

BIBLIOGRAPHY ON THE MULTIVARIATE NORMAL INTEGRALS AND RELATED TOPICS

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This bibliography lists references pertaining to the evaluation of the probability integrals of the multivariate normal and the multivariate t over rectangular domains. References on some other relevant topics are also listed. The references are divided in 12 different sections with a section for the miscellaneous and additions. Several references are relevant to more than one section and hence they have been cross-listed.

An earlier version of this bibliography was circulated to the members of the Committee on Mathematical Tables of the Institute of Mathematical Statistics and to the members of the Subcommittee on the Multivariate Normal. Many comments and suggestions were received from colleagues. However, any responsibility for any omissions or errors rests solely with the author. In spite of all the efforts to make it an exhaustive bibliography this claim may not be justified.

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See also [44] and [169].