Bianchi Type V Bulk Viscous Cosmological Models with Time Dependent Λ-Term

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Abstract: Spatially homogeneous and anisotropic Bianchi type V space-time with bulk viscous fluid source and time-dependent cosmological term are considered. Cosmological models have been obtained by assuming a variation law for the Hubble parameter which yields a constant value of deceleration parameter. Physical and kinematical parameters of the models are discussed. The models are found to be compatible with the results of cosmological observations. © Electronic Journal of Theoretical Physics. All rights reserved.

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1. Introduction

Observations during the last few years provided increasingly strong evidence that the universe at present is expanding with acceleration [3, 33, 36, 37]. In Einstein's theory of general relativity, to account for such an expansion, one needs to introduce some new energy density with a large negative pressure in the present universe, in addition to the usual relativistic or non-relativistic matter. This exotic matter causing cosmic acceleration is known as dark energy. The nature of dark energy is unknown and many radically different models related to this dark energy have been proposed [29, 38].

The simplest explanation of dark energy is provided by the cosmological constant Λ, but it needs to be severely fine-tuned due to the problem associated with its energy scale. The vacuum energy density observed today falls below the value of the vacuum energy density predicted by quantum field theory by many order of magnitude [54]. To explain the decay of the density, a number of dynamical models have been suggested

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in which cosmological term Λ varies with cosmic time t. These models give rise to an effective cosmological term which as long as the universe expands, decays from a huge value at initial times to the small value observed at present. Cosmological models with different decay laws for the variation of cosmological term were investigated during last two decades [1, 7, 9, 30, 45, 46, 49, 51].

In the investigation of most of the cosmological models, the source of the gravitational field is assumed to be a perfect fluid. But these models do not explain satisfactorily the early stages of evolution. Viscosity may be important in cosmology for a number of reasons. Dissipative mechanisms responsible for smoothing out initial isotropies and the observed high entropy per baryon in the present state of the universe can be explained by involving some kind of dissipative mechanisms e.g. bulk viscosity [52, 53]. Dissipative effects including bulk viscosity are supposed to play a very important role in the early evolution of the universe. During the neutrino decoupling stage, apart from streaming neutrinos moving with fundamental velocity, there is a part behaving like a viscous fluid co-moving with matter. Decoupling of radiation and matter during the recombination era is also expected to give rise to viscous effects. Moreover, a combination of cosmic fluid with bulk dissipative pressure can generate accelerated expansion [28]. Influence of viscosity on the nature of the initial singularity and on the formation of galaxies have been investigated [16, 28]. It has been shown that the coincidence problem can be solved by taking viscous effects into account [12, 13]. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabris et al. [17]. Santos et al. [39] have derived exact solution with bulk viscosity by considering the bulk viscous coefficient as power function of mass density. Johri and Sudarshan [21] have investigated the effect of bulk viscosity on the evolution of Friedmann models. Cosmological models with bulk viscosity have also been studied by Burd and Coley [8], Maartens [24], Pavon and Zimdahl [31], Pavon et al. [32].

In the construction of a cosmological model, assumption of homogeneity and isotropy of the universe are motivated by the cosmological principle and mathematical tractability of the resulting FRW models. However, the observed universe is obviously neither homogeneous nor isotropic. So these symmetries can only be approximate. There are theoretical arguments [11, 27] and recent experimental data regarding cosmic background radiation anisotropies which support the existence of an anisotropic phase that approaches an isotropic one [22]. These observations led us to consider more general anisotropic cosmologies, whilst retaining the assumption of (large scale) spatial homogeneity. Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than the FRW model play significant role in the description of early universe. Bianchi type V models being anisotropic generalization of open FRW models are interesting to study. These models are favoured by the available evidences for low density universes. Bianchi type V cosmological models have been investigated by Collins [15], Farnsworth [18], Maartens and Nel [25], Wainwright et al. [50]. Coley [14] has investigated Bianchi type V imperfect cosmological model. Bianchi type V bulk viscous cosmological models have also been studied by Bali and Singh [4], Pradhan and

Yadav [35], Singh and Chaubey [47].

In the present paper, we examine the possibility of the following three cases of phenomenological decay of Λ in the background of Bianchi type V space-time with bulk viscous fluid source:

Case 1: $\Lambda \sim H^2$

Case 2: $\Lambda \sim H$

Case 3: $\Lambda \sim \rho$.

Here H and ρ are, respectively, the Hubble parameter and matter energy density of the Bianchi type V space-time. The dynamical laws for decay of Λ have been widely studied by Arbab [1, 2], Carvalho et al. [9], Chen and Wu [10], Schutzhold [40, 41], Vishwakarma [48] to name only a few.

2. Metric and Field Equations

We consider the Bianchi type V space-time in orthogonal form represented by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\left\{B^2(t)dy^2 + C^2(t)dz^2\right\}. \tag{1}$$

We assume the cosmic matter consisting of bulk viscous fluid given by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p}g_{ij}, \tag{2}$$

with

$$\bar{p} = p - \zeta v^i_{;i}, \tag{3}$$

where ρ is the energy density of matter, p is the isotropic pressure, ζ is the coefficient of bulk viscosity and v_i, the four-velocity vector of the fluid satisfying $v_i v^i = -1$. The semicolon stands for covariant differentiation. On thermodynamical grounds bulk viscous coefficient ζ is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. But correction to the thermodynamical pressure p due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution does not change fundamentally by the inclusion of viscous term in the energy momentum tensor. The Einstein's field equations with time-varying cosmological term $\Lambda(t)$ are given by

$$R^j_i - \frac{1}{2}Rg^j_i = -8\pi GT^j_i + \Lambda g^j_i. \tag{4}$$

We use comoving system of reference so that $v_i = -\delta_{i4}$. The field equations (4) for the Bianchi type V space-time lead to

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{5}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \tag{6}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB}, \tag{7}$$

$$8\pi G\rho + \Lambda = -\frac{3\alpha^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}, \tag{8}$$

$$0 = \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}, \tag{9}$$

where an overhead dot$(\cdot)$ denotes ordinary differentiation with respect to cosmic time t. Covariant divergence of (4) gives

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{10}$$

We observe that for a constant Λ, equation (10) reduces to the equation of continuity. In view of energy conservation, equation (10) shows that a decaying vacuum term Λ transfers energy continuously to the matter component. The effective time-dependent cosmological term is regarded as second fluid component with energy density $\rho_v = \frac{\Lambda(t)}{8\pi G}$, where ρ_v is the vacuum energy density. We assume that the non-vacuum component of matter obeys the equation of state

$$p = \omega\rho, \quad \omega \in [0,1]. \tag{11}$$

To write metric functions explicitly, we introduce the average scale factor R of Bianchi type V space-time defined by $R^3 = ABC$. From equations (5)–(7) and (9), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R}, \tag{12}$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{k_1}{R^3}, \tag{13}$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{k_1}{R^3}, \tag{14}$$

where k_1 is constant of integration. Equations (12)–(14), on integration yield

$$A = m_1 R, \tag{15}$$

$$B = m_2 R \exp\left(-k_1 \int \frac{dt}{R^3}\right), \tag{16}$$

$$C = m_3 R \exp\left(k_1 \int \frac{dt}{R^3}\right), \tag{17}$$

where m_1, m_2 and m_3 are constants of integration satisfying $m_1 m_2 m_3 = 1$. Using suitable coordinate transformations, constants m_2 and m_3 can be absorbed. Therefore, m_2 and m_3 can be taken to be 1 implying $m_1 = m_2 = m_3 = 1$.

We introduce the dynamical scalars such as volume expansion θ and shear scalar σ as usual

$$\theta = v^i_{;i}, \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{18}$$

where σ_{ij} is the shear tensor defined by

$$\sigma_{ij} = \frac{1}{2}\left(v_{i;\alpha}h_j^\alpha + v_{j;\alpha}h_i^\alpha\right) - \frac{1}{3}\theta h_{ij}. \tag{19}$$

Here h_{ij} is the projection tensor given by

$$h_{ij} = g_{ij} + v_i v_j. \tag{20}$$

For the Bianchi type V metric, the dynamical scalars have the form

$$\theta = \frac{3\dot{R}}{R} \tag{21}$$

and

$$\sigma = \frac{k_1}{R^3}. \tag{22}$$

In analogy with FRW universe, we define generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R}, \tag{23}$$

$$q = -\frac{\ddot{R}}{RH^2}. \tag{24}$$

We can write equations (5)–(8) and (10) in terms of H, σ and q as

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{R^2} + H^2(2q-1) - \sigma^2, \tag{25}$$

$$8\pi G\rho + \Lambda = -\frac{3\alpha^2}{R^2} + 3H^2 - \sigma^2, \tag{26}$$

$$\dot{\rho} + 3(\rho + \bar{p})H + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{27}$$

From equation (26), we get

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{3\alpha^2}{R^2\theta^2} - \frac{\Lambda}{\theta^2}. \tag{28}$$

Therefore $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Thus a positive Λ restricts the upper limit of anisotropy whereas a negative Λ will increase the anisotropy. From the equations (25) and (26), we obtain

$$\frac{d\theta}{dt} = -4\pi G(\rho + 3p) - 2\sigma^2 - \frac{\theta^2}{3} + 12\pi G\zeta\theta + \Lambda, \tag{29}$$

which is the Raychaudhuri equation for the given distribution. We observe that for $\Lambda \leq 0$ and $\zeta = 0$, the universe will always be in decelerating phase provided the strong energy condition [19] holds. In this case, we have

$$\frac{d\theta}{dt} \leq -\frac{\theta^2}{3}, \tag{30}$$

which integrates to give

$$\frac{1}{\theta} \geq \frac{1}{\theta_0} + \frac{t}{3}, \tag{31}$$

where θ_0 is the initial value of θ. If $\theta_0 < 0$, θ will diverge ($\theta \to -\infty$) for $t < \frac{3}{|\theta_0|}$. From equation (29), one also concludes that the presence of viscosity and a positive Λ will slow down the rate of decrease of volume expansion. Again from equation (22), we get

$$\dot{\sigma} = -3\sigma H. \tag{32}$$

Thus, the energy density associated with the anisotropy σ decays rapidly in an evolving universe and it becomes negligible for infinitely large values of R. From equations (25) and (26), we obtain

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\pi G(\rho + 3p) - \frac{2}{3}\sigma^2 + 4\pi G\zeta\theta + \frac{\Lambda}{3}. \tag{33}$$

We observe that the positive cosmological term and bulk viscosity contribute positively in driving the acceleration of the universe. Also from equation (26), we get

$$\frac{3\dot{R}^2}{R^2} = \frac{3\alpha^2}{R^2} + \sigma^2 + 8\pi G\rho + \Lambda. \tag{34}$$

When $\Lambda \geq 0$, each term on the right hand side of (34) is non-negative. Thus $\dot{R}$ does not change sign and we get ever-expanding models. For $\Lambda < 0$, however, we can get universes that expand and then recontract. From equation (10), we obtain

$$R^{-3(\omega+1)}\frac{d}{dt}\left\{\rho R^{3(\omega+1)}\right\} = 9\zeta H^2 - \frac{\dot{\Lambda}}{8\pi G}. \tag{35}$$

Thus, decaying vacuum energy and viscosity of the fluid lead to matter creation.

Over the years, it has been difficult and fascinating problem for cosmologists to explain the expansion history of the universe. To describe the dynamics of the universe, Hubble parameter H and deceleration parameter q are important observational quantities. The present value H_0 of Hubble parameter sets the present time scale of the expansion while q_0, the present day value of deceleration parameter tells us that the expansion of the present universe is accelerating rather than going to decelerate as expected before the supernovae of Ia observations [3, 33, 36, 37]. From equations (12)–(14), we observe that scale factors are completely characterized by the Hubble parameter H. Therefore, we assume a relation between Hubble parameter H and average scale factor R given by

$$H = kR^{-m} \tag{36}$$

where $k > 0$ and $m \geq 0$ are constants. Such a relation has already been discussed by Berman [5], Berman and Gomide [6] in case of FRW models that yields a constant value of deceleration parameter. Models with constant deceleration parameter have also been studied by a number of authors [20, 26, 34, 42–44] for FRW and Bianchi cosmology. For the relation (36), deceleration parameter q comes out to be constant i.e.

$$q = m - 1. \tag{37}$$

The equation (37) shows that the universe is decelerating for $m > 1$ and it represents an accelerating universe for $m < 1$. When $m = 1$, we obtain $H = \frac{1}{T}$ and $q = 0$. Therefore galaxies move with constant speed and the model represents anisotropic Milne universe [23] for $m = 1$. For $m = 0$, we get $H = k$ and $q = -1$. Thus Hubble parameter H, being constant in time, equals to its present value H_0 and the model describes accelerated phase of the universe.

Equation (36) integrates to give

$$R = (mkt + t_1)^{\frac{1}{m}} \quad \text{for } m \neq 0 \tag{38}$$

and

$$R = \exp\{k(t - t_0)\} \quad \text{for } m = 0, \tag{39}$$

where t_1 and t_0 are constants of integration.

Equation (38) along with (15)–(17) gives

$$A = (mkt + t_1)^{\frac{1}{m}}, \tag{40}$$

$$B = (mkt + t_1)^{\frac{1}{m}} \exp\left\{\frac{-k_1(mkt + t_1)^{\frac{m-3}{m}}}{k(m-3)}\right\}, \tag{41}$$

$$C = (mkt + t_1)^{\frac{1}{m}} \exp\left\{\frac{k_1(mkt + t_1)^{\frac{m-3}{m}}}{k(m-3)}\right\}. \tag{42}$$

For this solution, the metric (1) assumes the following form after suitable transformation of coordinates

$$ds^2 = -dT^2 + (mkT)^{\frac{2}{m}} dX^2$$
$$+ (mkT)^{\frac{2}{m}} \exp\left\{2\alpha X - \frac{2k_1(mkT)^{\frac{m-3}{m}}}{k(m-3)}\right\} dY^2$$
$$+ (mkT)^{\frac{2}{m}} \exp\left\{2\alpha X + \frac{2k_1(mkT)^{\frac{m-3}{m}}}{k(m-3)}\right\} dZ^2. \tag{43}$$

Equations (15)–(17) with the use of equation (39) yield

$$A = \exp\{k(t - t_0)\}, \tag{44}$$

$$B = \exp\left\{k(t - t_0) + \frac{k_1}{3k}e^{-3k(t-t_0)}\right\}, \tag{45}$$

$$C = \exp\left\{k(t - t_0) - \frac{k_1}{3k}e^{-3k(t-t_0)}\right\}. \tag{46}$$

The line-element (1) for this solution can be written as

$$ds^2 = -dT^2 + e^{2kT} dX^2$$
$$+ \exp\left(2kT + 2\alpha X + \frac{2k_1}{3k}e^{-3kT}\right) dY^2$$
$$+ \exp\left(2kT + 2\alpha X - \frac{2k_1}{3k}e^{-3kT}\right) dZ^2. \tag{47}$$

3.   Discussion

We now discuss the models resulting from different dynamical laws for the decay of Λ.

3.1

For the model (43), average scale factor R is given by

$$R = (mkT)^{\frac{1}{m}}. \tag{48}$$

Volume expansion θ, Hubble parameter H and shear scalar σ for this model are:

$$\theta = 3H = \frac{3}{mT}, \tag{49}$$

$$\sigma = k_1(mkT)^{\frac{-3}{m}}. \tag{50}$$

We observe that the model is not tenable for $m = 0$ and $m = 3$. For $m < 3$, $\frac{\sigma}{\theta} \to 0$ as $T \to \infty$. Therefore, the model approaches isotropy asymptotically.

3.1.1   Case 1:

We consider

$$\Lambda = 3\beta H^2, \tag{51}$$

where β is a constant. From equations (11), (25) and (26), we obtain

$$8\pi G\rho = \frac{3 - 3\beta}{m^2 T^2} - \frac{3\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{k_1{}^2}{(mkT)^{\frac{6}{m}}}, \tag{52}$$

$$24\pi G\zeta = \frac{3(1+\omega)(1-\beta) - 2m}{mT} - \frac{(1+3\omega)m\alpha^2 T}{(mkT)^{\frac{2}{m}}}$$

$$+ \frac{(1-\omega)k_1{}^2 mT}{(mkT)^{\frac{6}{m}}}, \tag{53}$$

$$\Lambda = \frac{3\beta}{m^2 T^2}. \tag{54}$$

We observe that the model has singularity at $T = 0$. It starts with a big bang from its singular state at $T = 0$ and continues to expand till $T = \infty$. At $T = 0$, ρ, p, Λ, ζ are all infinite and they become negligible for large values of T. Therefore, for large times, the model represents a non-rotating, shearing and expanding universe having big bang start and approaches isotropy asymptotically.

3.1.2   Case 2:

We assume

$$\Lambda = aH, \tag{55}$$

where a is a positive constant. For this choice, we obtain

$$8\pi G\rho = \frac{3}{m^2T^2} - \frac{3\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{{k_1}^2}{(mkT)^{\frac{6}{m}}} - \frac{a}{mT}, \tag{56}$$

$$24\pi G\zeta = \frac{3(1+\omega) - 2m}{mT} - \frac{(1+3\omega)m\alpha^2T}{(mkT)^{\frac{2}{m}}}$$

$$+ \frac{(1-\omega)m{k_1}^2T}{(mkT)^{\frac{6}{m}}} - (1+\omega)a, \tag{57}$$

$$\Lambda = \frac{a}{mT}. \tag{58}$$

This model also has singularity at $T = 0$. It evolves from its singular state at $T = 0$ with ρ, p, Λ, ζ all diverging and expansion in the model becomes zero for $T \to \infty$. We observe that the vacuum energy in this case decays slowly than the case 1.

3.1.3   Case 3:

We now consider

$$\Lambda = 8\pi G\gamma\rho, \tag{59}$$

where γ is a constant. In this case, we obtain

$$8\pi G(1+\gamma)\rho = \frac{3}{m^2T^2} - \frac{3\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{{k_1}^2}{(mkT)^{\frac{6}{m}}}, \tag{60}$$

$$24\pi G(1+\gamma)\zeta = \frac{3(1+\omega) - 2m(1+\gamma)}{mT}$$

$$- \frac{(1+3\omega - 2\gamma)m\alpha^2T}{(mkT)^{\frac{2}{m}}}$$

$$+ \frac{(1-\omega + 2\gamma)m{k_1}^2T}{(mkT)^{\frac{6}{m}}}, \tag{61}$$

$$\left(1 + \frac{1}{\gamma}\right)\Lambda = \frac{3}{m^2T^2} - \frac{3\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{{k_1}^2}{(mkT)^{\frac{6}{m}}}. \tag{62}$$

This model also starts expanding with a big bang at $T = 0$ with ρ, p, Λ, ζ all infinite and expansion in the model ceases at $T = \infty$. The bulk viscosity coefficient, being infinitely large at the initial singularity decreases with time. Matter density ρ and cosmological term Λ also decrease in the course of expansion to become zero for large times.

3.2

For the model (47), average scale factor R, expansion scalar θ, Hubble parameter H, shear scalar σ and deceleration parameter q are given by

$$R = e^{kT}, \tag{63}$$

$$\theta = 3H = 3k, \tag{64}$$

$$\sigma = k_1 e^{-3kT}, \tag{65}$$

$$q = -1. \tag{66}$$

The energy density ρ and bulk viscosity ζ have the expressions:

$$8\pi G\rho = 3k^2 - 3\alpha^2 e^{-2kT} - k_1{}^2 e^{-6kT} - \Lambda \tag{67}$$
$$24\pi Gk\zeta = 3(\omega + 1)k^2 - (3\omega + 1)\alpha^2 e^{-2kT}$$
$$+(1 - \omega)k_1{}^2 e^{-6kT}(\omega + 1)\Lambda. \tag{68}$$

We observe that $\Lambda \sim H^2$ and $\Lambda \sim H$, give Λ to be constant because Hubble parameter H is constant. Therefore, in these cases we obtain the model similar to the model (40) considered by Singh and Baghel [44].

For the case $\Lambda = 8\pi G\gamma\rho$, from equations (67) and (68), we obtain

$$8\pi G(1 + \gamma)\rho = 3k^2 - 3\alpha^2 e^{-2kT} - k_1{}^2 e^{-6kT}, \tag{69}$$

$$24\pi G(1 + \gamma)k\zeta = 3(1 + \omega)k^2 - (1 + 3\omega - 2\gamma)\alpha^2 e^{-2kT}$$
$$+(1 - \omega + 2\gamma)k_1{}^2 e^{-6kT}, \tag{70}$$

$$\left(1 + \frac{1}{\gamma}\right)\Lambda = 3k^2 - 3\alpha^2 e^{-2kT} - k_1{}^2 e^{-6kT}. \tag{71}$$

The model has no initial singularity. Expansion in the model starts at $T = 0$ with ρ, θ, σ, Λ and ζ all finite. The expansion scalar θ is constant throughout the expansion. Therefore the model represents uniform expansion. For large values of T, matter density ρ, bulk viscosity ζ and cosmological term Λ remain non-zero constants and anisotropy σ/θ becomes zero. Therefore the model approaches isotropy. For this model, deceleration parameter $q = -1$. Therefore, the model represents an accelerating universe. Thus, the model behaves in accordance with cosmological observations which indicate that the universe has entered a phase of accelerating expansion. This model represents a non-singular, shearing and accelerating universe which becomes isotropic for large times.

Conclusion

Anisotropic Bianchi type V cosmological models with bulk viscous fluid and time varying cosmological term are investigated by assuming a variation law for the Hubble's parameter that yields a constant value of deceleration parameter. Universe models for $m \neq 0$ and $m = 0$ have been derived. Three different decay laws for the cosmological term have been discussed in the context of models obtained. We observe that for $m \neq 0$, the model starts with a big bang at $T = 0$ where cosmological parameters diverge. It becomes isotropic for large values of T, provided $m < 3$. The cosmological term Λ being infinite at the initial singularity becomes negligible for large times.

When $m = 0$, we obtain a non-singular model representing accelerated phase of the universe. It evolves with finite values of kinematical parameters and expands uniformly. The model approaches isotropy for large values of T. In this model, matter density ρ, bulk viscosity ζ and cosmological term Λ remain non-zero for $T \to \infty$.

From equation (37), one concludes that for $m > 1$, the model represents a decelerating universe and for $0 \leq m < 1$, it gives rise to an accelerating universe. When $m = 1$, we obtain $H = \frac{1}{T}$ and $q = 0$ so that every galaxy moves with constant speed. Therefore, for $m = 1$, we recovers an anisotropic Milne model [23].

Acknowledgments

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