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Bi-parental product algorithm for coded waveform design in radar

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Abstract. The problem of obtaining long sequences with finite alphabet and peaky aperiodic auto-correlation is important in the context of radar, sonar and system identification and is called the coded waveform design problem, or simply the signal design problem in this limited context. It is good to remember that there are other signal design problems in coding theory and digital communication. It is viewed as a problem of optimization. An algorithm based on two operational ideas is developed. From the earlier experience of using the eugenic algorithm for the problem of waveform design, it was realised that rather than random but multiple mutations, all the first-order mutations should be examined to pick up the best one. This is called Hamming scan, which has the advantage of being locally complete, rather than random. The conventional genetic algorithm for non-local optimization leaves out the anabolic role of chemistry of allowing quick growth of complexity. Here, the Hamming scan is made to operate on the Kronecker or Chinese product of two sequences with best-known discrimination values, so that one can go to large lengths and yet get good results in affordable time. The details of the ternary pulse compression sequences obtained are given. They suggest the superiority of the ternary sequences.

Keywords. Coded waveform design; global optimization; bi-parental products; Hamming scan.

1. Introduction

The term signal design has different meanings in coding theory, spread-spectrum communication, and radar. Here it is short for a coded waveform design problem in radar.

This involves obtaining sequences of finite lengths with small discrete alphabet, which are good approximations to white noise (Golay 1977). In particular, the alphabet $(-1, 0, 1)$ is considered. The sequences using them are called ternary sequences. Much of the earlier work deals with binary sequences with $(-1, 1)$ as the alphabet. The goodness of the degree of approximation is measured here in terms of the discrimination. An alternative measure is the merit factor (Golay 1977).

To set the notation and to concretize the ideas, let

$$\mathbf{s} = [s_0, s_1, \ldots, s_{n-2}, s_{n-1}] \tag{1}$$

be the sequence of length n, where the elements s_i are from any of the two alphabets referred to above. Then

$$\rho(k) = \sum_{i=0}^{n-1-k} s_i s_{i+k}, \quad k = 0, 1, \ldots, n-1, \tag{2}$$

is called the aperiodic auto-correlation of the sequence $\mathbf{s}$. The quantity,

$$D = \rho(0) \Big/ \left[\max_{k \neq 0} |\rho(k)|\right], \tag{3}$$

is called the discrimination.

The problem of obtaining sequences with high discrimination is a very difficult one. It is so because whereas for the binary sequences with good periodic auto-correlation, analytical design procedures or constructions (Baumert 1971; Golay 1983; Hoholdt *et al* 1985; Reddy & Rao 1986; Jensen *et al* 1991) based on powerful number-theoretical results are available, for sequences with good aperiodic autocorrelations it has still to be basically a search. Further, the search cannot be exhaustive, because the number of sequences to be searched for grows combinatorially, as the length increases. Here, the emphasis is on obtaining as good sequences as possible with certain reasonably efficient procedures without claiming that even higher discrimination values are not possible. Golay's notion of sieves (Golay 1977) which restricts the search to a subset of sequences which can either be designed or searched efficiently and has properties which are desirable for enhancing the peakiness is a similar one. The sieves promise some good sequences without claiming that all good or even the best sequences are necessarily retained.

There is extensive work on binary sequences (Barker 1953; Turyn 1963, 1968; Boehmer 1967; Baumert 1971; Golay 1972, 1977, 1982, 1983; Moharir 1975; Beenker *et al* 1985; Hoholdt *et al* 1985; Kerdock *et al* 1986; Reddy & Rao 1986; Bernasconi 1987, 1988; Hoholdt & Justesen 1988; Golay & Harris 1990; Newmann & Byrnes 1990; Jensen *et al* 1991; De Groot *et al* 1992), which however we do not attempt to review here. Beyond the Barker sequences (Barker 1953) of length 11 and 13, having the discrimination values of 11 and 13, the four best binary sequences having discrimination values of 14, 15, 16 and 17 (Kerdock *et al* 1986) have been obtained at the smallest lengths of 28, 51, 69 and 88. Complete enumeration is possible only up to small lengths (Kerdock *et al* 1986) and would not be regarded as too satisfactory, the ultimate aim being construction rather than search. Partial searches using the notion of sieves (Golay 1977) were extended to higher lengths. Starting with binary sequences with ideal periodic autocorrelation and searching for sequences with the best discrimination among their cyclic shifts, many good sequences have been obtained. To cite some examples (Reddy & Rao 1986), discriminations of 22.82, 26.85, 36.86, 41.45, 41.77 and 42.46 have been obtained at the lengths of 251, 349, 811, 829, 919 and 1019, respectively.

Optimization techniques have been used for the signal design problem (Bernasconi 1987, 1988; De Groot *et al* 1992), but the design criterion has been different, viz., merit factor.

Ternary aperiodic sequences have been proposed and listed earlier with merit factor as the criterion of goodness (Moharir 1974, 1976; Moharir *et al* 1985). They were obtained using sieves such as terminal admissibility (Moharir 1976), skew-symmetry (Golay 1977), terminal admissibility for skew-symmetric sequences (Moharir *et al* 1985) etc. This earlier work establishing the superiority of ternary pulse compression sequences over binary sequences, if merit factor is accepted as a valid desideratum, is extended further in this paper with discrimination as a chosen criterion.

2. The algorithm and associated concepts

2.1 *Hamming scan*

An improved version of the genetic algorithm (Holland 1992; Michalewicz 1992) called eugenic algorithm was used (Singh *et al* 1996) to see whether better and longer ternary sequences could be obtained. Success was met with along both these directions. Yet it was not possible to go to very large lengths, as the search time requirement increased very fast. It was regarded that one should devise an optimization algorithm which is more efficient, even though possibly more suboptimal. The Hamming scan is one such algorithm. Genetic algorithms use random but possibly multiple mutations. Mutation is a term metaphorically used for a change in an element in the sequence. Thus, a single mutation is at a Hamming distance of one from the original sequence. The Hamming scan looks at all the Hamming -1 neighbours and picks up the one with the largest discrimination. If it is better than the original sequence with the chosen definition of goodness, the algorithm is recursively continued therefrom, as long as improvement is possible. Thus, an entirely probabilistic mechanism of mutation is replaced by a locally complete search. The Hamming scan is expedited, and hence, made applicable at large lengths, by not calculating the aperiodic autocorrelation of a Hamming neighbour *ab initio*, recognizing the fact that as only one element is different, only its different contributions need to be taken into account. Let the element s_j be changed to c_j. As a result, let $\rho(k)$ change to $\rho'(k)$. Then it can be shown that

$$\rho'(0) = \rho(0) + (c_j^2 - s_j^2), \tag{4}$$

and

$$\rho'(k) = \rho(k) + (c_j - s_j)s_{j+k} + s_{j-k}(c_j - s_j), \quad k = 1, 2, \ldots, n-1; \quad j = 0, 1, \ldots, n-1. \tag{5}$$

In (5), there are two correction terms. They have to be implemented with care. One way is to assume that s_p is equal to zero if p is outside $(0, 1, \ldots, n-1)$. Alternatively, the correction term $s_{j-k}(c_j - s_j)$ is included only for $k = 1, 2, \ldots, n-j-1$ and the correction term $s_{j-k}(c_j - s_j)$ is included only for $k = j+1, j+2, \ldots, n-1$. This idea is certainly trivial, but it has led to significantly increased efficiency, and hence, to search at longer lengths than would otherwise have been possible.

For the ternary sequences the actual implementation can be somewhat different. The elements in the sequence are -1, 0 and 1. Each one of them can mutate in two possible ways. The mutations $-1 \to 0$, $0 \to -1$ and $1 \to -1$ are considered to give the lower strand of Hamming neighbours and the other mutations $-1 \to 1$, $0 \to 1$ and $1 \to 0$ are regarded as giving the upper strand of Hamming neighbours. The best neighbours along these two strands were found separately. The idea is that if one strand gives improvement, the other strand may not even be considered in order to save time. As the results are certainly path-dependent and optimality is not guaranteed, efficiency is a valid determinant.

The Hamming scan yielded some better ternary sequences in reasonable time than were obtainable with the eugenic algorithm. However, the Hamming scan also became unaffordable at larger lengths.

2.2 *Simon's principle through Kronecker and Chinese products*

That is when Simon's principle (Koestler 1969; Simon 1981) suggested newer possibilities. It states that bigger systems evolve faster, when developed through the metastable intermediate subsystems, than if they are constructed *ab initio* from the smallest components. In the present context, it took the form of using two sequences of the best discrimination values available and obtaining a sequence of much larger length from them, such that it already had better discrimination than would result from random choice. The actual mechanism is provided by bi-parental products (Moharir 1992) of two or more sequences. In particular, two products, viz., Kronecker product (Brewere 1978; Moharir 1992) and the Chinese product (Moharir 1977, 1992) are chosen. These products are said to be bi-parental because each element in the product depends exactly on one element each from the two component sequences.

The Kronecker product of two sequences

$$\mathbf{s_1} = [s_{01}, s_{11}, \ldots, s_{(p-1)1}], \qquad \mathbf{s_2} = [s_{02}, s_{12}, \ldots, s_{(q-1)2}], \tag{6}$$

of lengths p and q respectively, is a sequence $\mathbf{s}$ of length pq, defined as

$$\mathbf{s_k} = (s_{01}\mathbf{s_2}, s_{11}\mathbf{s_2}, \ldots, s_{(p-1)1}\mathbf{s_2}). \tag{7}$$

The Kronecker product is not commutative.

The Chinese product of the two sequences of (6) is defined only when p and q are relatively prime, that is, when they do not have any common prime factor. The Chinese product is defined as

$$s_{\mathrm{Ck}} = s_{1i}s_{2j}, \tag{8}$$

where

$$k = \begin{cases} i \bmod p \\ j \bmod q \end{cases} \tag{9}$$

and has the length pq. It is called the Chinese product because the solution of the congruence relation (9) is what the Chinese remainder theorem deals with. It is commutative.

Computationally, the Chinese product of the two sequences can be obtained easily by repeating the sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ of relatively prime lengths p and q respectively, q and p times and then taking an element-wise (Schur) product (Moharir 1992).

The autocorrelation of $\mathbf{s}_k$ can be expressed in terms of the autocorrelations of $\mathbf{s}_1$ and $\mathbf{s}_2$ as follows (Turyn 1968).

Theorem 1.

$$\rho_{s_k}(qk_1 + k_2) = \rho_{s_1}(k_1)\rho_{s_2}(k_2) + \rho_{s_1}(k_1 + 1)\rho_{s_2}(q - k_2),$$
$$k_1 = 0, 1, \ldots, p - 1; \quad k_2 = 0, 1, \ldots, q - 1. \tag{10}$$

The theorem shows that if the individual autocorrelations are good, so is the resultant autocorrelation, except at some lag values. It can further be shown that the discrimination of the Kronecker product of two sequences depends only on the discriminations of the component sequences. The attenuated minimum guarantee theorem below is important.

Theorem 2. *If the discriminations of the two sequences* $\mathbf{s}_1$ *and* $\mathbf{s}_2$ *are* D_1, *and* D_2 *respectively, with* $\min(D_1, D_2) = D_{\min}$ *and* $\max(D_1, D_2) = D_{\max}$, *then the discrimination* D_{K} *of both their Kronecker products* $\mathbf{s}_1 \times \mathbf{s}_2$ *and* $\mathbf{s}_2 \times \mathbf{s}_1$ *is bounded as*

$$D_{\min} \geq D_{\mathrm{K}} \geq \alpha D_{\min}, \quad \alpha = (D_{\max}/(1 + D_{\max})), \tag{11}$$

where α *may be called the attenuator.*

The following has been shown by Moharir (1977).

Theorem 3. *The periodic autocorrelation of the Chinese product of two sequences is a Chinese product of their periodic autocorrelations.*

A sequence obtained by a bi-parental product (Moharir 1992) of two or more sequences can later be further improved by a Hamming scan. The time requirement comes down considerably because the algorithm begins with a good starting point. The importance of nonlocal optimizers such as the genetic algorithm is that the end result depends on the starting point only weakly, and ideally, not at all. But this statement is concerned with only the ultimate reachability of the algorithm, and the utility of a good starting point in determining the time requirements should not be underestimated. This is particularly so when good starting points can be designed or devised. Second, global optimization also is a goal and not a reality. In any case, the Hamming scan is not a global optimization algorithm. Therefore, if a large starting discrimination can be obtained by a simple procedure, in addition to the time saved, many lower local optima of which the Hamming scan possibly could not come out would also be avoided.

The anabolic bi-parental product scheme really works well as can be seen from table 1. It analyses the role of the Kronecker and the Chinese products in the proposed algorithm. Some ternary sequences with very good discrimination were chosen. They were obtained from the ternary sequences with the best merit factors listed by Golay (1977) by recursive Hamming scan to improve the discriminations. Then their Kronecker and Chinese products were obtained. The initial discriminations D_1 and D_2 and the discriminations D_{K} and D_{C}

Table 1. Analysis of the efficacy of the bi-parental products, viz. Kronecker and Chinese, in designing ternary sequences with large discrimination. The lengths of the component sequences are n_1 and n_2. Their discriminations are D_1 and D_2. The discrimination of the product sequence is D. The bi-parental product efficiency is η and the exponent is γ. Whether the bi-parental product P is Kronecker or Chinese is indicated by K and C respectively.

n_1	n_2	n	D_1	D_2	P	D	η	γ
31	31	961	13.0000	13.0000	K	12.7547	0.9811	0.4963
9	33	297	7.0000	12.5000	K	7.0000	0.7483	0.4352
33	33	1089	12.5000	12.5000	K	12.5000	1.0000	0.5000
24	111	2664	20.0000	20.0000	K	19.7531	0.9877	0.4979
24	147	3528	20.0000	20.0000	K	19.6721	0.9836	0.4972
24	159	3816	20.0000	21.8333	K	20.0000	0.9571	0.4928
11	13	143	11.0000	13.0000	K	11.0000	0.9199	0.4832
					C	8.9375	0.7474	0.4413
13	15	195	13.0000	13.0000	K	13.0000	1.0000	0.5000
					C	12.0714	0.9286	0.4856
11	17	187	11.0000	11.0000	K	11.0000	1.0000	0.5000
					C	6.7222	0.6111	0.3973
11	23	253	11.0000	19.0000	K	11.0000	0.7609	0.4488
					C	7.7407	0.5354	0.3831
19	31	589	13.0000	13.0000	K	13.0000	1.0000	0.5000
					C	12.0714	0.9286	0.4856
13	33	429	13.0000	12.5000	K	12.5000	0.9806	0.4961
					C	10.1563	0.7967	0.4554
29	33	957	8.6667	12.5000	K	8.6667	0.8327	0.4609
					C	8.5526	1.2141	0.5381
24	145	3480	20.0000	23.2000	K	20.0000	0.9285	0.4879
					C	11.2621	0.5228	0.3944
24	155	3720	20.0000	22.1667	K	20.0000	0.9499	0.4916
					C	19.4161	0.9221	0.4867
15	17	255	13.0000	11.0000	K	11.0000	0.9199	0.4832
					C	11.0000	0.9199	0.4832
11	25	275	11.0000	10.5000	K	10.5000	0.9770	0.4951
					C	10.5000	0.9770	0.4951
13	31	403	13.0000	13.0000	K	13.0000	1.0000	0.5000
					C	13.0000	1.0000	0.5000
9	11	99	7.0000	11.0000	K	7.0000	0.7977	0.4480
					C	8.5556	0.9750	0.4942
9	23	207	7.0000	19.0000	K	7.0000	0.6070	0.3979
					C	8.3125	0.7208	0.4330

(*Continued on facing page*)

of these two products are listed. It can be seen that impressive starting points for the Hamming scan are available. The purpose of table 1 is, however, different. It compares the two products as the bases of the bi-parental product signal design. In table 1, six situations can be identified. In view of theorem 2, there is not much uncertainty about what the Kronecker product can achieve. For some entries in table 1, the component lengths are such that only the Kronecker product is defined and the Chinese product is not defined as the two lengths have a common factor. For these sets of component lengths, only the Kronecker product can be used. For the next set of entries, D_C is less than D_K. For the next four sets of entries, D_C equals D_K, lies between D_K and D_{max}, equals D_{max}, and exceeds

Table 1. (*Continued*)

n_1	n_2	n	D_1	D_2	P	D	η	γ
9	29	261	7.0000	8.6667	K	7.0000	0.8987	0.4740
					C	8.2727	1.0621	0.5147
13	29	377	13.0000	8.6667	K	8.6667	0.8165	0.4571
					C	11.2667	1.0614	0.5126
15	29	435	13.0000	8.6667	K	8.6667	0.8165	0.4571
					C	11.6552	1.0980	0.5198
25	29	725	10.5000	8.6667	K	8.6667	0.9085	0.4787
					C	9.1000	0.9539	0.4895
17	25	425	11.0000	10.5000	K	10.5000	0.9770	0.4951
					C	11.0000	1.0235	0.5049
21	25	525	8.5000	10.5000	K	8.5000	0.8997	0.4765
					C	10.5000	1.1114	0.5235
21	31	651	8.5000	13.0000	K	8.5000	0.8086	0.4548
					C	13.0000	1.2367	0.5452
17	21	357	11.0000	8.5000	K	8.5000	0.8790	0.4716
					C	13.3571	1.3814	0.5712
19	21	399	13.0000	8.5000	K	8.5000	0.8086	0.4548
					C	13.8125	1.3140	0.5580
17	27	459	11.0000	8.3333	K	8.3333	0.8704	0.4693
					C	11.4583	1.1968	0.5398
17	29	493	11.0000	8.6667	K	8.6667	0.8876	0.4738
					C	13.0000	1.3314	0.5628
19	29	551	13.0000	8.6667	K	8.6667	0.8165	0.4571
					C	14.0833	1.3268	0.5599
21	29	609	8.5000	8.6667	K	8.5000	0.9903	0.4977
					C	9.2083	1.0729	0.5164
27	29	783	8.3333	8.6667	K	8.3333	0.9806	0.4954
					C	9.4203	1.1085	0.5241
15	31	465	13.0000	13.0000	K	13.0000	1.0000	0.5000
					C	13.5200	1.0400	0.5076
25	33	825	10.5000	12.5000	K	10.5000	0.9165	0.4821
					C	13.1250	1.1456	0.5279
31	33	1023	12.5000	12.5000	K	12.5000	0.9806	0.4961
					C	15.4762	1.2141	0.5381

D_{max}. Thus, the Chinese product has the possibilities of providing better starting points than those provided by the Kronecker product, for the bi-parental product algorithm for signal design.

It is useful to define some quantitative measures for the bi-parental products in the context of the objective function chosen. The Kronecker efficiency η_K (for discrimination) is defined as

$$\eta_K = D_K/(D_1 D_2)^{1/2}. \tag{12}$$

The Kronecker exponent γ_K is defined by

$$D_K = (D_1 D_2)^{\gamma_k} \quad \text{or} \quad \gamma_K = \log D_K/\log(D_1 D_2). \tag{13}$$

Table 1 shows values of η_K and γ_K. It can be seen that they lie in narrower ranges than is suggested by theorem 2. A starting sequence of a particular large length can be obtained by factoring the length differently. In such cases two thumb rules could be used for guidance. the Chinese efficiency η_C and the Chinese exponent γ_C can be defined by replacing D_K by D_C in (10) and (11), where D_C is the discrimination obtained by using a Chinese product. Table 1 also shows η_K and γ_K. The ranges of these are larger than those of η_K and γ_K. The extension of the ranges on the higher sides must be viewed as advantageous.

2.3 *Hamming scan and Simon's principle*

Our interest was in obtaining ternary sequences with good discrimination values. The Kronecker product is not commutative. But the discriminations of $\mathbf{s}_1 \times \mathbf{s}_2$ and $\mathbf{s}_2 \times \mathbf{s}_1$ are the same. The Chinese product may have a superior, equal or inferior discrimination. Subsequently, however, they may evolve differently under a Hamming scan, so that the initial advantage of a superior discrimination may not last. When starting sequences of a particular length can be obtained by bi-parental products using different factors, further progress under a Hamming scan is also different. These are indicators that a Hamming scan does not give the global optimum. Let the ratio of the discrimination eventually obtained by recursively using a Hamming scan to the discrimination of the starting sequence obtained by a bi-parental product be called the Hamming gain for that product. It is not as constant as the bi-parental product efficiency or the exponent. Let both the components in the Kronecker product be Barker sequence of length 11. Then the discrimination of the resultant sequence of length 121 cannot be improved by the Hamming scan, meaning that the Hamming gain is just unity. But such situations are rather rare. Variability of the Hamming gain makes it difficult to use a good starting point as a very dependable criterion and offers no good thumb rules. One may make a choice on the basis of the average size of the improvement in the first few Hamming scans. But that is more a way of trying to minimize the computational effort than trying to estimate beforehand what discriminations may be achievable, as the improvement is not uniform over successive Hamming scans and the number of successful scans is also not known beforehand. It has been observed that the Hamming gain for the Chinese product is frequently less than that for the Kronecker product. It may also be noted that a sequence obtained by Hamming scan operating on a bi-parental product of two sequences, can later be used as a component in the same or different bi-parental product. All these possibilities have not been exhausted.

However, one more idea has been found to be very useful. After taking a bi-parental product of two good sequences, if the merit factor is improved by the recursive Hamming scan for some time and then the discrimination is taken to be the objective function to be

Table 2. The number of lengths N_T at which ternary sequences exceeding the discrimination threshold of D_T have been obtained.

D_T	N_T	D_T	N_T	D_T	N_T
48	2	44	7	40	27
36	47	34	75	32	119
30	170	28	242	26	304
24	344	22	367	20	389
18	414	16	434	14	464

Table 3. A list of ternary sequences, with discrimination values greater than 36, in the descending order of the discrimination.

Length	Discrimination	Length	Discrimination	Length	Discrimination	Length	Discrimination
1564	48.9048	924	42.4667	1584	40.2414	1001	37.6087
1131	48.2222	649	42.4615	590	40.0000	592	37.5455
1073	47.3125	975	41.7059	582	40.0000	555	37.4000
591	44.6667	1080	41.3889	584	39.8000	596	37.2727
599	44.4444	819	41.1538	1650	39.7667	593	36.9091
1632	44.2400	1188	40.9474	1680	39.6000	851	36.8125
1221	44.0588	1305	40.8889	1088	38.5714	1121	36.5789
1610	43.5385	726	40.8571	925	38.5625	1608	36.5556
1700	43.3214	623	40.7000	625	38.1818	1617	36.1389
1147	43.1579	858	40.6471	1071	37.9524	1053	36.1200
1518	43.0000	1750	40.6333	1173	37.7500	767	36.0000
999	42.5294	792	40.3846	1464	37.6429		

increased by the same procedure, superior discrimination values frequently result, than if the discrimination was increased from the start.

3. Results

Only the sequence having the largest discrimination obtained at any length has been retained. The numbers of lengths at which various discrimination thresholds have been exceeded are shown in table 2. Thus the discrimination thresholds of 32, 26 and 18 have been exceeded at more than 100, 300 and 400 lengths respectively. The details about the sequences having the discrimination values of greater than 36 are shown in table 3.

It is seen that as the length increases it is easier to reach higher discrimination values. This is a simple consequence of theorem 2. To see this point, assume that the work of obtaining ternary sequences with good discrimination values is conducted up to the length of 160. Then, the best sequences are of lengths 24, 111, 145, 147, 155 and 159, and have discriminations of 20, 20, 23, 20, 22.1667 and 21.8333 respectively. That is, they all have discriminations exceeding 20. Then, theorem 2 implies that a Kronecker product of any two of them must have a discrimination exceeding 19.05. In general, the actual discrimination achieved is frequently much better than $\alpha D_{\min}$, as can be seen from table 1. Hamming scan can then raise the discrimination even further. It is very rare that the Hamming gain is just 1. Thus, the highest discrimination obtained up to any length can almost always be exceeded at some higher lengths.

4. Conclusion

The bi-parental product algorithm has given ternary sequences with very good discriminations. The success of the algorithm indicates that the locally complete search may be preferable to the Monte Carlo search which depends rather excessively on the efficacy of chance in the matter of optimization. Whereas chance is a useful ally while dealing with combinatorially complex optimization problems, where feasible, it should be helped by design procedures (Singh *et al* 1996) which can minimise the search effort. The anabolic role of bi-parental products which permitted going to large lengths quickly is particularly noteworthy in this regard. The algorithm has taken the problem of aperiodic signal design one step closer to that of the periodic signal design. The latter has two features. One of them is the availability of the regular construction procedures. That goal is still far away for the former problem. The second feature is that the discovery of a new good sequence means automatic construction of good sequences at many larger lengths. The bi-parental product procedure in the algorithm gives that feature to the aperiodic signal design problem also, though in a weaker sense, as the Hamming scan, which includes choice, is still to be performed.

To the best of our knowledge, the ternary sequences obtained here are the best. Yet, the bi-parental product algorithm is not a global optimisation algorithm. Therefore, there could be procedures which can improve upon the sequences obtained here.

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