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PREFACE

This book is prepared in order to present a number of important new phenomena related to the description of nonlinear dynamical systems whose behavior is controlled by piecewise-smooth differential equations.

The practical significance of these phenomena is illustrated through a series of well-documented and realistic applications to switching power converters, relay systems, and different types of pulse-width modulated control systems. Other examples are derived from mechanical engineering, digital electronics, and economic business cycles theory.

Besides providing an overview of the bifurcation scenarios and transitions to chaos observed in piecewise-smooth dynamical systems, the book may also serve as a textbook in nonlinear dynamics for professionally engaged control and power engineers. With this purpose, the book includes two introductory chapters, that can help readers from these fields get accustomed with the concepts of modern nonlinear dynamics. To our knowledge, no other monograph with this focus is available.

Three basic bifurcation scenarios are known for the transition to chaos in smooth dissipative systems: The infinite cascade of period-doubling bifurcations of a cycle (Feigenbaum scenario), the transition through various types of intermittency as a stable cycle ceases to exist (Pomeau-Manneville scenario), and the transition via different forms of torus destruction (Ruelle-Takens-Newhouse scenario). However, these scenarios do not exhaust the possible mechanisms. There is a much broader class of bifurcation phenomena in systems described by non-smooth differential equations that still lack a detailed description and whose theoretical and numerical analysis presents an extremely difficult problem. At the same time, such systems are of significant practical interest, particularly in engineering.

All problems in mechanical engineering involving collisions, finite clearances, and stick-slip phenomena violate the requirements of smoothness to the equations of motion. A simple example is the so-called impact oscillator that is restricted in its motion by collisions with a fixed wall. This type of phenomenon arises, for instance, when the displacement of a vibrating steam pipe become large enough to engender collisions with other construction parts, in connection with pendulum type centrifugal vibration absorbers, in railroad dynamics, and for rotating machines with clearance in the bearings. Another well-known example is the mooring of an oiltanker at sea. In this case the lack of smoothness is related to a drastic change in the force as the mooring cable is stretched.

In microeconomics and management science, human decisions often involve logical functions: Depending on the relative costs of two different options, either one or the other will be chosen. In simulations of managerial systems with repeated decisions, these logical functions play the role of non-smooth restrictions, and we observe similar types of complex nonlinear dynamic phenomena as in other systems with non-smooth equations of motion. This includes, for instance, finite cascades of period-doublings, period-tripling bifurcations, new types of direct transitions to chaos, and resonance tongues delineated by border-collision bifurcations (rather than by saddle-node bifurcations as in smooth systems). An example from this realm is considered in Chapter 7.

However, the main part of the book is dedicated to the analysis of bifurcations and chaotic oscillations in switching power converters, and in relay and pulse-width modulated control systems. Systems of this type have a broad range of applications for power control and temperature regulation and they are also used to obtain highly stabilized electric and magnetic fields. Examples are power supplies for radio-electronics, computer equipment and spacecrafts, test stands for investigations of high- or low-temperature superconductivity, electron microscopes, and nuclear magnetic resonance tomographs.

In Chapter 3 we introduce a system of autonomous differential equations with discontinuous right-hand sides that can serve as a model of relay systems. We illustrate how investigation of this model can be reduced to the study of the properties of a multi-dimensional piecewise-smooth mapping, and describe a general approach to determining the periodic solutions and analyzing their local stability. The problem of determining the limit cycles is reduced to solving a system of transcendental equations with respect to the switching instants of the relay element. This approach allows us to obtain not only the stable but also the unstable periodic orbits.

The second part of Chapter 3 contains two practically important examples of relay control systems. As a first example we consider a DC/DC converter with relay control whose behavior is described by a four-dimensional set of piecewise-linear autonomous differential equations. As a second example we consider a system applied to regulate the motor torque of a DC electric drive. The mathematical model of this system is represented by a four-dimensional set of piecewise-smooth autonomous differential equations.

A separate section is devoted to a detailed description of the technical aspects of the implementation of relay control systems.

In Chapter 4 we perform a detailed analysis of complex behavior and transitions to chaos for two different types of relay control systems that one can meet in real-world technical applications. The aim is to investigate the structure of the two-parameter diagram of dynamical modes and to examine the regularities in the occurrence of periodic cycles. We also determine the main bifurcations and analyze the nature of the transitions to chaotic dynamics.

We show that relay systems can display situations where several locally stable limit cycles with different dynamic characteristics coexist for a wide range of parameters. These cycles typically arise in hard transitions, for example through saddle-node bifurcations, and with changing parameters they can undergo either a finite or an infinite sequence of period-doubling bifurcations, resulting in the transition to chaos. Moreover, the transition to chaos can take place through the appearance and subsequent destruction of a quasiperiodic motion on a two-dimensional torus. As a result we have parameter domains wherein, alongside with locally stable limit cycles, there are co-existing modes of chaotic or quasiperiodic oscillations. Under such conditions the action of external noise, even of low intensity, can induce a sudden transition from one dynamic state to another and, in particular, from regular to chaotic dynamics.

In Chapter 5 we investigate the main bifurcations and routes to chaos for pulse-modulated control systems. As a specific example we consider a DC/DC converter with pulse-width modulation of the second kind. The behavior of such a converter is described by a three-dimensional system of piecewise-smooth differential equations with an external periodic action.

The chapter contains a detailed analysis of border-collision bifurcations and transitions to chaos exhibited by multi-dimensional piecewise-smooth dynamical systems. We show that transitions from periodic to chaotic oscillations can occur through a sequence of border-collision bifurcations including period-doubling, -tripling, -quadrupling, quintupling, etc., bifurcations. More complex bifurcational transitions with fundamentally new properties are cited as examples, for instance, finite sequences of period-tripling bifurcations and soft transitions from one family of cycles to another family of cycles with multiple periods.

Chapter 6 contains the results of an investigation of a number of new phenomena that can arise in piecewise-smooth dynamical systems whose motion involves two (or more) periodic components. When two self-oscillatory systems interact (or a self-oscillatory system is subjected to an external periodic forcing), the total motion can be viewed as occurring on the surface of a two-dimensional torus and, in the absence of resonances, the motion is said to be quasiperiodic. One problem is here associated with the role that border-collision bifurcations play in the synchronization of the two modes, and in the transition to chaos when the torus finally breaks down.

As a specific example of a piecewise-smooth dynamical system with a quasiperiodic route to chaos we consider a three-dimensional piecewise smooth mapping describing the behavior of a DC/DC converter with pulse-width modulation of the first kind. One of the main ideas of Chapter 6 is to show that the transitions to chaos through two-frequency quasiperiodicity in piecewise smooth systems can differ fundamentally from the mechanisms described in the existing literature. At the same time we show that the boundaries of the Arnol'd tongues for non-smooth systems are formed by border-collisions bifurcations, rather than by the saddle-node bifurcations known from smooth systems. Moreover, the internal organization of the tongues is also quite different from the case of smooth systems.

In spite of its orientation towards engineering problems, the book addresses theoretical and numerical problems in connection with border-collision bifurcations and other peculiarities of piecewise-smooth systems in sufficient detail to be of interest to nonlinear scientists in general.

We would like to express our gratitude to our students and collaborators who have played a significant role in the often extremely demanding process of unraveling the complicated bifurcation structure of a nonlinear dynamic system. These thanks are primarily devoted to Vadim Rudakov, Sergei Pinaev, Elena Emelyanova, Mikael Togeby, Erik Reimer Larsen, Jakob Laugesen, Olga Sosnovtseva, Lasse Christiansen, Tue Lehn-Schiøler, Carsten Knudsen, Jesper Skovhus Thomsen, John D. Sterman, Maciej Szymkat, and Hiroshi Matsushita. Yuri and Vladimir Maistrenko, Laura Gardini, and Irina Sushko have contributed through a number of enlightening discussions of nonlinear dynamic phenomena in general and border-collision bifurcations in particular.

Most of all we would like to express our sincere appreciation to Evgeniy Soukhoterin who has been strongly involved in our recent work on study of quasiperiodic route to chaos in pulse-width modulated control systems. Evgeniy Soukhoterin has also contributed invaluably to the preparation of the present book. Without his engagement and care, the book might never have been published.

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The book is dedicated to our wives Inna and Margit.

Kursk and Lyngby, March 2003
Zhanybai Zhusubaliyev and Erik Mosekilde

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