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BIFURCATION AND CHAOS OF SLIGHTLY CURVED PIPES

B. Gültekin SINIR
Celal Bayar University, Civil Engineering Department, Manisa, Turkey
gultekin.sinir@bayar.edu.tr

Abstract- Non-linear vibrations of slightly curved pipes conveying fluid with constant velocity are investigated. The curvature is taken as an arbitrary function of the spatial variable. The initial displacement is considered due to the geometry of the pipe itself. The ends of the curved pipe are assumed to be immovable simple supports. The equations of motion of pipes are derived using Hamilton's principle and solved by Galerkin method. The bifurcation diagrams are presented for various amplitudes of the curvature function and fluid velocity. The periodic and chaotic motions have been observed in the transverse vibrations of slightly curved pipe conveying fluid.

Key words-Slightly curved pipe, conveying fluid, nonlinear vibration, Galerkin method, bifurcation

1. INTRODUCTION

The understanding of the vibrations of pipes conveying fluid is important as pipes with internal fluid flow are encountered in many engineering installations, particularly in power-generating, chemical and petrochemical industries in the form of process piping, heat exchanger tube bundles, hydraulic oil tubes, lubrication pipes and cooling water pipes in nuclear power plants. These structures are subjected to flow-induced vibration due to turbulence in the flow or resonance with some periodicity in the flow. If these oscillations are not prevented, they can result in leakage, hazards and accidents. Due to their technological importance, the dynamics of pipes conveying fluid have been investigated by many researchers. Effects of some factors such as parametric excitation in the form of flow fluctuation [1, 2], external excitations [3], support conditions [4], articulated, inclined or continuous nature of pipe [5, 6], additional system configurations like lumped mass [7, 8, 9], elastic foundations [10] and different forms of nonlinearities in the system arising from various sources [11, 12, 13, 14] have been examined. An extensive review is given by Païdoussis [15]. This review discusses various aspects such as mathematical modeling, solution methodology, effect of system parameters like boundary conditions, fluid pipe mass ratio , gravity effect, parametric instabilities of the system due to pulsatile flow, fluid friction effects, the mechanisms of instabilities, destabilizing effects of dissipation, effects of elastic constraints, motion limiting constraints, lumped mass, attached nozzles, elastic foundation and various other parameters on the dynamics of the system. Païdoussis' two well known books should also have been mentioned on fluid structure interactions [16, 17].

Many papers have considered straight pipes to analyze pipe vibrations and stability. Curved pipes, on the other hand, have attracted relatively little attention. However, a construction cannot be considered to be either completely straight or vertical. Since most piping systems are composed of both straight and curved pipes, considerable research concerning the dynamics of curved pipes is required to reduce the vibrations of systems and guarantee their stability. Therefore certain curved systems need to be modeled. An early analytical model for the dynamics and stability of a curved pipe conveying fluid has been suggested by Chen [18, 19], in which the centre line of the curved pipe is assumed to be inextensible. However, if the system has immovable end conditions, this causes axial extensions in the beam/pipe which introduces integral type cubic non-linearities into the equations of motions. The effect of stretching has also been included in the vibration of slightly curved beams/pipes or shallow arcs. Pakdemirli and Nayfeh [20] have investigated a beam-mass-spring system where the non-linearities arise due to stretching. Özkaya et al. [21] have investigated a concentrated mass on a Euler-Bernoulli beam supported by immovable end conditions leading to stretching during the vibration. Öz *et al.* [22] have examined effect of curvature on the vibrations of the beam and found that the effect of curvature is of softening type. Hill and Davis [23], as well as Doll and Mote [24], have found that if the centerline is extensible, a fluid-conveying curved pipe does not lose stability, even for high fluid velocities. However, Misra *et al.* [25, 26] have examined the dynamics for two cases of curved pipes with both extensible and inextensible centerlines. They have found that the dynamic analysis of curved pipes with an extensible centerline is more reasonable than analysis with an inextensible centerline. The study on the extensible curved pipe by Misra *et al.* [26] has been based on the assumption of small pipe deformation, which is valid only when fluid velocity is relatively low. With this assumption, the steady -state equilibrium equations of the curved pipe become linear and the equations of motion are linearized around an equilibrium configuration. When the pipe deformation is not small, the natural frequencies computed with these equilibrium and linearized equations can differ largely from the actual natural frequencies. Jung and Chung [27] have derived new nonlinear equilibrium equations and linearized the equations of motion around the equilibrium configuration of the curved pipe. The derived equilibrium equations are different from the corresponding equations presented by Misra *et al.* [26]. There are two available classes of methods for determining approximate solutions of nonlinear systems: numerical and analytical methods. With numerical methods, the nonlinear partial differential equations and boundary conditions are replaced with a set of nonlinear algebraic equations. By employing Galerkin method, one reduces the original partial-differential equations and boundary conditions to a system of ordinary –differential equations for the time-dependent functions [28-33].

In this study, non-linear vibrations of slightly curved pipes conveying fluid with constant flow velocity are investigated. The curvature is taken as a sinusoidal function of the spatial variable. The curvature, which may be regarded as an initial displacement, originates from the geometry of the pipe itself; therefore, is not due to buckling of pipe. It is classified as either imperfections of the pipe or slightly curved pipe. The ends of the curved pipe are assumed to be located on immovable simple supports. The immovable end supports result in the extension of the pipe during the vibration and some additional non-linear terms to include such extensions are added to the equations of motion.

Galerkin method is applied to truncate the integro-nonlinear partial differential equation into a set of ordinary differential equations. The dynamical behavior is identified based on the numerical solutions of the ordinary differential equations. The bifurcation diagrams are presented for the case where the fluid velocity and the amplitude of the curvature function are varied while other parameters are fixed. Based on the numerical simulations, the existence of the periodic and chaotic motions in the transverse vibrations of the slightly curved pipes has been observed for constant internal fluid velocity.

2. EQUATION OF MOTIONS

An Euler-Bernoulli beam/pipe with an initial geometric imperfection is considered. It is noted that the theory of the non-linear planar vibrations of a one dimensional continuous system, geometrically imperfect pipes coincides with the theory of shallow arches. A geometrically imperfect pipe is, in fact, a shallow arch because the rise-to-span ratio of the imperfect pipe is assumed to be small.

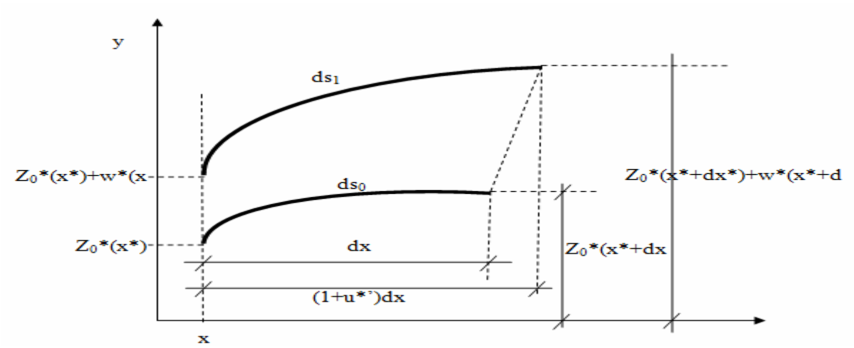


Figure 1. A schematic of an initially imperfect pipe

To derive the equation of motion governing the transverse vibrations of the beam, it is considered a differential element, located at a pointed distance x^* from the origin, as shown in Fig. 1. The elongation of the differential element is given by

$$e = \frac{ds_0 - ds_1}{ds_0} \tag{1}$$

where

$$ds_0 = \sqrt{1 + Z_0^{*\prime 2}}\, dx^* \tag{2}$$

$$ds_1 = \sqrt{\left(1 + u^{*\prime 2}\right) + \left(Z_0^{*\prime} + w^{*\prime}\right)^2}\, dx^* \tag{3}$$

$Z_o^*(x^*)$ is the arbitrary initial rise function. $(\ )'$ represents derivatives with respect to the spatial variable. x^* is the spatial variable along the projected length. u^* and w^* are longitudinal and transverse displacements, respectively. Retaining up to the quadratic terms and removing the higher-order terms, and expanding into a binomial series with some mathematical manipulation, approximate elongation and curvature of the midplane are given by,

$$e \approx u^{*\prime} + \frac{1}{2} w^{*\prime 2} + Z_o^{*\prime} w^{*\prime} \tag{4}$$

$$\kappa \approx w^{*\prime\prime} \tag{5}$$

respectively. Here, the effect of geometric imperfection on bending moment and fluid flow is negligible since the rise-to-span ratio of the imperfect pipe is assumed to be small.

t^* denotes time and $(^*)$ represents derivatives with respect to time. v^* is constant velocity; ρ_f, density and A_f is the cross-sectional area of the fluid. L is the projected length; $\rho_p A_p$, mass per unit length, P^* is the pretension at $x^*=L$ and E_pI_p is the bending stiffness of the pipe. The extensional stiffness is sufficiently large so that the longitudinal deformation is negligible. Variation of cross-sectional dimensions during vibration is not considered. Gravity, pressure, and fluid friction effects are neglected. The total kinetic energy of the system [13 and 28] is

$$T = \frac{1}{2}\rho_f A_f \int_0^L \left[(v^*(1+u^{*\prime}) + \dot{u}^*)^2 + (\dot{w}^* + w^{*\prime} v^*)^2 \right] dx^* + \frac{1}{2}\rho_p A_p \int_0^L (\dot{w}^*)^2 dx^* \tag{6}$$

where the first term denotes the kinetic energy of the fluid and the last term is the pipe's. The bending moment $M(x^*)$ at any location x^* is given by

$$M(x) = E_p I_p \kappa \tag{7}$$

The elastic potential energy of the system [13 and 28]

$$U = \frac{1}{2} E_p A_p \int_0^L e^2 dx^* - P^* u(L, t^*) + \frac{1}{2}\int_0^L M(x)\kappa dx^* \tag{8}$$

The first term is the elastic potential energy of the pipe due to elongation, and second term is due to bending. Hamilton's extended principle can be written as

$$\int_{t_1^*}^{t_2^*} (\delta T - \delta U + \delta W_{nc}) dt^* \tag{9}$$

where, W_{nc} is the nonconservative work done. The first variation of nonconservative force is

$$\int_{t_1^*}^{t_2^*} \delta W_{nc} dt^* = \int_{t_1^*}^{t_2^*} c\dot{w}^* \delta w^* dt^* \tag{10}$$

where c is the viscous damping coefficient.

Substituting Eqs. (4-5) and (7) into Eq. (8), and then substituting Eqs.(6) and (8) into Eq. (9), and applying Hamilton's extended principle, the equation of motions for the transverse vibration of the slightly curved pipe conveying fluid is derived as

$$\begin{aligned} &(\rho_p A_p + \rho_f A_f)\ddot{w}^* + c\dot{w}^* + \rho_f A_f \left(2\dot{w}^{*\prime} v^* + w^{*\prime\prime} v^{*2} \right) - P^* w^{*\prime\prime} + E_p I_p w^{*IV} \\ &= \frac{E_p A_p}{L} \int_0^L \left(Z_o^{*\prime} w^{*\prime} + \frac{1}{2} w^{*\prime 2} \right) dx^* \left(w^{*\prime\prime} + Z_o^{*\prime\prime} \right) \end{aligned} \tag{11}$$

Dropping the internal fluid force and pretension terms, the equation of motion becomes the same as the equation of motion in [22] without foundation terms and in [34] without both lateral and axial load terms. Another justification is that assuming constant fluid velocity and elastic material, the nonlinear equation of motion for the straight pipe given by Holmes [35] turns into the equation of motion (11) without curvature term.

The boundary conditions for the pinned-pinned pipe are

$$w^*\left(0,t^*\right)=0, \quad w^*\left(L,t^*\right)=0$$
$$w^{*''}\left(0,t^*\right)=0, \quad w^{*''}\left(L,t^*\right)=0 \tag{12}$$

and the boundary conditions for clamped-clamped supports

$$w^*\left(0,t^*\right)=0, \quad w^*\left(L,t^*\right)=0$$
$$w^{*'}\left(0,t^*\right)=0, \quad w^{*'}\left(L,t^*\right)=0 \tag{13}$$

For the sake of convenience, the equation is made dimensionless by introducing the following terms;

$$w=\frac{w^*}{r}, x=\frac{x^*}{L}, Z_0=\frac{Z_0^*}{r}, \; t=\frac{t^*}{L^2}\sqrt{\frac{E_pI_p}{\rho_pA_p+\rho_fA_f}}, \; v=L\sqrt{\frac{\rho_fA_f}{E_pI_p}}v^*, \; P=P^*\frac{L^2}{EI} \tag{14}$$

where r is the radius of gyration of the pipe cross-section. Substituting dimensionless quantities (14) into Eq. (11), the dimensionless form of the equation and boundary conditions can be obtained as follows,

$$\ddot{w}+\mu\dot{w}+2v\sqrt{\beta}\dot{w}'+\left(v^2-P\right)w''+w^{iv}-\int_0^1\left(Z_0{}'w'+\frac{1}{2}w'^2\right)dx\left(w''+Z_0{}''\right)=0 \tag{15}$$

and boundary conditions for pinned-pinned and clamped-clamped supports as,

$$w(0)=w(1)=w''(0)=w''(1)=0 \tag{16.a}$$

$$w(0)=w(1)=w'(0)=w'(1)=0 \tag{16.b}$$

respectively. Here new dimensionless parameters are defined as

$$\beta=\frac{\rho_fA_f}{\rho_pA_p+\rho_fA_f} \quad \text{and} \quad \mu=\frac{cL^2}{\sqrt{\left(\rho_pA_p+\rho_fA_f\right)E_pI_p}} \tag{17}$$

where β is the ratio of fluid mass to the combined mass of the fluid and the pipe, and μ is the dimensionless viscosity coefficient.

3. SOLUTION METHODS

Galerkin method is widely used to discretize the equations of motions of distributed-paremeter systems. The discretization techniques replace the integro-nonlinear partial differential equation by a set of ordinary differential equations. The discretized set of the equations is truncated to a finite set. However, the number of modes needed in the discretization should carefully be selected so that the modes disregarded have a negligible effect on the predicted response. It was pointed out by Païdoussis and Issid [30] that the instability boundaries for pinned-pinned and clamped-clamped pipes could be determined with the two-mode expansion with adequate precision, and their experimental results were found to be at least in good qualitative agreement with those based on theoretical solutions [31]. G.L. Kuiper et al. [3], also has shown that two-mode approximation describes the dynamic behaviour of the system qualitatively correctly. Even in a quantitative sense this regarded as a reasonable approximation.

As a first step towards solving the partial differential equation of motion, (15), it is transformed into a set of second-order ordinary differential equations using Galerkin's technique

$$w(x,t)=\sum_{n=1}^{N} q_n(t)Y_n(x) \tag{18}$$

where $q_n(t)$; *i=1; 2; . . . ;N,* are the generalized coordinates and $Y_n(x)$ are the eigenfunctions of the supported (pinned–pinned or clamped-clamped) beam.
Taking the appropriate derivatives and substituting them into Eq. (15), the residual can be obtained as follows.

$$\sum_{n=1}^{N}\left(\begin{array}{l}\ddot{q}_n(t)Y_n(x)+\left(2v\sqrt{\beta}Y_n'(x)+\mu Y_n(x)\right)\dot{q}_n(t)\\ +\left(\left(v^2-P\right)Y_n''(x)+Y_n^{iv}(x)\right)q_n(t)\\ -\left(q_n(t)Y_n''(x)+Z_0''\right)\int_0^1 (Z_0' q_n(t)Y_n'(x)+\frac{1}{2}\left(q_n(t)Y_n(x)\right)^2)dx\end{array}\right)=R(x,t) \tag{19}$$

Application of Galerkin's Method requires that

$$\int_0^1 R.Y_j(x)dx=0 \qquad j=1,2,......,N \tag{20}$$

where, $Y_j(x)$ is eigenfunction of the vibrating beam. The eigenfunctions are said to be orthogonal for the boundary conditions of pinned–pinned and clamped-clamped

In coming two sections of the paper, the dynamics of pinned-pinned and clamped-clamped pipes will be illustrated.

3.1. Analysis of pinned-pinned pipe

In the case of a pinned-pinned beam, the identical eigenfunctions of a bar in transverse vibration, $Y_j(x)$, are used; i.e.,

$$Y_n(x)=\sqrt{2}\sin(n\pi x) \tag{21}$$

Substituting Eq. (21) into Eq. (19) and using the relation (20), and then integrating for a given *n* yields and *n* number ordinary differential equation. In present study, we have choosen *n=2*. Choosing a sinusoidal curvature function satisfying boundary conditions of pinned- pinned support [22],

$$Z_0(x)=b.\sin(\pi x) \tag{22}$$

the 2 term Galerkin approximation of Eq. (15) is obtained as follows

$$\begin{aligned}&\ddot{q}_1+\mu\dot{q}_1-\frac{16}{3}v\sqrt{\beta}\dot{q}_2+\left(\pi^4-\left(v^2-P\right)\pi^2+\frac{1}{2}\pi^4 b^2\right)q_1\\&+\frac{3}{4}\pi^4 b q_1^2+\pi^4 b q_2^2+\pi^4 q_2^2 q_1+\frac{\pi^4}{4}q_1^3=0\end{aligned} \tag{23.a}$$

$$\ddot{q}_2+\mu\dot{q}_2+\frac{16}{3}v\sqrt{\beta}\dot{q}_1+\left(16\pi^4-4\left(v^2-P\right)\pi^2\right)q_2+2\pi^4 b q_1 q_2+\pi^4 q_1^2 q_2+4\pi^4 q_2^3=0$$

(23.b)

Linear analysis of straight pipe conveying fluid [36], considering small motions, can predict the point where the stability is lost at the first time, but cannot provide any definitive prediction of its post-critical behavior. Linear theory predicts that, in general, the pipe is stable at low-flow velocities; then, the cylinder is subjected to divergence (buckling) in its first mode as the flow velocity increases. Linear theory also predicts the occurrence of second-mode buckling of the system, and the existence of coupled-mode flutter (so-called ‘‘Païdoussis flutter’’) at higher flow velocities in some cases [14].

3.2. Analysis of clamped-clamped pipe

The eigenfunction of a clamped-clamped beam is more complicated than pinned-pinned, that is,

$$Y_n(x) = \cosh\lambda_n x - \cos\lambda_n x - \frac{\cosh\lambda_n - \cos\lambda_n}{\sinh\lambda_n - \sin\lambda_n}\left(\sinh\lambda_n x - \sin\lambda_n x\right) \tag{24}$$

Here, λ_n are roots of

$$\cosh\lambda_n x . \cos\lambda_n x = 1 \tag{25}$$

The lowest two roots are obtained from the numerical solutions of Eq. (25) as, $\lambda_1 = 4.7300$ and $\lambda_2 = 7.8532$

The appropriate curvature function for clamped-clamped boundary conditions is assumed to be;

$$Z_0(x) = b\frac{1}{2}\left(1 - \cos 2\pi x\right) \tag{26}$$

Using eigenfunction relation, and applying two term Galerkin’s discretization, a set of ordinary differential equations is obtained as

$$\begin{aligned}&\ddot{q}_1 + \mu\dot{q}_1 - 6.6919v\sqrt{\beta}\dot{q}_2 + \left(500.5639 - 12.2937\left(v^2 - P\right) + 60.2277b^2\right)q_1 \\ &+ 143.2107bq_1^2 + 179.3202bq_2^2 + 284.1379q_2^2 q_1 + 75.7059q_1^3 = 0\end{aligned} \tag{27.a}$$

$$\begin{aligned}&\ddot{q}_2 + \mu\dot{q}_2 + 6.6773v\sqrt{\beta}\dot{q}_1 + \left(3803.5371 - 45.9404\left(v^2 - P\right)\right)q_2 \\ &+ 356.6222bq_1q_2 + 282.9056q_1^2 q_2 + 1061.7961q_2^3 = 0\end{aligned} \tag{27.b}$$

These nonlinear ordinary differential equations are solved using *dsolve* command of Maple software program. The *dsolve* command with options numeric and method=dverk78 finds a numerical solution using a seventh-eighth order continuous Runge-Kutta method.

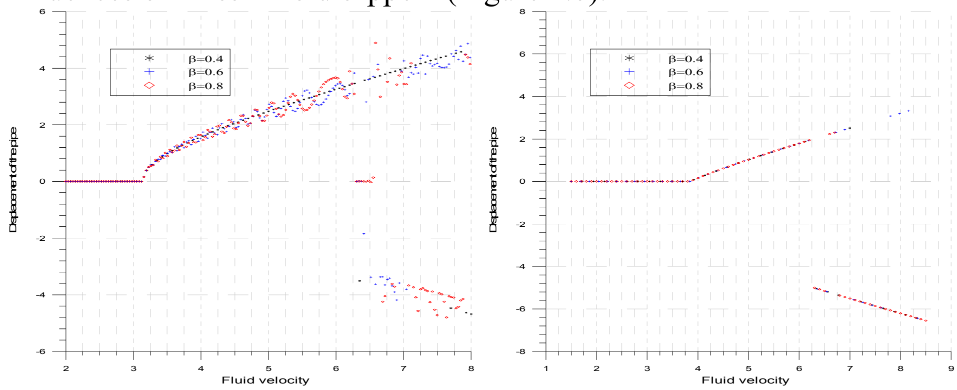
4. NUMERICAL RESULTS

In order to identify the dynamical behavior, bifurcation and phase trajectory diagrams, and the time histories for the dimensionless transverse deflections and velocities of the pipe are obtained. The bifurcation diagrams summarized the nonlinear analysis of the slightly curved pipe is plotted as a function of the dimensionless flow velocity. Solutions are obtained using a seventh-eighth order continuous Runge-Kutta method with MAPLE software.

In the following sections, the influence of different parameters on the stability and the amplitude of the buckled solution of the system is examined for the following parameters: *L/r=10* and *x=0.5* for all results and *t=80* for bifurcation diagrams. Here, at the bifurcation diagrams, flow velocity, *v*, is used as the independent varied parameter.

4.1.1 Influence of mass ratio

It has to be noticed that fluid mass ratio, *β,* has no influence on the first bifurcation point but has on variation of bifurcation diagram over critical velocity, when curvature amplitude, *b,* is zero (Figure 2.a). However, when the system has the curvature, the influences of mass ratio disappear (Figure 2.b).

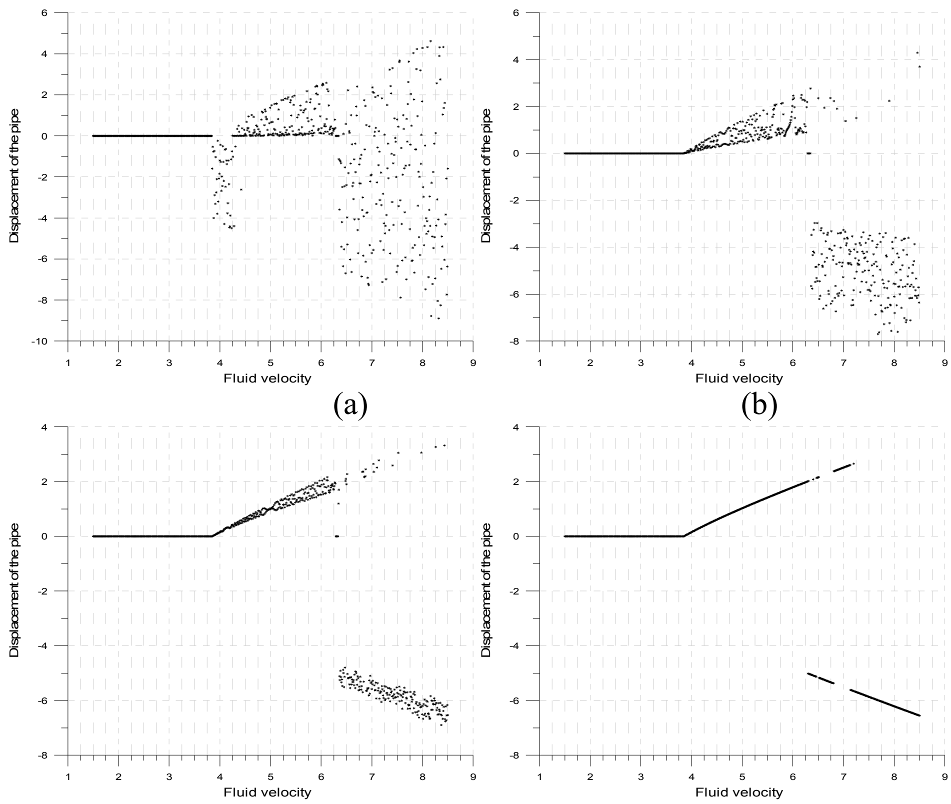


(a) (b)

Figure 2. Bifurcation diagrams of a pinned-pinned pipe obtained with $N=2$ and with different values of β, ranging from 0.2 to 0.8, (a) $b=0.0,\ \mu=0.5,\ P=0$ (b) $b=1.0,\ \mu=0.5,\ P=0.$

4.1.2 Influence of viscous damping term

Figures 3.a and b, show the bifurcation diagrams of the system with varying damping term, μ, for $\beta=0.4$ and $b=1.0.$ It should be noticed that μ has no influence on the first bifurcation point. However, the bifurcation diagrams turn from chaotic motions to pitchfork bifurcation motions with increasing damping term. The chaotic motions may occur due to interactions between first and second or any other modes, when $\mu=0$.



(a) (b)

(c) (d)

Figure 3. Bifurcation diagrams of a simply-supported pipe obtained for *b=1.0, β=0.4, P=0* and different values of μ, ranging from 0.0 to 0.5, (a) $\mu=0.0$, (b) $\mu=0.01$, (c) $\mu=0.05$, (b) $\mu=0.5$.

4.1.3 Influence of curvature amplitude

The bifurcation diagrams of the displacement of the system with varying curvature amplitude are given in Figure 4.a for simply supported pipe and in Figure 4.b for pined-pinned supported pipe. Here it is assumed that damping coefficient, μ, is 1.0, corresponding to $\beta=0.8$.

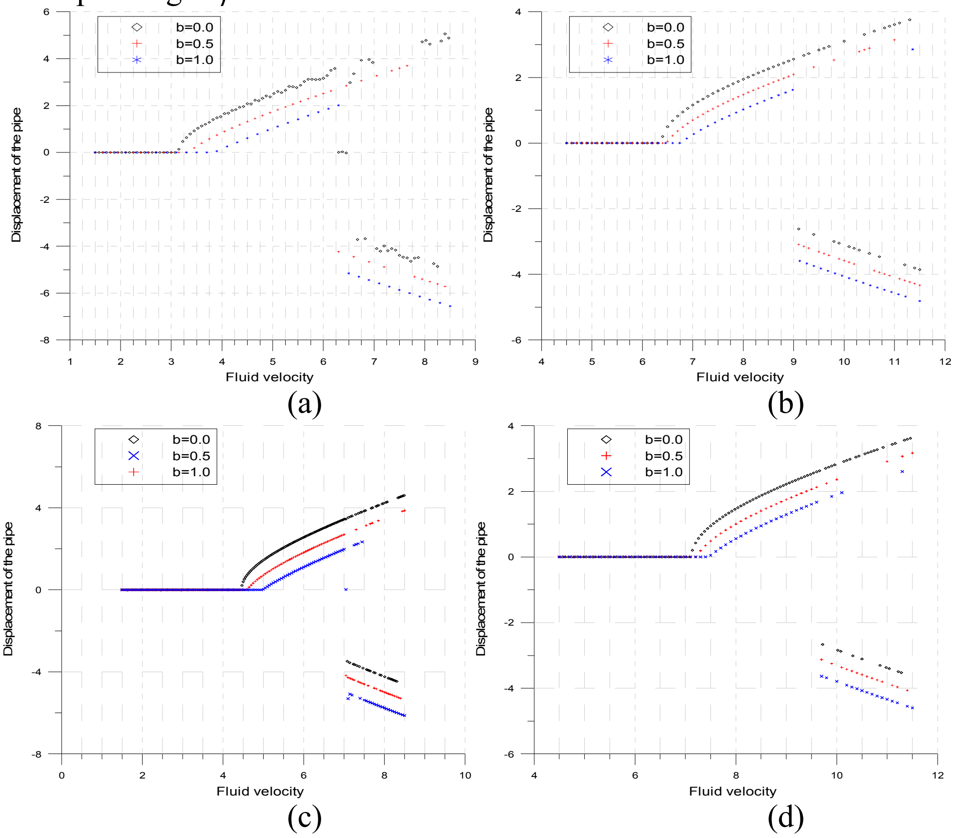
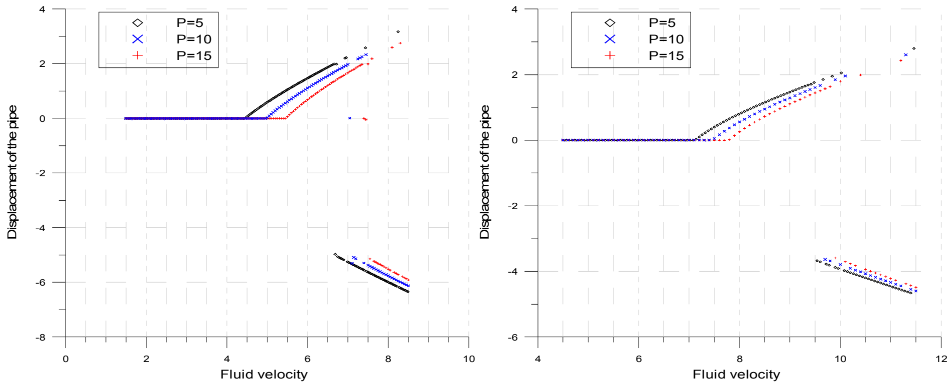


Figure 4. Bifurcation diagrams of a slightly curved pipe obtained with *β=0.8, c=1.0* with different values of curvature amplitude, b, ranging from 0.0 to 1.0, and *P=0* (a) for pinned-pinned supported pipe (b) for clamped-clamped supported pipe, and *P=10* (c) for pinned-pinned supported pipe (d) for clamped-clamped supported pipe

Bifurcation diagrams with flow velocity as the independent variable show that the system loses stability via a supercritical pitchfork bifurcation leading to divergence. It is seen that this particular system loses stability by divergence at a nondimensional flow velocity $v=\pi$ for simply supported case and $v=2\pi$ for clamped-clamped pipe, in conformity with linear theory, when the pipe is straight. When the pipe posses a geometric imperfection, it has the curvature amplitude. It is seen that, the first bifurcation point (divergence) occurs at progressively higher flow velocities with increasing curvature amplitude, b. It is seen that increasing P is the similar cases to increasing b.

At higher flow velocities, a secondary Hopf bifurcation leads to flutter. Coupled-mode flutter, however, associated with another loss of stability of the trivial equilibrium, as predicted by linear theory, does not arise. At approximately the same v for all curvature amplitude, the non-trivial static solution becomes unstable by a Hopf bifurcation, leading to flutter. This value of v is about 2π for pinned-pinned pipe and about 9.0 for clamped-clamped pipe.



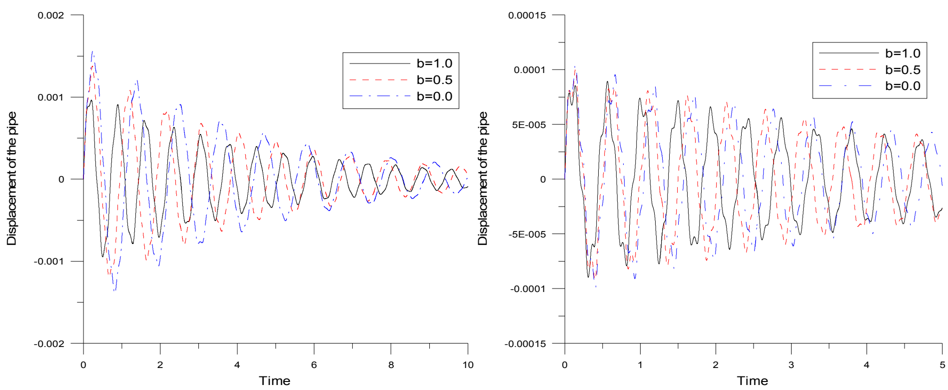
(a) (b)

Figure 5. Bifurcation diagrams of the displacement of the slightly curved pipe obtained with $\beta=0.8$, $c=1.0$ and $b=1.0$ with different values of the pretension, P (a) for pinned-pinned supported pipe (b) for clamped-clamped supported pipe

4.1.3 Influence of externally imposed tension

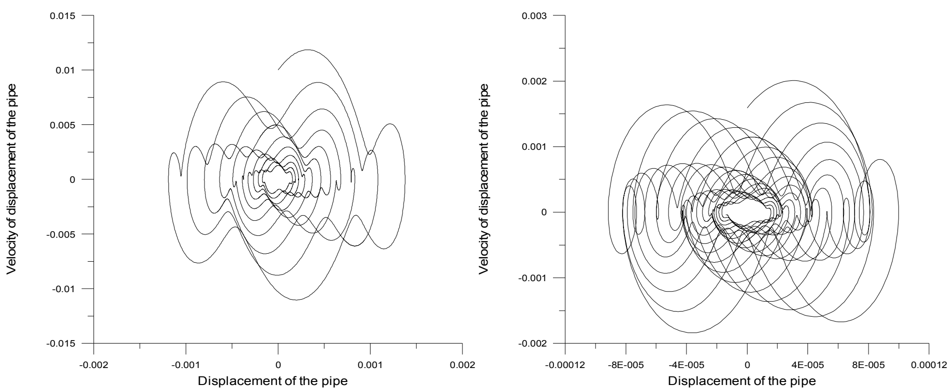
An externally imposed tension represents a pre-strain in the longitudinal direction of the pipe. Figure 5 shows the bifurcation diagrams of the system with varying P for $\beta=0.8$, $c=1.0$ and $b=1.0$ When a larger tension is applied on a pipe, higher flow velocities are needed to cause instability; hence, the critical flow velocity (for divergence) increases. This is because in a stretched pipe, the lateral displacement will be reduced. One would expect the same influence on the behaviour of the system for the curvature amplitude, b, and pretension, P, both of them representing a prestrain in the longitudinal direction of the pipe.

Figures 6 and 7 show the time history and phase plane of the periodic response of the system. With increasing curvature amplitude, b, at a fixed flow velocity, the amplitude of displacement and also period of the system decrease. This is because the lateral displacement will be reduced in a stretched pipe.



(a) (b)

Figure 6. Time histories with μ=0.5 and β=0.8 (a) for pinned-pinned pipe with v=2.5 (b) for clamped-clamped pipe with v=5.0



(a) (b)

Figure 7. Phase plane with b=0.5, μ=0.5 and β=0.8 (a) for pinned-pinned pipe with v=2.5 (b) for clamped-clamped pipe with v=5.0

5. CONCLUSION

In this study, a slightly curved pipes conveying fluid have been considered. The end supports are immovable causing axial stretching during the vibrations. The non-linearity arises due to stretching and curvature. The equations of motion have been written for an arbitrary curvature function and small transverse displacement. The fluid velocity is assumed to be constant. The nonlinear integro-partial differential equation governing the motion is discretized using Galerkin's method. Taking two terms has lead to non-linear ordinary differential equations.

The nonlinear dynamical behaviors for two different support conditions are numerically investigated by means of bifurcation diagrams, the phase portrait and the time history. From the numerical results obtained in this study, following conclusions can be drawn;

- ✓ the equilibrium or periodicity occur with the fluid velocity smaller then the critical velocity;
- ✓ the effect of curvature amplitude increasing the critical flow velocities and the natural frequencies of the system, and decreasing the amplitude of the resultant motions;
- ✓ the system loses stability by a pitchfork bifurcation leading to divergence at higher flow then the critical flow;
- ✓ with further increases in the flow velocity, the system becomes unstable by a Hopf bifurcation, leading to flutter.

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