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Bianchi Type-III Inflationary Universe with Constant Deceleration Parameter in General Relativity

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Abstract. Bianchi type-III inflationary universe is investigated in the presence of mass less scalar field with a flat potential. To get an inflationary universe a flat region in which potential V is constant is considered. Some physical and kinematical properties of the universe are also discussed.

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1 Introduction

Inflation, the stage of accelerated expansion of the Universe, first proposed in the beginning of the 1980s, nowadays receives a great deal of attention. Guth [1] proposed inflationary model in the context of grand unified theory (GUT), which has been accepted soon as the model of the early Universe. Barrow and Turner [2] showed that the large anisotropy prevents transition into an inflationary era according to Guth's original inflationary scenario. Inflationary models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The standard explanation for the flatness of the universe is that it has undergone at an early stage of the evolution a period of exponential expansion named as inflation.

Scalar fields are the simplest classical fields and there exists an extensive literature containing numerous solutions of the Einstein equation where the scalar field is minimally coupled to the gravitational field. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology.

Many authors Burd and Barrow [3], Wald [4], Barrow [5], Ellis and Madsen[6], Heusler[7] studied several aspects of scalar field in the evolution of the universe and FRW models. Using the concept of Higgs field ϕ with potential $V(\phi)$ has a flat region and the ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy.

The role of self-interacting scalar fields in inflationary cosmology in four–dimensional space-time has been investigated by Bhattacharjee and Baruah [8], Bali and Jain [9], Rahman *et al.* [10], C.P. Singh *et al.* [11], Reddy and Naidu [12], Reddy *et al.* [13]. In recent years, Katore *et al.* [15], Reddy and Naidu [16] have studied the cosmological models with constant deceleration parameter of the universe in the context of different aspects of different space-time. Recently Reddy [17] has discussed Bianchi type-V inflationary universe in general relativity. Very recently, Katore *et al.* [18] have discussed Kantowaski-Sachs inflationary universe in general relativity.

In this paper, we have investigated Bianchi type-III cosmological model in the presence of mass less scalar field with a flat potential in general relativity. To get a determinate solution, we have considered a flat region in which the potential is constant. We have also assumed a relation between metric coefficients for this purpose.

2 Metric and the Field Equations

We consider the Bianchi type-III metric of the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2dz^2, \tag{1}$$

where A, B, C are the functions of t only.

The non-vanishing components of the Einstein tensor for the metric (1) are

$$\begin{aligned}
G_1^1 &= -\Big(\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C}\Big),\\
G_2^2 &= -\Big(\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C}\Big),\\
G_3^3 &= -\Big(\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{1}{A^2}\Big),\\
G_4^4 &= -\Big(\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{1}{A^2}\Big),\\
G_4^1 &= \Big(\frac{B_4}{B} - \frac{A_4}{A}\Big).
\end{aligned} \tag{2}$$

Here the subscript 4 denotes the differentiation with respect to t.

In this case of gravity minimally coupled to a scalar field $V(\phi)$, the Lagrangian is

$$L = \int \Big[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\Big]\sqrt{-g}\, d^4x, \tag{3}$$

which on variation of L, with respect to the dynamical fields, to Einstein field equations

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{4}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right] g_{ij}\,. \tag{5}$$

The field equation is

$$\phi^{i}_{;i} = -\frac{dV}{d\phi}\,, \tag{6}$$

where comma and semicolon indicate ordinary and covariant differentiation respectively.

Other symbols have their usual meaning and units are taken so that

$$8\pi G = C = 1.$$

Now the field equations (4) for the metric (1) with the help of equation (5) are given by

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} + \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{1}{A^2} + \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{9}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{1}{A^2} - \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{10}$$

$$\frac{B_4}{B} - \frac{A_4}{A} = 0, \tag{11}$$

and (6), for the scalar field, takes the form

$$\phi_{44} + \phi_4\Big(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\Big) = -\frac{dV}{d\phi}. \tag{12}$$

From (11), without loss of generality, we get

$$A = B. \tag{13}$$

Using (13), the above set of equations reduces to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{14}$$

$$2\frac{B_{44}}{B} + \Big(\frac{B_4}{B}\Big)^2 - \frac{1}{B^2} + \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{15}$$

$$\Big(\frac{B_4}{B}\Big)^2 + 2\frac{B_4}{B}\frac{C_4}{C} - \frac{1}{B^2} - \frac{1}{2}\phi_4^2 + V(\phi) = 0, \tag{16}$$

$$\phi_{44} + \phi_4\Big(2\frac{B_4}{B} + \frac{C_4}{C}\Big) = -\frac{dV}{d\phi}\,. \tag{17}$$

Here the subscript 4 denotes the differentiation with respect to t.

3 Solution of the Field Equations and the Model

Stein–Schabas [19] has shown that Higgs field ϕ with potential $V(\phi)$ has flat region and the field evolves slowly but the universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Thus, we are interested here, in inflationary solutions of the field equations.

The flat region is considered where the potential is constant, *i.e.*,

$$V(\phi) = \text{const} = v_0. \tag{18}$$

Also, the set of equations being highly non-linear, we assume a relation between metric coefficients given by

$$B = \mu C\,, \tag{19}$$

where μ is constant.

We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's parameter, proposed by Bermann that yields constant deceleration parameter models of the universe.

We consider only constant deceleration parameter model defined by

$$q = -\left[\frac{RR_{44}}{(R_4)^2}\right] = \text{const}, \tag{20}$$

where $R = (B^2Ce^x)^{1/3}$ is the over all scale factor. Here the constant is taken as negative (*i.e.*, it is an accelerating model of the universe.)

The solution of (20) is given by

$$R = (\alpha t + \beta)^{1/1+q}, \tag{21}$$

where $\alpha \neq 0$ and β are constants of integration. This equation implies that the condition of expansion is $1 + q > 0$.

Field equations (14)-(17) with the help of (19) and (21), now admit an exact solution given by

$$A = B = \mu^{1/3} e^{-x/3} (\alpha t + \beta)^{1/1+q}, \tag{22}$$

$$C = \mu^{-2/3} e^{-x/3} (\alpha t + \beta)^{1/1+q}, \tag{23}$$

$$\phi = k_1 e^x \left(\frac{1+q}{q-2}\right) (\alpha t + \beta)^{q-2/1+q} + \phi_0, \tag{24}$$

where k_1 and ϕ_0 are constants, $1 + q > 0$.

Hence, Bianchi type-III cosmological model corresponding to the above solutions, can be written (through a proper choice of co-ordinates and constants of integration) as

$$ds^2 = dT^2 - \mu^{\frac{2}{3}} e^{\frac{-2x}{3}} T^{\frac{2}{1+q}} dX^2 - \mu^{\frac{2}{3}} e^{\frac{-4x}{3}} T^{\frac{2}{1+q}} dY^2 - \mu^{\frac{-4}{3}} e^{\frac{-2x}{3}} T^{\frac{2}{1+q}} dZ^2. \tag{25}$$

It is interesting to note that this model is free from singularities.

4 Some Physical and Kinematical Properties

The physical quantities that are important in cosmology are proper volume V^3, the expansion scalar θ, shear scalar σ and Hubble's parameter H. They have the following expressions for the model (25):

$$\text{Special volume} = T^{3/_{1+q}}, \tag{26}$$

$$\text{Scalar expansion } \theta = \frac{1}{3} u^i_{;i} = \frac{\alpha}{(1+q)T}, \tag{27}$$

$$\text{Shear scalar } \sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{\alpha^2}{6(1+q)^2 T^2}, \tag{28}$$

$$\text{Hubble's parameter } H = \frac{R_4}{R} = \left(\frac{\alpha}{1+q}\right) \frac{1}{T}. \tag{29}$$

It is observed that at initial epoch $(T = 0)$, the spatial volume will be zero. For large values of T, the spatial volume tends to infinity. The volume increases as the time increases, *i.e.*, the model is expanding. It is observed that when $T \to 0$ the expansion scalar θ tends to infinity. While for large value of T, the expansion scalar θ becomes zero. Also for T tends to zero the scalar field ϕ diverges.

It is observed that when $T \to 0$ the shear scalar σ tends to infinity. While for large value of T, the shear scalar σ becomes zero, *i.e.*, the shear dies out as T increases.

At the initial epoch $(T = 0)$, the Hubble parameter becomes infinite and for large value of T, H becomes zero. Also, the ratio $\sigma^2/\theta^2 \neq 0$ for large T, which imply that the model does not approach isotropy.

5 Conclusion

In this paper, we have obtained a Bianchi Type-III inflationary universe in the presence of mass less scalar field with flat potential in general relativity. It can be observed that for large T, the parameters ϕ, θ, σ, H vanish and diverge when $T \to 0$. It is observed that the model is free from singularities. The model is expanding and does not approach isotropy at late times. Our investigations for Bianchi type-III resembles to the investigations of D.R.K. Reddy *et al.* [12], Reddy *et al.* [16], Katore *et al.* [18] whereas our investigation differs to the results obtained by Reddy *et al*. [13], Katore *et al*. [20] for shear scalar, expansion scalar and Reddy [17] for isotropy. The inflationary model obtained here has considerable astrophysical significance. For example, classical scalar fields are essential in the study of the present day cosmological models. In view of the fact that there is an increasing interest, in recent years, in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary Universe and they help us to describe the early stages of evolution of the Universe.

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