The Economic Journal, **103** (*July*) 908–915. Published by Blackwell Publishers, 108 Cowley Road, Oxford OX4 1JF, UK and 238 Main Street, Cambridge, MA 02142, USA.

BIAS IN ESTIMATING THE ALMOST IDEAL DEMAND SYSTEM WITH THE STONE INDEX APPROXIMATION*

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The Almost Ideal (AI) demand system is often estimated with the Stone index approximation as suggested by Deaton and Muellbauer (1980). So far the evidence about the effect of this approximation on the parameter estimates of the AI model is mainly empirical and inconclusive. Deaton and Muellbauer (1980) estimate an eight-commodity system using aggregate annual UK data for the period 1954–74 and report that the differences between the parameters obtained from the linearly approximated and exact budget share equations are empirically unimportant. Browning and Meghir (1991) estimate a seven-commodity system using microdata drawn from the UK *Family Expenditure Survey* (FES) over the period 1979–84 and report that the parameter differences between the approximated and exact models are important. Neither of these two studies nor any other study that I am aware of provides analytical or numerical comparison of results obtained with and without the Stone index approximation.

Exactly how the parameters of the AI demand system are affected by the Stone index approximation is important because the popularity of this demand system is largely due to the fact that, while based on a flexible cost function, its shares can be estimated as linear equations. I examine this question by comparing analytical expressions and empirical findings obtained from the AI model with and without the Stone index approximation. The results suggest that this approximation can result in biased parameter estimates, particularly when the AI model is applied to microdata. This is important as it is at the micro level that the linear approximation is most useful because it facilitates the cure of measurement errors through simple IV methods and eases computational difficulties arising from the large number of variables and sample size. The use of microdata is encouraged by results showing that the restrictions required for aggregate demand analysis (Muellbauer, 1975) are not supported by empirical evidence, see for example Blundell *et al.* (1989) and Nicol (1989). The paper also shows that the price parameter bias can be largely corrected through a simple re-parameterisation. The empirical analysis is based on UK individual household and aggregate data from the FES, 1970–86.

Section I examines the bias caused by the Stone index approximation and

* I would like to thank Vanessa Fry, Costas Meghir and an anonymous referee for comments and Richard Blundell and Guglielmo Weber for helpful discussions. I also thank the Department of Employment for providing the *Family Expenditure Survey* data used in this study. I am solely responsible for the analysis and interpretation of the data.

proposes a solution. Section II uses micro and macro estimates to provide empirical illustration. Section III concludes the paper.

I. THE PARAMETER BIAS FROM USING THE STONE INDEX APPROXIMATION

Consider the case where the AI model is applied to microdata, and write the share of good i in the budget of household h in period t as

$$w_{iht} = \alpha_{ih} + \Sigma_j \gamma_{ij} \ln p_{jt} + \beta_{ih} [\ln y_{ht} - \ln a_h(p_t)], \tag{1}$$

where p_{it} is the market price of good i in period t and y_{ht} the expenditure of household h in the same period; also

$$\ln a_h(p_t) = \alpha_{oh} + \Sigma_k \alpha_{kh} \ln p_{kt} + 0{\cdot}5 \Sigma_k \Sigma_j \gamma_{kj} \ln p_{jt} \ln p_{kt} \tag{2}$$

and α_{oh}, α_{ih} and β_{ih} are functions of the observed characteristics of household h. The restrictions $\Sigma_i \alpha_{ih} = 1$, $\Sigma_i \gamma_{ij} = 0$ and $\Sigma_i \beta_{ih} = 0$ are required for adding up, $\Sigma_j \gamma_{ij} = 0$ for price homogeneity and $\gamma_{ij} = \gamma_{ji}$ for symmetry.

Following the advice of Deaton and Muellbauer (1980), the demand system (1) is often estimated linearly by replacing the 'exact' index (2) with the Stone index $\ln P_{ht} = \Sigma_k w_{kht} \ln p_{kt}$. Adding and subtracting this index to (2) the budget shares can be written

$$w_{iht} = \alpha'_{ih} + \Sigma_j \gamma_{ij} \ln p_{jt} + \beta_{ih} (\ln y_{ht} - \ln P_{ht}) + \beta_{ih} B_{ht}, \tag{3}$$

where $B_{ht} = \Sigma_j (w_{jht} - \alpha_{jh} - 0{\cdot}5 \Sigma_k \gamma_{kj} \ln p_{kt}) \ln p_{jt}$ and $\alpha'_{ih} = \alpha_{ih} - \beta_{ih} \alpha_{oh}$. Therefore the Stone index approximation can be seen as an omitted variables problem, equivalent to estimating (3) without the 'price index' B_{ht}. As such it can result in biased parameters depending on the significance of β_{ih} and the covariance between B_{ht} and other variables in the budget share equations.

In the context of micro demand analysis B_{ht} varies directly with prices and, through α_{jh} and w_{jht}, with household characteristics and log expenditure. Thus, all the parameters in the budget shares are subject to potential omitted variable bias. In the case of the price parameter, it is possible to assess the direction of this bias by re-writing (3) as

$$w_{iht} = \alpha'_{ih} + \Sigma_j [\gamma_{ij} + (w_{jht} - \alpha_{jh}) \beta_{ih} - 0{\cdot}5 \beta_{ih} \Sigma_k \gamma_{kj} \ln p_{kt}] \ln p_{jt} + \beta_{ih} \ln (y_{ht}/P_{ht}). \tag{4}$$

Furthermore: (i) at reference prices $(w_{jht} - \alpha_{jh})/\beta_{jh} = (\ln y_{ht} - \alpha_{oh})$ thus β_{jh} and $(w_{jht} - \alpha_{jh})$ have the same sign, given that α_{oh} is the minimum $\ln y_{ht}$; and (ii) the value of $0{\cdot}5 \Sigma_i \gamma_{ij} \ln p_{it}$ (reflecting substitution effects) is likely to be small, given that $\Sigma_i \gamma_{ij} = 0$.[1] Thus, subject to $0{\cdot}5 \Sigma_i \gamma_{ij} \ln p_{it} \simeq 0$, the bias will be positive for own price parameters. Also, because $(w_{jht} - \alpha_{jh})$ has the same sign as β_{jh}, the cross price effects will be positively biased when β_{ih} and β_{jh} have the same sign (i.e. either both luxuries or both necessities) and negatively biased when β_{ih} and β_{jh} have the opposite sign.

[1] The α_{jh} parameters in (4) correspond to the intercepts of the budget share equations when $\alpha_{oh} = 0$, not to the estimated intercepts of the same equation, α'_{ih}, which are affected by assumptions about the size of α_{oh}. In empirical applications α_{oh} is fixed to a level somewhat below the logarithm of the minimum expenditure in the sample (Deaton and Muellbauer, 1980). In this case $(w_{jht} - a_{jh}) = (w_{jht} - \alpha'_{jh} + \beta_{jh} \bar{\alpha}_{oh})$, where $\bar{\alpha}_{oh}$ is the chosen α_{oh}.

The price parameter bias caused by the Stone index approximation may be more serious in micro rather than aggregate demand analysis because composition has an important effect on the consumption pattern of the household and this effect is correlated with that of the level of consumer budget: families with less and younger members allocate a relatively large proportion of their budget to necessities so that α_{ih} tends to be positive when β_{ih} is negative and vice versa (Pashardes, 1991). The empirical evidence in the next section appears to support this conjecture which is also consistent with the difference between the results obtained by Deaton and Muellbauer (1980) on macrodata and those obtained by Browning and Meghir (1991) on microdata.

An interesting question is whether the Stone index approximation can also affect the adding-up, homogeneity and symmetry restrictions. With regard to adding up and homogeneity, the answer to this question is negative: $\Sigma_i \beta_{ih} = 0$ preserves adding-up while $\Sigma_j(w_{jht} - \alpha_{jh}) = 0$ and $\Sigma_j \gamma_{ij} = 0$ preserve homogeneity. Symmetry, however, is not preserved: to the extent that they absorb the omitted variable bias, as in (4), the logarithmic price effects become

$$\tilde{\gamma}_{ijht} = \gamma_{ij} + \beta_{ih}\beta_{jh}[\ln y_{ht} - \ln a_h(p_t)] + 0.5\beta_{ih}\Sigma_j \gamma_{ij} \ln p_{jt}. \tag{5}$$

While the first two terms on the right hand side are symmetric, the last one is not.[2]

Furthermore, when $0.5\Sigma_j \gamma_{ij} \ln p_{jt} \simeq 0$ then $\tilde{\gamma}_{ijht} \simeq \gamma_{ij} + (w_{jht} - \alpha_{jh})\beta_{ih}$ so that the price parameter bias caused by the Stone index approximation can be largely corrected through a simple re-parameterisation. For instance, for the (reference) household with $\alpha_{ih} = \alpha_i$ and $\beta_{ih} = \beta_i$ the uncompensated price elasticities of the AI model at base prices are given by

$$e_{ij} = (1/w_i)(\gamma_{ij} - \beta_i \alpha_j) - \delta_{ij}, \tag{6}$$

where δ_{ij} is the Kronecker delta. The proposed re-parameterisation implies that when the model is estimated with the Stone index approximation, the same elasticities are

$$\tilde{e}_{ij} = (1/w_i)(\tilde{\gamma}_{ij} - \beta_i w_j) - \delta_{ij}. \tag{7}$$

How close $\tilde{e}_{ij}$ can be to e_{ij} is illustrated empirically in the next section.

II. EMPIRICAL RESULTS

This section compares parameter estimates of the AI model obtained with and without the Stone index approximation. A demand system of seven non-durables (food, alcohol, fuel-light, clothing, transport, other goods and services) is estimated in each case using individual household and aggregate data from the *Family Expenditure Survey* (FES), 1970–86. The prices are taken from the Retail Price Index published in the *Employment Gazette*.

For the micro model I have adopted the empirical specification

$$w_{iht} = \alpha_i(t) + \Sigma_k \delta_{ik} z_{kht} + \Sigma_j \gamma_{ij} \ln p_{jt} + \Sigma_s \beta_{is} D_{sht}[\ln y_{ht} - \ln a_h(p_t)], \tag{8}$$

where $\ln a_h(p_t) = \alpha_0 + \Sigma_i[\alpha_i(t)_h + \Sigma_k \delta_{ik} z_{kht}] \ln p_{it} + 0.5\Sigma_i \Sigma_j \gamma_{ij} \ln p_{jt} \ln p_{it}$ and

[2] Equation (5) suggests that the Stone index approximation can also cause price parameter instability.

Table 1

Parameter Estimates of the Almost Ideal Demand System

	Micro model				Macro model			
	Stone index		Exact index		Stone index		Exact index	
	Params	t-st	Params	t-st	Params	t-st	Params	t-st
Food								
Intercept	0·5201	63·3	0·5318	62·2	0·6417	9·4	0·6458	10·0
Pfood	0·1016	8·7	−0·0576	−4·0	0·1175	5·1	0·0788	2·8
Palcl	0·0092	1·2	0·0630	7·4	0·0013	0·1	−0·0013	−0·1
Pfuel	−0·0269	−4·0	−0·0946	−12·4	−0·0096	−0·8	−0·0336	−2·4
Pclth	−0·0107	−0·9	0·0343	2·5	−0·0397	−2·3	−0·0279	−1·4
Ptrpt	−0·0210	−1·5	0·0558	3·7	−0·0117	−0·5	−0·0026	−0·1
Pothr	−0·0534	−5·7	−0·0566	−5·3	−0·0576	−3·6	−0·0509	−2·7
Pserv	0·0012	0·1	0·0557	5·4	−0·0000	−0·0	0·0377	1·7
Log*Ex*	−0·1458	−19·2	−0·1385	−19·7	−0·0715	−4·5	−0·0724	−4·8
Alcohol								
Intercept	0·0053	0·8	0·0052	0·7	0·0866	2·4	0·0884	2·7
Pfood	0·0092	1·2	0·0630	7·4	0·0013	0·1	−0·0013	−0·1
Palcl	−0·0572	−6·6	−0·0770	−7·7	−0·0272	−2·0	−0·0268	−2·0
Pfuel	0·0540	8·5	0·0738	10·9	0·0294	2·9	0·0271	2·5
Pclth	0·0119	1·3	−0·0019	−0·2	0·0142	1·2	0·0152	1·3
Ptrpt	0·0121	1·0	−0·0172	−1·4	0·0244	1·4	0·0253	1·5
Pothr	−0·0254	−2·6	−0·0211	−2·1	−0·0306	−1·8	−0·0305	−1·8
Pserv	−0·0047	−0·5	−0·0194	−2·2	−0·0114	−0·9	−0·0089	−0·6
Log*Ex*	0·0509	8·5	0·0435	8·1	−0·0034	−0·4	−0·0038	−0·5
Fuel								
Intercept	0·1307	26·8	0·1388	26·9	0·2275	5·7	0·2383	6·2
Pfood	−0·0269	−4·0	−0·0946	−12·4	−0·0096	−0·8	−0·0336	−2·4
Palcl	0·0540	8·5	0·0738	10·9	0·0294	2·9	0·0271	2·5
Pfuel	0·0321	4·1	0·0091	1·1	0·0241	1·8	0·0095	0·6
Pclth	−0·0035	−0·4	0·0172	2·0	−0·0164	−1·3	−0·0088	−0·7
Ptrpt	−0·0524	−5·2	−0·0188	−1·8	−0·0449	−2·7	−0·0401	−2·3
Pothr	0·0218	2·6	0·0204	2·4	0·0235	1·6	0·0278	1·8
Pserv	−0·0250	−3·6	−0·0071	−0·9	−0·0060	−0·5	0·0182	1·3
Log*Ex*	−0·0665	−15·5	−0·0675	−16·4	−0·0398	−4·3	−0·0423	−4·8
Clothing								
Intercept	0·0102	1·1	−0·0026	−0·3	0·0790	1·5	0·0727	1·5
Pfood	−0·0107	−0·9	0·0343	2·5	−0·0397	−2·3	−0·0279	−1·4
Palcl	0·0119	1·3	−0·0019	−0·2	0·0142	1·2	0·0152	1·3
Pfuel	−0·0035	−0·4	0·0172	2·0	−0·0164	−1·3	−0·0088	−0·7
Pclth	0·0229	1·4	0·0105	0·6	0·0785	4·0	0·0751	3·6
Ptrpt	−0·0284	−1·8	−0·0525	−3·3	−0·0551	−2·7	−0·0573	−2·7
Pothr	0·0427	3·5	0·0422	3·4	0·0541	3·0	0·0515	2·8
Pserv	−0·0349	−3·3	−0·0499	−4·6	−0·0356	−2·3	−0·0477	−2·8
Log*Ex*	0·0371	4·6	0·0446	6·1	0·0193	1·6	0·0208	1·9
Transport								
Intercept	0·1626	12·4	0·1595	12·1	0·0913	1·0	0·0861	1·1
Pfood	−0·0210	−1·5	0·0558	3·7	−0·0117	−0·5	−0·0026	−0·1
Palcl	0·0121	1·0	−0·0172	−1·4	0·0244	1·4	0·0253	1·5
Pfuel	−0·0524	−5·2	−0·0188	−1·8	−0·0449	−2·7	−0·0401	−2·3
Pclth	−0·0284	−1·8	−0·0525	−3·3	−0·0551	−2·7	−0·0573	−2·7
Ptrpt	0·0658	2·1	0·0266	0·9	0·0748	1·5	0·0719	1·5
Pothr	−0·0071	−0·4	−0·0026	−0·2	−0·0080	−0·3	−0·0086	−0·3
Pserv	0·0310	2·0	0·0087	0·6	0·0207	0·7	0·0116	0·4
Log*Ex*	0·0935	7·6	0·0925	9·0	0·0187	0·9	0·0199	1·0

Table 1 (*cont.*)

	Micro model				Macro model			
	Stone index		Exact index		Stone index		Exact index	
	Params	t-st	Params	t-st	Params	t-st	Params	T-st
Other								
Intercept	0·0962	12·9	0·0931	12·1	0·1085	2·2	0·0825	1·8
Pfood	−0·0534	−5·7	−0·0566	−5·3	−0·0576	−3·5	−0·0509	−2·7
Palcl	−0·0254	−2·6	−0·0211	−2·1	−0·0306	−1·8	−0·0305	−1·8
Pfuel	0·0218	2·6	0·0204	2·4	0·0235	1·6	0·0278	1·8
Pclth	0·0427	3·5	0·0422	3·4	0·0541	3·0	0·0515	2·8
Ptrpt	−0·0071	−0·4	−0·0026	−0·2	−0·0080	−0·3	−0·0086	−0·3
Pothr	0·0182	1·0	0·0165	0·9	0·0248	0·8	0·0230	0·7
Pserv	0·0032	0·3	0·0011	0·1	−0·0063	−0·3	−0·0124	−0·6
Log*Ex*	−0·0126	−2·2	−0·0079	−1·5	0·0015	0·1	0·0076	0·7
Services								
Intercept	0·0744	8·5	0·0740	8·3	−0·2344	−2·9	−0·2139	−3·1
Pfood	0·0011	0·1	0·0556	5·4	−0·0000	−0·0	0·0377	1·7
Palcl	−0·0048	−0·6	−0·0194	−2·2	−0·0114	−0·9	−0·0089	−0·6
Pfuel	−0·0250	−3·6	−0·0071	−1·0	−0·0060	−0·5	0·0182	1·2
Pclth	−0·0348	−3·3	−0·0498	−4·6	−0·0356	−2·3	−0·0477	−2·8
Ptrpt	0·0308	2·0	0·0087	0·6	0·0207	0·7	0·0116	0·4
Pothr	0·0034	0·3	0·0012	0·1	−0·0063	−0·3	−0·0124	−0·6
Pserv	0·0292	2·1	0·0109	0·8	0·0392	1·3	0·0019	0·1
Log*Ex*	0·0434	5·5	0·0333	4·8	0·0751	3·9	0·0701	4·3
System diagnostics								
R^2	0·154		0·153			0·921		0·921
MSE	0·905		0·885			0·902		0·902
Homogeneity								
F(7, 441E3)	0·075		0·043		F(7, 321)	0·037		0·113
Symmetry								
F(15, 441E3)	5·20		4·87		F(15, 321)	1·60		1·48
No. of obs.	63,019		62,019			57		57

* Acronyms: *Pfood*: log price of food, *Palcl*: log price of alcohol, *Pfuel*: log price of fuel, *Pclth*: log price of clothing, *Ptrpt*: log price of transport, *Pothr*: log price other goods, *Pserv*: log price of services and Log*Ex*: log real expenditure.

$\alpha_i(t)$ is a linear function of seasonal and monthly dummies and trend. The variable z_{kht} denotes the k characteristics of household h in period t and D_{sht} denotes dummy variables allowing the β_i parameters to vary with each household characteristic s in period t.

The system of equations (8) is first estimated by replacing $\ln a_h(p_t)$ by the Stone index and applying 3SLS to the resulting linear system. As in Blundell *et al.* (1989), real log expenditure is instrumented to avoid measurement error bias from infrequency of purchase (Keen, 1986) and the parameters β_i are allowed to vary with the presence of children in the family, the occupation of head and seasonal dummies. The estimated intercepts, price and expenditure parameters and t ratios obtained from this estimation are reported in columns 1 and 2 of Table 1, under the heading 'Stone index'. (The parameter estimates of the large number of household characteristics included in the micro models

Table 2

*Elasticity Comparisons with the Parameter Estimates of the Micro Model**

(Numbers in parentheses are asymptotic standard errors)

	Uncompensated price elasticities							Budget elast.
	Food	Alchl	Fuel	Cloth	Trspt	Other	Servs	
				(*a*) Stone index estimates				
Food	−0·173	−0·076	0·109	−0·082	−0·164	−0·086	−0·081	0·554
	(0·05)	(0·03)	(0·02)	(0·05)	(0·09)	(0·05)	(0·04)	(0·02)
Alchl	−0·822	−1·670	0·469	0·270	−0·368	−0·504	0·083	1·806
	(0·24)	(0·15)	(0·13)	(0·25)	(0·45)	(0·27)	(0·21)	(0·07)
Servs	0·536	0·472	−0·321	−0·124	−0·782	0·367	−0·431	0·284
	(0·19)	(0·12)	(0·10)	(0·20)	(0·35)	(0·21)	(0·17)	(0·05)
Fuel	−0·567	0·207	−0·198	−0·728	−0·193	0·364	−0·272	1·387
	(0·16)	(0·10)	(0·08)	(0·16)	(0·30)	(0·18)	(0·14)	(0·07)
Cloth	−0·614	0·164	−0·468	−0·112	−0·536	−0·104	0·253	1·416
	(0·09)	(0·06)	(0·05)	(0·09)	(0·17)	(0·10)	(0·08)	(0·05)
Trspt	−0·408	−0·262	0·243	0·398	−0·088	−0·813	0·014	0·916
	(0·15)	(0·09)	(0·08)	(0·16)	(0·29)	(0·17)	(0·14)	(0·95)
Other	−0·557	0·069	−0·412	−0·241	0·371	−0·044	−0·662	1·476
	(0·13)	(0·08)	(0·07)	(0·14)	(0·25)	(0·15)	(0·12)	(0·05)
				(*b*) Exact index estimates				
Food	−0·550	0·065	−0·048	−0·005	0·025	−0·110	0·061	0·561
	(0·05)	(0·03)	(0·02)	(0·05)	(0·09)	(0·05)	(0·04)	(0·02)
Alchl	−0·065	−1·957	0·732	0·140	−0·035	−0·404	−0·127	1·716
	(0·24)	(0·15)	(0·13)	(0·25)	(0·45)	(0·27)	(0·21)	(0·07)
Servs	−0·096	0·674	−0·519	0·031	−0·446	0·330	−0·249	0·276
	(0·19)	(0·12)	(0·10)	(0·20)	(0·35)	(0·21)	(0·17)	(0·05)
Fuel	−0·333	0·109	−0·076	−0·782	−0·371	0·359	−0·385	1·479
	(0·16)	(0·10)	(0·08)	(0·16)	(0·30)	(0·18)	(0·14)	(0·07)
Cloth	−0·237	0·009	−0·307	−0·201	−0·730	−0·063	0·133	1·391
	(0·09)	(0·06)	(0·05)	(0·09)	(0·17)	(0·10)	(0·08)	(0·05)
Trspt	−0·504	−0·208	0·208	0·397	0·033	−0·840	0·006	0·975
	(0·15)	(0·09)	(0·08)	(0·16)	(0·29)	(0·17)	(0·14)	(0·05)
Other	−0·118	0·053	−0·277	−0·323	0·204	−0·041	−0·812	1·419
	(0·13)	(0·08)	(0·07)	(0·14)	(0·25)	(0·15)	(0·12)	(0·05)

* At average budget shares and reference prices.

are available from the author on request.) In columns 3 and 4 of Table 1, under the heading 'Exact Index', are the parameter estimates of the exact model.[3] The symmetry and homogeneity restrictions are imposed on both models for comparability.

The last four columns of Table 1 report results obtained from the same data but averaged to the quarterly level. Simple quarterly averages are constructed here because the objective is to replicate published (not consistently averaged) macrodata. The budget share equations are estimated by Seemingly Unrelated

[3] The exact model is estimated using an iterative procedure as follows. Multiply (1) by $\ln p_{it}$, sum across i, add α_0 to both sides and rearrange terms to obtain

$$\ln a_h(p_t) = (\alpha_{oh} + \ln P_{ht} - \ln y_{ht} \Sigma_i \beta_{ih} \ln p_{it} - 0{\cdot}5 \Sigma_i \Sigma_j \gamma_{ij} \ln p_{jt} \ln p_{it}) / (1 - \Sigma_i \beta_{ih} \ln p_{it}).$$

Using the parameters estimated with the Stone index approximation, obtain a first round estimate of $\ln a_h(p_t)$ to begin the iterations. Convergence (warranted by the fact that at each step we minimise a quadratic equation) has been achieved in 4–5 iterations.

Regression (SUR) because aggregate expenditure can be assumed to be exogenous (Blundell *et al.* 1989).

The absolute value of the substitution term $0.5\Sigma_j \gamma_{ij} \ln p_{jt}$ corresponding to the parameter estimates in Table 1 is small, on average below 0.0004 for all models. Consequently the effect of the Stone index approximation is in line with the discussion in Section I: it biases the price parameters according to the sign and size of the parameter product $\beta_i \beta_j$ (see equation (5)) and is generally more serious in the case of micro demand analysis, especially in equations where the effects of the (demographic) characteristics of the household are highly correlated with the expenditure effects (e.g. food).

The system diagnostics reported in Table 1 show that homogeneity is accepted in all cases but symmetry is only accepted for the macro model.[4] They also suggest that the impact of the Stone index approximation is too small to affect the outcome of the symmetry and homogeneity tests critically. This is consistent with the discussion in Section 1 and the finding here that the term $0.5\Sigma_j \gamma_{ij} \ln p_{jt}$ is very small.

Table 2 shows how misleading the price elasticities can be when estimates based on the Stone index approximation are used in formula (6) as if they were not biased. The elasticities in this table correspond to the parameter estimates of the micro model (where the approximation bias is found to be more serious). In section (*a*) they are computed using the parameter estimates obtained with the Stone index (column 1 in Table 2) and in section (*b*) with the parameter estimates obtained with the exact index (column 3 in Table 2). The results support the argument that the Stone index approximation leads to under-

Table 3

*Elasticities when the Stone Index Estimates are Used in the Adjusted Formula**

(Numbers in parentheses are asymptotic standard errors)

	Uncompensated price elasticities							Budget elast.
	Food	Alchl	Fuel	Cloth	Trspt	Other	Servs	
Food	−0.551	0.057	−0.040	0.014	0.019	−0.108	0.056	0.554
	(0.03)	(0.03)	(0.02)	(0.05)	(0.09)	(0.05)	(0.04)	(0.02)
Alchl	−0.140	−1.909	0.739	0.097	0.038	−0.465	−0.165	1.813
	(0.18)	(0.14)	(0.12)	(0.24)	(0.47)	(0.27)	(0.21)	(0.09)
Servs	−0.070	−0.684	−0.561	0.030	−0.489	0.332	−0.210	0.280
	(0.14)	(0.11)	(0.09)	(0.19)	(0.37)	(0.21)	(0.16)	(0.05)
Fuel	−0.240	0.093	−0.068	−0.811	−0.351	0.383	−0.391	1.390
	(0.12)	(0.09)	(0.08)	(0.16)	(0.31)	(0.18)	(0.14)	(0.08)
Cloth	−0.261	0.040	−0.328	−0.201	−0.706	−0.084	0.124	1.414
	(0.07)	(0.05)	(0.04)	(0.09)	(0.18)	(0.10)	(0.08)	(0.07)
Trspt	−0.480	−0.237	0.215	0.416	−0.053	−0.817	0.041	0.917
	(0.11)	(0.09)	(0.08)	(0.15)	(0.30)	(0.17)	(0.13)	(0.05)
Other	−0.155	−0.073	−0.253	−0.343	0.176	−0.020	−0.808	1.473
	(0.10)	(0.08)	(0.07)	(0.13)	(0.27)	(0.15)	(0.12)	(0.07)

* At average budget shares and reference prices.

[4] It is, however, rather difficult to give a meaningful interpretation to this result because the price effects in the macro model are, generally, less well determined than those in the micro model: in this sense the restrictions on the macro parameters are easier to accept.

statement (in absolute value) of the own price elasticities. The cross-price elasticities are also biased as discussed earlier. In some cases they even change sign, e.g. food and alcohol change from substitutes to complements and food and fuel from complements to substitutes.[5]

To assess the extent to which the re-parameterisation proposed at the end of Section I can cure the biases above, Table 3 reports the elasticities obtained when the parameter estimates of the micro model obtained with the Stone index approximation are used in the adjusted formula (7). The re-parameterisation brings the elasticities close to those obtained from the exact model (Table 2, section (*b*)), as one would expect from the small size of the estimated substitution term $0{\cdot}5\Sigma_j \gamma_{ij} \ln p_{jt}$.

III. CONCLUSION

The commonly used Stone index approximation for linear estimation of the AI model can bias the parameters estimates of the budget share equations. In general, this approximation can result in understated (in absolute value) own price elasticities and cross-price elasticities of goods which are either both luxuries or both necessities. The cross-price elasticities of other goods are overstated. The empirical analysis in the paper suggests that the bias is more serious when the budget share equations are estimated from micro rather than aggregate data. It is also found that, in the absence of strong substitution effects, the price parameter bias can be largely corrected through simple re-parameterisation.

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Date of receipt of final typescript: December 1992

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[5] Elasticities obtained from aggregate data by authors not aware of the bias caused by the Stone index approximation, e.g. Blanciforti and Green (1983), may suffer from a smaller margin of error than that suggested by Table 2.

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[Footnotes]

[5] **An Almost Ideal Demand System Incorporating Habits: An Analysis of Expenditures on Food and Aggregate Commodity Groups**
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