (\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)). \quad (70)$$

Note that  $\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)$  is a lower triangular matrix and that  $\underline{\psi}_{SS,h} = \underline{T}_{\psi_{SS,h}} \underline{e}_{N_h}$ , with  $\underline{T}_{\psi_{SS,h}}$  in (26) is also a lower triangular matrix.

Next, the commutative property of lower triangular matrices is exploited to express  $\underline{\xi}_l$  in  $\underline{\theta}$ . To this end, the Kronecker product rule and the relation  $\underline{\alpha} = \underline{R}_l \underline{\theta}$  are used:

$$\underline{\xi}_l = \underline{C}_l \underline{D}_F \underline{\psi}_{SS,h} \quad (71)$$

$$= \underline{D}_F (\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)) \underline{\psi}_{SS,h} \quad (72)$$

$$= \underline{D}_F (\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)) \underline{T}_{\psi_{SS,h}} \underline{e}_{N_h} \quad (73)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} (\underline{C}_l \otimes (\underline{e}_F \underline{e}_F^\top)) \underline{e}_{N_h} \quad (74)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{\alpha} \otimes \underline{e}_F \\ \underline{0}_{(N_h - F n_\alpha) \times 1} \end{bmatrix} \quad (75)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} (\underline{I}_{n_\alpha} \underline{\alpha}) \otimes (\underline{e}_F \mathbf{1}) \\ \underline{0}_{(N_h - F n_\alpha) \times 1} \end{bmatrix} \quad (76)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_\alpha} \otimes \underline{e}_F \\ \underline{0}_{(N_h - F n_\alpha) \times n_\alpha} \end{bmatrix} \underline{\alpha} \quad (77)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_\alpha} \otimes \underline{e}_F \\ \underline{0}_{(N_h - F n_\alpha) \times n_\alpha} \end{bmatrix} \underline{R}_l \underline{\theta} \quad (78)$$

$$= \underline{\Phi}_l \underline{\theta}, \quad (79)$$

which concludes the proof of (62). Relations (23) and (24) directly follow from this result.

## APPENDIX C PROOF LEMMA 7

Substitution of (23) and (24) in (16) and using (18) yields

$$\underline{\varepsilon}_* = \underline{\psi}_{SS,*} - \underline{A} \begin{bmatrix} \underline{V}_{FF,LoS,l} \\ \underline{\rho}_{\psi,LoS,l} \end{bmatrix} \quad (80)$$

$$= \underline{\psi}_{SS,*} - \underline{A} \begin{bmatrix} \underline{\Phi}_{FF,l} \underline{\theta}_{FF} \\ \underline{D}_F \underline{\psi}_{SS,h} + \underline{\Phi}_{\psi,l} \underline{\theta}_\psi \end{bmatrix} \quad (81)$$

$$= \underline{\psi}_{SS,*} - \underline{A} \begin{bmatrix} \underline{0}_{N_l} \\ \underline{D}_F \underline{\psi}_{SS,h} \end{bmatrix} - \underline{A} \begin{bmatrix} \underline{\Phi}_{FF,l} \underline{\theta}_{FF} \\ \underline{\Phi}_{\psi,l} \underline{\theta}_\psi \end{bmatrix} \quad (82)$$

$$= \underline{\psi}_{SS,*} - \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{S}_{LoS,l} \begin{bmatrix} \underline{I}_{N_l} & \underline{C}_{FB,LoS,l} \end{bmatrix} \times \begin{bmatrix} \underline{0}_{N_l} \\ \underline{D}_F \underline{\psi}_{SS,h} \end{bmatrix} - \underline{A} \begin{bmatrix} \underline{\Phi}_{FF,l} \underline{\theta}_{FF} \\ \underline{\Phi}_{\psi,l} \underline{\theta}_\psi \end{bmatrix} \quad (83)$$

$$= \underline{\psi}_{SS,*} - \underline{G}_{LoS,*} \underline{H}_{F_l} \underline{S}_{LoS,l} \underline{C}_{FB,LoS,l} \underline{D}_F \underline{\psi}_{SS,h} - \underline{A} \begin{bmatrix} \underline{\Phi}_{FF,l} & 0 \\ 0 & \underline{\Phi}_{\psi,l} \end{bmatrix} \begin{bmatrix} \underline{\theta}_{FF} \\ \underline{\theta}_\psi \end{bmatrix} \quad (84)$$

$$= \underline{b} - \underline{A} \underline{\Phi} \underline{\theta}, \quad (85)$$

with  $\underline{b}$ ,  $\underline{\Phi}$ ,  $\underline{\theta}$  as given in Lemma 7.

## APPENDIX D PROOF LEMMA 12

It is shown that for the general parameterization

$$C_h(\underline{\theta}) = \sum_{i=0}^{n-1} \theta[i] \left( \frac{f_h(z-1)}{z} \right)^{i+1} \quad (86)$$

it holds

$$\underline{\xi}_l = \underline{D}_F \underline{C}_h \underline{\psi}_{SS,h} = \underline{D}_F \underline{\Phi}_h \underline{\theta}. \quad (87)$$

Relations (42) and (43) directly follow from this result.

The proof is similar to that of Theorem 6. First,  $C_h$  is expressed in terms of FIR parameters  $\underline{\alpha}$ :

$$C_h(\underline{\theta}) = \sum_{i=0}^{n-1} \theta[i] \left( \frac{f_h(z-1)}{z} \right)^{i+1} = \sum_{i=0}^{n_\alpha-1} \alpha[i] z^{-i}, \quad (88)$$

where  $\underline{\alpha} = \underline{R}_h \underline{\theta}$  with  $\underline{R}_h \in \mathbb{R}^{n_\alpha \times n}$  as in (46) and  $n_\alpha = n + 1$ . The finite-time description of  $C_h$  in terms of  $\underline{\alpha}$  is given by

$$\underline{C}_h = \begin{bmatrix} \alpha[0] & 0 & 0 & \dots \\ \alpha[1] & \alpha[0] & 0 & \dots \\ \vdots & \alpha[1] & \alpha[0] & \dots \\ \alpha[n_\alpha-1] & \vdots & \alpha[1] & \ddots \\ 0 & \alpha[n_\alpha-1] & \vdots & \ddots \\ 0 & 0 & \alpha[n_\alpha-1] & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \quad (89)$$

Next, it is exploited that the lower triangular matrices  $\underline{C}_h$  and  $\underline{T}_{\psi_{SS,h}}$  commute:

$$\underline{\xi}_l = \underline{D}_F \underline{C}_h \underline{\psi}_{SS,h} \quad (90)$$

$$= \underline{D}_F \underline{C}_h \underline{T}_{\psi_{SS,h}} \underline{e}_{N_h} \quad (91)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \underline{C}_h \underline{e}_{N_h} \quad (92)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{\alpha} \\ \underline{0}_{(N_h-n_\alpha) \times 1} \end{bmatrix} \quad (93)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_\alpha} \\ \underline{0}_{(N_h-n_\alpha) \times n_\alpha} \end{bmatrix} \underline{\alpha} \quad (94)$$

$$= \underline{D}_F \underline{T}_{\psi_{SS,h}} \begin{bmatrix} \underline{I}_{n_\alpha} \\ \underline{0}_{(N_h-n_\alpha) \times n_\alpha} \end{bmatrix} \underline{R}_h \underline{\theta} \quad (95)$$

$$= \underline{D}_F \underline{\Phi}_h \underline{\theta}. \quad (96)$$

Relations (42) and (43) directly follow from this result.

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#### REFERENCES

- [1] T. Chen and B. A. Francis, *Optimal Sampled-Data Control Systems*. London, Great Britain: Springer, 1995.
- [2] K. J. Åström and B. Wittenmark, *Computer Control System Theory and Design*, 3rd ed. Prentice Hall, 1997.
- [3] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, 2nd ed. West Sussex, United Kingdom: John Wiley & Sons, Ltd., 2005.
- [4] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 7th ed. Upper Saddle River, New Jersey: Pearson, 2015.
- [5] M. Steinbuch, R. J. E. Merry, M. L. G. Boerlage, M. J. C. Ronde, and M. J. G. van de Molengraft, "Advanced Motion Control Design," in *The Control Handbook: Control System Applications*, 2nd ed., W. S. Levine, Ed. CRC Press, 2010, ch. 27.
- [6] D. Bristow, M. Tharayil, and A. Alleyne, "A Survey of Iterative Learning Control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.
- [7] D. P. Glasson, "Development and applications of multirate digital control," *IEEE Control Systems Magazine*, vol. 3, no. 4, pp. 2–8, 1983.
- [8] J. Salt and M. Tomizuka, "Hard disk drive control by model based dual-rate controller. Computation saving by interlacing," *IFAC Mechatronics*, vol. 24, no. 6, pp. 691–700, 2014.
- [9] J. Salt and P. Albertos, "Model-Based Multirate Controllers Design," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 6, pp. 988–997, 2005.
- [10] S. Lall and G. Dullerud, "An LMI solution to the robust synthesis problem for multi-rate sampled-data systems," *Automatica*, vol. 37, no. 12, pp. 1909–1922, 2001.
- [11] W. Ohnishi, T. Beauduin, and H. Fujimoto, "Preactuated multirate feedforward for a high-precision stage with continuous time unstable zeros," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 10907–10912, 2017, 20th World Congress.
- [12] X. Chen and H. Xiao, "Multirate forward-model disturbance observer for feedback regulation beyond Nyquist frequency," *Systems & Control Letters*, vol. 94, pp. 181–188, 2016.
- [13] J. Ding, F. Marcassa, S.-C. Wu, and M. Tomizuka, "Multirate Control for Computation Saving," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 1, pp. 165–169, 2006.
- [14] S.-H. Lee, "Multirate Digital Control System Design and its Application to Computer Disk Drives," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 1, pp. 124–133, 2006.
- [15] H. Fujimoto, Y. Hori, and A. Kawamura, "Perfect Tracking Control Based on Multirate Feedforward Control with Generalized Sampling Periods," *IEEE Transactions on Industrial Electronics*, vol. 48, no. 3, pp. 636–644, 2001.
- [16] D. Antunes and W. P. M. H. Heemels, "Frequency-Domain Analysis of Control Loops With Intermittent Data Losses," *IEEE Transactions on Automatic Control*, vol. 61, no. 8, pp. 2295–2300, 2016.
- [17] T. Oomen and C. R. Rojas, "Sparse iterative learning control with application to a wafer stage: Achieving performance, resource efficiency, and task flexibility," *IFAC Mechatronics*, vol. 47, pp. 134–147, 2017.
- [18] J. Valencia, E. E. van Horssen, D. Goswami, W. M. Heemels, and K. Goossens, "Resource Utilization and Quality-of-Control Trade-off for a Composable Platform," in *Proceedings of the 2016 Design, Automation and Test in Europe Conference & Exhibition*, Dresden, Germany, 2016, pp. 654–659.
- [19] J. van Zundert, T. Oomen, D. Goswami, and W. Heemels, "On the Potential of Lifted Domain Feedforward Controllers with a Periodic Sampling Sequence," in *Proceedings of the 2016 American Control Conference*, Boston, Massachusetts, 2016, pp. 4227–4232.
- [20] S. Fraser, M. Attia, and M. Osman, "Modelling, Identification and Control of Thermal Deformation of Machine Tool Structures, Part 1: Concept of Generalized Modelling," *Journal of Manufacturing Science and Engineering*, vol. 120, no. 3, pp. 623–631, 1999.
- [21] A. Aminifar, E. Bini, P. Eles, and Z. Peng, "Analysis and design of real-time servers for control applications," *Transactions on Computers*, vol. 65, no. 3, pp. 834–846, 2016.
- [22] O. Lindgärde and B. Lennartson, "Performance and Robust Frequency Response for Multirate Sample-Data Systems," in *Proceedings of the 1997 American Control Conference*, Albuquerque, New Mexico, 1997, pp. 3877–3881.
- [23] M. Cantoni and K. Glover, "Frequency-Domain Analysis of Linear Periodic Operators with Application to Sampled-Data Control Design," in *Proceedings of the 36th Conference on Decision and Control*, San Diego, California, 1997, pp. 4318–4323.
- [24] H. Sandberg, E. Möllerstedt, and B. Bernhardsson, "Frequency-Domain Analysis of Linear Time-Periodic Systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 12, pp. 1971–1983, 2005.
- [25] T. Oomen, M. van de Wal, and O. Bosgra, "Design framework for high-performance optimal sampled-data control with application to a wafer stage," *International Journal of Control*, vol. 80, no. 6, pp. 919–934, 2007.
- [26] J. van Zundert and T. Oomen, "LPTV Loop-Shaping with Application to Non-Equidistantly Sampled Precision Mechatronics," in *Proceedings of the 15th International Workshop on Advanced Motion Control*, Tokyo, Japan, 2018, pp. 467–472.
- [27] T. Oomen, J. van de Wijdeven, and O. Bosgra, "Suppressing intersample behavior in iterative learning control," *Automatica*, vol. 45, no. 4, pp. 981–988, 2009.
- [28] R. Munnig Schmidt, G. Schitter, and J. van Eijk, *The Design of High Performance Mechatronics*. Amsterdam, The Netherlands: Delft University Press, 2011.
- [29] J. van Zundert, J. Verhaegh, W. Aangenent, T. Oomen, D. Antunes, and W. Heemels, "Feedforward for Multi-Rate Motion Control: Enhanced Performance and Cost-Effectiveness," in *Proceedings of the 2015 American Control Conference*, Chicago, Illinois, 2015, pp. 2831–2836.
- [30] B. Bamieh, J. B. Pearson, B. A. Francis, and A. Tannenbaum, "A lifting technique for linear periodic systems with applications to sampled-data control," *Systems & Control Letters*, vol. 17, no. 2, pp. 79–88, 1991.
- [31] M. van de Wal, G. van Baars, F. Sperling, and O. Bosgra, "Multivariable  $\mathcal{H}_\infty/\mu$  feedback control design for high-precision wafer stage motion," *Control Engineering Practice*, vol. 10, no. 7, pp. 739–755, 2002.
- [32] M. F. Heertjes, B. van der Velden, and T. Oomen, "Constrained Iterative Feedback Tuning for Robust Control of a Wafer Stage System," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 1, pp. 56–66, 2016.
- [33] H. Butler, "Adaptive Feedforward for a Wafer Stage in a Lithographic Tool," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 3, pp. 875–881, 2013.
- [34] M. Groot Wassink, M. van de Wal, C. Scherer, and O. Bosgra, "LPV control for a wafer stage: beyond the theoretical solution," *Control Engineering Practice*, vol. 13, no. 2, pp. 231–245, 2005.
- [35] H. Butler, "Position Control in Lithographic Equipment: An Enabler for Current-Day Chip Manufacturing," *IEEE Control Systems Magazine*, vol. 31, no. 5, pp. 28–47, 2011.
- [36] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1993.
- [37] J. Bolder, T. Oomen, S. Koekebakker, and M. Steinbuch, "Using iterative learning control with basis functions to compensate medium deformation in a wide-format inkjet printer," *IFAC Mechatronics*, vol. 24, no. 8, pp. 944–953, 2014.
- [38] J. Bolder and T. Oomen, "Rational Basis Functions in Iterative Learning Control - With Experimental Verification on a Motion System," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 2, pp. 722–729, 2015.

- [39] F. Boeren, D. Bruijnen, N. van Dijk, and T. Oomen, "Joint input shaping and feedforward for point-to-point motion: Automated tuning for an industrial nanopositioning system," *IFAC Mechatronics*, vol. 24, no. 6, pp. 572–581, 2014.
- [40] E. Evers, M. van de Wal, and T. Oomen, "Synchronizing Decentralized Control Loops for Overall Performance Enhancement: A Youla Framework Applied to a Wafer Scanner," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 10845–10850, 2017, 20th World Congress.
- [41] P. Lambrechts, M. Boerlage, and M. Steinbuch, "Trajectory planning and feedforward design for electromechanical motion systems," *Control Engineering Practice*, vol. 13, no. 2, pp. 145–157, 2005.



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