

Bidder Behavior in Multiunit Auctions: Evidence from Swedish Treasury Auctions

Kjell G. Nyborg

London Business School and Centre for Economic Policy Research

Kristian Rydqvist

Norwegian School of Management and Centre for Economic Policy Research

Suresh M. Sundaresan

Columbia University

We analyze a unique data set on multiunit auctions, which contains the actual demand schedules of the bidders as well as the auction awards in over 400 Swedish Treasury auctions. First, we document that bidders vary their prices, bid dispersion, *and* the quantity demanded in response to increased uncertainty at the time of bidding. Second, we find that bid shading can be explained by a winner's curse-driven model in which each bidder submits only one bid, despite the fact that the bidders in our data set use much richer bidding strategies.

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Third, we explore the extent to which the received theories of multiunit auctions are able to offer insights into the bidder behavior we observe. Our empirical evidence is consistent with some of the predictions of the models of auctions that emphasize private information, the winner's curse and the champion's plague. While the models of multiunit auctions serve as useful guideposts, our empirical findings also point to several new areas of research in multiunit auctions that are of policy and theoretical interest.

I. Introduction

We analyze the performance of multiunit auctions in treasury markets using a unique data set provided by the Swedish National Debt Office.¹ The data set contains the actual bid distributions (demand schedules) submitted by each bidder as well as the auction awards to the bidders in over 400 Swedish Treasury auctions that were held during the period 1990–94. Our data set consists entirely of discriminatory price auctions. Using this data set, we make the following contributions.

First, we document the ways in which bidders respond to increased uncertainty and auction size. Here we focus not only on bid shading but also on the richer strategy space available to bidders in multiunit auctions. In particular, in treasury auctions, bidders can submit multiple bids for multiple units, each bid consisting of a price-quantity pair. The Treasury hits the highest bids first until supply is exhausted, and in discriminatory price auctions, winning bidders pay what they bid. Hence bidders can adjust the total amount they bid for and the dispersion of their bids in response to market conditions. By documenting these bidding strategies, we make a connection between our data and the predictions of received theories of multiunit auctions, which focus on such bidding strategies over and beyond bid shading. In this context, our sample period is particularly interesting: it includes several auctions that were held in periods of extreme volatility when interest rates went as high as 500 percent.

Second, we provide evidence that bidders rationally adjust for the winner's curse. In particular, we test the *quantitative* implications of an equilibrium in which each bidder submits a single bid for 100 percent of the auctioned securities at a price that is a function of his private information. This model allows us to quantify the amount of bid shading at the cost of additional, restrictive assumptions. We find that observed bid shading is remarkably close to the predicted values of the single-bid equilibrium, despite the fact that the bidders in our data set use

¹ For conformity with the literature on treasury auctions, we refer to it as the Swedish Treasury.

much richer strategies. We interpret this as evidence that the winner's curse is an important concern for bidders in Swedish Treasury auctions, and quite possibly in other countries as well.

Third, we use the data to explore the extent to which received theories of multiunit auctions are able to offer some insights into the observed multidimensional aspects of bidding behavior. In particular, in multiunit auctions, the winner's curse may be more nuanced than in single-unit auctions: Ausubel (1997) suggests that in multiunit auctions, the more a bidder wins, the worse news it is for him. Ausubel refers to this as the "champion's plague." In multiunit auctions with private information, bidders would be expected to respond to a more severe champion's plague by submitting lower-demand schedules. We find corroborating evidence of this hypothesis in our data. An alternative view is that the primary driver of bidder behavior is risk aversion. For instance, Wang and Zender (1998) show that risk-averse investors shade their bids more when there is greater uncertainty, even in the absence of private information. But our results do not provide much evidence supporting the risk aversion hypothesis.

The theoretical research in multiunit auctions is relatively new. While our data appear to be broadly consistent with some (but not all) of the implications of models of multiunit auctions, we feel that there are currently few tractable models of multiunit auctions that place sharp restrictions on data like ours. This is important in setting directions for future theoretical work in multiunit auctions since the data that we have are about as detailed as one can possibly get concerning treasury auctions. We therefore believe that our findings will stimulate further research in the theory of multiunit auctions.

The paper is organized as follows. Section II describes the Swedish Treasury market. In Section III we provide a description of the data set and some summary statistics. Section IV reviews the empirical implications of auction theory under private information. We develop empirical implications of the single-bid equilibrium and then turn to the richer predictions of the champion's plague. Section V provides descriptive statistics of the data. We break the data up into two exogenous variables (auction size and volatility) and five endogenous variables (discount, intrabidder dispersion, quantity demanded, profit per unit bought, and award concentration). We then run regressions of these endogenous variables on the two exogenous variables. We also test the predictions of the single-bid equilibrium on average discounts. The results are broadly consistent with rational adjustments to the winner's curse/champion's plague, and they also suggest that risk aversion is perhaps not so important. Finally, we illustrate the multidimensionality of actual bidding strategies by looking at individual demand schedules

within auctions. Section VI discusses risk aversion, presents conclusions, and points out some directions for future research.

II. The Swedish Treasury Market

We provide a summary of the institutional aspects of Swedish Treasury markets in this section. The Swedish Treasury sells Treasury bills, which are discount securities, and Treasury bonds, which pay coupons once a year. The Treasury bills are short-term securities that mature within 14 months, whereas the Treasury bonds are long-term securities that mature between six and 16 years from the original issue date.

The Swedish Treasury auctions are sealed, multiple-bid, discriminatory price auctions. The bids are submitted electronically before 12:30 on the auction day, and the awards are announced at 13:30 (Treasury bills) and 14:00 (Treasury bonds). Since July 1994, all results are released at 13:00. Any number of bids may be submitted.² Each awarded bidder pays what he bids. Only authorized bond dealers may submit bids in the auction. Investors who wish to purchase Treasury securities in the auction must bid through a dealer. There are 14 primary dealers in the Swedish Treasury market except during a short period in 1994, when there were 15 dealers.

Swedish Treasury auctions differ from U.S. Treasury auctions in five major ways.³ These differences naturally circumscribe the inferences of our study as they relate to the U.S. Treasury markets. First, no Swedish auctions are conducted under the uniform pricing format. Second, non-competitive bids are filled at the highest winning bid as opposed to the stop-out yields as in the United States. As a result, noncompetitive bids are not part of the Swedish Treasury auction experience. Third, the Swedish Treasury reserves the right to withdraw securities from the auction after bids have been submitted. The rejection is ad hoc and is not related to an explicit formula. Fourth, there is no 35 percent rule that limits the maximum awards to one dealer. In fact, a single dealer collects all the awards in approximately 10 percent of the auctions. Finally, there is no when-issued trading prior to the auction in the Swedish market. Investors who wish to trade in the securities before the auction can do so in the secondary market, where identical securities are traded both before and after the auction, because most of the time the auction is a reopening of an existing security. A when-issued market has not been developed for the few occasions on which a new security is auctioned.

² Bids for Treasury bills must be separated by one yield basis point and, from April 1994, by 0.1 basis point. Bids for Treasury bonds must be separated by three decimals on the price per 100 kronor of face value.

³ See, e.g., Nyborg and Sundaresan (1996) for a study of both discriminatory and uniform U.S. Treasury auctions.

TABLE 1
SWEDISH TREASURY AUCTIONS, 1990–94

	1990	1991	1992	1993	1994	Subtotal
Treasury bills	46	60	72	78	78	334
Treasury bonds	9	18	26	50	50	153
Subtotal	55	78	98	128	128	487

There are two differences between purchasing the security in the auction and purchasing the same security in the secondary market. First, bidders in the auction do not know until after the announcement of the auction results if they are awarded. This is an advantage of purchasing in the secondary market. Second, securities can be purchased in the auction without paying a commission or a spread to the dealer, who simply passes the bid to the Treasury. This is an advantage of purchasing in the auction. The settlement in the auction is matched with the settlement in the secondary market.⁴

In table 1, we show the breakdown of the auctions over time. Treasury bill auctions account for nearly 70 percent of the auctions, and the number has increased over time. The auction schedule is regular and is announced in advance. The revenue target for the next six months is announced at the beginning of each semester. The terms of the auction (security and volume) are announced two weeks in advance. Every second Thursday, three different Treasury bills are auctioned simultaneously. The Treasury bills mature in approximately 90, 180, and 360 days, respectively. On the subsequent Monday (four days later), two different Treasury bonds are auctioned.⁵

The median issue volume is 3,000 million kronor and ranges from 500 to 8,000 million kronor. This is approximately 5 percent of the supply in U.S. Treasury auctions (Sundaresan 1994). The bid-to-cover ratios average 2.41 and range from 0.55 to 8.38. The average is lower than in the United States, indicating that there is less competition in the Swedish Treasury market. The lowest bid-to-cover ratio of 0.55 means that the auction is undersubscribed. Table 2 shows the number of auctions that are undersubscribed or in which bids are rejected by the Treasury. Note that 45 auctions (roughly 10 percent of the auctions) have rejected bids. Twenty-nine auctions were undersubscribed and 16

⁴ Treasury bills are settled two workdays after the auction, and Treasury bonds five workdays later.

⁵ Before July 1991, two Treasury bill maturities (180 and 360 days) were auctioned simultaneously, and before November 1992, only one Treasury bond was auctioned at a time. The 334 Treasury bill auctions were held on 125 auction days, 37 auction days with two securities and 88 auction days with three securities, and the 153 Treasury bonds were held on 101 auction days, 49 auction days with one security and 52 auction days with two securities.

TABLE 2
ISSUE CUTS

	Undersubscribed	Oversubscribed	Subtotal
No rejected bids	6	436	442
Rejected bids	29	16	45
Subtotal	35	452	487

NOTE.—The number of undersubscribed auctions and auctions with rejected bids. Undersubscribed auctions are defined as auctions with a bid-to-cover ratio less than one, inclusive of rejected bids.

oversubscribed. By comparison, Sundaresan (1994) reports no incidences of undersubscribed auctions in the United States. This suggests that demand uncertainty has been a serious problem for Swedish Treasury departments.

Treasury policy is to concentrate borrowing on a few securities by reopening existing instruments. Table 3 reports the frequency of reopenings. While nearly 95 percent of the Treasury bonds are reopened, a little less than 80 percent of the Treasury bills are reopened. The purpose of reopening is to enhance the liquidity of the secondary market in certain maturities. A disadvantage is that only a few instruments are available to investors. All Treasury bills are scheduled to mature on the Wednesday of the third week of the month, so there are never more than 12 maturity dates within the year.⁶ The supply of Treasury bonds is also scarce. There are 12 Treasury bonds outstanding during the sample period.

III. Data

A. Bid Distribution Data

For this study, the Swedish Treasury has created a tape with all the individual bids. Each row contains the price per 100 kronor of face value, the yield to maturity, the volume (million kronor of face value), and a four-digit dealer code. The dealer code remains constant within the auction, but it varies randomly from auction to auction. The randomization serves to protect the identity of the bidder. The bid data are confidential and not available even to the bond dealers. Bids may be made by the dealers or by their customers, but only the dealers know whether the bids are customer-based.

The tape contains all the bids for 320 of the 334 Treasury bill auctions (96 percent) and for 138 of the 153 Treasury bond auctions (90 per-

⁶ A new “month” is opened when the Treasury issues, say, a 374-day Treasury bill. The same maturity is reopened two weeks later with the issuance of a 360-day Treasury bill. The maturity is opened a third and a fourth time when a 180-day and a 166-day Treasury bill are issued, and a fifth and sixth time when a 100-day and an 86-day Treasury bill are issued. All Treasury bills that mature within a year are reopenings.

TABLE 3
REOPENINGS

	New	Reopen	Subtotal
Treasury bills	60	274	334
Treasury bonds	7	146	153
Subtotal	67	420	487

cent). Nineteen Treasury bill and Treasury bond auctions in August–September 1994 are missing, as are the bids in a few other auctions. This information has been lost from the Treasury. The total number of auctions is 458, the number of demand schedules 5,845, and the number of bids 28,761. On average, there are 4.9 bids per demand schedule. The mode is two bids for Treasury bills and four bids for Treasury bonds. Most demand schedules contain more than one bid, but only a few contain more than 15 bids. For comparison, Gordy (1999) reports that the average number of bids is 2.8 in Portuguese Treasury bill auctions. The thrust of these summary findings is that *bidders employ multiple bids and hence submit a demand schedule as a rule*. The distribution of the number of dealers who actually participate in the auctions is tight. The number of bidders is six in one auction, 10 in seven auctions, 11 in 33 auctions, 12 in 136 auctions, 13 in 177 auctions, 14 in 100 auctions, and 15 in four auctions.

B. Secondary Market Prices

To estimate auction revenue and the extent to which bidders shade their bids, we need not only the bids made by individual bidders but also secondary market prices to serve as benchmarks for the true value of the auctioned securities.

Every Treasury bill maturity (one per month within the year) and the most frequently issued Treasury bonds, designated as benchmark bonds, are traded actively in the secondary market. Trading is organized as a telephone market. Only the dealers know their transactions. The dealers are obliged to provide bid and ask yield quotes on an electronic screen, which is available to the other dealers and to the public; but the quotes are not binding for any quantity, so the posted quotes are indicative only of the real quotes. Additionally, the quoted spread between the bid and the ask is a time-series constant, which is equal to five yield basis points for Treasury bills and three basis points for Treasury bonds and, from July 1994, three yield basis points for all securities.

Time-stamped transaction yields and volumes are not publicly available, so we use the indicative yield quotes for estimating the true value of the securities. Short-term yield quotes are provided to us by a primary

dealer (Öhman Fondkommission), which collects and distributes bid quotes at the end of the day at 16:05. (We use the median quote of 14 dealers.) Long-term yields are taken from the FINDATA tape, which contains end-of-day bid yields from two financial newspapers.⁷ When a security is reopened, yield quotes are available both before and after the auction. When a new security is auctioned, yield quotes are available only after the auction.⁸ All our data sources provide the bid quotes only because trading is at, or near, the bid. Below, we shall therefore use the bid quotes as proxies for the true secondary market prices of the auctioned securities.

IV. Empirical Implications of Multiunit Auction Theory under Private Information

Treasury auctions are normally thought of as common-value auctions. The usual argument is that primary dealers buy in the auction primarily to resell in the postauction secondary market (Bikhchandani and Huang 1993). More generally, as long as liquidity in the secondary market is deep, the common-value model should be a good approximation. In this section, we draw out some empirical predictions of the common-value model, which we later shall use as a benchmark when assessing bidder behavior and auction performance in Swedish Treasury auctions. Here, we focus on the impact of private information.

When bidders have different estimates about the postauction price, auction theory shows that the winner's curse becomes a preponderant consideration. The winner's curse has its basis in the difference between unconditional and conditional expectations and was originally studied in the context of single-unit auctions by, among others, Wilson (1977) and Milgrom and Weber (1982). In particular, if bidders' signals about the postauction price are affiliated, as they are in the common-value model, once a bidder learns that he has won an auction, his updated estimate of the value of the object is reduced relative to his unconditional expectation. The reason is essentially that, by winning, the bidder learns that his original estimate was the highest among all bidders. In other words, the winner learns that he most probably had overvalued the auctioned object. Rational bidders take the winner's curse into account and submit bids at prices that are lower than their estimates, and

⁷ FINDATA is a firm that collects and distributes financial information in Sweden. The newspapers are *Svenska Dagbladet* and *Finanstidningen*.

⁸ In 27 Treasury bill auctions and 22 Treasury bond auctions, most in the beginning of the sample period, yield quotes from the secondary market are not available until the next day or, in some cases, several days or weeks after the auction. In these cases, we interpolate the secondary market yield for the auctioned security from available term structure data on the auction day.

the less precise bidders' signals are, the more they will shade down their bids. As a result, the expected selling price is inversely related to uncertainty. In short, when uncertainty is higher, the winner's curse is higher—leading bidders to bid more cautiously, thus lowering expected revenue to the seller.

In multiunit auctions, Ausubel (1997) has shown that the winner's curse is potentially more nuanced. This has its basis in the fact that in multiunit auctions, the auctioned assets can be shared among several buyers. Bidders now form conditional expectations after the auction on the basis of how many units they won. When bidders' values are interdependent, the more a bidder wins, the worse news it is for him. Ausubel refers to this as the "champion's plague." Provided that other bidders submit downward-sloping demand schedules or individually demand less than the auctioned supply, a bidder can adjust for this champion's plague by reducing quantity at a given price, in other words, by submitting a lower demand schedule.

The champion's plague suggests that it is individually rational for a bidder to submit a downward-sloping demand schedule or reduce demand if other bidders also do so. But if bidders are risk-neutral and have constant marginal valuations, the only discriminatory price auction equilibrium that has been demonstrated in the literature when there are no quantity restrictions is the "single-bid" equilibrium, where each bidder demands 100 percent of the auctioned supply at a single price, which depends on his information set (see, e.g., Back and Zender 1993; Ausubel and Cramton 1998). In this case, the discriminatory price auction reduces to a single-unit, first-price auction.

In the remainder of this section, we shall first develop some empirical predictions of the single-bid equilibrium. While this equilibrium can be rejected in our sample on the grounds that bidders typically submit multiple bids, it is interesting to ask whether bid shading in practice is quantitatively similar to what we would expect in the single-bid equilibrium. If we find similarity, then that would suggest that bidders submit demand schedules centered around the single price they would have bid at in a first-price auction. Second, we discuss the richer predictions of the champion's plague. Here, since we do not have an explicit equilibrium to work with, we focus on drawing out qualitative empirical predictions. In subsequent sections, we shall empirically examine the quantitative predictions of the single-bid equilibrium and the qualitative predictions of the champion's plague.

A. *The Single-Bid Equilibrium*

This subsection presents the single-bid equilibrium under specific parametric assumptions, which allow us to draw out testable predictions.

We consider a common-value auction with n risk-neutral bidders who have proprietary information of equal precision. Each bidder i receives a private signal, s_i , with mean v (the true value of the auctioned objects) and standard deviation σ . That is, $\tilde{s}_i = \tilde{v} + \tilde{\epsilon}_i$, where, for all i , $\tilde{\epsilon}_i$ has mean zero and variance σ^2 , and the $\tilde{\epsilon}$'s are independent of each other and of \tilde{v} .

It has been shown by Wilson (1977) that first-price auction equilibrium bidding strategies can be characterized by a first-order ordinary differential equation. Under the assumption that the $\tilde{\epsilon}_i$'s are normally distributed and the prior distribution of \tilde{v} is diffuse, Levin and Smith (1991) have solved Wilson's differential equation, showing that all symmetric equilibrium bidding strategies in which each bidder demands 100 percent of the auctioned assets at a single price are given by the following equation:

$$p(s) = s - \alpha\sigma - Ce^{-E(\tilde{z}_{(n)})s/\sigma}, \tag{1}$$

where C is an arbitrary nonnegative constant,

$$\alpha = E(\tilde{z}_{(n)}) + \frac{\text{Var}[\tilde{z}_{(n)}]}{E(\tilde{z}_{(n)})}, \tag{2}$$

and $\tilde{z}_{(n)}$ is the maximum of n independent draws from the standardized normal distribution. The last two terms in (1) are the amount of bid shading, which depends only on the precision of signals and the number of bidders. Bid shading increases with σ , whereas the effect of the number of bidders is ambiguous.

In these symmetric equilibria, the winning bidder will be the bidder with the highest estimate, which will be denoted by $\tilde{s}_{(n)}$. The price paid by the winning bidder equals his bid, which will be denoted by $\tilde{p}_{(n)}$. Given the bidding strategy (1), the winning bidder's expected profit (or rents) conditional on v is

$$E(\Pi|v) = v - E(\tilde{p}_{(n)}) = v - E(\tilde{s}_{(n)}) + \alpha\sigma + E(Ce^{-E(\tilde{z}_{(n)})\tilde{s}_{(n)}/\sigma}).$$

Since

$$E(\tilde{s}_{(n)}) = v + \sigma E(\tilde{z}_{(n)}),$$

the expected profit of the winning bidder can be written as

$$E(\Pi|v) = \beta\sigma + E(Ce^{-E(\tilde{z}_{(n)})\tilde{s}_{(n)}/\sigma}), \tag{3}$$

where

$$\beta = \frac{\text{Var}[\tilde{z}_{(n)}]}{E(\tilde{z}_{(n)})}.$$

While (3) has been derived conditional on v , neither σ nor β is a function

of v . Thus (3) also represents unconditional expected profits when $C = 0$.

B. Multiple Units and the Champion's Plague

As outlined earlier, Ausubel's (1997) champion's plague argument hinges critically on bidders' submitting downward-sloping demand schedules or individually demanding less than the auctioned supply. This is ensured in Ausubel's model by the assumption that bidders are restricted from bidding for more than a fraction of the auctioned supply. However, throughout our sample period, in Swedish Treasury auctions there are no quantity restrictions. Indeed, it happens occasionally that a single bidder buys the entire auction. Nevertheless, bidders do submit downward-sloping demand schedules. So, empirically, the winner's curse, in its champion's plague guise, should be important in Sweden insofar as bidders have private information.

In treasury auctions, one of the choice variables of a bidder is how much to bid for. When there is no reservation price, the champion's plague does not have anything to say about quantity demanded since a bidder would always be happy to buy at a price marginally larger than zero. However, when there is a reservation price, as there implicitly is in Sweden, an implication of the champion's plague is that a bidder would reduce his overall demand when uncertainty increases. The implication is that if private information and the winner's curse are important, we should see uncertainty having a positive effect on bid shading and a negative effect on quantity demanded.

Another choice variable for bidders in treasury auctions is how much to disperse bids. In a multiunit auction, a bidder would like to win some extra units when bidding by the other bidders suggests that the value of the auctioned object is high. In particular, if the stop-out price is high, the inference is that other bidders have information that the secondary market will be strong and the bidder would like to revise his bids upward to win more units. Conversely, if the stop-out price is low, the bidder would like to revise his bids downward to win fewer units. This is, of course, not allowed under the discriminatory price auction rules, but a bidder can achieve a similar effect by increasing the variance of his bids. If the stop-out bid turns out to be higher than his average bid, the bidder will tend to win more units with a high-variance bid distribution than with a low-variance bid distribution. Conversely, if the stop-out price bid turns out to be lower than his average bid, the bidder will tend to win fewer bids with a high-variance distribution than with a low-variance distribution. The implication is that we would expect to see that individual bidders increase the dispersion of their bids when uncertainty increases.

In summary, the champion's plague suggests a richer response to uncertainty in a multiunit context than the winner's curse in a single-unit context. It suggests that it is individually rational for a bidder to submit a downward-sloping demand schedule or reduce demand if other bidders also do so. In particular, if bidders rationally adjust for the winner's curse/champion's plague, we would expect that more uncertainty leads to more bid shading, less quantity demanded, and more intrabidder dispersion. However, it is possible that the champion's plague effect on quantity and intrabidder dispersion requires risk aversion or decreasing marginal valuations to generate downward-sloping demand schedules in the first place. We shall examine risk aversion in a later section.

V. Empirical Findings on Bidder Behavior and Auction Performance

In this section, we report on the broad patterns in bidder behavior and, in particular, how bidders respond to, and how auction performance is related to, uncertainty and auction size. We interpret our findings in light of the theory reviewed above and, in particular, we test the single-bid equilibrium's ability to explain the magnitude of observed bid shading.

A. Descriptive Statistics

Table 4 reports summary statistics on bid shading, intrabidder dispersion, and quantity demanded, in the sample as a whole and within 10 duration bands. Summary statistics on bidders' profit per unit, award concentration, volatility, and auction size are also reported.

Bid shading is measured by the *discount*, which for bidder i in auction j is defined as $\delta_{ij} \equiv P_j - p_{ij}$, where p_{ij} is the quantity-weighted average bid by bidder i in auction j , and P_j is the secondary market price at the end of the day of auction j . If all of a bidder's bids turned out to be winning bids, the bidder would pay p_{ij} per unit. The discount is a measure of how much a bidder's bids are shaded relative to the postauction price. It provides us with a normalized measure of the price level of a bidder's bids.

The extent to which a bidder disperses his bids is measured by the *intrabidder dispersion*, σ_{ij} , which is the quantity-weighted standard deviation of bidder i 's bids in auction j . Similarly, q_{ij} is the quantity demanded by bidder i in auction j as a fraction of auction size. In sum, we use the vector $\{\delta_{ij}, \sigma_{ij}, q_{ij}\}$ to describe dealer i 's demand schedule.

In terms of measuring revenue in the auction, a drawback of taking the average of the discounts is that it takes account of both winning

TABLE 4
DESCRIPTIVE STATISTICS

	EXOGENOUS		BIDDING VARIABLES			PERFORMANCE	
	Volatility (η_j) (1)	Size (Q_j) (2)	Discount (δ_{ij}) (3)	Dispersion (σ_{ij}) (4)	Quantity (q_{ij}) (5)	Profit (Π_j) (6)	Award Concentration (7)
Mean	.267	3.295	.092	.046	.188	.020	.388
Standard deviation	.323	1.730	.317	.092	.225	.270	.340
Minimum	.040	.500	-1.564	.000	.001	-1.067	.015
Maximum	2.803	8.000	3.318	2.145	2.150	2.667	1.000
Observations	458	458	5,831	5,831	5,831	458	458
Duration:							
.25	.116	2.572	.014	.011	.224	-.001	.547
.50	.172	3.039	.011	.016	.182	-.014	.378
1	.189	3.991	.034	.031	.174	-.014	.307
2	.259	3.210	.073	.040	.208	-.001	.459
3	.334	3.912	.189	.067	.197	.061	.382
4	.408	3.443	.154	.086	.198	.017	.354
5	.546	3.285	.252	.122	.176	.086	.351
6	.600	3.159	.478	.162	.190	.204	.374
7	.509	3.300	.442	.148	.180	.195	.414
8	.888	3.087	.295	.149	.146	.109	.277

NOTE.—Discount, dispersion, and profit are percentages of face value, quantity is the fraction of auction size, award concentration is the fraction awarded to the five highest bids, volatility is the daily price standard deviation, size is expressed per billions of kronor, and duration is measured in years. For the 10 duration bands, only means are reported.

and losing bids. Therefore, to measure revenue, we define the *profit* per unit sold in auction j , Π_j , to be the difference between the postauction price and the quantity-weighted average winning bid. A small *profit* to bidders translates into a high revenue for the seller. Our second auction performance measure is *award concentration*, which is the fraction of awards captured by the five highest individual bids.

In table 4 and throughout the paper, all prices are expressed as a percentage of face value. The lowest bid in each auction is excluded from all estimations. This is a simple solution to an errors-in-variables problem resulting from bids that are thrown in at low prices well before the end of the auction.⁹

Table 4 exhibits two exogenous variables: uncertainty and auction size. We measure uncertainty as the auction day volatility using an ARCH(2) process of bond returns.¹⁰ The one-day volatility, η_j , increases monotonically with duration and has an average of 0.267 percent and a maximum of 2.8 percent. This is a very large number: it is 10.5 standard deviations larger than the mean and has the same order of magnitude as the stock return volatility of a small firm. The auction size, Q_j , has an average of 3.3 billion kronor and varies from 500 million to 8 billion, but does not vary systematically with duration.

The time-series distribution of the two exogenous variables and the issue cuts are shown in figure 1. The six plots are organized as the Treasury bills to the left and the Treasury bonds to the right. Panel *a* shows the distribution of volatility, panel *b* the supply, and panel *c* the issue cuts after the bids have been submitted, that is, the sum of undersubscription and rejected bids. Panel *a* reveals a volatility spike in 1992. The spike is related to the breakdown of the fixed exchange rate regime. The lining up of the plots on top of each other shows that auction sizes during this high-volatility period were unusually large and that most of the issue cuts clustered during this period. The auctions that were held during this extreme volatility period are studied in detail below.

The three bidding variables are described in the middle section of table 4. Column 3 shows that *the average bid is shaded below the aftermarket price*. The average discount is 0.092 percent of face value and is broadly increasing in duration. A noticeable exception is the last duration band, which has a much lower discount than the penultimate duration band. Column 4 shows that *individual bidders typically disperse their bids*. The magnitude of intrabidder dispersion is about one-fifth of the daily vol-

⁹ For example, one dealer submitted bids for 10 million kronor at an annual yield of 99.99 percent in each of three contemporary Treasury bill auctions. In another case, one dealer submitted in writing (not electronically) five bids for 1,000 million kronor each at 2–3 kronor below all other bids.

¹⁰ The details of the time-series modeling are in the Appendix.

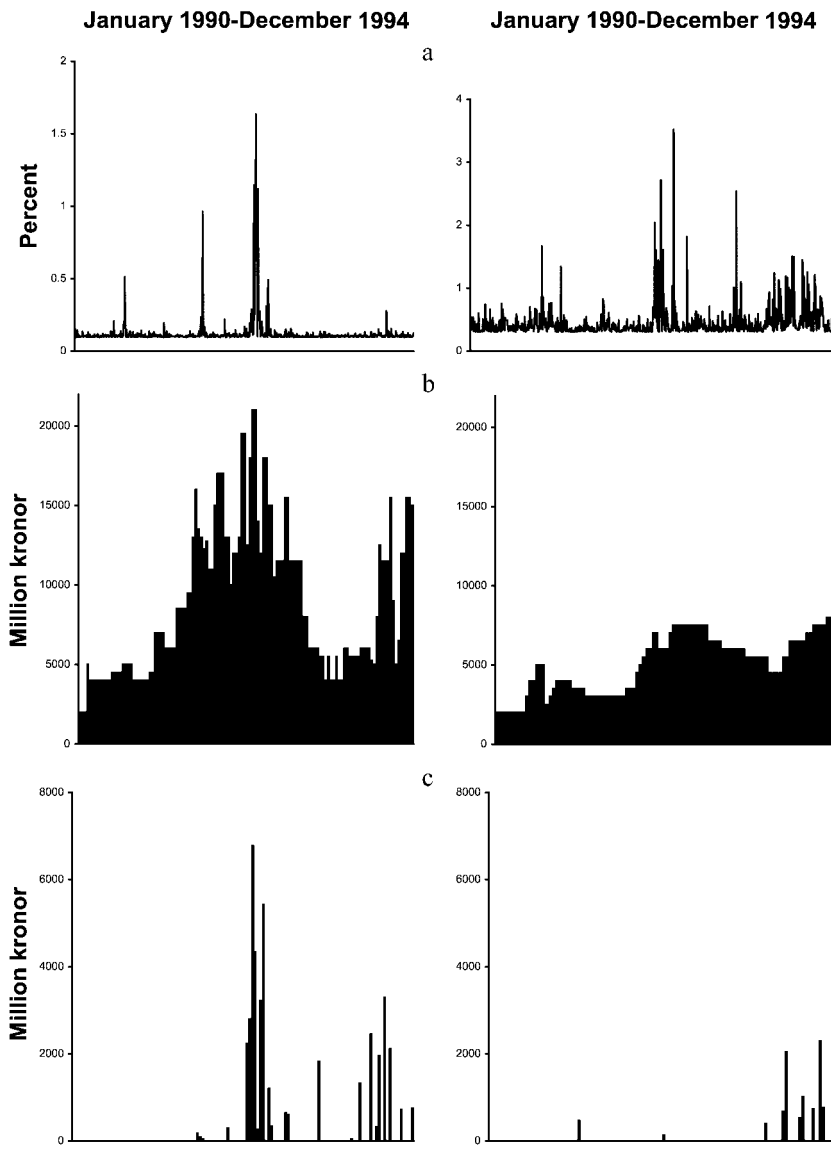


FIG. 1.—*a*, Volatility, *b*, auction size, and *c*, issue cuts. Treasury bills are on the left and Treasury bonds on the right.

atility, and it increases with duration. The smallest dispersion is zero when the bidder submitted only one bid. The largest dispersion is 22.8 standard deviations away from the mean. Column 5 shows that a bidder's average demand is 18.8 percent of the auctioned securities. The largest demand schedule is for 215 percent of supply. This might have been submitted in anticipation of prorating at the stop-out price, or it could reflect a large amount of customer bids. In contrast to the other two variables, there is no obvious relationship between duration and quantity demanded.

The two performance variables are displayed in columns 6 and 7 of table 4. Bidders' average profit per unit purchased is positive, 0.020 percent of face value, and increases with duration. But the standard deviation is so large that we cannot be confident that the average profit is significantly different from zero (t -statistic 1.6). The average profit is negative for the four shortest duration bands and positive for the six longest. The lack of statistical significance and the negative profits for the short duration bands should be interpreted with caution. First, price movements during the 3.5 hours from the time of the auction (12:30) to the closure of the secondary market (16:05) raise the standard error of the reported per unit profit. Second, auction profits have been calculated by subtracting transactions prices in the auction from bid quotes in the aftermarket. If secondary market transactions, on average, are carried out within the bid-ask spread, our reported profits underestimate true auction profits. If we had used ask quotes, average profits would have been positive for each duration band and statistically significant in the sample as a whole.

Column 7 of table 4 reveals that, on average, the five highest individual bids are awarded a total of 38.8 percent of all sold securities per auction. The cross-section variation is large, however, going from 1.5 percent to 100 percent, with a single bidder obtaining all the awards in approximately 10 percent of the auctions. Like the other quantity-related variables (auction size and quantity demanded), but unlike the price-related variables (volatility, discount, dispersion, and profit), award concentration displays no clear duration pattern, except that the awards are much more concentrated for the Treasury bill with the shortest duration than for the other securities.

B. Regression Analysis

In this subsection, we examine the qualitative predictions of the champion's plague as discussed in Section IV. We do so by regressing the endogenous variables on volatility, which here proxies for the precision of bidders' signals. The idea is that volatility is positively correlated with the precision of bidders' signals, so that a relatively high volatility is

TABLE 5
REGRESSION ANALYSIS

	BIDDING VARIABLES				PERFORMANCE	
	Discount ($\hat{\delta}_j$) (1)	Average Discount ($\hat{\delta}_j$) (2)	Dispersion (σ_{ij}) (3)	Quantity (q_{ij}) (4)	Profit (Π_j) (5)	Award Concentration (6)
Constant	-.0194 (-11.7)*	-.0194 (-3.5)*	-.0093 (-24.7)*	.2635 (40.6)*	-.0121 (-2.5)*	.6405 (19.5)*
Volatility	.4380 (32.1)*	.4404 (9.7)*	.2029 (65.1)*	-.0315 (-3.5)*	.1080 (2.7)*	-.0906 (-2.0)*
Size	-.0015 (-3.7)*	-.0015 (-1.1)	.0006 (6.7)*	-.0201 (-12.0)*	-.0006 (-.5)	-.0694 (-8.1)*
R^2	.151	.172	.442	.028	.016	.140
Observations	5,831	458	5,831	5,831	458	458

NOTE.—Discount, dispersion, and profit are percentages of face value, quantity is the fraction of supply, award concentration is the fraction awarded to the five highest bids, volatility is the daily price standard deviation, and supply is expressed in billions of kronor. *t*-statistics are in parentheses. The discount, dispersion, and profit regressions are weighted by volatility.

* Significant at the 5 percent level or better.

indicative of a large winner's curse/champion's plague and should therefore be associated with relatively cautious bidding. In our regressions, we control for auction size. This variable may also have an impact on bidder behavior, particularly if some bidders are capacity-constrained. The maximum fraction of the auction size that such bidders can demand is obviously larger in a small auction than in a large auction.

The three price regressions (shading, dispersion, and profit) are weighted with volatility, whereas the two quantity regressions (quantity demanded and award concentration) are estimated with ordinary least squares. Each equation is estimated separately, as opposed to joint estimation in a system, since we lack a theory that imposes constraints on the estimation.

In the discount regression,

$$\delta_{ij} = \gamma_0 + \gamma_1 \eta_j + \gamma_2 Q_j + \epsilon_{ij}, \quad (4)$$

some of the residuals may be cross-sectionally dependent, since a change in the aftermarket price P_j affects the discount estimates of all bids in auction j . To control for this, we also run the discount regression with the average discount in auction j ,

$$\delta_j \equiv \frac{1}{n_j} \sum_{i=1}^{n_j} \delta_{ij}, \quad (5)$$

as the independent variable. Throughout the paper, we use the equally weighted average discount. The results using quantity-weighted averages are qualitatively similar.

The regression results are reported in table 5. The bidding variable

regressions are to the left and the performance regressions to the right. In both discount regressions, we see that the *discount increases significantly with volatility*. A one-percentage-point increase in daily price volatility results in a typical increase in the discount by 0.44 percent of face value. Since variations in volatility typically have an order of magnitude of 0.1 percent, with extreme cases being well beyond 1 percent, volatility has an economically significant impact on discounts. The positive correlation between auction discounts and uncertainty confirms the empirical findings by Cammack (1991), Spindt and Stolz (1992), and others. It is consistent with the hypothesis that bidders have private information and rationally adjust for the winner's curse.¹¹

The coefficient on auction size is negative but economically small—a 1 billion kronor increase in auction size reduces the discount by only 0.0015 percent—and its statistical significance is questionable, as seen from the *average discount* regression. Our finding that discounts exhibit so little sensitivity to auction size suggests that the aggregate demand function is highly elastic.

The regression on the dispersion of bids is displayed in column 3 of the table. Like the discounts, *intrabidder dispersion increases with uncertainty*. Unlike the discounts, intrabidder dispersion also increases with auction size, but the size effect is much smaller in magnitude than the volatility effect. An increase in volatility by 0.10 percentage points increases intrabidder dispersion by 0.02 percentage points, which is large compared to the average intrabidder dispersion of 0.046 percent in table 4. On the other hand, an increase in auction size by 1 billion kronor would not raise intrabidder dispersion by more than 0.0006 percentage points.

Next, we turn to the regression on quantity demanded per bidder, as a fraction of supply, which is seen to decrease significantly with both uncertainty and auction size. A one-percentage-point increase in volatility tends to decrease the relative demand per bidder by 3 percent of supply. This translates into a reduction in the bid-to-cover ratio of around 0.39, with 13 bidders in the auction. The average bid-to-cover ratio is 2.41. A billion kronor increase in auction size tends to decrease the relative demand per bidder by 2 percent of supply, which translates into a reduction in the bid-to-cover ratio of around 0.26 when there are 13 bidders. However, the absolute quantity demanded per bidder increases with auction size. For example, recall from table 4 that the

¹¹ Since volatility is a generated regressor, γ_1 may be negatively biased, but this goes against our hypothesis that increased uncertainty leads to increased bid shading and therefore strengthens the empirical results. Our regression findings are also supported by regressions in which duration is the independent variable (not reported). Duration is not a generated regressor but has the disadvantage of not picking up changes in interest rate volatility.

average auction is about 3.3 billion kronor and the average quantity demanded per bidder is about 18.8 percent of auction size. Hence, in a 3.3 billion kronor auction, the typical demand per bidder is around 620 million kronor. In a 4.3 billion kronor auction, the typical demand per bidder increases to around 722 million kronor.

The performance regressions are displayed in columns 5 and 6 of table 5. The profit regression parallels the discount regressions. An increase of 0.1 percentage point in volatility increases bidders' profits by a statistically significant 0.011 percent of face value. On the other hand, auction size has no effect on per unit profits. Like the discount regressions, these findings suggest that the winner's curse is empirically important.

Finally, we turn to the award concentration regression. The coefficients on volatility and auction size are both negative and statistically significant. This is not surprising given the previous findings that volatility has a positive effect on intrabidder dispersion and a negative effect on the relative quantity demanded per bidder. Additionally, an increase in auction size tends to decrease relative quantity demanded. Both an increase in intrabidder dispersion and a decrease in the relative quantity demanded per bidder tend to decrease award concentration. Hence, to the extent that dispersed awards are desirable, our regression results suggest that the Treasury could decrease award concentration without lowering the price per unit sold by holding large auctions. The drawback of a large auction is the risk that the entire issue will not be sold.

In sum, the evidence shows that bidders react to a high level of uncertainty by lowering the prices they bid at, lowering demand, and increasing the dispersion of their bids. This is consistent with rational adjustments to the winner's curse/champion's plague, as discussed in Section IV. Additionally, bidders react to large auctions by lowering the relative quantity demanded but increasing the absolute quantity demanded. But the size of the auction has no effect on the price levels at which bids are placed and only a marginal effect on intrabidder dispersion.

C. Extreme Uncertainty: Case Study

The volatility plot in figure 1 revealed a volatility spike in September 1992, which was related to the breakdown of the fixed exchange rate regime. In an attempt to defend the Swedish kronor, the Central Bank raised the interest for overnight loans from 75 percent to 500 percent on September 16, 1992. The next day, the Swedish Treasury held three auctions of short-term securities. Comparing bidder behavior in these three auctions to behavior in more normal times provides an oppor-

tunity to examine how bidder behavior is affected by extreme uncertainty. The comparison is carried out in table 6.

Columns 1 and 2 compare the exogenous variables. While the auctions in our sample with the highest price volatilities tend to be auctions of longer-dated securities, the table shows that price volatility is much higher for the auctions on September 17, 1992, than for other auctions of similar maturities. The auction sizes are also larger than normal.

Columns 3–5 look at the bidding variables. Intradidder dispersion on September 17, 1992, was much higher than usual—about 40–100 times as high. Measured in yields, the spread between the lowest and the highest yield in the auction went from 24.25 percent to 50 percent for the 90-day Treasury bill and from 22 percent to 38 percent for the 180-day Treasury bill. As a result of the wide intradidder dispersion, awards were also highly dispersed.

Quantity demanded was substantially smaller than normal for the two shorter maturities. Bid-to-cover ratios were only 0.74 (90-day) and 0.85 (180-day), and low bids, amounting to 28 percent and 20 percent of supply, were rejected by the Treasury. So only 46 percent (90-day) and 65 percent (180-day) of the announced volume were placed, respectively. However, the Treasury bill with the longest duration was fully subscribed.

The results on bid shading and profits are less clear-cut, in part because we lack reliable quotes from the secondary market for September 17, 1992. Practitioners have informed us that there were no trades in the secondary market on that day. Additionally, the high rejection rate for bids for the two shorter maturities contributes to deflating winning bidders' profit per unit bought relative to a case in which no bids were rejected.

In sum, the bidding at these three auctions reinforces our regression findings that bidders respond to uncertainty by decreasing prices, increasing intradidder dispersion, and reducing quantity demanded.

D. Estimation of the Single-Bid Equilibrium

The previous subsection shows that auction discounts and profits vary with uncertainty, but not with auction size, suggesting that the winner's curse plays an important role in determining bidder behavior. In this subsection, we examine whether the winner's curse can explain the magnitude of, and the variation in, observed *average discounts* and auction profits by using the single-bid equilibria given by (1) as a benchmark.

We handle the multiplicity of equilibria by initially focusing on the

TABLE 6
AUCTIONS ON SEPTEMBER 17, 1992

	EXOGENOUS		BIDDING VARIABLES			PERFORMANCE	
	Volatility (η_j) (1)	Size (Q_j) (2)	Discount (δ_{ij}) (3)	Dispersion (σ_{ij}) (4)	Quantity (q_{ij}) (5)	Profit (Π_j) (6)	Award Concentration (7)
90-Day Treasury Bill							
September 17, 1992	1.312	8.000	.616	.470	.056	.410	.041
All other	.099	2.506	.006	.005	.226	-.006	.553
<i>t</i> -test difference	6.5*	2.4*	3.2*	3.6*	-9.7*	n.a.	n.a.
180-Day Treasury Bill							
September 17, 1992	1.801	7.000	-.062	.590	.070	-.771	.196
All other	.155	3.006	.012	.011	.183	-.008	.380
<i>t</i> -test difference	7.5*	2.3*	.3	4.5*	-7.3*	n.a.	n.a.
360-Day Treasury Bill							
September 17, 1992	1.211	6.000	.143	.855	.145	-.643	.015
All other	.180	3.975	.033	.021	.174	-.009	.309
<i>t</i> -test difference	8.4*	1.3	.6	5.7*	1.1	n.a.	n.a.

NOTE.—Discount, dispersion, and profit are percentages of face value, quantity is the fraction of supply, award concentration is the fraction awarded to the five highest bids, volatility is the daily price standard deviation, and supply is expressed in billions of kronor.

* Significant at the 5 percent level or better.

linear equilibrium, where $C = 0$. The nonlinear equilibria, where $C > 0$, are discussed toward the end of the subsection. When $C = 0$, we have

$$p(s) = s - \alpha\sigma. \tag{6}$$

The term $\alpha\sigma$ is the amount of bid shading, which depends only on the precision of signals and the number of bidders; it increases with σ and, for $n \geq 4$, it also increases with the number of bidders. For $n = 14$, $\alpha = 1.8840$.

While we can clearly reject the single-bid equilibrium as a full description of bidder behavior, here we address the more limited empirical question as to whether the price levels that bidders submit bids at are in line with what we would expect from (6). We use discounts to measure and normalize price levels. Under the null hypothesis that a bidder's quantity-weighted average bid in auction j is given by (6), the average discount can be written as

$$\delta_j = \alpha_j\sigma_j + \frac{1}{n_j} \sum_{i=1}^{n_j} (P_j - s_{ij}),$$

where s_{ij} is bidder i 's (unobservable) signal in auction j .¹² Hence, the expected discount in auction j across bidders is

$$E(\delta_j) = \alpha_j\sigma_j \tag{7}$$

with the additional assumption that the secondary market bid price is an unbiased estimator of the average signal.

Equation (7) says that the predicted discount for auction j is linearly related to the standard deviation of bidders' signals, which must be estimated. Under the hypothesis that bidders use the linear bidding strategy (6) under which (7) has been derived, the standard deviation of the bids is the best unbiased estimator of the unobservable standard deviation of the signal distribution. Hence, we estimate σ_j as the equally weighted standard deviation of each bidder's quantity-weighted average bid, which we refer to as the *interbidder dispersion* of bids:

$$\hat{\sigma}_j = \sqrt{\frac{1}{n_j - 1} \sum_{i=1}^{n_j} (p_{ij} - \bar{p}_j)^2}.$$

The theoretical prediction on bid shading from equation (7) can be tested with the regression model

$$\delta_j = \gamma_0 + \gamma_1\alpha_j\hat{\sigma}_j + \epsilon_j, \tag{8}$$

where α_j is calculated from (2) using the actual number of participating

¹² The average across dealers is equally weighted since the theoretical predictions are based on symmetric bidders. Equally weighted or quantity weighted makes no qualitative difference to the regression results in this subsection.

TABLE 7
TEST OF SINGLE-BID EQUILIBRIUM

	DISCOUNT		PROFIT	
	(1)	(2)	(3)	(4)
Constant	-.0045 (.0021)	-.0139 (.0050)	-.0046 (.0020)	-.0127 (.0045)
Predicted	.860 (.099)	1.137 (.143)	1.192 (.900)	3.334 (1.263)
Ftest	11.0 (.000)	5.3 (.006)	4.8 (.009)	4.1 (.018)
R ²	.143	.231	.004	.032
Observations	458	211	458	211

NOTE.—Weighted least squares regression of auction discount and profit on predicted discount and profit, respectively. One regression treats each auction as one independent observation ($N=458$); the other treats each auction day average as one independent observation ($N=211$). Standard errors are reported in parentheses below the coefficients; p -values are below the F -statistic, which tests the joint hypothesis that the constant is zero and the slope coefficient is one.

bidders in each auction.¹³ The null hypothesis is that $\gamma_0 = 0$ and $\gamma_1 = 1$.

We also test the quantitative implications of the profit equation (3) for $C = 0$. The corresponding regression model is

$$\Pi_j = \gamma_0 + \gamma_1 \beta_j \hat{\sigma}_j + \epsilon_j, \quad (9)$$

where Π_j is the difference between the postauction price and the quantity-weighted average winning bid, and β_j is calculated from (3) using the actual number of bidders in each auction. For $n = 14$, $\beta = 0.1807$. Again, we want to test whether $\gamma_0 = 0$ and $\gamma_1 = 1$.

To eliminate heteroskedasticity, regressions (8) and (9) are estimated with weighted least squares using the independent variable in each regression as the weight. The estimated coefficients along with the standard errors are reported in table 7. In each case, the first regression treats all 458 auctions as independent observations, and the second regression treats the 211 auction day averages as independent observations.

The discount regressions show that observed discounts react to uncertainty much as predicted by the single-bid equilibrium. In both regressions, the slope coefficients are close to—and not statistically different from—one. The intercept is negative and statistically different from zero but is close to zero in magnitude.

In the profit regressions, the estimated coefficients also remain close to the predicted values, and we cannot reject that the slope coefficient

¹³ The results are not sensitive to whether we use the actual number of bidders, the average number of bidders (13), or the number of primary dealers (14). For $N = 12, 13$, and 14, which is the case in more than 90 percent of all auctions, α is 1.828, 1.857, and 1.884, respectively.

is different from one. However, for $N = 458$, the standard error of the slope coefficient is so large that we cannot be statistically confident that observed profits increase with the level of uncertainty. While they are small, the intercepts are negative and significant. Formally, we can reject the joint hypothesis that $\gamma_0 = 0$ and $\gamma_1 = 1$ for both the discount and the profit regressions, but this does not change the conclusion that bid levels (midpoints of demand schedules) are, on average, in line with what is predicted by the single-bid equilibrium.¹⁴

Another implication of (6) is that price levels should be normally distributed, just as signals are. A priori, the assumption that signals are normally distributed may appear unreasonable since the values of Treasury securities are bounded below by zero and above by the sum of their face value and coupons. On the other hand, normality may be a reasonable approximation since dealers' signals are aggregations of several pieces of information and since the precision of bidders' signals appears to be high— $\hat{\sigma}$ ranges from 0.0032 to 0.9445 per 100 kronor of face value—which is not surprising given the sort of securities we are dealing with and given the fact that many of the auctions are reopenings. To examine whether price levels are normally distributed, we first compute the standardized (quantity-weighted) average bid for bidder i in auction j as follows:

$$z_{ij} = \frac{p_{ij} - \bar{p}_j}{\hat{\sigma}_j \sqrt{(n_j - 1)/n_j}}$$

Figure 2 depicts the relative frequency distribution of this statistic across 17 intervals around the mean. While the average bid distribution in figure 2 looks approximately normal, formally a Jarque-Bera test rejects normality with a $\chi^2(2)$ statistic of 7.72. This appears to be a result of excess kurtosis rather than skewness. The standardized bid statistic z_{ij} has a skewness of -0.0429 with a standard error of 0.0321 and excess kurtosis of 0.1574 with a standard error of 0.0641. Although this means that we can reject another implication of the single-bid equilibrium (6), the closeness with which it seems to describe average bid shading is quite remarkable, particularly given that we know that bidders use much richer strategies.

¹⁴ A critical input in the tests above is the estimate of the common value. We have used the quoted market bid price at the end of the auction day. If trades take place within the quoted bid-ask spread, this underestimates the true aftermarket price. In this case, both the intercept and the slope coefficients are biased downward. Suppose that the true common value is systematically higher than the quoted bid price and that the difference between the quoted price and the true price increases with uncertainty. This possibility is suggested by the fact that quoted bid-ask spreads increase with duration. This would imply that both the intercept and the slope coefficients in table 7 are underestimated. Reestimating the regressions using the midpoint between the quoted bid and ask or using the ask side of the market raises both the intercept and the slope coefficients.

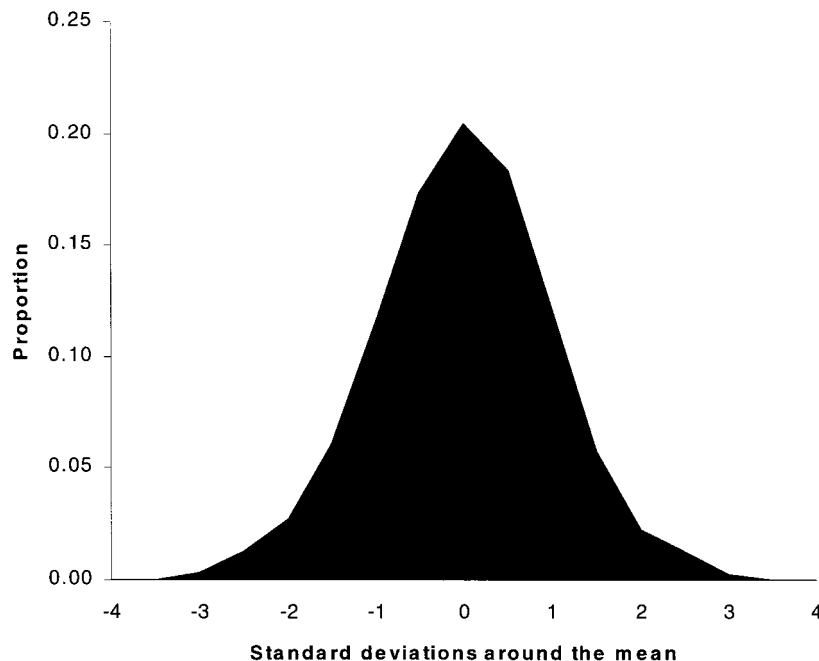


FIG. 2.—Relative frequency of standardized bids across 17 intervals around the average bid.

The absence of skewness in the bid distribution also suggests that the linear equilibrium describes bid shading better than any of the nonlinear equilibria described by (1). For if $C > 0$ and signals are normally distributed, the exponential in the nonlinear term implies that the bid distribution should exhibit negative skewness. Additionally, if the true strategy used by dealers involves $C > 0$, then the linear model would underestimate bid shading, in which case in (8) we would expect γ_0 to be positive and γ_1 to be larger than one. The fact that this is not the case also supports the hypothesis that $C = 0$ fits the data best.

The reason why the linear equilibrium fits better than the nonlinear equilibrium may be related to the very high magnitude and variation in the signal to noise ratio in our data. The nonlinear term in (1) is $e^{-E(\tilde{z}_{(n)})s/\sigma}$, where s/σ is the signal-to-noise ratio. Using the secondary market price as a proxy for s and the interbidder dispersion in place of σ , we can compute that the nonlinear term ranges from e^{-152} to $e^{-52,531}$ or 10^{-66} to $10^{-22,814}$. The large variation here poses problems for the hy-

pothesis that C is positive and constant across auctions.¹⁵ For if C is around 10^{63} to 10^{66} , then the nonlinear term would have an economic impact in only a few auctions. As C gets larger, say around $10^{10,000}$, the model would be observationally equivalent to the linear model in auctions with low uncertainty; but in auctions with high uncertainty, most bidders would either drop out or place extremely low bids. We do not see this in the data. Even in the undersubscribed auctions, of which there were only 35 during the sample period, the bid-to-cover ratio goes from 54.5 percent to 99.8 percent. An alternative hypothesis is that C varies across auctions according to σ and is always just large enough to have an impact on bidding strategies yet small enough not to lead to absurdly low bids. But this would lead to negative skewness in the bid distribution, which, as we have seen above, is just not there.

E. Bidding on Many Dimensions and the Champion's Plague

So far we have seen that bidders typically submit multiple bids and that variations in bid shading, intrabidder dispersion, and quantity demanded across auctions appear to be consistent with the hypothesis that bidders have private information and rationally adjust for the winner's curse/champion's plague. Thus, in a given auction, we might expect bidders who are more confident that secondary market demand will be strong not only to bid at higher prices but also to demand more and disperse less than less confident bidders.

To examine this, table 8 reports on the interrelationships among price levels (p_{ij}), intrabidder dispersion (σ_{ij}), and quantity demanded (q_{ij}). Panel A tabulates average quantity demanded and intrabidder dispersion at ordered price levels. That is, for each auction we first order price levels from the lowest to the highest:

$$p_{1j} \leq p_{2j} \leq \dots \leq p_{n-1j} \leq p_{nj}.$$

The idea is that bidders with relatively strong signals about the secondary market submit collections of bids with a higher quantity-weighted average price than bidders with relatively weak signals. We then compute and tabulate the cross-sectional average intrabidder dispersion and quantity demanded for each of these ordered price levels. *The table shows that intrabidder dispersion decreases with the price level, whereas quantity demanded increases.* On average, the highest bidder in an auction disperses only half as much, and demands about twice as much, as the lowest bidder.

Panel B of table 8 corroborates the findings in panel A. For each

¹⁵ The extremely small magnitude also means that estimating C by regression analysis is practically impossible.

TABLE 8
PRICE LEVEL, DISPERSION, QUANTITY, AND AWARD RATIO

Ordered Price Levels	Dispersion (σ_{ij}) (1)	Quantity (q_{ij}) (2)	Award Ratio (3)
A. Mean			
p_n	.036	.377	.893
p_{n-1}	.040	.259	.706
p_{n-2}	.037	.220	.611
p_{n-3}	.040	.171	.561
p_{n-4}	.041	.155	.495
p_{n-5}	.047	.156	.443
p_6	.048	.162	.367
p_5	.047	.159	.324
p_4	.047	.159	.281
p_3	.054	.138	.237
p_2	.053	.144	.186
p_1	.074	.151	.133
B. Correlation			
Mean	-.239	.245	
Standard deviation	.372	.347	
Minimum	-.939	-.826	
Maximum	.712	.899	
Positives	28%	77%	
t-test	-13.8*	15.1*	

NOTE.—In panel A, bidders are ordered from the one with the highest price level (p_n) to the lowest (p_1). For each price level, we report the average (across all auctions) intrabidder dispersion, overall quantity demanded, and award ratio (quantity awarded divided by overall quantity demanded). In panel B, we report the average intra-auction correlation coefficient between price level and dispersion, and between price level and quantity demanded. The *t*-statistic tests the hypothesis that the average correlation is zero.

* Significant at the 5 percent level or better.

auction, we have calculated the correlations between p_{ij} , q_{ij} , and σ_{ij} . The table reports the cross-sectional average correlations as well as the percentage of auctions with positive correlations. We find a significant positive correlation between price and quantity and a significant negative correlation between price and dispersion.

Finally, column 3 of table 8 reports the typical ratio of quantity awarded to quantity demanded for different price levels.¹⁶ Not surprisingly, high bidders have higher award ratios than low bidders. On average, the highest bidder is awarded 89.3 percent of his demand, the second-highest bidder 70.6 percent, and the lowest bidder 13.3 percent. These numbers support the idea of the champion's plague that the more a bidder wins, the worse news it is. It is noteworthy that the average award ratio for the highest bidder is economically close to 100 percent, which would be the award ratio in the single-bid equilibrium. This may

¹⁶ In the few cases in which one bidder demands more than the available volume, we rescale the quantity demanded to equal the auctioned volume.

help explain our finding in Section VD that the single-bid equilibrium explained the magnitude of average discounts so well.

Table 8 confirms what is quite intuitive, namely, that cautious bidding involves reducing prices and demand while increasing dispersion. An interesting question is whether these results could be generated in a model such as the one we used to develop the single-bid equilibrium, where bidders receive one-dimensional signals of equal precision. If so, our results suggest that higher signals map not only into higher prices but also into higher demand and lower dispersion. An alternative hypothesis would be that some bidders have more precise information than others about secondary market demand. We also have found an average correlation between quantity and dispersion of .199 (with 72 percent positive), which suggests the somewhat more nuanced hypothesis that bidders' signals may be two-dimensional. Which of these is the most accurate model is an important question for future research.

VI. Discussion

A. Risk Aversion

One interpretation of our findings is that bidder behavior is driven by private information and rational adjustments to the winner's curse/champion's plague. An alternative view is that dispersion is driven by risk aversion. Discriminatory price auction models by Scott and Wolf (1979), Gordy (1994), and Wang and Zender (1998) show that risk-averse bidders may disperse bids if the stop-out price is uncertain, which it could be in these models because of either exogenous supply uncertainty or private information.

To examine the risk aversion hypothesis, we can look at how bidders respond to auction size. In large auctions, more aggregate security risk must be shared among the bidders. Hence, if risk aversion is a key driver of bidder behavior, we would expect to see discounts and intrabidder dispersion to be increasing in auction size. Looking at the discount regression in table 5, we see that dispersion does indeed increase with auction size. However, the magnitude of the size effect on dispersion is very small. A 1 billion kronor increase in the auction size raises dispersion by only 0.0006 percentage points, which is only 1.3 percent of the mean dispersion in the entire sample. Even more problematic is the discount regression. The coefficient on auction size is negative, which is the opposite of what risk aversion would predict. We therefore feel that it is unlikely that risk aversion is the key driver of bidder behavior in our sample.¹⁷ The reason why risk aversion seems so unimportant

¹⁷ An explicit test of Wang and Zender (1998) gives a similar result.

may be that the auctioned securities are only a small part of primary dealers' portfolios. However, we cannot entirely rule out that there is a small degree of risk aversion among bidders and that this can help explain the reaction of quantity demanded and dispersion to auction size. Alternatively, this may be a result when some bidders face capacity constraints.

B. Conclusion

We have documented that bidders respond to uncertainty along three dimensions: as uncertainty increases, bidders reduce the price levels at which they bid, they reduce quantity demanded, and they increase intrabidder dispersion of their bids. Auction size seems to be a less important factor than price uncertainty in how it affects actual bidder behavior. As auction size increases, bidders increase their demands, but on a less than one-for-one basis. Auction size appears not to affect the price levels bidders bid at and only marginally affects intrabidder dispersion. Our broad interpretation is that the observed bidder behavior is consistent with an adjustment for the presence of the winner's curse/champion's plague. This interpretation is also supported by the finding that the price levels at which bidders bid are in line with the theoretical predictions of the single-bid equilibrium in the common-value model.

But perhaps more important, within an auction, bidders who submit bids at low price levels also tend to demand less and disperse their bids more than high bidders. In other words, cautious bidding involves not only reducing prices but also reducing demand and increasing dispersion. This shows that bidders make use of the expanded strategies that are typically available in a multiunit auction. We have presented evidence in the paper that existing theories of multiunit auctions provide useful guideposts to interpret our evidence.

Our findings also suggest that it will be of interest to build models of multiunit auctions in which endogenous supply uncertainty is present. Many Treasury departments reserve the right to either reject the bids or change the amount that they intend to auction. The data on bidding that empiricists collect presumably reflect optimal adjustments by bidders and the Treasury to reflect this supply uncertainty.

Our inquiry has uncovered an interesting puzzle. We have shown that there is considerable intrabidder dispersion. At the same time, our analysis has not uncovered evidence that bidders act in a risk-averse manner in the way they adjust prices. Although auction theory suggests that it may be a rational response to the winner's curse/champion's plague to reduce quantity and increase dispersion when other bidders also disperse their bids, equilibria in which non-capacity-constrained, risk-neutral bidders disperse their bids have not been demonstrated in the lit-

erature. A challenge for the multiunit auction theory is therefore to establish under what conditions such equilibria exist.

Appendix

Volatility Estimation

We estimate conditional volatility with an ARCH(2) model using daily bond price data. Let P_t be the bond price at time t and A the one-day accrued interest for a coupon bond. We assume that bond returns follow a random walk with constant drift a :

$$\frac{P_t - P_{t-1} + A}{P_{t-1}} = a + e_t$$

The cross-section and time-series data are pooled. The cross section includes Treasury bills with 90, 180, and 360 days to maturity and the seven benchmark Treasury bonds in 1990–94. The volatility of the error term is

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \phi_1 \text{DUR}_t + \nu_t$$

with estimated coefficients in table A1.

TABLE A1
VOLATILITY COEFFICIENTS

α_0	α_1	α_2	ϕ_1
-.0048	.5037	.2264	.0277
(.0002)	(.0261)	(.0176)	(.0007)

When a new security is auctioned, there are no bond prices from the secondary market before the auction. In those cases, we use the prices of the most similar security, that is, the traded Treasury bill, which matures 30 days before the new Treasury bill maturity (57 auctions), and the traded Treasury bond with duration that most closely mimics the duration of the new Treasury bond (seven auctions). When a new Treasury bond is auctioned, we use the average winning auction yield to compute duration.

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