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# Bi-Objective Optimization Problems—A Game Theory Perspective to Improve Process and Product

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**Abstract:** Cost-effective manufacturing processes or products are no longer the only requirements for business sustainability. An approach based on Game Theory is suggested to find solutions for bi-objective problems. In particular, Stackelberg's technique is employed and complemented with the Factors Scaling tool to help the users in defining its strategy for optimizing process and product quality characteristics. No subjective information (shape factors, weights, and/or any other preference information) is required from the users, and basic computational background is enough for implementing it. Two case studies provide evidence that the suggested easy-to-use approach can yield nondominated solutions from a small number of Leader–Follower cycles, what reinforces its usefulness for bi-objective optimization problems.

## 1. Introduction

Businesses sustainability is no longer supported exclusively on cost-effective manufacturing processes or products. High quality (reliability, durability, performance, robustness, . . . ) is also a usual criterion that supports costumers' decision when they buy a product, however, consumers' behavior has been changing and cannot be ignored. As example, nowadays, consumers are discarding products for purely fashion (technological, shape, functional, . . . ) reasons. The safety and the environmental impact of products are other current consumers' concerns, who are looking for more secure and eco-friendly products. This means that, more than ever, products must be designed, and manufacturing processes must be managed, to satisfy all consumers' needs and concerns or, if possible, exceed their expectations.

For that purpose, organizations must use efficient and not just effective approaches, methodologies, and/or tools. However, to identify and to select the right or most appropriate ones is not an easy task. In fact, the variety of existing approaches, methodologies, and/or tools put forward in the literature for designing and improving or optimizing process's efficiency and product's characteristics (in terms of quality, safety, and environmental impact) is huge. This is not an issue in itself, but one cannot ignore that among the major reasons for the termination or failure of improvement projects are related with practitioners' qualifications, skills, knowledge, and training on the use of some tools [1–3].

In (non)manufacturing settings, practitioners tend to reject what they do not understand or what they have difficulty in using when they want or need to improve process and product. Sorensen [4], McDermott et al. [5], and Chen et al. [6] are authors who highlight this and, as Vining et al. [7] suggested, academia and researchers should always keep one eye on the practicability of the methodologies that are developed and put forward in the literature. Unfortunately, the trial-and-error, one-factor-at-a-time, and brute force are still common practices in industry, as well as in the academy, to design and optimize processes and products. A more recommended practice is to use the Design of Experiments (DoE).

This is a flexible and cost-effective tool that has been often and successfully used in various science fields, namely in the chemical and aerospace industries, as examples.

It is widely accepted that process efficiency and product characteristics (reliability, durability, performance, robustness, ... , safety and/or environmental impact) can be improved and optimized in a structured, faster, and cheaper way at the design and manufacturing stages if data are generated, collected, and analyzed with the right tools, namely the DoE. Comprehensive guidelines to help researchers and practitioners in planning, conducting, and analyzing experiments were reported in Refs. [2,8–10]. To understand some of the challenges and fundamental gaps which need to be tackled for applying DoE in the service industry, the reader is referred to Antony et al. [11].

A more sustainable knowledge about the process and/or the product is, in general, a result of a sequential experimentation approach that may consist of three experimental phases (screening, characterization, and optimization) whose objectives are the following:

1. Screening phase: Identification of active variables (those with practical or significant effect on response).
2. Characterization phase: Understanding the relationship between the active variables and the response, by guiding those variables, and their interactions, to the region where the response yields the most favorable results.
3. Optimization phase: Modelling one or more responses, usually by second order models, and identifying optimal settings for the active variables.

This sequential learning process is inherent to the called Response Surface Methodology (RSM), and its usefulness has been stressed in the literature [2,12]. Nonetheless, case studies where the three experimental phases are adopted are rarely reported in the literature or illustrated in textbooks. An exception was reported by Lv et al. [13]. Screening–optimization, characterization–optimization, and optimization experimental phases are often reported in the literature.

Many case studies reported in the literature aim to select an optimal location in the input variables space that simultaneously yields the most balanced value for several objectives (responses) which are, in general, in conflict. In the RSM, to aggregate the multiple responses into a composite (single objective) function and then proceed to its optimization is a current practice. The most popular aggregate functions among researchers and practitioners are the desirability function-based and the loss function-based optimization functions. For an extensive review on these aggregate functions, the reader is referred to Refs. [14,15]. Other desirability function-based and loss function-based optimization functions were proposed in Refs. [16,17]. The applicability and computational aspects of various optimization functions in different decision-making contexts were discussed by Ardakani et al. [18], where the foremost approaches are categorized and integrated as well. The working ability of several easy-to-use optimization functions, namely, mathematical programming-based and global criterion-based, were evaluated and compared with that of a theoretically sound (more sophisticated) method by Costa and Lourenço [19].

Dual response optimization (DRO) is a particular case of multiple response optimization (MRO), and its goal is to optimize two responses, namely, two mean responses or the mean and standard deviation of the process or product quality, environmental and/or safety characteristic. Various methods have been put forward in the literature for this purpose in the RSM framework, including priority-based, mean squared-based, desirability-based, goal programming-based, and global criterion-based methods. A review on these methods was made by Costa [20].

MRO and DRO remain appealing research fields because real-life problems are, by nature, multiobjective, and finding a balanced solution for more than one objective is a need. Examples of recent works on MRO and DRO were reported in the literature in Refs. [21–24]. In this paper, a novel approach is presented based on game theory to find compromise or equilibrium solutions for optimization problems without requiring either subjective information or advanced statistical and computational background from the user (analyst

or decision maker). In particular, Stackelberg's technique is employed and complemented with an easy-to-use tool to improve its efficiency in solving the called DRO problems.

The remainder of this article is structured as follows: Section 2 includes an overview on game theory and the Stackelberg technique; Section 3 presents the results from two popular case studies in the RSM literature; the Results and Discussion are presented in Section 4 and Conclusions are in Section 5.

## 2. Game Theory—An Overview

The usefulness of game theory is widely recognized in various science domains, and the Royal Swedish Academy of Sciences awarded the Nobel Prize in economics to several game theorists. As examples, John Harsanyi, John Nash, and Reinhard Selten were awarded, in 1994, for their analysis of equilibria in the theory of non-cooperative games, followed by Aumann and Schelling in 2005, who contributed to a better understanding of conflict and cooperation through game theory analysis [25]. In 2007, three authors, namely Leonid Hurwicz, Eric Maskin, and Roger Myerson, received the prize for laying the foundations of mechanism design theory (design of games). Five years later, Lloyd Shapely and Alvin Roth were recognized for their theoretical contributions to the economics of market and non-market mechanisms.

John von Neumann and Oscar Morgenstern are considered the founders of modern game theory, and their work differs from that of other authors, both in terms of the generality of their approach and the ambitiousness of their project [25]. By 1948, they had developed many fundamental elements for a theory of games: the extensive and normal forms linked by the concept of a strategy, the use of fixed-point theorems to prove existence of solutions for games with randomization, and a general derivation of the expected utility criterion for individual decision making [26].

Game theory has been often used in various social science domains like political science, psychology, economics, sociology, and successfully extended to many other knowledge domains, including finance, accounting, and marketing domains [27–31]. It remains an open research field and it is employed in real-life problems as diverse as arms control policies, dating and marriage, college selection, human organ transplant, environmental cooperation, asset pricing, security at infrastructures (airports, wireless networks, . . . ), and supply chains design. As a study of strategic interactions in a decision-making process, game theory is a mathematical tool that provides a common language to formulate, structure, analyze, and understand the interaction among rational agents (players) and their strategies or decisions/actions. The scenarios examined include the following:

- Cooperative or non-cooperative—when binding agreements are stablished and decisions made by the players lead to outcomes that are expected to be satisfactory, and accepted, by all of them, it is a cooperative situation. Otherwise, in games where the players decide on their own strategy to maximize their outcome, the game is classified as non-cooperative.
- Symmetric or asymmetric—when the payoffs of a strategy only depend on previous strategy played, it means that it is not influenced by the player employing that strategy, the game is of symmetric-type. If the payoff is influenced by the player employing that strategy, namely, when the players adopt different roles and goals, the game is classified as asymmetric.
- Zero-sum or non-zero-sum—when the sum of the payoffs to the players equals zero, it is a zero-sum game type. In non-zero-sum games, the gains/losses of one player do not result in the same losses/gains of the other players, which means that a non-zero-sum game may result in a win–win situation.
- Simultaneous or sequential—when all the players play simultaneously, with no previous knowledge of the decisions or strategies of the other players, the game is of simultaneous type. In a sequential game, each player makes a decision and plays after the other player(s) have played, without having detailed information about the strategies or outcomes of the other previous player(s). In this sense, a special case of a

sequential game is the called hierarchical game, where a (secondary) player, called the Follower, only plays after the main player (or Leader) have played.

- Perfect information or imperfect information—The perfect information case occurs when all the players make their decisions based on the same inputs whereas in the imperfect information case, the inputs available to one player are inaccessible to the other ones.

### 2.1. Equilibrium Concepts

The key elements of a game are the following:

- The players: who is interacting;
- The strategies: the options that each player can choose, considering the order they play;
- The payoffs: how the strategies translate into outcomes, considering the players' preferences over possible outcomes;
- The information or beliefs: what the players know or believe about the problem under analysis and the actions they observe before making decisions;
- the rationality: how the players think and act.

In practice, the input information in many games consist of:

- A finite number of players;
- A set of rational strategies from each player;
- A payoff function for each player, whose outcome depends on the strategy selected by him/her or on the strategy(ies) of the other players.

It is widely accepted that in multiobjective optimization problems, including those solved in the game theory setting, the most favorable solutions are those that belong to the nondominated solutions set. The nondominated (optimal) solutions set is called the Pareto front. However, when a multidisciplinary team, distinct entities, or organizations with conflicting interests (payoff functions) have to select a solution for a multiobjective problem using game theory, nondominated or Pareto solutions are not always achieved. In practice, equilibrium solutions are expected because, at some phase of a game, a player may see no interest or any chance for improving its payoff function (outcome) by changing its strategy. As it is pointed out by Monfared et al. [32], equilibrium solutions are expected for MRO problems with multiple sovereign decision makers whose objectives are conflicting. Nevertheless, they argued that practitioners should always look for Pareto Optimal Equilibrium (POE) solutions.

Nash and Stackelberg equilibrium solutions are expected outputs for various scenarios in games, and their characterization from a mathematic (theoretic) point of view is often revisited in the literature (see Refs. [33,34] as examples). Nash equilibrium solutions are outputs in games where no single player dominates the decision-making process. When there is a hierarchy among the players (someone holds a dominant position or plays first than the other player(s)), the called Stackelberg solution is the best outcome for the game. A Stackelberg equilibrium solution is an expected result in games with two players who have conflicting objectives, where the player who holds the dominant position is called the Leader and the player who reacts (rationally) to the Leader decision is called the Follower. A Stackelberg–Nash equilibrium solution is appropriate in games where one or more players, with higher hierarchy, enforce their strategy to lower hierarchy players who react playing a nonzero sum finite game according to the strategy established by the Leader(s). Examples of the growing interest and relevance of Leader–Follower games are reported in Refs. [35–43] so it is pertinent to confirm that the Stackelberg technique can yield POE solutions for (games) DRO problems carried out in the RSM framework.

### 2.2. Stackelberg Games and DRO

Dual response problems are usual in industrial settings as any process or product has technical characteristics whose mean value must be equal to or as close as possible to a desired target and the variation around that mean value must be minimal. The

simultaneous optimization of the models fitted to mean ($f_1(X)$) and standard deviation ($f_2(X)$) responses,

$$\underset{X}{Optimize} \ \{f_1(X), f_2(X)\} \tag{1}$$

where $X$ is a vector of input variables ($x_i$ with $i$ = 1, 2, ... , n), either converting or not converting those models into a composite function, is a current practice in DRO carried out under the RSM framework and is an active research field [44,45]. A distinct approach is followed in game theory, namely in the Stackelberg technique, whose procedure is exemplified in the next subsection.

In a Stackelberg (hierarchical) or Leader–Follower game, assuming that $f_1(X)$ and $f_2(X)$ represent the polynomial models fitted to estimated mean ($\hat{\mu}$) and estimated standard deviation ($\hat{\sigma}$) responses of a process or product quality characteristic, two types of games can be formulated: 1—the Leader optimizes the estimated mean response and the Follower optimizes the estimated standard deviation response; 2—the Leader optimizes the estimated standard deviation response and the Follower optimizes the estimated mean response. In each type of game, there are three possible scenarios, because the Leader's or the Follower's objective can be to minimize, set on target, or to maximize the estimated mean response function, such as presented in Tables 1 and 2.

**Table 1.** Leader–Mean games.

| Minimize the estimated mean | Set the estimated mean on target | Maximize the estimated mean |
| --- | --- | --- |
| | Minimize the estimated standard deviation | |

**Table 2.** Leader–Standard Deviation games.

| | Minimize the estimated standard deviation | |
| --- | --- | --- |
| Minimize the estimated mean | Set the estimated mean on target | Maximize the estimated mean |

Leader–Mean type game consists of optimizing $\{f_1(X), f_2(X)\} = \{\hat{\mu}(X), \hat{\sigma}(X)\}$ while the Leader–Standard Deviation type game consists of optimizing $\{f_1(X), f_2(X)\} = \{\hat{\sigma}(X), \hat{\mu}(X)\}$. Nevertheless, notice that, in a Leader–Mean type game, when the objective is to set the estimated mean response on target minimizing the estimated standard deviation, the Leader optimizes $f_1(X) = (\hat{\mu}(X) - \tau)$ or $(\hat{\mu}(X) - \tau)^2$, and the Follower optimizes $f_2(X) = \hat{\sigma}(X)$, where $\tau$ represents a target value and X represents a vector of input variables. The corresponding inverted game is a Leader–Standard Deviation type game, where the Leader optimizes $f_1(X) = \hat{\sigma}(X)$ and the Follower optimizes $f_2(X) = (\hat{\mu}(X) - \tau)$ or $(\hat{\mu}(X) - \tau)^2$.

*2.3. Stackelberg Game: An Exemplification of the Protocol*

In a Stackelberg (hierarchical) game, the Leader defines its strategy and is the first one to play (maximize or minimize $f_1(X)$ using, as example, the Excel®-Solver tool). Then, the Follower reacts by using its own strategy, which depends on the Leader's strategy. This sequential process stops when the Leader cannot improve his/her payoff, which means that an equilibrium solution is found [34]. As an example, let's assume that a player (player 1) acts as the Leader and aims at optimizing $f_1(X)$ by processing one or more input variables ($x_l$) from X, while player 2 acts as the Follower and its purpose is to optimize $f_2(X)$ by processing the remaining input variables ($x_j$) from X. The rational strategy played is represented by $(x_l, x_j) \in (Sx_l, Sx_j)$, where $Sx_l$ and $Sx_j$ are the rational strategies of the player 1 and 2, respectively. As shown in Figure 1, at each iteration, player 1 (the Leader) minimizes $f_1(X)$ by varying $x_l$, while $x_j$ remains unchanged, and then player 2 minimizes $f_2(X)$ by varying $x_j$, while $x_l$ remains unchanged (fixed by the player 1). The game ends when no player can achieve any additional gain or a better outcome, which means that a Stackelberg equilibrium point is found. This solution is called a Pareto-optimal equilibrium point if the Stackelberg equilibrium point is also a nondominated (Pareto) solution, that

is, a solution where any improvement in $f_1(X)$ cannot be achieved without degrading the value of $f_2(X)$.
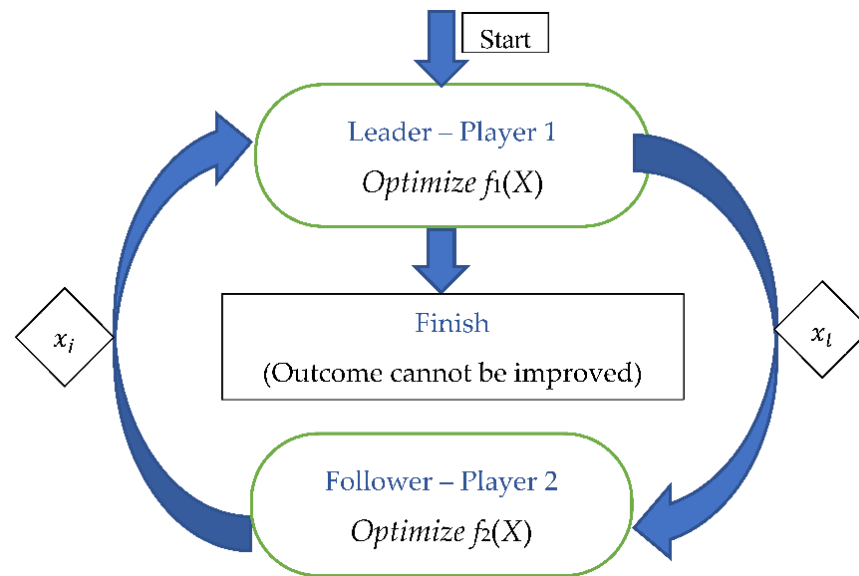


**Figure 1.** Stackelberg Game.

*2.4. Variables Selection*

In a Leader–Follower game it is not previously known which variables should be processed by the leader and the Follower. A time-consuming alternative is to test all the players' rational strategies that result from the combinations of input variables. However, when the number of input (controllable) variables is large and, consequently, the rational strategy set is large as well, to find a Stackelberg equilibrium point can be time consuming for those who have limited computational background for automating the optimization procedure. Therefore, to minimize the number of rational strategies played, and this way to improve the effectiveness of a Stackelberg game, tools like those reported by Hamby [46] can be used. Here, the Factors Scaling tool was selected and employed due to its application easiness and efficacy.

The Factors Scaling tool can be applied to any kind of experiment that uses a multifactorial design, with multiple responses and multiple input variables [47]. This tool enables to combine main effects, quadratic effects, and interactions into a meaningful summary that allows the experimenter/data analyst to identify the most influential factors for each response. In a sequential (Stackelberg) game, each player has a set of variables under his control to optimize his objective function without interference from the other player, so it is essential that only the most influential variables, those with the strongest effects in the objective or optimization function, be used by the players.

Let us assume that a full quadratic model is fitted to a response, such as defined in (2),

$$E(Y_k) = \alpha_k + \sum_{n=1}^{N} \beta_{nk} X_n + \sum_{n=1}^{N} \gamma_{nk} X_n^2 + \sum_{n=1}^{N} \sum_{m=n+1}^{N} \delta_{nmk} X_n X_m \qquad (2)$$

where $Y_k$ denotes a vector of $k$ estimated responses, $X_n$ ($n = 1, \ldots, N$) denotes the input variables $\{X_1, \ldots, X_N\}$ in the model fitted to k-th response, $\alpha_k$ represents the intercept of *k-th* model, $\beta_{nk}$ represents the regression coefficients of the main effect of $X_n$, $\gamma_{nk}$ represents the regression coefficients of the quadratic effect of $X_n$, and $\delta_{nkm}$ represents the regression coefficients of the interaction effect between $X_n$ and $X_m$ ($n = 1, \ldots, N - 1$, $m = n + 1, \ldots, N$). According to Otava and Mylona [47] the 'relative importance' of the factors within a response is

$$MPI_{nk}^{std} = MPI_{nk} / (\max_{n=1, \ldots, N} |MPI_{nk}|) \qquad (3)$$

where MPI$_{nk}$ (see Equation (4)) denotes the maximal absolute change in Y$_k$ that can be induced by increasing X$_n$ by a value H, for a certain starting value of X$_n$ and some favourable setting of the other factors X$_1$, ... , X$_{n-1}$, X$_{n+1}$, ... , X$_N$ included in the model fitted to response.

$$|MPI_{nk}| = H|\beta_{nk}| + H^2|\gamma_{nk}| + (H*H)[\sum_{m=n+1}^{N}|\delta_{nmk}| + \sum_{m=1}^{n-1}|\delta_{nmk}|] \qquad (4)$$

In Equation (4), H can take a value equal to one (half-range on range $[-1, 1]$) or equal to 1.682 (half-range on range $[-1.682, 1.682]$ of the experimental space).

It is also important to highlight that this tool allows the players to know the most influential factors across multiple responses and lead to a better definition of its strategy, but it does not replace the required statistical evaluation/validation of the models fitted to responses [48,49]. Nevertheless, this acquired understanding is a significant contribution to the game's effectiveness, saving time and money in the optimization process by minimizing the number of strategies the players can use.

## 3. Case Studies

Two DRO problems are presented below to illustrate and evaluate the game theory usefulness for this type of problem, namely the usefulness of the Stackelberg technique and the Factors Scaling tool. Mean and standard deviation responses are, by nature, conflicting responses so the Stackelberg technique is appropriate for solving this type of DRO problem as a game. Leader–Mean and Leader–Standard deviation approaches (see Tables 1 and 2) are tested in both case studies to better understand the Stackelberg technique capability to yield POE solutions.

### 3.1. Case Study 1—Etching Process

This case study was developed in a semiconductor manufacturing industry and introduced by Robinson et al. [50]. It relates to the fabrication of wafers in an etching process, and its objective was to determine the factor settings that yield a mean resistivity of 350 ohm-cm with little to no variation. The effect of three control factors, the gas flow rate ($x_1$), the temperature ($x_2$) and the pressure ($x_3$), was analyzed based on data generated from a central composite design with four centre runs. The models fitted to mean and standard deviation responses are the following:

$$f_1(x) = \hat{\mu} = 255.71 + 23.69x_1 - 49.06x_2 - 35.14x_3 - 16.57x_2^2 + 27.75x_3^2 - 25.54x_1*x_3 \qquad (5)$$

$$f_2(x) = \hat{\sigma} = 79.97 + 2.50x_1 - 14.81x_2 + 1.72x_3 - 9.41x_1^2 - 9.80x_2^2 + 44.56x_3^2 - 12.43x_2*x_3 \qquad (6)$$

These two functions and the three variables enable the formulation of 12 Stackelberg games: six games, where the Leader optimizes the mean selecting one or two variables and the Follower optimizes the standard deviation function using the remaining variable(s), and six games where the Leader optimizes the standard deviation selecting one or two variables and the Follower optimizes the mean function using the remaining variable(s). Examples of these games are:

- Leader optimizes $\hat{\mu}(x_2)$, keeping the settings of variables $x_1$ and $x_3$ fixed, and the Follower optimizes $\hat{\sigma}(x_1, x_3)$, keeping the setting of variable $x_2$ fixed;
- Leader optimizes $\hat{\sigma}(x_1, x_3)$, keeping the setting of variable $x_2$ fixed, and the Follower optimizes $\hat{\mu}(x_2)$, keeping the settings of variables $x_1$ and $x_3$ fixed.

When two games yield the same solution, only one game and the symbol of the respective solution is shown so only eight (Stackelberg) solutions are represented by the respective symbols in Figure 2. This figure also includes the Pareto frontier for this problem. Tables 3 and 4 are examples of the optimization procedure with the Stackelberg technique for the presented game, and one can see that the final solution is achieved after one cycle Leader–Follower. The same happens in the games whose solutions are represented by a

cross, a left-pointing triangle, and a hexagram. In the remaining games, the solution is achieved after two cycles of Leader–Follower.
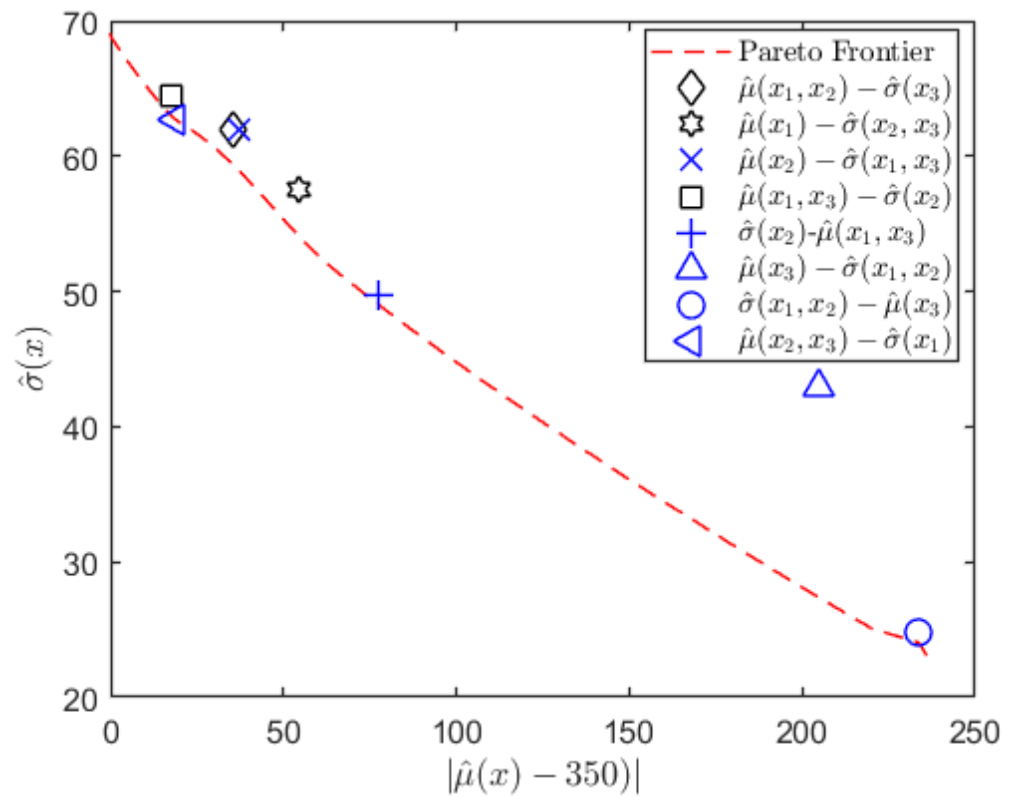


**Figure 2.** Pareto frontier and Stackelberg solutions.

**Table 3.** Game $\hat{\mu}(x_2, x_3) - \hat{\sigma}(x_1)$.

| Iteration | Output Improvement | Variables | | | Leader–Mean | Follower |
| | | $x_1$ | $x_2$ | $x_3$ | $\|\hat{\mu}(x_2,x_3)-\tau\|$ | $\hat{\sigma}(x_1)$ |
|---|---|---|---|---|---|---|
| Start | | 1.682 | 0.000 | 0.000 | 54.434 | 57.572 |
| Leader | Yes | 1.682 | −0.329 | −0.250 | 18.815 | 62.718 |
| Follower | No | 1.682 | −0.329 | −0.250 | 18.815 | 62.718 |

**Table 4.** Game $\hat{\sigma}(x_1) - \hat{\mu}(x_2, x_3)$.

| Iteration | Output Improvement | Variables | | | Leader–Standard Deviation | Follower |
| | | $x_1$ | $x_2$ | $x_3$ | $\hat{\sigma}(x_1)$ | $\|\hat{\mu}(x_2,x_3)-\tau\|$ |
|---|---|---|---|---|---|---|
| Start | | 1.682 | 0.000 | 0.000 | 54.434 | 57.572 |
| Leader | No | 1.682 | 0.000 | 0.000 | 54.434 | 57.572 |
| Follower | Yes | 1.682 | −0.329 | −0.250 | 18.815 | 62.718 |
| Leader | No | 1.682 | −0.329 | −0.250 | 18.815 | 62.718 |

Results represented in Figure 2 show the Stackelberg technique ability to capture POE solutions or solutions at a marginal distance from Pareto frontier. In fact, only the solution represented by the triangle cannot be considered a nondominated solution. This case study also shows that inverting the players hierarchy in a game may yield different solutions. For example, the solution achieved when the Leader optimizes $\hat{\sigma}(x_1, x_2)$ and the Follower optimizes $\hat{\mu}(x_3)$ is better than that achieved when the Leader optimizes $\hat{\mu}(x_3)$ and the Follower optimizes $\hat{\sigma}(x_1, x_2)$.

Concerning the variables selection, this case study provides evidence that the Factors Scaling tool results are useful because the two most favorable solutions, those represented by the left-pointing triangle and the square, are achieved when the Leader manipulates the variable with the highest influence on his/her optimization function (see Table 5). Two competitive solutions, namely, the solutions represented by the diamond and the cross, are also achieved when the Follower manipulates the variable with the highest influence on his/her optimization function, what suggests that competitive solutions are achieved when both the Leader and the Follower manipulate the variable with the highest influence on his/her optimization function.

**Table 5.** Input variables influence on responses.

| | **Mean** | | |
| --- | --- | --- | --- |
| | $x_1$ | $x_2$ | $x_3$ |
| $\left| MPI_{n1}^{std} \right|$ | 0.5568 | 0.7421 | 1.0000 |
| | **Standard Deviation** | | |
| | $x_1$ | $x_2$ | $x_3$ |
| $\left| MPI_{n2}^{std} \right|$ | 0.2029 | 0.6310 | 1.000 |

### 3.2. Case Study 2—Cutting Machine

To optimize the metal removal rate of a metal cutting machine, the effects of speed ($x_1$), depth ($x_2$), and feed ($x_3$) in the cutting process were evaluated from an experimental study [51]. The models fitted to the mean and standard deviation responses, presented below, are obtained from results of a central composite design with eight factorial points, six axial points, and six centre points replicated three times (see Shin and Cho [51]). In the optimization procedure, it was assumed that the mean response ($\hat{\mu}$) is of the nominal-is-best type, with target equal to 71.14, and the variance ($\hat{\sigma}^2$) is of the smaller-the-better type.

$$f_1(x) = \hat{\mu} = 79.8568 + 1.2073x_1 - 0.1540x_2 + 0.0706x_3 - 1.4708x_1x_2 + 0.7542x_1x_3 + 0.8708x_2x_3 - 2.0421x_1^2 - 0.1922x_2^2 - 0.4456x_3^2 \quad (7)$$

$$f_2(x) = \hat{\sigma}^2 = 2.8165 + 0.1040x_1 + 0.3436x_2 - 0.1484x_3 + 0.6371x_1x_2 - 0.1763x_1x_3 + 0.9729x_2x_3 - 0.2595x_1^2 - 0.1087x_2^2 + 0.0203x_3^2 \quad (8)$$

Following the rational of the previous case study, whenever a game and its inverse game (inverted game hierarchy) solutions are equal, only one game and the respective solution symbol is graphically represented. Figure 3 shows the Stackelberg solution for nine games, and the Pareto frontier. This provides evidence that inverting the player's hierarchy in a game does not always mean that the same solution will be achieved. Examples of these games are: 1—the Leader optimizes $\hat{\mu}(x_1, x_3)$ and the Follower optimizes $\hat{\sigma}^2(x_2)$, whose solution is represented by the cross; 2—the Leader optimizes $\hat{\sigma}^2(x_2)$ and the Follower optimizes $\hat{\mu}(x_1, x_3)$, whose solution is represented by the left-pointing triangle. Tables 6 and 7 are examples of games development, and one can see that final solutions are achieved only after one cycle Leader–Follower, respectively. The number of cycles in the other games ranges from one, in the games whose solutions are represented by the triangle and the plus sign, to eight cycles, in the game whose solution is represented by the hexagram.

This case study also enables the affirmation that the Stackelberg technique can capture POE solutions in DRO problems developed under the RSM framework, such as that shown in Figure 3. Notice that eight games (those whose solutions are represented by the plus sign, diamond, hexagram, cross, and triangle) yield nondominated solutions, six of them have the mean on target, those represented by the plus sign, hexagram, diamond, and cross. The solutions represented by the circle and the left-pointing triangle are at a negligible distance from the Pareto frontier so they can also be considered as alternatives to POE solutions, in contrast to those solutions represented by the square and the right-pointing triangle, which are dominated solutions.
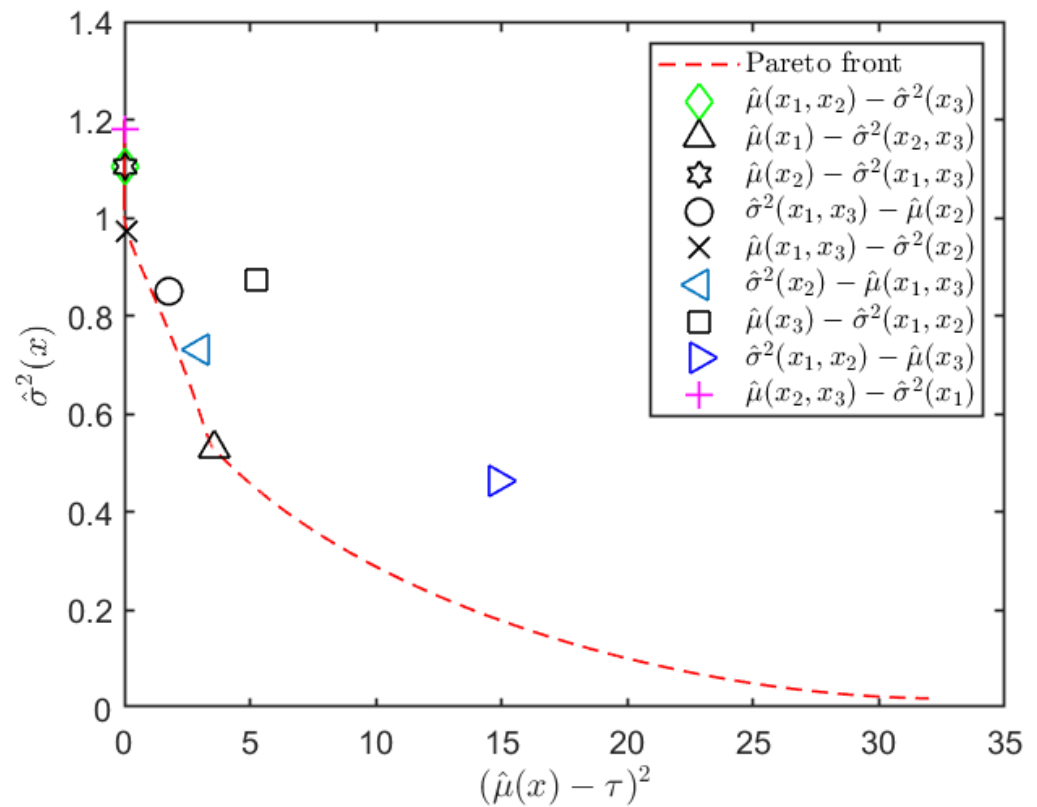
**Figure 3.** Pareto frontier and Stackelberg solutions.

**Table 6.** Game $\hat{\mu}(x_1, x_3) - \hat{\sigma}^2(x_2)$.

| Iteration | Output Improvement | Variables | | | Leader–Mean | Follower |
|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $(\hat{\mu}(x_1,x_3)-\tau)^2$ | $\hat{\sigma}^2(x_2)$ |
| | Start | −1.682 | 0.000 | 0.000 | 0.826 | 0.907 |
| Leader | Yes | −1.682 | 0.000 | 0.413 | 0.114 | 0.972 |
| Follower | No | −1.682 | 0.000 | 0.413 | 0.114 | 0.972 |

**Table 7.** Game $\hat{\sigma}^2(x_2) - \hat{\mu}(x_1, x_3)$.

| Iteration | Output Improvement | Variables | | | Leader–Variance | Follower |
|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $\hat{\sigma}^2(x_2)$ | $(\hat{\mu}(x_1,x_3)-\tau)^2$ |
| | Start | −1.682 | 0.000 | 0.000 | 0.907 | 0.826 |
| Leader | Yes | −1.682 | 0.413 | 0.000 | 0.588 | 3.367 |
| Follower | Yes | −1.682 | 0.413 | 0.226 | 0.732 | 3.024 |
| Leader | No | −1.682 | −0.413 | 0.226 | 0.732 | 3.024 |

In this case study, the Factors Scaling tool usefulness is confirmed. A POE solution is achieved when the Leader and the Follower manipulate the variables with the highest influence on their functions (see Table 8). Solutions represented by the triangle and the cross are examples of solutions that validate it. Nevertheless, exceptions to this guideline may also generate competitive solutions. As an example, in this case study, even when none of the players use the variable with the highest influence on its optimization function, a POE solution was achieved. For example, in the game $\hat{\mu}(x_2, x_3) - \hat{\sigma}^2(x_1)$, where both players play with the variable with the lowest influence on their function, a POE solution was achieved (that represented by the plus signal (+) in Figure 3).

**Table 8.** Input variables influence on responses.

| | Mean | | |
|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ |
| $\left|MPI^{std}_{n1}\right|$ | 1.0000 | 0.5267 | 0.4238 |
| | Variance | | |
| | $x_1$ | $x_2$ | $x_3$ |
| $\left|MPI^{std}_{n2}\right|$ | 0.5901 | 1.0000 | 0.6541 |

## 4. Results Discussion

There are various methods for solving DRO problems, each one having its own merits and weaknesses [20,52,53]. Thus, selecting a method is not easy for practitioners, namely, for those with limited mathematical and statistical background as they cannot obtain the necessary expertise to easily select a method suited for their needs, under the assumption that practitioners have access to the various academic journals where those methods are published. In addition, the overload of subjective information (shape factors, weights, or other preference information) required from the decision makers and the high level of computational, mathematical, and statistical expertise necessary for using some methods are major reasons by which they are not appealing to or used by practitioners.

In practice, practitioners with limited background in computation, mathematics, and statistics may overcome the difficulties they face in the implementation of theoretically sound methods, though there may not be any advantage in using these methods. In fact, easy-to-implement methods can generate solutions and depict Pareto frontiers similar to those achieved by theoretical sound methods [19,54].

The Stackelberg technique presented and illustrated here does neither require any advanced statistical and computational background nor subjective information from the users, who can implement it in an Excel spreadsheet. In addition, it can capture POE solutions, and, in some games, this is reached after a small number of cycles Leader–Follower, though Stackelberg technique is not designed to depict Pareto frontiers. It is also important to point out that to capture a set of solutions evenly distributed along a Pareto frontier, which may include convex and nonconvex regions, besides an appropriate starting point for input variables, the curvature of objective functions has to be manipulated through a shape factor or weight [54–57], which is not also incorporated in the Stackelberg technique.

Notice that to depict a Pareto frontier is not easy for practitioners who do not have advanced computational, mathematical, and statistical expertise and is always a laborious task, which also includes a purge in the generated solutions, because all optimization methods generate dominated solutions among the nondominated ones. Some nondominated solutions may lead to operation conditions more hazardous, more costly, or more difficult to implement and control, so solutions closer to the nondominated ones cannot be discarded/rejected without a further analysis. How good a solution is may depends on either economical and technical issues or decision-makers' preference [58]. In this setting, the presented case studies provide evidence of the Stackelberg technique usefulness, at least as an exploratory tool for DRO, in the sense that some solutions are the best possible results for a 'bargain' between players with conflicting objectives. Nevertheless, there is no guarantee it yields one or more competitive solutions for any case study.

Concerning the selection of input variables by the Leader, both case studies suggest that the Factors Scaling tool can provide helpful guidelines. In fact, in the second case study, nondominated solutions were achieved after a small number of cycles when the Leader and the Follower play with the variables with the highest influence on their optimization functions. This is the ideal situation. However, it may not always be possible, as shown in case study 1, and it does not mean that nondominated solutions will not be achieved. In case study 1, POE solutions are achieved when the Leader or the Follower manipulates the

variable with the highest influence on its optimization function. Even when this may not be possible to occur, and the players manipulate the variables with low influence on the function that they want to optimize (maximize or minimize), in some cases, as shown in case study 2, a nondominated solution may be achieved as well.

Besides the selection of input variables by the Leader and the Follower be usually made without any technical justification, there is also no tool or method to identify the best starting point for input variables to run a game. To use the Factors Scaling toll is a recommended practice to overcome the first aforementioned problem. Concerning the second one, several starting points must be tested because different starting points yield a different set of solutions for the games. These problems occur in game theory/Stackelberg technique, as well as in other approaches for DRO or MRO [59–61].

## 5. Conclusions

To satisfy the needs and concerns of the consumers directly affects the sustainability of businesses. Nowadays, managers, engineers, and other professionals have this awareness and are under pressure to adapt their decisions and actions to an increasingly volatile, uncertain, complex, and ambiguous business context. The optimization of process efficiency and product characteristics is a current practice reported in the literature, and a variety of approaches, methodologies, and/or tools have been used for that purpose. However, formulating DRO problems as a game is an approach to solve real problems in (non)manufacturing settings that has not received the deserved attention. Thus, this paper extends the game theory to DRO and exposes the strengths and weaknesses of the Stackelberg technique using two case studies. The presented easy-to-understand and easy-to-use approach can be useful for practitioners who do not have a strong background on computation, mathematics, and statistics and look for equilibrium solutions, including POE solutions, for DRO problems.

To select a case study for assessing methods or techniques performance is a difficult task to be carried out in a fair basis, if at all possible, because there is a lack of definition upon the case studies that must be used to evaluate and provide a clear understanding of the working abilities of the methods and/or techniques used. The presented case studies provide evidence of the game theory/Stackelberg technique's working ability for solving real-life problems developed under the RSM framework where conflicting objectives exist. Stackelberg solutions may be compared with those of the Pareto frontier, and it is shown that Stackelberg technique can capture POE solutions. Another novelty of this paper is the Factors Scaling tool integration into the Stackelberg technique, and results confirm its usefulness in the selection of the input variables for both players. Nevertheless, to better understand the working ability of the presented approach to yield solutions for DRO problems that may be implemented in practice, namely, POE solutions, other scenarios (more case studies) must be tested in future work. To extend the game theory to MRO problems is another open research field.

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