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**Institutions:** Humboldt University of Berlin, Florida State University

**Published on:** 01 Apr 2004 - Economic Inquiry (Blackwell Publishing Ltd)

**Topics:** English auction, Generalized second-price auction, Vickrey auction, Auction theory and Revenue equivalence

# Bidder Preferences among Auction Institutions*

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November 14, 2002

**Abstract**

This study examines bidder preferences between alternative auction institutions. In particular we seek to experimentally characterize the degree to which bidders prefer an ascending auction over a sealed bid auction. We find very strong *ceteris paribus* preferences for the ascending institution with bidders choosing it overwhelmingly often when entry prices for the two auctions are the same. When the entry prices of the two auctions differ, many subjects can be shown to be willing to pay far more to enter the ascending auction than is explainable by their risk attitudes when accounting for their expectations about the risk preferences of their opponents.

**JEL Codes:** C91, D44

**Key Words:** bidder preferences, private values, sealed bid auctions, ascending auctions

*The authors would like to thank various participants in seminars at Florida State University, University College London, Max Planck Institute for Research into Economic Systems, and ESA Meeting 2002 for comments and advice. We thank Mark Isaac and Sabine Kröger in particular for many helpful conversations and suggestions. Financial Support from the *Deutsche Forschungsgemeinschaft*, SFB 373 ("Quantifikation und Simulation ökonomischer Prozesse"), Humboldt-University Berlin, is gratefully acknowledged. Complete sets of instructions for the experiments in this paper are available upon request to the authors.

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# 1 Introduction

Auctions have become a pervasive method of exchange in the on-line world as each day thousands of auctions take place online and this trade volume totals to billions of dollars worth of goods per year[1]. This large volume of auction transactions implies the existence of a large number of sellers competing for buyers. The obvious implication is that the competition among the sellers for the pool of potential buyers can be fierce and any competitive edge a seller can find could be important. One such competitive edge a seller might exploit is using an auction design that attracts bidders away from their competitors.

When designing a real auction or modeling a theoretical one, this entry decision of prospective bidders is rarely considered. Most auction analysis is performed assuming that a certain number of bidders will participate for certain or perhaps that the number of bidders is unknown and randomly determined. It should be clear however, that the most crucial part of a successful auction is encouraging as many bidders as possible to participate. In general this should be expected to have a positive effect on revenue (at least in non-common value environments) and in certain types of auctions it may help to combat the possibility of bidder collusion. Since there are typically competing auctions available for similar goods or even outside options that bidders can pursue when auctions are for unique goods, it is important to understand how the aspects of an auction format can effect a persons decision to enter.

Consider a bidder who is faced with the choice of entering one of two auctions for similar or even identical objects. How does this bidder make his decision of which auction to enter? The obvious answer is that the bidder will enter into the auction that maximizes their expected utility so long as that expected utility is greater than some reservation value. The real question, then, is how are these expected utilities constructed? Profit from participating in the auction is an obvious argument. There are also a number of environmental considerations that might effect this decision that would be difficult to account for precisely such as the reputation and trustworthiness of the

[1]For a survey of the on-line auction activity see Lucking-Reiley (2000).

auctioneer, quality of the advertisement for the auction and things of this nature. It is also possible though that the format of the auction itself can have an impact on the preferences of the bidder. This latter point will be the issue of this study. The particular focus will be looking at bidder preferences between the two most common standard auction formats used in the field; the sealed bid first price (will be abbreviated as just the "sealed bid" auction) and the ascending or English auction. The other reason we are interested in comparing these two auction formats rather than between the ascending and second price or first price and descending is that it seems reasonable to expect bidders to have preferences between the two due to the strategic differences between them. Such differences lead to substantial differences in terms of the difficulty of deciding how to bid and also in the possibility that an outcome that leads to regret as in a first price auction one can lose to a bid that is below one's value potentially causing a bidder to regret having bid so low while this should not happen in an ascending auction.

If one considers the situation of a bidder choosing between two auctions that differ only by whether the auction is being conducted according to an ascending or sealed bid format, it is not immediately obvious which would be the most preferred even assuming a standard symmetric independent private values environment. Were the bidders risk neutral, then of course revenue equivalence would hold and the bidders would be indifferent. If the bidders are risk averse, the situation is more complex. As shown in Milgrom and Weber (1982), RA bidders will bid higher in a sealed bid auction than risk neutral bidders and therefore expect to make a lower surplus than if they participated in an ascending auction where they will bid identically to risk neutral bidders. That would imply a preference for the ascending since the surplus if they win is higher. On the other hand, the surplus in the ascending auction is more variable than the surplus in the sealed bid and a RA decision maker dislikes a variable outcome causing the sealed bid to be more attractive. These conflicting attractions lead to a lack of a general conclusion about which format a RA bidder would prefer. Matthews (1987) presents a solution to this dilemma by showing that if a bidder possesses decreasing absolute risk averse (DARA) preferences, they will prefer the ascending, increasing absolute risk averse (IARA) the sealed bid and constant absolute risk averse (CARA) preferences will lead to indifference. These results will serve as a useful back-drop to the analysis below.

There are several other empirical and theoretical papers that look at the issue of endogenous entry choices in regard to auctions, Bajari and Hortacsu (2000), Lucking-Reiley (1999), Harstad (1990), Engelbrecht-Wiggans (1993), Levin and Smith (1994), McAfee (1993) and McAfee and McMillan (1987) to name a few. These papers are examining entry decisions in a context quite different than we are concerned with here. In general, they are looking at either the decision of whether to enter an auction or not, or at the choice of which auction to enter based upon the entry price or reserve price being the main or even only characteristic upon which the auctions differ. Our point of concern is to look at the entry decision when the main characteristic that distinguishes two auctions is the format being used to conduct the auction and examine the preferences that underlie those decisions. We note that in Klemperer (2002), the author discusses such preferences and proposes that ascending auctions can actually discourage disadvantaged or weak bidders from entering. We only consider symmetric bidders so the relative strengths of bidders is not an issue. Klemperer's claim is investigated in Goeree and Offerman (2002) where they find that if weak bidders are allowed to choose sequentially among themselves between entering an auction or playing a fixed lottery, they tend to enter first price auctions more than ascending.

The most closely related prior study is Ivanova-Stenzel and Sonsino (2001). In this paper the authors conduct an experiment comparing the outcomes in a "one-bid" auction, i.e. a standard sealed bid first price auction, to a "two-bid" auction, a modified version where subjects may submit two bids: a high bid and a low one and the winner pays his low bid if this was higher than all other bids. One of the issues they examined was which auction format the bidders preferred. They accomplished this by allowing bidders in one part of the experiment to repeatedly choose between participating in a one-bid auction competing with one other bidder or a two-bid auction against one other player. The results showed a strong preference for the two-bid auction.

The one shortcoming in the methodology in Ivanova-Stenzel and Sonsino (2001) as a means of measuring bidder preferences is that it only allowed for comparing auctions on what might be called a *ceteris paribus* basis. That is, with all things being equal, which auction would the bidders prefer? If one wishes to use the results from a study of this sort to argue in favor of adopting a new design, it would be useful to take the investigation a step further and measure the intensity of this preference or find an answer to the question "how much are bidders willing to pay for their preferred auction format?" The current study will extend the methodology in Ivanova-Stenzel and Sonsino (2001) to explore this additional question in the context of bidder preferences between the one-bid version of the first price auction and the standard ascending or English auction.

The outcome of these experiments will show the existence of very strong preferences for the ascending auction. When subjects are given a choice between the first price and the ascending auction on a *ceteris paribus* basis, they overwhelmingly choose the ascending auction. As expected, the surplus achieved by the winners in the ascending auctions far exceeds the surplus obtained in the sealed bid auctions. When the subjects are asked to pay to get into an ascending auction, however, some are evidencing a willingness to pay that far exceeds what appear to be the most appropriate theoretical predictions.

Section 2 of the paper will explain the design and conduct of the experiments. Section 3 contains the analysis of results and section 4 will conclude. There is also an appendix to the paper which contains some technical details relating to a few of the computations made in the analysis.

## 2 Design of Experiments

The experiments for this study were divided into two distinct phases. In the first phase, the learning or training phase, the subjects played both a sealed bid and an ascending auction for 10 consecutive rounds with each auction being conducted with two bidders. Each round consisted of the bidder playing one of each auction type with the same value. The bidders possessed private values which were randomly drawn from the set $V = \{0, 1, 2, , ...99, 100\}$ with all values $v_i \in V$ being equally likely. After seeing their value draw, subjects were asked to submit their bid to be used in the sealed-bid auction. Subjects could choose integer bids between 0 and 150, which did allow them to overbid their highest possible value. All values were denoted in a fictitious currency termed ECU for Experimental Currency Unit. Before bidders were informed about the results of the sealed bid auction, they participated in a Japanese or ascending clock auction[2]. At the end of the round the bidders observed a feedback–window specifying the results from both auction formats indicating whether or not they won, the price paid by the winner in each, the private value of the buyer, their own profit in the auction and their total profit in the current round. They were not given cumulative profit numbers, only numbers from the current round. There were 10 participants in each experiment session and in each round, subjects were randomly re-paired to bid against a new opponent. In a given round, subjects competed against the same opponent in both the ascending and sealed bid auctions.

The idea for this phase was to allow subjects time to figure out how to bid in these auctions as well as to understand the formats well enough for them to form preferences between them. The reason for having subjects play both auctions with the same value was an attempt to minimize any negative impressions a bidder might receive about an auction format due to a random series of bad draws on one format while getting good draws in the other. In four out of the six sessions, at the end of the learning phase there was a summary screen detailing the average profit achieved by the winner across both auction types. This screen was eliminated in two of the sessions. The purpose of including this information screen was to aid subjects in learning about the average actual profitability for participating in the two mechanisms and it was removed in the two sessions to determine if it had any effect.

In the second phase of the experiment, the preference assessment phase, the participants played an extended auction-selection game for 30 rounds. In a single round of this phase, bidders were asked to choose to enter either an ascending or sealed bid auction, knowing that regardless of which they chose they would be competing against one other bidder[3]. In each round, both auction formats

[2]The price started at 0 and began increasing at the rate of 1 ECU every 2 seconds. The auction concluded when one of the bidders clicked on a button to indicate they were withdrawing from the auction with the remaining bidder winning the auction at the price the first bidder dropped out at.

[3]To guarantee that an even number of subjects participated in each mechanism, only 9 out of the 10 participants were able to choose an auction type in each round. The 10-th participant was automatically assigned to whichever

had an entry price attached to choosing them which the bidder had to pay regardless of whether or not they won. This choice of which auction to enter was made before observing their realized private value for the auction. In the first 10 rounds of this phase, the entry prices for both auction formats (sealed bid and ascending) were the same (1.40 ECU). The preferred auction design was identified for each individual as the one they chose in at least 5 rounds of these 10 rounds. In the remaining 20 rounds, the entry price for the preferred auction format was varied in each round using a grid consisting of entry prices ranging from .7 to 14 ECU with an increment of .7. This range was decided upon based on two pilot sessions to identify a reasonable range of values that yielded fine enough resolution to identify bidder preferences while still being wide enough to allow the observation of the maximum willingness to pay of most subjects. To avoid the possibility that the subjects would see the experiment as a simple grid exercise and become bored or disinterested, the grid was not presented in an ascending order, rather the order was randomized. To make the grid structure even less apparent, we added an $\epsilon$ to each element of the grid, where $\epsilon$ is a random variable normally distributed on the range $(-.05, .05)$.[4] There are other ways of eliciting a subject's willingness to pay such as running a second-price sealed bid auction for the right to enter each format that some might be inclined to find more straightforward. There are, however, two main advantages of our approach. The first is that it allows us to conduct consistency checks on the elicited WTP through using this randomized grid. We are also able to use our results to specifically test the effect of entry prices on entry which is in itself an important issue for applied auction design.

After subjects made their choices concerning the auction type, the round was played with 20% probability. This was a session wide determination, not specific to any particular player. At the end of each auction that was actually conducted, subjects were informed whether or not they won the auction, the price paid by the winner, the entry price they paid, their private (reselling) value

auction type had an odd number of people selecting it. The identity of the "10-th" player was changed in each round, so that each subject played the balancing role once every ten rounds or three times among the 30 rounds.

[4] The actual entry price order all subjects saw was {8.39, 2.10, 0.70, 4.92, 12.61, 1.42, 6.27, 4.20, 9.79, 11.15, 13.27, 5.59, 11.90, 9.07, 2.80, 10.49, 7.01, 3.50, 13.98, 7.74}.

and their own payoff in the current round. Note that an entry price was only charged to a subject if the auction was conducted. In the rounds in which no auction was conducted, no entry prices were charged.

All experimental sessions were conducted with the use of a computer based software system, created with z-Tree (Fischbacher (1998)). All experiments were conducted at Humboldt-University, Berlin and most participants were students of economics or business administration. They had been invited by leaflets to participate in an experiment announced to last about two hours which turned out to be approximately accurate. The conversion rate of the ECU earned by each subject into cash was: 1 ECU = 0.04 EUR or about US$0.035 (at the time the experiment was conducted). In addition, subjects were paid a fixed participation fee of 2.50 EUR or about $2.20. Subjects' total earnings ranged between 7.35 EUR ($6.47) to 26.60 EUR ($23.41) with a mean of 16.83 EUR ($14.81).

# 3 Results

There are three basic questions that arise from these experiments which are 1. What did people prefer?, 2. How much were they willing to pay for that preference? and 3. What can account for that willingness to pay? Each of these will be answered in order.

## 3.1 Which institution did subjects prefer?

When the entry prices for the two auction institutions were equal, subjects overwhelmingly preferred the ascending auction. There was only one subject choosing the sealed bid exclusively while 39 chose the ascending exclusively. In fact only 5 out of the 60 subjects chose the sealed bid more often than the ascending. The average number of times the ascending was chosen was 7.87 with a median of 9 while the numbers were 1.13 and 0 for the sealed bid.

## 3.2 How much were they willing to pay?

There are several different ways to look at how much the subjects were willing to pay for their most preferred auction. Since only five subjects evidenced a preference for the sealed bid, we will be ignoring willingness to pay for it and will concentrate on analyzing willingness to pay for the ascending.

One characterization of this would be to construct a pseudo demand curve showing how many people are willing to pay each possible entry price for the ascending auction. Such a construct is shown in figure 1. This pseudo demand curve exhibits the standard characteristics of a normal demand curve. It shows that as the price rises, fewer people are willing to pay for the ascending auction. It is a bit jagged, however, indicating that there are some bidders making choices that are not purely monotonic. For example, a subject may have declined to pay a price of 2.1 for the ascending auction, but agreed to pay a price of 2.8 or 3.5. This is also partially an artifact of not allowing one person to choose at each price. For example, at one point we may observe 35 subjects willing to pay a price of 2.1 and 36 willing to pay 2.8 because the person held out at 2.1 was willing to pay that price and higher, then at the price of 2.8, he was allowed to choose and accepted that price while one of the people not willing to pay at 2.1 were now held out.

Theoretically, expected profit would have been the same between both institutions had the subjects bid as risk neutral expected utility maximizers. In reality, average profit to winners from the ascending auctions was 37.02 and 19.78 for sealed bid[5]. This implies a difference of 17.24. If subjects expected to win half the time, this implies an expected profit differential of 17.24/2= 8.62. The theoretical average profits, or the profits that would have been obtained had bidders bid according to risk neutral Nash equilibrium bidding strategies, were 37.40 and 33.63 (37.64 and 33.28 for phase 1)[6].

[5] These numbers are average profits to winners from all auctions excluding any entry price payments, if we just look at phase 1 numbers, the results are 37.02 and 19.32.

[6] The theoretically expected profits for the ascending auction are a bit higher than they should be. This was due to a degree of correlation somehow getting into the values for the two bidders. It is unlikely this was detectable by the subjects and should have had little impact on their choice behavior. The most likely impact would have been to increase the level of the price that leads to a switch-over from the ascending auction to the sealed bid, but results

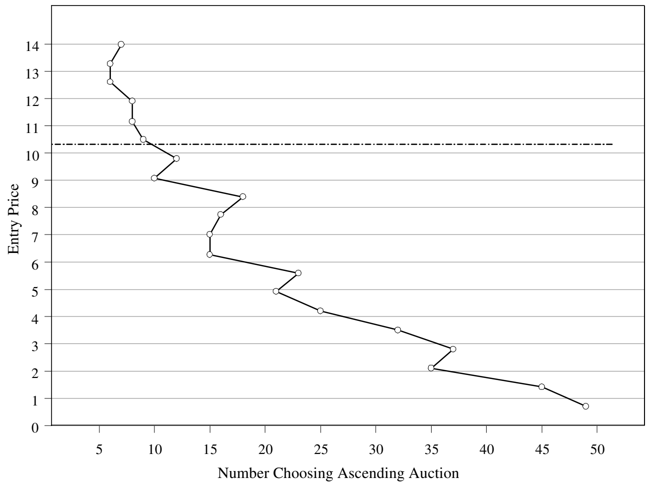


**Figure 1:** Pseudo demand curve indicating the number of subjects choosing the ascending auction format at each entry price.

If bidders were concerned only with expected profitability, they should have been willing to pay up to 8.62 on top of the entry price for the sealed bid auction, 1.4, or a total entry price of 10. 02 to be in the ascending versus the sealed bid auction. The horizontal dotted line in the demand curve graph represents this cut-off price. At this line, expected profits from the two formats are equal. Below this line, the subjects will be making less on average from participating in the sealed bid auction. This indicates that fewer than 10 out of 60 people were willing to pay more than this to participate in the ascending auction, most were only willing to pay far less.

To get a more precise picture of the willingness to pay of the subjects requires estimating their switch-over price or the price at which the subjects would switch from choosing the ascending auction to the sealed bid. The best way to understand this process is to visualize the price of the two auctions starting off the same, subjects choosing the ascending, and then the price of the ascending slowly rising. At some point, the subject will being choosing the sealed bid. We want to find the price that best describes the point at which each subject finds the ascending auction no

will show that experiential variables such as this have no effect on the switch-over price.

longer worthwhile to choose. To to that we will propose that there exists some price differential, $\delta_i$, that will lead to the subject to switch from choosing the ascending to the sealed bid. They will bid on the ascending so long as the actual price differential $p_t(A) - p_t(SB)$ is less than this threshold. If we make a reasonable specification of probabilistic, rather than deterministic choice, $\delta_i$ can be estimated for each subject by finding the $\delta_i$ that solves the following:

$$\max_{\delta_i} \sum_{t=1}^{T} [2\rho^*(t) - \sum_{j=1}^{2} \rho_j^2] \quad (1)$$

$$\rho(c_t) = \begin{cases} \frac{e^{\delta_i - (p_t(A) - p_t(SB))}}{1 + e^{\delta_i - (p_t(A) - p_t(SB))}} & if \quad c_t = A \\ 1 - \frac{e^{\delta_i - (p_t(A) - p_t(SB))}}{1 + e^{\delta_i - (p_t(A) - p_t(SB))}} & if \quad c_t = SB \end{cases}$$

This specification is essentially minimizing the mean squared deviation of the predictions. Since many of the predicted probabilities will be close to 0 and 1, this specification should be expected to be superior to a standard log-likelihood specification[7].

The $\delta_i$ for each subject represents the price differential that will make them prefer to choose the sealed bid institution. To obtain the actual price for the ascending auction at which this switch-over should be observed we must add 1.4 as this is the static price for the sealed bid auction. The results from such an estimation are summarized in figure 2. Note that the few negative observations in the graph represent those bidders who preferred the sealed bid auction in periods 1-10. The average price that lead people to switch from the ascending to the sealed bid is 5.95 (6. 61 considering only those evidencing a preference for the ascending auction in periods 1-10) while the average profit difference between the two institutions was 8. 62 leading to a implied switch-over price of 10. 02 if the subjects were only concerned about average profits. This leads to the same conclusion that was implied by the pseudo demand curve above which is that subjects were willing to pay significantly less than would be implied by the expected payoff differential alone. The expected

[7] See Selten (1998) and Friedman (1983) for a discussion of the problems of using a log-likelihood function for this sort of a problem.

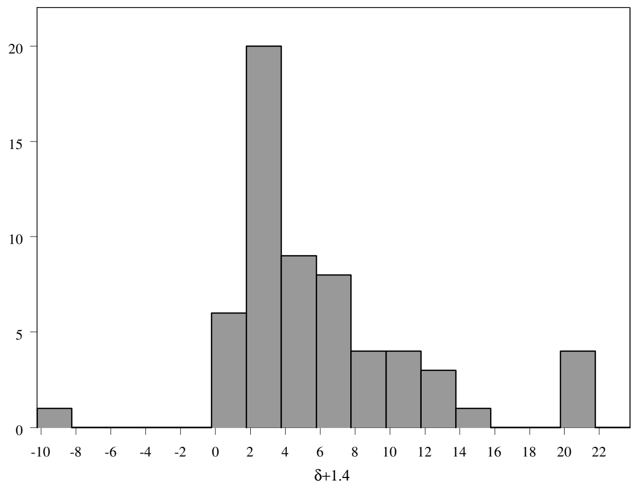


**Figure 2:** Estimated price at which subjects chose to switch from ascending to sealed bid.

payoff differential is of course a very crude measure of what a bidder should be willing to pay and will be improved on in the next section.

Based upon the demand curve seen in figure 1 it is clear that the subjects were not displaying purely monotonic preferences. It is important then to get some characterization of the degree to which the choices of the subjects were consistent and purposeful instead of random. If bidders were perfectly consistent in their choices, we would expect to see one of two patterns to their choices. One is maintaining a constant choice throughout the second phase, such as a choice path consisting of all A's. A second would be choosing the ascending auction up to some price and then switching once and for all to the sealed bid, which we might represent as an A-SB path. It might also be reasonable to find that preferences are somewhat probabilistic and when the two auctions are roughly equal in expected utility the subjects choose randomly. This would lead to a reasonable expectation that say for a price of 2.8 a bidder is observed choosing the ascending, 3.5 sealed bid, switching back to the ascending at the next price and sealed bid at the one above that and then staying constant for the rest. This would be three switches and can be described as an A-SB-A-SB

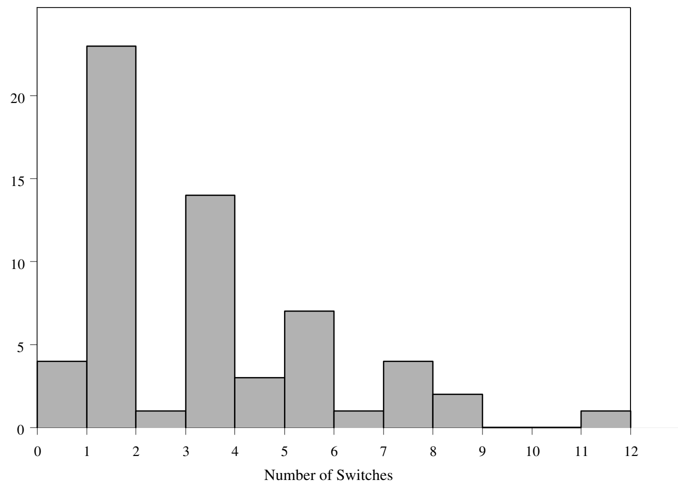


**Figure 3:** Histogram showing the number of subjects switching each number of times. Note that the mass on a number is shown to the right of it.

path. In fact the median number of switches was 3 (average 2.92, mode 1) with the histogram in figure 3 detailing the full distribution of the number of times subjects switched their choice from one auction format to the next as the entry price increased. The way this graph is set up, the mass or line to the right of a number is the number of subjects switching that number of times.

The way to interpret this graph is that the more switches a person has, the more random is their decision making. The distribution follows a relatively standard exponential decay with half of the mass on 1 and 3 and then trailing off sharply after that. Those switching more than 5 or 6 times are choosing fairly randomly and there are a very small number of even switches.

Of course, observing even just three switches does not necessarily imply roughly consistent choices as described above since three switches may also be the result of a far less consistent choice sequence. An example would be something like the following:

| **Entry Price** | 2.8 | 3.5-7.7 | 8.4-9.1 | 10.5- |
|---|---|---|---|---|
| **Choice** | A | SB | A | SB |

Whereas the initial example of a three switch bid path seems perfectly reasonable and consistent with a probabilistic choice model, this one is a little more difficult to rationalize that way. Why would a person not pay 3.5 for the ascending auction but pay 8.4? If most of the switching behavior looked like this second example, it would be difficult to conclude that subjects were being systematic about their choices. As it turns out, of the 14 subjects who switched three times, that is had an A-SB-A-SB path, 7 chose the ascending only once in the second A phase while 5 chose it twice.[8] Either of which are fairly consistent with the idea of a random mistake.

Another measure of consistency could be subjects' choices in the second part of the preference testing phase for the entry price that was less than 1.4. Recall that in the first 10 periods, the entry prices for both auction types was 1.4 and the static price of the less preferred auction remained 1.4 in the next 20 periods. If a subject evidenced a preference for the ascending auction for equal prices, consistent choice behavior would imply that they make the same choice if the entry price on the ascending auction is lower than the entry price on the sealed bid. There was one such entry price in the grid to check for this property and this held true for 94% of all subjects.

A final measure of consistency of choices can be derived by looking at how many times the estimated $\delta_i$'s predict the choice of the subject accurately. The mean number of correct predictions is 24.7 and median is 25. The number of choices each bidder had was 27. So for half of the subjects, our estimate is only misses at most 2 out of 27 choices. Overall we can conclude that the observed choice sequences are in most cases purposeful and consistent with a probabilistic choice model.

### 3.3 What can account for the observed willingness to pay?

There are a couple of obvious things to check to see if they can account for the observed differences in willingness to pay. One might think that preferences over auction institutions would be formed by outcomes from the learning phase of the experiment. Subjects who ended up with higher profits

[8] For the two remaining subjects with three switches as well as the three subjects with four switches no reasonable explanation was found. Those switches were not influenced either by the fact that the auction before the switch was actually played or if the subjects had won the previous auction (if it was played) or not (which results in negative payoffs).

| | Value | Std. Error | t-value | Pr(>\|t\|) |
|---|---|---|---|---|
| **Intercept** | -15.32 | 20.03 | -0.76 | 0.45 |
| **Treatment** | -3.94 | 1.94 | -2.03 | 0.05 |
| **Avg Sealed bid Profit** | 0.11 | 0.29 | 0.37 | 0.71 |
| **% Sealed bid wins** | -0.57 | 6.25 | -0.09 | 0.93 |
| **Avg A Profit** | 0.15 | 0.26 | 0.60 | 0.55 |
| **% A wins** | -2.25 | 10.29 | -0.22 | 0.83 |
| **Session Sealed bid Profit** | -0.82 | 0.87 | -0.95 | 0.35 |
| **Session A Profit** | 1.70 | 1.34 | 1.27 | 0.21 |
| $\overline{R}^2$ = **0.134** | | **F-Stat 1.16** | **p-val 0.34** | |

**Table 1:** Regression results of regressing these parameters on observed switch-over price.

in the ascending auction may have been more willing to pay for it or perhaps those who won more often in the sealed bid auction would be less willing to pay for the ascending auction and so forth. While the design of the experiment attempts to control for these effects, the correlations between the outcomes of the two auction types are not as perfect as one might expect (coefficient of correlation is .65 between average profit in both institutions and .57 for probability of winning). Another likely possibility would be the treatment effect of whether or not the subjects saw the summary statistics from the results in the training phase. While it is unclear how this might effect the outcome, it seems possible that it could. The results from a regression of such things on the observed switch-over prices, $\delta_i + 1.4$, are contained in table 1.

As the table shows, only the treatment variable is potentially important as all the others are statistically insignificant. As it turns out though, the hairline significance of the treatment variable is being driven by a few outliers. By chance, the one very low willingness to pay and the few very high ones were observed in different treatments. If these observations are left out, the significance disappears. As already noted, some of these regressors are strongly correlated but the significance of each parameter does not change if this regression is performed with subsets of the variables to eliminate the problem. The combined regression was presented simply for compactness of presentation.

These results show that the heterogeneity in observed willingness to pay does not seem to be derived from the fact that different bidders experienced different results during the training phase. That is actually an encouraging result as theoretically, these variables should not have any impact on these decisions. This leaves us with the likelihood that the observed differences were based upon unobserved heterogeneity in preferences of the bidders rather than upon the observed heterogeneity of their experiences. One possible source of this preference heterogeneity is in the risk preferences of the subjects.

A casual examination of the bids observed in the sealed bid auctions, reveals the standard pattern observed in most sealed bid auction experiments which is bids far in excess of those predicted by the risk neutral Nash equilibrium. This might suggest that bidders possessed some form of risk averse preferences as represented in their bidding behavior and this risk aversion may have influenced their choice of auction formats. Since the constant relative risk averse (CRRA) utility function, $u(x) = x^{\alpha}$, also satisfies DARA, we would seem justified in the use of this utility function to represent the preferences of the bidders since as we previously discussed Matthews (1987) shows that bidders possessing DARA preferences will prefer the ascending auction just as our subjects did. Using this utility function, we can then estimate the risk preferences of the bidders based upon their bids in the sealed bid auctions and generate predictions of the switch-over prices that bidders would have had, were their choices guided by the same risk attitudes they exhibited in their bidding behavior. This predicted switch-over price is computed by finding an entry price for the ascending auction that makes the expected utility of participating in the two different auction formats equal given that the entry price for the sealed bid auction was 1.4. Due to the length of the equations for performing these calculations, they and partial derivations can be found in the appendix.

There are two important issues involved in performing these calculations. The most obvious involves the fact that when someone loses an auction yet still pays an entry price, their utility is $(-e)^{\alpha}$ where $e$ is the entry price they paid. Since taking roots of negative numbers leads to problems, we must use some measure of wealth to add into the utility function to insure a positive argument. What to use for this measure of wealth is certain to be a controversial issue. We will primarily use two different measures of wealth with the first being the cash balances of the subject at the end of the first phase and the second being that amount divided by ten. This latter measure is essentially constructed to be the smallest wealth measure we can use, the amounts typically correspond to about \$0.50-\$1.00, and still be able to evaluate the equations. We will refer to this as the "no wealth" case for convenience as this is the closest approximation to that situation that can be obtained.

The second issue deals with the expectations subjects have regarding the risk attitudes of their opponents in sealed bid auctions. We will examine two specifications with the first being that subjects assume any potential opponent will have the same risk preference as themselves (the "equivalent opponent" case) and the second in which they assume their opponent will possess the average degree of risk aversion in the population (the "average opponent" case). The equivalent opponent case might be taken as a simple heuristic for when the subject has no information on likely opponents and therefore assumes they are the same as themselves. Since the auction choice behavior occurs after observing aggregate results from the first phase of the auction though, it might be more reasonable to assume that bidders have developed some intuition about the degree of risk aversion of likely opponents or even just the average rate at which they bid below value. While we could use the actual empirical distribution as their beliefs for this case, that would be overly cumbersome and likely not obtain better results than modeling subjects as though they assume their opponent possesses the average level of risk aversion in the population.

As derived in the appendix, the bid functions for sealed bid auctions under these two belief systems are largely the same, $b(v) = \frac{1}{1+\alpha}v$. The only difference comes in when bidders who are more risk averse than the average would be bidding more than the maximum the person with average risk aversion would be willing to bid, $b(100) = \frac{1}{1+\bar{\alpha}}(100)$, allowing $\bar{\alpha}$ to represent the average level of risk aversion in the population. The key difference in these two cases that will drive the results is that the expected probabilities of winning for a given value are different between the two cases. In the equivalent opponent case, a bidder's expected probability of winning is just $F(v)$. Since the bid function is monotonic in value, they win if their value is higher than their opponents. If they believe they are facing someone of average risk aversion, $\bar{\alpha}$, then the bid function is no longer purely monotonic in value. Someone with a lower value could bid higher than someone with a higher value if the former is more risk averse. Thus the expected probability of winning becomes $F(\frac{1+\bar{\alpha}}{1+\alpha}v)$. Those more risk averse than average ($\alpha < \bar{\alpha}$) will expect to win with higher probability relative to $F(v)$ and those less risk averse expect to win with lower probability.

Risk aversion parameters can be estimated by running a standard OLS regression with the equation $b_i = \beta + \gamma v_i$ where $\gamma = \frac{1}{1+\alpha}$ as is developed in Cox, Roberson, and Smith (1982) and Cox, Smith, and Walker (1988). Doing so yields a distribution of risk aversion parameters with an average of 0.66[9] which is what will be used as $\bar{\alpha}$ in the calculations below. For the equivalent opponent model this is a correctly specified estimation. For the average opponent case, the estimated bid function is an approximation only, due to the existence of the flat region for bids above $\frac{1}{1+\bar{\alpha}}(100) = \frac{1}{1+.66}(100) = 60.241$. A bidder will be predicted to bid 60.241 anytime their value is such that $\frac{1}{1+\alpha}(v) \geq 60.241$. For some extremely risk averse bidder, say $\alpha = .3$, this threshold value is, e.g., 78.313. For only about half of the subjects is this ever an issue, those more risk averse than average, and even for them, this flat portion of the bid function covers a relatively small range of the value space. Thus the bias introduced by ignoring this in the estimations should not be expected to be severe while keeping the same risk aversion values throughout the analysis aids in continuity of exposition.

One might suggest that since we have put wealth into the utility function for auction choice, we should also have wealth in the utility function for bidding in sealed bid auctions and thus for estimating risk aversion. We have done so, but will not present either the methods or details of the results here to conserve space. We have chosen to omit the results for two reasons. First is that the general nature of our conclusions do not appear to change. Second, the average degree of risk aversion is estimated to be -19.05 (requires changing the utility function to $u(x) = x^{\alpha}/\alpha$ to allow for negative values of $\alpha$) assuming subjects consider their wealth to be their cash balance at the time of the bid plus show-up fee in the experiment. We believe this is simply an incomprehensibly large

[9] As a technical note, there were only 8 out of the 60 bidders who possessed intercepts that were statistically significant at the 5% level. Excluding these 8 yields an average $\alpha$ of 0.60.

degree of risk aversion and indicates a specification bias in the utility function from the addition of wealth. This is related to, though importantly not the same as, the problems with the standard risk aversion model discussed in Rabin and Thaler (2001)[10].

Based on the discussion above, we could construct four different models for generating predicted switch-over prices by combining the two different wealth specifications and two different belief specifications. We will concentrate on only two of those and just describe generally the results from the other two. The first model we will examine is the Wealth-Equivalent Opponent model and the second will be No Wealth-Average Opponent model. We can treat these as assumptions about how subjects form their decisions and compare the predictions of these two models to their actual decisions to determine which is a better model of subject behavior.

Using the Wealth-Equivalent Opponent model to generate predicted switch-over prices yields results that can be seen in figure 4. This figure shows a scatterplot of the actual observed switch-over prices, the previously estimated $\delta_i + 1.4$ values, plotted against the estimated levels of risk aversion with a regression line. Overlaid onto this is the scatterplot of the predicted switch-over prices according to this model against the estimated risk aversion levels. This graph and the subsequent analysis leaves out those 5 subjects who indicated a preference for the sealed bid auction in periods 1-10 and those few bidders who were found to have risk loving preferences or $\alpha > 1$. Excluding both sets of bidders leaves 46 in the sample set. The relationship between risk aversion and predicted switch-over price is negative as bidders who are highly risk averse should expect to make very little money in a sealed bid auction should therefore be willing to pay more to enter an ascending auction. A regression of the predicted switch-over price on the estimated risk aversion parameter, $\alpha$, yields

[10]It is important to note that the specific problem reported in Rabin and Thaler (2001) does not actually apply to the risk aversion model as applied to bidding behavior. The easiest way to see this is that the loss aversion model proposed to "fix" the problem is the same as the risk aversion model we have used in this context as losses are not possible. More generally, both the results reported here and those in the Rabin and Thaler (2001) paper seem to suggest that the more reasonable interpretation of both is not that the risk aversion model is not applicable but rather that the problem is in the assumption that people always consider their external wealth position in any decision. Removing this assumption leaves us with reasonable levels of risk aversion here and would do the same for the examples in Rabin and Thaler (2001). Of course a complete discussion of this issue goes well beyond the scope of this paper and those unconvinced by this short note are directed to Cox and Sadiraj (2002) for a more in-depth examination of these issues.

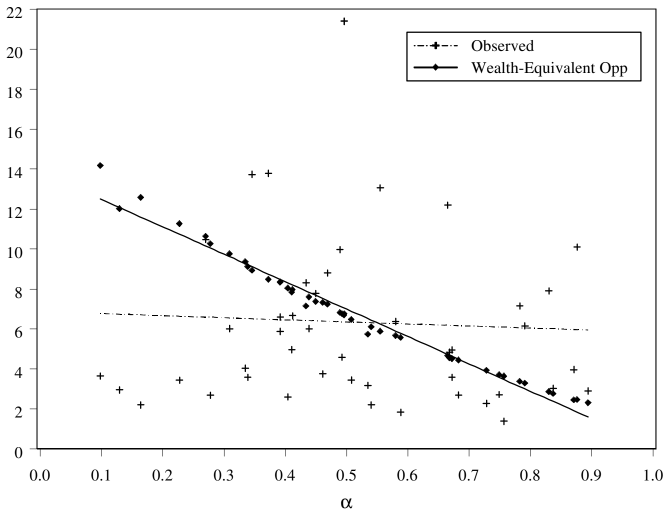


**Figure 4:** Scatterplot of both observed and predicted switchover prices using the Wealth-Equivalent Opponent model against estimated risk aversion parameter for each individual.

a highly significant coefficient of -13.72[11].

Comparing these results to the relationship between the observed switch-over prices and the estimated risk aversion levels should make it clear that the Wealth-Equivalent Opponent model does not explain the data well at all. Recall that these were the prices at which the bidders actually did switch from choosing the ascending auction to the sealed bid. A regression of the observed switch-over price on the estimated level of risk aversion yields a coefficient of -1.03 which has a resulting p-value of 0.76[12] indicating a lack of significance. These results indicate that there is no observed relationship between actual switch-over prices and estimated degree of risk aversion. Further, there are very few bidders who possessed switch-over prices that were predicted well by this model.

Figure 5 shows the corresponding scatterplot from generating the predicted switch-over prices with the No Wealth-Average Opponent model. The primary difference in the predicted relationship

[11] The t-statistic on this coefficient is -42.42 and p-value is 0.0000. For completeness, the value of the intercept is 13.85 with a t-statistic of 76.75 and p-value of 0.0000.

[12] The intercept in the regression had a value of 6.87, t-statistic of 3.66 and p-value of 0.0007.

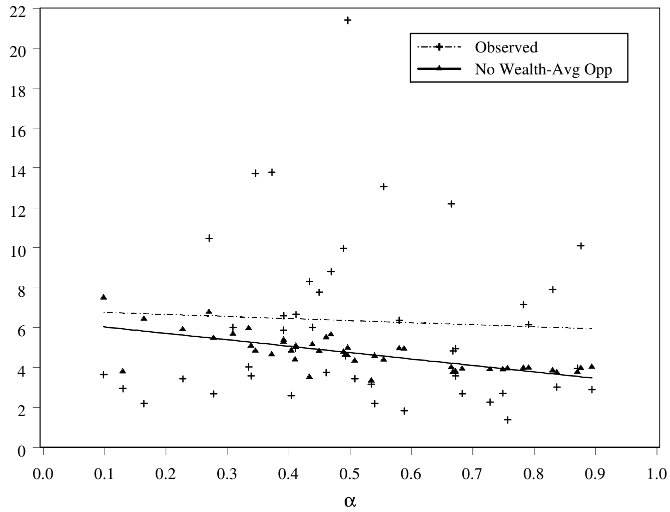


**Figure 5:** Scatterplot of both observed and predicted switchover prices using the No Wealth-Average Opponent model against estimated risk aversion parameter for each individual.

from the previous one is that the regression line is rotated significantly to the left, making the overall relationship much flatter. The regression coefficient on the risk aversion parameter is now -3.21 with a P-value of 0.0000. Bidders who are quite risk averse are predicted to be willing to pay much less while bidders close to risk neutral are predicted to pay a little more. The reason for this difference is found in the different expected probability of winning resulting from the change in beliefs. For bidders who are quite a bit more risk averse than the average, they expect their probability of winning to be high. While they expect to make little surplus, the expected probability of getting the surplus is very high and this causes the sealed bid auction to be very appealing relative to the more risky yet lucrative ascending auction. Thus even highly risk averse bidders would not be willing to pay much to enter into the ascending auction. Overall 24(11) of the 46 bidders have predicted and actual switch-over prices that are different by less than 2(1) ECU according to the No Wealth-Average Opponent model which compares to only 17(10) for the Wealth-Equivalent Opponent model. Formal significance tests of these relationships are difficult due to the derivation of the predicted switch-over prices, but this comparison should be indicative that the NoWealth-

Average Opponent model would perform a bit better.

For completeness, we can point out that the predicted switch-over prices derived from the other two potential models (No Wealth-Equivalent Opponent and Wealth-Average Opponent) are practically identical to each other and lead to regression lines that are about half way in rotation between the other two.[13]

From examining these figures it would appear that the No Wealth-Average Opponent model works fairly well at matching the behavior of one class of the subjects, but there is another class of subjects who exhibit a willingness to pay far in excess of what their risk preferences alone would predict.

Combining these results with the results from estimating risk aversion levels allowing for bidders to consider their wealth position seems to indicate support for the hypothesis that bidders tend to ignore their wealth position when making both bidding and auction entry decisions. At first glance this would appear to be contradicted by the results found in the more careful analysis of the effect of wealth on bidding behavior in Ham, Kagel, and Lehrer (2002). While an analysis of the bids in our experiments also demonstrates a statistically significant relationship between cash balances and bidding behavior it is either a: much smaller than would be predicted by theory assuming bidders consider only the wealth earned in the experiment or b: much larger than would likely be predicted by theory assuming bidders consider the entirety of their external wealth. The first is supported by the fact that when wealth is 0 or small as it would be in the first part of an experiment, bidders with negative risk aversion parameters, as are found when wealth is in the bid function, either bid above their value for values of $-1 < \alpha < 0$ or below 0 for $\alpha < -1$, with the bid level changing significantly as wealth is accumulated. None of these predictions are observed to any significant degree. The latter claim is supported by the fact that if a subject has several hundred dollars in external wealth, the dollar or so at stake in each auction would be trivial in comparison so the impact on total wealth and thereby the impact on bidding behavior of winning a dollar in the

[13] We can also note that if we use a wealth specification of the subjects' phase I wealth multiplied by 100 to capture the possibility that the subjects consider their external wealth in making these decisions, the corresponding regression lines rotate to the right and shift up, getting farther from matching the data.

experiment should be virtually undetectable. This suggests that while bidders likely do somehow allow their cash balances to impact their behavior it does not appear to do so in quite the same way or degree as this model predicts. Thus we do not view our results and those in Ham, Kagel, and Lehrer (2002) as contradictory on this matter, as we are not suggesting that there is truly no impact on decisions from wealth, only that the impact is in a different form or degree than would be appropriate to model in this structure.

The additional result appears to be that subjects do form expectations in regard to something that approximates the degree of risk aversion of their opponent and this impacts their choice of which auction to enter. We, of course, find it highly implausible that the subjects view their beliefs as beliefs about the likely risk aversion of their opponent, but we do find it plausible that they form beliefs about the likely bidding behavior, perhaps bid/value ratio, of their opponent for which we are able to use risk aversion as a suitable proxy. This is quite reasonable since after each sealed bid auction they are informed about both the bid and value of the winner. We are certainly aware of and sympathetic to the many problems that have been noted in the literature in regard to using risk aversion as a model of behavior in auctions, but it is the simplest way of specifying a common decision structure across both choice environments and seems to fit with the observed behavior quite well.

A plausible explanation for the fact that many bidders appear to be willing to pay more for the ascending auction than they "should" is that for various reasons participating in the sealed bid auction incurs a certain amount of disutility. This disutility could come from the extra effort that is required to figure out how to determine a bidding strategy, the mental anguish over seeing someone else win with a bid lower than your value or any number of other factors.[14] We can use the results above to obtain an estimate of this disutility. We will model this disutility using a parameter $\lambda$ that will be considered to be the equivalent of an extra entry price for the sealed bid auction in the utility function. Thus when a bidder wins a sealed bid auction, their utility is $(W + v - b - e + \lambda)^{\alpha}$

[14] We did conduct a short post-experimental questionnaire on this issue and subjects gave the following main reasons for preferring the ascending auction: (i) higher payoffs, (ii) easier decision-problem, (iii) decision independent from what others do, (iv) avoidance of the risk to lose by bidding too low, and (v) no uncertainty.

and when the bidder loses, $(W - e + \lambda)^\alpha$. We could of course frame this willingness to pay as based upon extra utility from participating in the ascending auction, but the previous formulation seems more natural and identification problems preclude simultaneously identifying both. To find the value of $\lambda$ for each bidder, we will use the No Wealth-Average bidder model and solve for the value of $\lambda$ that makes the bidder indifferent between the sealed bid auction at an entry price of 1.4 and the ascending auction at their observed switch-over price, $\delta_i + 1.4$ given their estimated degree of risk aversion. Details can again be found in the appendix.

For some bidders, $\lambda$ will be negative indicating a disutility for the sealed bid auction while for others $\lambda$ could be positive. These will obviously correspond to bidders who were found to be willing to pay more/less than the theoretical prediction. A histogram summary of the $\lambda's$ found in the population can be seen in figure 6. Most of the mass in the population is between 2 and -2. There are only a few subjects who possess a $\lambda > 2$ (8) while quite a number possess a $\lambda < -2$ (15) and some significantly so. This matches with the results found in figure 5 showing relatively few and only minor over-predictions of switch-over prices yet a number of large under-predictions.

It is difficult to understand exactly what this $\lambda$ means in its raw form. It is therefore useful to translate $\lambda$ into a different measure of the degree to which subjects prefer the ascending auction to the sealed bid. The most natural such measure would be to find the $n$-bidder ascending auction that is utility equivalent to the 2-bidder sealed bid auction. To do that we solve for the $\widehat{N}$ that makes the expected utility of being in a sealed bid auction, assuming the computed value of $\lambda$ for the bidder, equal to the expected value of being in an $n$-bidder ascending auction. Details on how this calculation is done are found in the appendix. There were 25 bidders found to have a $\lambda < 0$ out of the 46 who were found to be risk averse and not prefer the sealed bid auction. Of these 25 bidders, only 2 were found to possess an $\widehat{N} = 2$, 14 had an $\widehat{N} = 3$, 3 with $\widehat{N} = 4$, 1 with $\widehat{N} = 5$, 1 with $\widehat{N} = 9$ and 4 bidders were found to have values of $\widehat{N}$ that were essentially arbitrarily large[15]. This measure of the degree to which a bidder prefers an ascending auction is quite strong due to

[15] Possible values out to 40 were checked and the ascending still generated higher utility. Since expected values vary so little at this point and beyond it was considered not worthwhile to search at higher values.

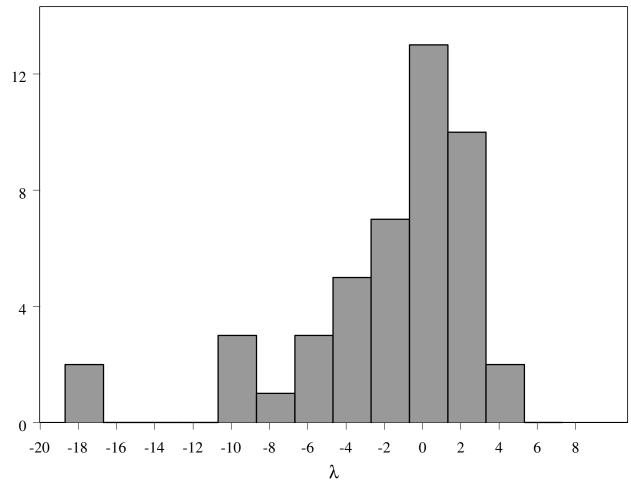


**Figure 6:** Histogram of the estimated values of $\lambda$, or value of participating in a sealed bid auction, found for the subjects.

the very large jump in expected value between a 2 and 3 bidder auction. Thus the fact that so many bidders would be willing to participate in ascending auctions with larger numbers of bidders even when they could participate in a 2 bidder sealed bid auction indicates quite strong preferences for the ascending auction.

# 4 Conclusion

In this study we attempt to do two things. First, we try to develop a methodology capable of measuring bidder preferences for various attributes of an auction design. Second, we apply this methodology to looking at bidder preferences between sealed bid and ascending auctions. The methodology involves the use of a two phase experiment protocol. In the first phase, the learning or training phase, subjects participate in both auction types, to make sure that the subjects have some idea for how the relevant characteristics of the different auction formats effect the outcome. To enhance learning, after each auction, the results of the winner for both auction types were published.

At the end of the learning phase in some of the sessions there was a summary screen detailing the average profit achieved by the winners across both auction types while in other sessions this screen was not included. Having sessions run with and without this screen allowed us to determine whether it had any effect on the relevant behavior. Theoretically these feedback differences are irrelevant and the results demonstrate that they appear to be so. The second phase, the preference assessment phase, begins by trying to detect which mechanism each subject prefers under a *ceteris paribus* assumption by allowing subjects to choose to enter into either an ascending or sealed bid auction for the same price. The phase continues by using these initial choices to identify the preferred format and then assess the strength of this preference by randomly varying the entry price for the more preferred format to find the switch-over point or entry price such that below it the subject would choose their more preferred option, but above it they switch to the less preferred, but cheaper, option.

The results show that when the entry prices for the two auction institutions are equal, subjects overwhelmingly prefer the ascending auction. However, the revealed willingness to pay for that preference in the second phase is much lower than the average realized profit differential between the two auction mechanisms. The average price that lead subjects to switch from the ascending to the sealed bid auction is 5.95 while the average profit differential between the two institutions is 8. 62 leading to a implied switch-over price of 10.02, accounting for the static entry price on the sealed bid auction. One hypothesis of subject behavior is that they are more or less risk neutral and bidding above the risk neutral Nash equilibrium level in the sealed bid auctions is a function of some phenomenon other than risk aversion. Were that true, it would have been reasonable to expect bidders to have paid up to the 10.02 level for the ascending auction since below that level the expected profit from the ascending is greater than the expected profit from the sealed bid. Thus we were forced to search for an alternative explanation.

We first tried to explain the observed switch-over prices either through experiential data concerning the subjects experiences in the learning phase. Data on the individual outcomes from phase I had no real explanatory power over the observed switch-over prices. We therefore tried modeling the subjects choosing as if they possessed risk averse preferences. When modeling the subjects behavior as though they assumed their opponent behaved as themselves and that they considered their wealth position when deciding on which auction format to enter, the predicted results from these hypotheses bore little resemblance to the actual results in the experiment. When modeling the subjects as though they believed they were facing a bidder of average risk aversion and considered just enough wealth to pay the entry price, the behavior of one group of the subjects became predicted rather well while the rest of the subjects were all found to be willing to pay far more than they "should" have based on this decision model.

We then went on to try to characterize what this relatively high willingness to pay on the part of the subjects really meant. To an auction designer, the important implication would be that bidders would be willing to enter an ascending auction with a larger number of bidders than a smaller sealed bid auction as if they would do so, the auctioneer might expect to make more revenue with the ascending auction. The results indicated that most of the bidders would be willing to enter into ascending auctions with at least 3-4 bidders instead of a 2 bidder sealed bid auction. Considering the sizable expected value difference in going from a 2 bidder to a 3 or 4 bidder auction, this suggests very strong preferences. Were the sealed bid auction a 4-bidder auction, the implication is that a sizable portion of the subjects would be willing to participate in ascending auctions of much larger size rather than participate in the sealed bid auction. This is an important result for auctioneers who want to attract bidders to their auctions.

There is, of course, a way of looking at our results that would suggest much less significance to them. This is derived from the results found in Isaac and James (2000) as in that paper the authors attempt to estimate the risk aversion parameters possessed by subjects using two different procedures (sealed bid auctions and the Becker-DeGroot-Marschak procedure) and find the estimations obtained using both of these procedures to be quite different. The implication is that either risk preferences are not invariant between choice mechanisms or perhaps that the bidding behavior in auctions is truly not a function of risk aversion. Either would suggest that our attempt to measure risk preferences in the sealed bid auction behavior and use that to predict behavior in the auction choice setting is doomed to failure. These experiments do not allow us to adequately deal with such an argument, but in future work we will be subjecting the predictions on entry behavior this model suggests to detailed tests to determine if they are robust.

One might also reasonably propose that the auction choice behavior we observe in these experiments is derived from loss averse preferences. Since subjects must pay the entry fee in the event they enter an auction and lose, subjects might avoid entering an auction with a high entry fee even if the expected value from doing so is higher than entering the other to avoid the large possible loss. We have conducted a series of sessions with a variant of our experimental design that is identical to the version explained here except that the entry price has been replaced by a percentage tax on the surplus of the winner which can never cause a loss. In these experiments, the average profit in the learning phase for the first price auctions was 16.48 and for the ascending it was 34.18. Thus subjects should have been willing to pay up to a tax of $1 - 16.48/34.18$ or 52% to be in the ascending as all lower taxes leave them better off on average in the ascending with no possibility of losses. We observed only 13 out of 50 subjects willing to pay .48 and 9 out of 50 willing to pay .52 and the overall structure of the willingness to pay the tax looks strikingly similar to what was shown in figure 1. This is only a preliminary look at the issue and it will receive additional scrutiny in future study. This should, however, be a very strong indication that loss aversion is not likely to be the true cause of the phenomenon we study here.

There is a final important insight that can be derived from the observed willingness to pay of most of the bidders in regard to entry prices for their preferred auction. Many optimal auction designs (see, e.g., Bulow and Roberts (1989), Riley and Samuelson (1981)) rely upon the use of properly chosen entry prices for full surplus extraction. The theories generally rely on the assumption of a fixed pool of bidders. Our results cast doubt on the true optimality of those designs for cases in which bidders have alternative options. If the bidders have an alternative option and if the entry price is too high, they may well take advantage of it. The results have not been included above to conserve space, but if we were to consider these experiments an attempt to determine if adding an entry fee to the ascending auction would increase revenue, the answer would be a clear "no". Total revenue in the ascending auction is almost monotonically decreasing in the entry price. The reason for that is that while per auction revenue is increasing in the entry price, the auctioneer will be able to conduct fewer and fewer of these auctions as the entry price rises since more bidders will be pursuing their outside options. The latter effect outweighs the former leading to total revenue decreasing as the entry price rises. These results show that entry prices do not have to be very high in order to scare off would-be bidders even when those bidders have a significant preference for the more expensive mechanism. These results confirm the theoretical predictions derived by Levin and Smith (1994) that the optimal design for the seller in private value auctions is one without an entry fee, though for different reasons. It has yet to be determined definitively, however, if the preferences observed in the current study do lead to an auctioneer being able to increase revenue by attracting more bidders to an ascending auction. This too will be verified in future work.

## APPENDIX A: Computation of Expected Utilities

For the data analysis below we will need to be able to compute the expected utility of the bidders for choosing to enter a sealed bid auction and their expected utility for entering an ascending auction given entry prices for both. The use of entry prices poses something of a complication to this as if the subjects lose the auction, or win with a low surplus, they face the possibility of a loss. We can therefore not represent a subjects utility from winning the auction as $u(v-b-e)$ and must introduce wealth into the equation to avoid the negativity problem and use $u(W+v-b-e)$. However, when bidders are deciding what to bid in the sealed bid auction we will be ignoring wealth effects.

### Sealed Bid

The first thing that we need to do is derive the bidding function under the two different models of beliefs: equivalent opponent and average opponent. As shown in Cox, Roberson, and Smith (1982), the optimal bid function for a risk averse bidder in the SIPV environment assuming that values are uniformly distributed on the range $[0,100]$ can be defined by $b^*(v) = v\left(\frac{N-1}{N-1+\alpha}\right)$ so long as

$b < 100\left(\frac{N-1}{N}\right)$. Above this point, the bid function becomes non-linear. This non-linearity is due to allowing that bidders may believe that their opponents have different risk aversion parameters than themselves. If, however, bidders assume their opponents have the same risk aversion parameters as themselves, then the bid function is

$$b^*(v) = v\frac{N-1}{N-1+\alpha} \tag{2}$$

for the entire range of possible values. Thus this is the bid function for the equivalent opponent model. For the average opponent model, assuming the average risk aversion parameter is $\bar{\alpha}$, the bid function is:

$$b^*(v) = \begin{cases} v\frac{N-1}{N-1+\alpha} & \text{for} \quad v \in [0, \min(\frac{N-1+\alpha}{n-1+\bar{\alpha}} * 100, 100)] \\ 100\frac{N-1}{N-1+\bar{\alpha}} & \text{for} \quad v \in [\min(\frac{N-1+\alpha}{N-1+\bar{\alpha}} * 100, 100), 100] \end{cases} \tag{3}$$

**Equivalent bidder model:** For $n = 2$ if a bidder draws value $v$, they will bid $\frac{v}{1+\alpha}$. We will add a parameter $\lambda$ to allow for a possible extra boost to utility or perhaps a disutility from participating in sealed bid auctions. The utility from the outcome when the bidder wins will be $\left(W + v - \frac{v}{1+\alpha} - e + \lambda\right)^\alpha = \left(W + \frac{\alpha}{1+\alpha}v - e + \lambda\right)^\alpha$ and loses $(W - e + \lambda)^\alpha$. Expected utility will be $\left(W + \frac{\alpha}{1+\alpha}v - e + \lambda\right)^\alpha$ times probability that they win, $F(v) = \frac{v+1}{101}$ as we are using the discrete uniform distribution on $[0, 100]$, plus $(W - e + \lambda)^\alpha$ times probability that you don't all times the probability of that particular $v$ occurring, $\frac{1}{101}$. This gives us that the expected utility from participating in a sealed bid auction can be defined by the following equation:

$$EU_S^E(W, e, \alpha, \lambda) = \sum_{v=0}^{100} \left( \begin{array}{c} \left(W + \frac{\alpha}{1+\alpha}v - e + \lambda\right)^\alpha \frac{v+1}{101} * \frac{1}{101} \\ + (W - e + \lambda)^\alpha (1 - \frac{v+1}{101}) * \frac{1}{101} \end{array} \right) \tag{4}$$

**Average bidder model:** The expected utility under this model is computed much the same way except for two details. One is the technical detail coming from the two part bid function. The second is derived from the fact that the probability of winning is no longer just $F(v)$. Now if you bid $b_i$ assuming that your opponent is bidding $\frac{N-1}{N-1+\bar{\alpha}}v_j$, then

$$\begin{aligned} \Pr(win) &= \Pr(b_i > b_j = \frac{N-1}{N-1+\bar{\alpha}}v_j) = \Pr(\frac{N-1+\bar{\alpha}}{N-1}b_i > v_j) \\ &= F(\frac{N-1+\bar{\alpha}}{N-1}b_i)^{n-1} = \left(\frac{\frac{N-1+\bar{\alpha}}{N-1}b_i + 1}{101}\right)^{n-1} \end{aligned}$$

If $b_i = v\frac{N-1}{N-1+\alpha}$ then this becomes

$$\left(\frac{\frac{N-1+\bar{\alpha}}{N-1}b_i + 1}{101}\right)^{N-1} = \left(\frac{\left(\frac{N-1+\bar{\alpha}}{N-1}\right)v\left(\frac{N-1}{N-1+\alpha}\right)+1}{101}\right)^{n-1} = \left(\frac{v\left(\frac{N-1+\bar{\alpha}}{N-1+\alpha}\right)+1}{101}\right)^{N-1}$$

At least this is your probability of winning along the sloped portion of the bid function. Along the flat portion of the bid function, your belief is that your opponent will never bid above $100\frac{N-1}{N-1+\bar{\alpha}}$ thus if you bid that plus $\epsilon$, you expect to win with a probability of 1.

The expected utility of participating in a sealed bid auction assuming you are facing a bidder with risk aversion of $\bar{\alpha}$ is then:

$$\begin{aligned} EU_S^A(W,e,\alpha,\lambda) = &\sum_{v=0}^{\min(\frac{1+\alpha}{1+\bar{\alpha}}(100),100)} \left( \begin{array}{c} \left(W + \frac{\alpha}{1+\alpha}v + \lambda - e\right)^{\alpha} * \frac{\frac{1+\bar{\alpha}}{1+\alpha}v+1}{101} * \frac{1}{101} + \\ (W+\lambda-e)^{\alpha} * (1 - \frac{\frac{1+\bar{\alpha}}{1+\alpha}v+1}{101}) * \frac{1}{101} \end{array} \right) + \\ &\sum_{v=\min(\frac{1+\alpha}{1+\bar{\alpha}}(100),100)}^{100} \left(\left(W + v - \frac{1}{1+\bar{\alpha}}(100) + \lambda - e\right)^{\alpha} * 1 * \frac{1}{101}\right) \end{aligned}$$

**Ascending**

In an ascending auction the risk attitude of your opponent is irrelevant thus either model of expectations produces the same result. For a given value $v$, a bidder will pay the next highest value if they do win or their utility will be $(W + v - v_{(2)} - e)^{\alpha}$. The distribution of $v_{(2)}$ is given by the standard distribution of an order statistic for the $k$'th highest draw, $f_{(k)}(x) = \frac{(n)!}{(n-k)!(k-1)!}F(x)^{n-k}[1-F(x)]^{k-1}f(x)$, for the case in which $x$ is drawn from the discrete uniform distribution on $[0, v]$, $k = 1$ and $n = N - 1$, $N$ being the actual number of bidders in the auction. This simplifies to $(N-1)\left(\frac{x+1}{v+1}\right)^{N-2}\frac{1}{v+1}$. To find the expected value of being in the ascending auction, we must first compute the expected surplus that occurs when the bidder wins multiplied by the probability of winning and then we add on a second term for the case in which the bidder loses. Since we will be allowing for the possibility of more than 2 bidders in the ascending auctions, we have to generalize the format. Assuming that $\mu$ represents the extra utility or disutility from participating in the ascending auction, this leads to

$$EU_A(W, \alpha, e, \mu, N) = \sum_{v=0}^{100} \left( \begin{array}{c} \left( \sum_{x=0}^{v} (W + v - x - e + \mu)^{\alpha} * (N-1) \left(\frac{x+1}{v+1}\right)^{N-2} \frac{1}{v+1} \right) \\ * \left(\frac{v+1}{101}\right)^{N-1} * \frac{1}{101} \\ + (W - e + \mu)^{\alpha} \left(1 - \left(\frac{v+1}{101}\right)^{N-1}\right) * \frac{1}{101} \end{array} \right) \tag{5}$$

**Comparing**

In the experiments, it was observed that for equal entry prices most bidders preferred the ascending auction. The entry price for the sealed bid auction is then held static at 1.4 while the entry price of the ascending auction is increased until we find a point that the bidder prefers to choose the sealed bid auction. Given a level of risk aversion, $\alpha$, and a level of wealth, $W$, it is possible to compute predictions as to what this price should be from both models by solving the following equations for $e$, assuming that $\lambda$ and $\mu$ are both 0.

$$EU_S^E(W, 1.4, \alpha, 0) = EU_A(W, e, \alpha, 0, 2) \tag{6}$$

$$EU_S^A(W, 1.4, \alpha, 0) = EU_A(W, e, \alpha, 0, 2) \tag{7}$$

As described in the body of the paper, the No Wealth-Average bidder model is then used to identify the level of (dis)utility from participating in the sealed bid auctions. This is the entry price equivalent of what would make the expected utility of participating in a sealed bid auction equal to the expected utility of being in the ascending at the entry price the subject was observed to pay, $\delta_i + 1.4$. This is done by finding the value of $\lambda$ that solves the following:

$$EU_S^A(W, 1.4, \alpha, \lambda) = EU_A(W, \delta_i + 1.4, \alpha, 0, 2) \tag{8}$$

Finally, the $n-$bidder ascending auction a bidder finds equivalent to participating in a two bidder sealed bid auction assuming entry prices of 0 for both is found by solving the following for $\widehat{N}$ :

$$EU_S^A(W, 0, \alpha, \lambda) = EU_A(W, 0, \alpha, 0, \widehat{N}) \tag{9}$$

# References


Bajari, P. and A. Hortacsu (2000). Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions. Working Paper.

Bulow, J. I. and D. J. Roberts (1989). The Simple Economics of Optimal Auctions. *Journal of Political Economy 97*, 1060–1090.

Cox, J. C., B. Roberson, and V. L. Smith (1982). Theory and Behavior of Single Object Auctions. In *Research in Experimental Economics*, Volume 2, pp. 1–43. JAI Press Inc.

Cox, J. C. and V. Sadiraj (2002). Risk aversion and expected utility: Coherence for small- and large-stakes gambles. University of Arizona Working Paper.

Cox, J. C., V. L. Smith, and J. M. Walker (1988). Theory and Individual Behavior of First Price Auctions. *Journal of Risk and Uncertainty 1*, 61–99.

Engelbrecht-Wiggans, R. (1993). Optimal Auctions Revisited. *Games and Economic Behavior 5*, 227–239.

Fischbacher, U. (1998). z-Tree: Zurich Toolbox For Readymade Economic Experiments. Zurich University.

Friedman, D. (1983). Effective Scoring Rules for Probabilistic Forecasts. *Management Science 29*(4), 447–454.

Goeree, J. K. and T. Offerman (2002). First-Price Auctions to Sell Heterogeneous Licenses. CREED Working Paper.

Ham, J., J. Kagel, and S. Lehrer (2002). Randomization, Endogeniety and Laboroatory Experiments: The Role of Cash Balances in Private Value Auctions. Working Paper, Ohio State University.

Harstad, R. M. (1990). Alternative Common Values Auction Procedures: Revenue Comparisons with Free Entry. *Journal of Political Economy 98*, 421–429.

Isaac, R. M. and D. James (2000). Just Who Are You Calling Risk Averse. *Journal of Risk and Uncertainty 20*(2), 177–187.

Ivanova-Stenzel, R. and D. Sonsino (2001). Comparative Study of One-Bid versus Two-Bid Auctions. Working Paper, Humboldt University.

Klemperer, P. (2002). What Really Matters in Auction Design. *Journal of Economic Perspectives 16*(1), 169–190.

Levin, D. and J. L. Smith (1994). Equilibrium in auctions with Entry. *American Economic Review 84*(3), 585–599.

Lucking-Reiley, D. (1999). Experimental Evidence on the Endogenous Entry of Bidders in Internet Auctions. Working Paper, University of Arizona.

Lucking-Reiley, D. (2000). Auctions on the Internet: What's being auctioned and How? *Journal of Industrial Economics 48*(3), 227–252.

Matthews, S. (1987). Comparing Auctions For Risk Averse Buyers: A Buyer's Point of View. *Econometrica 55*(3), 633–646.

McAfee, P. (1993). Mechanism Design by Competing Sellers. *Econometrica 61*(6), 1281–1312.

McAfee, P. and J. McMillan (1987). Auctions with Entry. *Economics Letters 23*, 343–347.

Milgrom, P. R. and R. J. Weber (1982). A Theory of Auctions and Competitive Bidding. *Econometrica 50*(5), 1089–1122.

Rabin, M. and R. H. Thaler (2001). Risk Aversion. *Journal of Economic Perspectives 15*(1), 219–232.

Riley, J. G. and W. F. Samuelson (1981). Optimal Auctions. *American Economic Review 71*, 381–392.

Selten, R. (1998). Axiomatic Characterization of the Quadratic Scoring Rule. *Experimental Economics 1*, 43–62.