Bilateral Contract Networks for Peer-to-Peer Energy Trading

Thomas Morstyn, *Member, IEEE*, Alexander Teytelboym, and Malcolm D. McCulloch, *Senior Member, IEEE*

***Abstract*—This paper proposes bilateral contract networks as a new scalable market design for peer-to-peer energy trading. Coordinating small-scale distributed energy resources to shape overall demand could offer significant value to power systems, by alleviating the need for investments in upstream generation and transmission infrastructure, increasing network efficiency and increasing energy security. However, incentivising coordination between the owners of large-scale and small-scale energy resources at different levels of the power system remains an unsolved challenge. This paper introduces real-time and forward markets, consisting of energy contracts offered between generators with fuel-based sources, suppliers acting as intermediaries and consumers with inflexible loads, time-coupled flexible loads and/or renewable sources. For each type of agent, utility-maximising preferences for real-time contracts and forward contracts are derived. It is shown that these preferences satisfy full substitutability conditions essential for establishing the existence of a stable outcome—an agreed network of contracts specifying energy trades and prices, which agents do not wish to mutually deviate from. Important characteristics of energy trading are incorporated, including upstream–downstream energy balance and forward market uncertainty. Full substitutability ensures a distributed price-adjustment process can be used, which only requires local agent decisions and agent-to-agent communication between trading partners.**

***Index Terms*—Bilateral contracts, energy trading, electricity markets, game theory, market design, matching markets, microgrids, peer-to-peer trading, prosumers, smart grid, trading networks, transactive energy.**

NOMENCLATURE

$\lvert\cdot\rvert$	Number of elements of a set
$[\cdot]^+$	$\max\{\cdot, 0\}$
$+1_{\{\cdot\}}$	Indicator function for the specified condition
α_{fi}	Prosumer i's flexible load utility
$\underline{\alpha}_{fi}$	Initial α_{fi} for a time-coupled flexible load
$\bar{\alpha}_{fi}$	Final α_{fi} for a time-coupled flexible load
β	Element of $\{0, 1, 2\}$
Δp	Price adjustment increment
$\kappa(\cdot)$	Selected contracts of an arrangement
$\nu_i(t)$	Mean real-time price prediction error
$\sigma_{d_{ri}}$	Standard deviation of prosumer i's net demand
$\sigma_{p_{Ri}}$	Standard deviation of the real-time energy price
$\tau(\cdot)$	Underlying trades for a set of contracts
Ψ_m^*	Selected trades at a competitive equilibrium
Ω_m	Trades in market m
Ω_{mi}^B	Agent i's potential upstream trades
Ω_{mi}^S	Agent i's potential downstream trades
$\mathcal{A}$	Set of agents
$a(\cdot)$	Set of agents associated with a set of contracts
$b(\cdot)$	Buyer of a trade or contract
$C_{mi}(\cdot)$	Agent i's choice correspondence
$C_{mi}^B(\cdot)$	Agent i's chosen upstream contracts
$C_{mi}^S(\cdot)$	Agent i's chosen downstream contracts
c_y'	Net upstream trades
c_z'	Net downstream trades
c_{g1i}	Generator i's linear cost coefficient
c_{g2i}	Generator i's quadratic cost coefficient
c_{t1i}	Supplier i's linear cost coefficient
d_{fi}	Prosumer i's net flexible demand
d_{fi}'	d_{fi}, adjusted for the time-coupled flexible load
$\bar{d}_{fi}$	Prosumer i's flexible load power limit
$D_{fi}(t)$	Remaining time-coupled flexible load
d_{ri}	Prosumer i's net inflexible demand
d_{ri}'	d_{ri}, adjusted for the time-coupled flexible load
$\mathbb{E}[\cdot]$	Expectation operator
g_i	Generator i's unconstrained optimal output
$\bar{g}_i$	Generator i's maximum capacity
$\mathcal{G}$	Set of generator agents
k_{di}^{ra}	Net demand risk-aversion coefficient
k_{pi}^{ra}	Energy price risk-aversion coefficient
m	Forward or real-time market ($m \in \{F, R\}$)
$\mathcal{P}$	Set of prosumer agents
p	Vector of contract prices
p^b	Vector of buyer prices
p_ω	Price (or prices) of a trade (or set of trades)
p_ω^b	Buyer price of a trade
p_ω^s	Seller price of a trade
p_i^{nom}	Nominal average contract price
p_{Ri}	Real-time energy price prediction
$\mathbb{R}$	Set of real numbers
r_i	Prosumer i's risk-aversion exponent
$R_{mi}^B(\cdot)$	Agent i's rejected upstream contracts

Manuscript received September 1, 2017; revised November 24, 2017; accepted December 19, 2017. Date of publication February 2, 2018; date of current version February 18, 2019. This work was supported in part by the Engineering and Physical Sciences Research Council under Grant EP/N03466X/1, and in part by the Oxford Martin Programme on Integrating Renewable Energy. Paper no. TSG-01273-2017. *(Corresponding author: Thomas Morstyn.)*

T. Morstyn and M. D. McCulloch are with the Department of Engineering Science, University of Oxford, Oxford OX1 2JD, U.K. (e-mail: thomas.morstyn@eng.ox.ac.uk; malcolm.mcculloch@eng.ox.ac.uk).

A. Teytelboym is with the Department of Economics, University of Oxford, Oxford OX1 3UQ, U.K. (e-mail: alexander.teytelboym@economics.ox.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSG.2017.2786668

1949-3053 © 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

$R^S_{mi}(\cdot)$	Agent i's rejected downstream contracts
$\mathcal{S}$	Set of supplier agents
$s(\cdot)$	Seller of a trade or contract
$[t_{si}, t_{fi}]$	Range of trading intervals for prosumer i to fulfil its time-coupled flexible load
t'_{fi}	Final flexible interval for a time-coupled load
$u_{mi}(\cdot)$	Agent i's valuation function
$U_{mi}(\cdot)$	Agent i's utility function
$U^{ra}_{Fi}(\cdot)$	Prosumer i's risk averse forward market utility
$V_{mi}(\cdot)$	Agent i's indirect utility function
X_m	Set of contracts in market m
X_{mi}	Agent i's contracts in market m
Y_m	Set of upstream contracts in market m
Y_{mi}	Agent i's upstream contracts
Y^*_{mi}	Agent i's selected upstream contracts
$\mathbb{Z}$	Set of integers
Z_m	Set of downstream contracts in market m
Z_{mi}	Agent i's downstream contracts
Z^*_{mi}	Agent i's selected downstream contracts.

I. Introduction

POWER systems are undergoing a fundamental transition due to two technology trends: 1) The adoption of distributed energy resources, including renewable sources and flexible loads such as smart appliances, electric vehicles and heat pumps [1]. 2) Communications and control extending to the individual consumer level via smart meters and energy management systems [2]. This has allowed traditionally passive small-scale electricity consumers to become prosumers, proactive-consumers that actively manage their consumption and production of energy [3].

In liberalised electricity markets, large-scale generators, suppliers and industrial consumers can trade energy in wholesale markets organised by the transmission system operator (TSO) [4]. However, it is considered impractical for small-scale distribution level prosumers to participate in wholesale markets. This is due to their relatively small individual impact on transmission system operations, and the cost and complexity of the communications/processing infrastructure that would be required to integrate them into the TSO's dispatch and settlement procedures [5].

Instead, small-scale prosumers are serviced in retail markets, where they are individually metered by large suppliers. However, individually metered prosumers are only incentivised to optimise their own local energy usage. If properly coordinated to shape overall demand, distributed energy resources could offer significant value to power systems, by alleviating the need for investments in upstream generation and transmission infrastructure, increasing network efficiency and increasing energy security [6]. However, incentivising coordination between the owners of large-scale and small-scale energy resources at different levels of the power system remains an unsolved challenge.

Time-of-use retail prices can incentivise prosumers to shift flexible loads to periods when low net demand is expected, incentivise demand response [7]. The strategic interactions between prosumers and a supplier that sets prices to maximise profits can be modelled by a Stackelberg game [8], [9]. A limitation is that new, possibly worse, demand peaks can be created if prosumers all take advantage of the same low price periods [10].

Distributed energy resources that are owned and operated by a single entity can be optimally scheduled [11], and distributed optimisation can be used to improve scalability [12]. However, an optimisation approach is not directly applicable if distributed energy resources have different owners. Although, optimisation dual price variables can be interpreted as competitive energy prices [13]–[15], they will not incentivise coordination if individual agents have market power.

Peer-to-peer (P2P) energy trading markets have been proposed to incentivise efficient prosumer distributed energy resource utilisation [16]. Sectors amenable to P2P trading are characterised by a lack of economies of scale, and diversity of demand [17]. The emergence of prosumers has made these conditions applicable to power systems.

Market designs to incentivise local energy trading between small-scale generators and consumers are presented in [18]–[23]. These designs are based on central auctions or profit-sharing mechanisms from cooperative game theory. A limitation of these designs is that they do not consider strategic decision making by upstream agents. The designs in [18] and [19] consider isolated systems, with a single market allowing all generators and consumers to trade with one another. This can be suitable for microgrids, but limits scalability. The designs in [20]–[23] assume there is a large upstream wholesale market/supplier, which buys/sells energy at fixed prices. As distributed energy resources begin to provide significant capacity, local energy trading is expected to have an increasingly large effect on upstream market prices.

Another important consideration is the need for forward energy trading, for example in a day-ahead market. Forward and real-time energy contracts are considered in [23], but it is noted the analysis only applies when there is a single consumer. At the transmission level, forward energy trading allows agents to hedge against real-time price risk and provides the TSO with information for reliability unit commitment. Forward energy trading is expected to become important at the local level, as microgrids and actively managed distribution networks become more prevalent [24].

Recently, there have been significant theoretical advances in the design of many-to-many matching markets with contracts. Matching markets consider the formation of mutually beneficial trading arrangements between groups of agents, based on agent-to-agent negotiation [25]. Key considerations are the existence of stable outcomes (networks of agreed contracts which agents do not wish to mutually deviate from), and scalable strategies for finding them. In [26], full substitutability is presented as a key stability condition for agents operating in a hierarchical supply chain. An agent's preferences satisfy full substitutability if: 1) when offered additional upstream contracts, the agent continues to accept previously accepted downstream contracts, and continues to reject previously rejected upstream contracts, and 2) when offered additional downstream contracts, the agent continues to accept previously

accepted upstream contracts, and continues to reject previously rejected downstream contracts. In [27] and [28], markets with cyclical contractual relationships are considered, where a pair of agents may simultaneously buy inputs and sell outputs to one another. This closely resembles the aim of a P2P energy trading market, which should allow structured trading between generators, suppliers and prosumers, which may each buy and sell energy.

This paper proposes bilateral contract networks as a new scalable market design for P2P energy trading. Real-time and forward markets are introduced, consisting of energy contracts offered between generators with traditional fuel-based sources, suppliers acting as intermediaries and prosumers with inflexible loads, time-coupled flexible loads and/or renewable sources. For each type of agent, utility-maximising preferences for real-time contracts and forward contracts are presented. It is shown that these preferences satisfy full substitutability conditions essential for establishing the existence of a stable outcome. The agent preferences are subject to several simplifying assumptions, but capture important characteristics particular to energy trading, including upstream-downstream energy balance and forward market uncertainty. The market power created by the structure of trading relationships between the agents is explicitly accounted for. Full substitutability ensures a distributed price-adjustment process can be used, which only requires local agent decisions and agent-to-agent communication between trading partners.

The rest of this paper is organised as follows. Section II describes the proposed market design and embeds it within the theoretical framework of many-to-many matching markets with contracts. Section III presents the preferences for each type of agent, and a scalable process allowing the agents to select utility-maximising contract bundles. Section IV describes the price-adjustment process. In Section V, simulation case studies are presented demonstrating the operation of the forward and real-time markets. Section VI concludes the paper. In the Appendix, it is shown that the agent preferences satisfy full substitutability, establishing the existence of a stable outcome.

II. Contract Networks for Energy Trading

This paper proposes forward and real-time P2P energy markets, each consisting of energy contracts traded between autonomous agents. Three types of agents are considered: 1) 'prosumers', which have a combination of inflexible loads, flexible loads and/or local renewable sources, 2) 'suppliers', which act as intermediaries and 3) 'generators' with traditional fuel-based sources. In the literature, agents fulfilling the suppliers' intermediary role are sometimes referred to as aggregators (e.g., [29]).

The potential trading relationships between the agents are described by a contract network. Each energy contract specifies the price of a discrete quantity of energy offered by one agent to another. Physical and informational restrictions on which agents can trade with one another are reflected by the topology of the contract network.

A forward market and real-time market are introduced for each energy trading interval during the day (e.g., each hour). In the forward market, agents buy and sell energy contracts based on their expectations of the real-time energy price and their net demand. In the real-time market, loads and renewable generation are no longer uncertain. The agents must generate energy, or buy it upstream, to meet their local loads and downstream obligations from the forward market.

The proposed market design operates on the tertiary power system control layer, which is used to coordinate energy flows on a relatively slow time-scale (e.g., one hour intervals). Lower level primary and secondary control layers operating at faster time-scales will still be necessary to maintain power quality and stability [30]. In particular, the lower level control layers need to maintain supply-demand balance when there are renewable generation or load variations which occur within individual trading intervals.

A key market design objective is to find a scalable price-adjustment process that ensures the agents reach agreement on a set of contracts which constitute a stable outcome – a set of contracts which no group of agents wish to deviate from. As with other energy market designs, losses are not explicitly considered, and need to be accounted for by a separate settlement process [31].

Let $\mathcal{A}$ be the set of agents. Let X_m be the set of contracts for market m, where $m \in \{F, R\}$, indicating the forward market and real-time market respectively. A contract $x \in X_m$ is a pair (ω, p_ω), where $\omega \in \Omega_m$ is the underlying trade and p_ω is the price specified by the contract for the trade. For contracts $X'_m \subseteq X_m$, let $\tau(X'_m) := \{\omega \in \Omega_m | (\omega, p_\omega) \in X'_m\}$ be the set of underlying trades.

Each trade ω has a buyer $b(\omega) \in \mathcal{A}$ and a seller $s(\omega) \in \mathcal{A}$. These are also the buyer and seller for each contract involving ω, i.e., for contract $x = (\omega, p_\omega)$ we denote its buyer $b(x) := b(\omega)$ and seller $s(x) := s(\omega)$. The agents involved with a set of contracts $X'_m \subseteq X_m$ are given by $a(X'_m) := \{\cup_{x \in X'_m} b(x), s(x)\}$. Agent i has upstream contracts $Y_{mi} := \{y \in X_m | i = b(y)\}$, and downstream contracts $Z_{mi} := \{z \in X_m | i = s(x)\}$.

Each agent i has a valuation function u_{mi} over sets of trades in market m, which can be nonlinear. It is assumed that the agents' preferences are linear with respect to contract prices, resulting in quasi-linear utility functions over contracts. For contracts $A_{mi} = Y_{mi} \cup Z_{mi}$, agent i's utility function is,

$$U_{mi}(A_{mi}) = u_{mi}(\tau(A_{mi})) + \sum_{(\omega, p_\omega) \in Z_{mi}} p_\omega - \sum_{(\phi, p_\phi) \in Y_{mi}} p_\phi. \tag{1}$$

The agent's utility function gives rise to a choice correspondence, which specifies the agent's preferences over a set of contracts,

$$C_{mi}(A_{mi}) = \text{argmax}_{B_{mi} \subseteq A_{mi}} U_{mi}(B_{mi}). \tag{2}$$

For upstream contracts $Y_m \subseteq X_m$ and downstream contracts $Z_m \subseteq X_m$, agent i's chosen sets of upstream and downstream contracts are defined respectively as,

$$C^B_{mi}(Y_m | Z_m) := C_{mi}(Y_{mi} \cup Z_{mi}) \cap Y_m, \tag{3}$$

$$C^S_{mi}(Z_m | Y_m) := C_{mi}(Z_{mi} \cup Y_{mi}) \cap Z_m. \tag{4}$$

Similarly, the rejected sets of upstream and downstream contracts are defined respectively as,

$$R^B_{mi}(Y_m|Z_m) \coloneqq Y_{mi} \setminus C^B_{mi}(Y_{mi}|Z_{mi}), \tag{5}$$
$$R^S_{mi}(Z_m|Y_m) \coloneqq Z_{mi} \setminus C^S_{mi}(Z_{mi}|Y_{mi}). \tag{6}$$

A set of contracts A_m is feasible if each trade $\omega \in \tau(A_m)$ is associated with at most one contract, and therefore has a unique price. A market outcome is a feasible set of contracts, $A_m \subseteq X_m$. An arrangement $[\Psi_m|p]$ is a set of selected trades $\Psi_m \subseteq \Omega_m$ and a vector p specifying a unique price for each trade in the market Ω_m. Let $\kappa([\Psi_m|p])$ be the set of selected contracts associated with the arrangement $[\Psi_m|p]$.

The following definitions for a stable outcome and a competitive equilibrium, from [27], are used.

Definition 1: An outcome A_m is *stable* if it is:

1) *Individually Rational:* $A_{mi} \in C_{mi}(A_m)$ for all $i \in \mathcal{A}$;
2) *Unblocked:* There is no feasible non-empty blocking set $B_m \subseteq X_m$. B_m is a blocking set if: a) $B_m \cap A_m = \varnothing$, and b) for all $i \in a(B_m)$, $B_{mi} \subseteq C_{mi}(B_m \cup A_m)$.

Individual rationality means that no agent would prefer to drop a contract from the outcome. Unblocked means there is no subset of contracts from the outcome which all involved agents would choose to drop in favour of a different (blocking) set of contracts that is not included in the outcome.

Definition 2: A *competitive equilibrium* is an arrangement $[\Psi^*_m|p^*]$, $\Psi^*_m \subseteq \Omega_m$, such that for all $i \in \mathcal{A}$, $\kappa([\Psi^*_{mi}|p^*]) = C_{mi}(\kappa([\Omega_m|p^*]))$.

In other words, a competitive equilibrium specifies a full set of prices p^* for trades in the market, and a set of trades Ψ^*_m which are mutually selected by the agents at these prices.

A key result from [27] is that when the preferences of the agents satisfy *full substitutability*, competitive equilibria are guaranteed to exist, and these competitive equilibria coincide with the set of stable outcomes. In addition, it is shown that when the agent preferences are not fully substitutable, stable outcomes and competitive equilibria may not exist.

Definition 3: The preferences of agent i are *fully substitutable* if, for all upstream contracts $Y'_m \subseteq Y_m$ and downstream contracts $Z'_m \subseteq Z_m$, the agent's choice correspondence is:

1) *Same-Side Substitutable:*
   a) $R^B_{mi}(Y'_m|Z_m) \subseteq R^B_{mi}(Y_m|Z_m)$,
   b) $R^S_{mi}(Z'_m|Y_m) \subseteq R^S_{mi}(Z_m|Y_m)$.
2) *Cross-Side Complementary:*
   a) $R^B_{mi}(Y_m|Z_m) \subseteq R^B_{mi}(Y_m|Z'_m)$,
   b) $R^S_{mi}(Z_m|Y_m) \subseteq R^S_{mi}(Z_m|Y'_m)$.

If an agent has fully substitutable preferences and is offered additional upstream (downstream) contracts, then it will 1) continue to reject other upstream (downstream) contracts it previously rejected, and 2) continue to accept downstream (upstream) contracts it previously accepted.

In this setting, full substitutability rules out several important classes of agent preferences. In particular, it rules out economies of scale for consumption and for production [27]. However, a range of important classes of economic preferences can still be captured.

III. Agent Preferences and Contract Selection

In this section, preferences are presented for prosumers, suppliers and generators, for both the forward and real-time markets. Then, a scalable process allowing agents to select utility-maximising bundles of contracts is presented. In the Appendix, it is shown that the agent preferences satisfy full substitutability, establishing the existence of a stable outcome.

A. Prosumers

Let $\mathcal{P} \subseteq \mathcal{A}$ be the set of prosumers. The prosumers may have inflexible loads, flexible loads and/or renewable sources. The prosumers must ensure their local loads and downstream energy contracts are satisfied by local generation and/or upstream contracts. In the real-time market, prosumer i knows its net demand $d_{ri} \in \mathbb{Z}$ ($d_{ri} < 0$ corresponds to net generation). Note that d_{ri} is normalised by the energy per contract. The prosumer receives utility $\alpha_{fi} \geq 0$ for each unit of energy above its net demand, up to its flexible load limit $d_{fi} \in \mathbb{Z}_{\geq 0}$. Note that any utility function with diminishing returns can be used while maintaining full substitutability. For the forward and real-time markets respectively, let Y_{Fi}, Y_{Ri} be the chosen upstream contracts, and let Z_{Fi}, Z_{Ri} be the chosen downstream contracts. In the real-time market, prosumer i's preferences are described by a utility function,

$$U_{Ri}(Y_{Ri}, Z_{Ri}) = \begin{cases} \sum_{(\omega,p_\omega)\in Z_{Ri}\cup Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Ri}\cup Y_{Fi}} p_\phi \\ \quad + \alpha_{fi}\min\{d_{fi}, |Y_{Ri}| + |Y_{Fi}| - |Z_{Ri}| - |Z_{Fi}| - d_{ri}\}, \\ \text{if } |Y_{Ri}| + |Y_{Fi}| - |Z_{Ri}| - |Z_{Fi}| \geq d_{ri}, \\ -\infty, \ \text{otherwise.} \end{cases} \tag{7}$$

In the forward market, the prosumer treats its net demand (load less renewable generation), d_{ri}, and the real-time price of energy, $p_{Ri} \geq 0$, as independent random variables. Note that the prosumers assume upstream and downstream energy contracts will have the same real-time price. Given a real-time price of energy of p_{Ri}, and a certain number of forward upstream and downstream contracts, prosumer i will pursue the following utility-maximising contracting strategy in the real-time market,

$$|Y_{Ri}| = \begin{cases} [|Z_{Fi}| - |Y_{Fi}| + d_{ri}]^+, & \text{if } p_{Ri} > \alpha_{fi}, \\ \left[|Z_{Fi}| - |Y_{Fi}| + d_{fi} + d_{ri}\right]^+, & \text{if } p_{Ri} \leq \alpha_{fi}, \end{cases}$$
$$|Z_{Ri}| = \begin{cases} [|Y_{Fi}| - |Z_{Fi}| - d_{ri}]^+, & \text{if } p_{Ri} > \alpha_{fi}, \\ \left[|Y_{Fi}| - |Z_{Fi}| - d_{fi} - d_{ri}\right]^+, & \text{if } p_{Ri} \leq \alpha_{fi}, \end{cases}$$

where $[\,\cdot\,]^+ = \max\{\cdot, 0\}$. Therefore, if prosumer i's forward market preferences are to maximise its expected utility in the real-time market, they are described by,

$$\begin{aligned} U_{Fi}(Y_{Fi}, Z_{Fi}) = &\sum_{(\omega,p_\omega)\in Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Fi}} p_\phi + \alpha_{fi} d_{fi} \mathbb{E}\Big[\mathbf{1}_{\{p_{Ri}\leq\alpha_{fi}\}}\Big] \\ &- d_{fi}\mathbb{E}\big[p_{Ri}\mathbf{1}_{\{p_{Ri}\leq\alpha_{fi}\}}\big] \\ &+ \mathbb{E}\big[p_{Ri}\big](|Y_{Fi}| - |Z_{Fi}| - \mathbb{E}[d_{ri}]). \end{aligned} \tag{8}$$

$\mathbf{1}_{\{p_{Ri}\leq\alpha_{fi}\}}$ is the indicator function, which is equal to 1 if $p_{Ri} \leq \alpha_{fi}$ and 0 otherwise.

Given their small-size, prosumers may be risk averse, and therefore may prefer to buy forward contracts to hedge against potential real-time market price risk. A risk averse version of (8) is given by,

$$U_{Fi}^{ra} = U_{Fi} + \left(k_{pi}^{ra}\sigma_{p_{Ri}} + k_{di}^{ra}\sigma_{d_{ri}}\right)\left(|Y_{Fi}|^{r_i} - |Z_{Fi}|^{\frac{1}{r_i}}\right) \quad (9)$$

$\sigma_{p_{Ri}}$ and $\sigma_{d_{Ri}}$ are the standard deviations of p_{Ri} and d_{Ri}. Increasing $k_{pi}^{ra} \geq 0$, $k_{di}^{ra} \geq 0$ and $r_i \geq 1$ increases the level of risk aversion, i.e., the extent to which higher real-time price variability and demand variability increases the prosumer's willingness to buy upstream forward contracts, and decreases its willingness to sell downstream forward contracts.

Theorem 1: The prosumer forward and real-time market preferences, described by $U_{Ri}, U_{Fi}, U_{Fi}^{ra}, i \in \mathcal{P}$, are fully substitutable.

The prosumer preferences can be updated between trading intervals to account for time-coupled flexible loads, which require a certain amount of energy over a number of trading intervals, considering a maximum power limit. This could be used for several different types of flexible loads including electric hot water tanks and electric vehicle charging [32]. Let $t \in [t_{si}, t_{fi}]$ be the current trading interval, where $[t_{si}, t_{fi}]$ is the range of trading intervals over which the time-coupled flexible load must be satisfied. Let $D_{fi}(t)$ be the remaining energy that must be delivered, and let $\bar{d}_{fi}$ be the flexible load maximum power limit.

With significant time remaining for the flexible load to be satisfied, the prosumer will only be willing to buy energy if it is below a minimum price $\underline{\alpha}_{fi}$ it expects to have to pay if it delays buying energy. However, as the deadline approaches, the amount the prosumer is willing to pay will increase, up to a maximum price it expects to pay when there is no flexibility $\bar{\alpha}_{fi}$.

Therefore, a time-coupled flexible load can be incorporated into prosumer i's preferences at trading interval t, by calculating an updated required demand $d'_{ri}(t)$, flexible demand $d'_{fi}(t)$ and flexible load utility $\alpha_{fi}(t)$, according to,

$$d'_{ri}(t) = \begin{cases} d_{ri}(t), & t < t'_{fi}, \\ d_{ri}(t) + \min\{D_{fi}(t), \bar{d}_{fi}\}, & \text{otherwise}, \end{cases}$$

$$d'_{fi}(t) = \begin{cases} \min\{D_{fi}(t), \bar{d}_{fi}\}, & t < t'_{fi}, \\ 0, & \text{otherwise}, \end{cases}$$

$$\alpha_{fi}(t) = \begin{cases} \dfrac{\underline{\alpha}_{fi}\left(t'_{fi} - t\right) + \bar{\alpha}_{fi}(t - t_{si})}{t'_{fi} - t_{si}}, & t < t'_{fi}, \\ 0, & \text{otherwise}, \end{cases}$$

$$t'_{fi} = t_{fi} - \frac{D_{fi}(t)}{\bar{d}_{fi}}.$$

B. Suppliers

Let $\mathcal{S} \subseteq \mathcal{A}$ be the set of suppliers. Suppliers act as intermediaries, buying and selling energy from other agents. The downstream energy contracts sold by a supplier must be matched by upstream energy contracts it has bought. It is assumed supplier i incurs linear transaction costs for energy contracts it buys, modelled by a coefficient $c_{t1i} \geq 0$. In the real-time market, supplier i's preferences are described by a utility function,

$$U_{Ri}(Y_{Ri}, Z_{Ri}) = \begin{cases} \sum_{(\omega,p_\omega)\in Z_{Ri}\cup Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Ri}\cup Y_{Fi}} p_\phi \\ -c_{t1i}(|Y_{Ri}| + |Y_{Fi}|), \text{ if } |Y_{Ri}| + |Y_{Fi}| \geq |Z_{Ri}| + |Z_{Fi}|, \\ -\infty, \quad \text{otherwise}. \end{cases} \quad (10)$$

In the forward market, the supplier assumes upstream and downstream real-time energy contracts will be available at a price of $p_{Ri} \geq 0$, which is treated as a random variable. Real-time upstream contracts will be bought to balance the supplier's downstream obligations from the forward market. Therefore, supplier i's preferences to maximise its expected utility in the real-time market are described by the following forward market utility function,

$$U_{Fi}(Y_{Fi}, Z_{Fi}) = \sum_{(\omega,p_\omega)\in Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Fi}} p_\phi + \mathbb{E}\big[p_{Ri}\big](|Y_{Fi}| - |Z_{Fi}|) - c_{t1i}\big(|Y_{Fi}| + [|Z_{Fi}| - |Y_{Fi}|]^+\big). \quad (11)$$

Theorem 2: The supplier forward and real-time market preferences, described by $U_{Ri}, U_{Fi}, i \in \mathcal{S}$, are fully substitutable.

C. Generators

Let $\mathcal{G} \subseteq \mathcal{A}$ be the set of generators. The generators have traditional fuel-based sources. The generators must ensure their downstream contracts are satisfied by local generation and/or upstream contracts. Traditional generators with fuel-based sources are often modelled as having increasing marginal costs [33]–[38]. This is suitable for two standard situations: 1) a single generation unit with increasing costs due to increased marginal fuel consumption at higher output powers and 2) multiple generation units, which are dispatched from lowest marginal cost to highest marginal cost. In this paper, a quadratic cost function is adopted, with linear and quadratic coefficients $c_{g1i}, c_{g2i} \geq 0$ [36]. Note that any function with constant or increasing marginal costs can be used while maintaining full substitutability. However, decreasing marginal costs (economies of scale) are restricted. Generator i has a maximum generation capacity $\bar{g}_i$. In the real-time market, generator i's preferences are described by a utility function,

$$U_{Ri}(Y_{Ri}, Z_{Ri}) = \begin{cases} \sum_{(\omega,p_\omega)\in Z_{Ri}\cup Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Ri}\cup Y_{Fi}} p_\phi \\ -c_{g1i}[|Z_{Ri}| + |Z_{Fi}| - |Y_{Ri}| - |Y_{Fi}|]^+ \\ -c_{g2i}\big([|Z_{Ri}| + |Z_{Fi}| - |Y_{Ri}| - |Y_{Fi}|]^+\big)^2, \\ \text{if } \bar{g}_i \geq |Z_{Ri}| + |Z_{Fi}| - |Y_{Ri}| - |Y_{Fi}| \geq 0, \\ -\infty, \text{ otherwise}. \end{cases} \quad (12)$$

In the forward market, the generator assumes upstream and downstream real-time energy contracts will be available at a price of $p_{Ri} \geq 0$, which is treated as a random variable. Given a real-time price of energy of p_{Ri}, and a certain number of upstream and downstream contracts, generator i will pursue the

following utility-maximising contracting strategy in the real-time market,

$$|Y_{Ri}| = \left[|Z_{Fi}| - |Y_{Fi}| - \min\{\bar{g}_i, \max\{g_i, 0\}\}\right]^+,$$
$$|Z_{Ri}| = \left[|Y_{Fi}| - |Z_{Fi}| + \min\{\bar{g}_i, \max\{g_i, 0\}\}\right]^+,$$
$$g_i = \frac{p_{Ri} - c_{g1i}}{2c_{g2i}}.$$

Therefore, generator i's preferences to maximise its expected utility in the real-time market are described by the following forward market utility function,

$$U_{Fi}(Y_{Fi}, Z_{Fi}) = \sum_{(\omega,p_\omega)\in Z_{Fi}} p_\omega - \sum_{(\phi,p_\phi)\in Y_{Fi}} p_\phi + \mathbb{E}\big[p_{Ri}\big](|Y_{Fi}| - |Z_{Fi}|) + \mathbb{E}\Big[\Big(g_i p_{Ri} - c_{g1i} g_i - c_{g2i} g_i^2\Big)\mathbf{1}_{\{\bar{g}_i \geq g_i \geq 0\}} + \Big(\bar{g}_i p_{Ri} - c_{g1i}\bar{g}_i - c_{g2i}\bar{g}_i^2\Big)\mathbf{1}_{\{g_i \geq \bar{g}_i\}}\Big]. \tag{13}$$

Theorem 3: The generator forward and real-time market preferences, described by $U_{Ri}, U_{Fi}, i \in \mathcal{G}$, are fully substitutable.

D. Contract Selection

At each iteration of the price-adjustment process for the forward and real-time P2P energy markets, each agent is offered a set of contracts by its neighbours in the contract network. Since the agent preferences satisfy full substitutability, the agent utility functions exhibit the single improvement property [39]. The single improvement property states that an agent with a suboptimal bundle of contracts can be made better off by taking one of the following options:

1) Accepting a new upstream or downstream contract.
2) Relinquishing an upstream or downstream contract.
3) Simultaneously accepting, or relinquishing, both an upstream and downstream contract.

Also, from (7)–(13), the marginal utility an agent receives for accepting a contract only depends on its price and the number of previously accepted upstream and downstream trades. Therefore, the following process can be used by the agents to select an optimal bundle of contracts: First, the agent accepts its minimum required number of upstream and downstream contracts. For example, in the real time market, prosumer i must accept at least $[|Z_{Fi}| - |Y_{Fi}| + d_{ri}]^+$ upstream contracts to meet its inflexible load and forward market obligations. Upstream contracts should be selected from lowest price to highest, and downstream contracts from highest price to lowest.

Then, the agent continues to select contracts until it cannot increase its utility further. Iteratively, the agent should consider the marginal utility provided by selecting either 1) the lowest price available upstream contract, 2) the highest price available downstream contract or 3) both the lowest price upstream contract and highest price downstream contract. The option (or multiple options in the case of ties) providing the greatest increase in utility should be taken. Once the agent cannot further increase its utility, a utility-maximising bundle of contracts has been found.

The process is scalable, since, at each iteration, only the lowest price remaining upstream contract and highest price remaining downstream need to be considered, and the number of available contracts is reduced.

IV. Price-Adjustment Process

Since the agents have fully substitutable preferences, the distributed price-adjustment process from [27] can be used to find a stable outcome. The price-adjustment process is completed for the forward and real-time markets, so that in each market, the agents agree on a network of contracts which none of them wish to mutually deviate from. The price-adjustment process is constructed with buyers making progressively higher offers for trades to potential sellers, and sellers choosing to accept or reject the offers they receive.

Let Δp be the increment of the price-adjustment process, which is the minimum difference in price that can be specified between contracts. For market $m \in \{F, R\}$:

1) Each agent $i \in \mathcal{A}$ starts by specifying the upstream trades it may be willing to make with sellers. Let the full set of trades in the market be given by Ω_m. For agent i, the set of potential upstream trades is given by $\Omega^B_{mi} = \{\omega \in \Omega_{mi} | i = b(\omega)\}$ and the set of potential downstream trades is given by $\Omega^S_{mi} = \{\omega \in \Omega_{mi} | i = s(\omega)\}$.
2) Each trade $\omega \in \Omega_m$ has a buyer price p^b_ω and a seller price p^s_ω. Initially, $p^b_\omega = p^s_\omega = 0$.
3) Iteratively:
   a) Each agent $i \in \mathcal{A}$ constructs a set of upstream contracts to choose from: $Y_{mi} = \{(\cup_{\omega\in\Omega^B_{mi}}(\omega, p^b_\omega)\}$.
   b) Each agent $i \in \mathcal{A}$ constructs a set of downstream contracts to choose from: $Z_{mi} = \{(\cup_{\omega\in\Omega^S_{mi}}(\omega, p^s_\omega)\}$.
   c) Each agent $i \in \mathcal{A}$ selects its favourite set of upstream contracts $Y^*_{mi} = C^B_{mi}(Y_{mi}|Z_{mi})$ and downstream contracts $Z^*_{mi} = C^S_{mi}(Z_{mi}|Y_{mi})$.
   d) For each trade $\omega \in \Omega_m$, the buyer and seller prices are adjusted as follows:
      if $\omega \in Y^*_{mb(\omega)}$ and $\omega \in Z_{ms(\omega)} \setminus Z^*_{ms(\omega)}$ **then**
         if $p^b_\omega > p^s_\omega$ **then**
            $p^s_\omega \leftarrow p^s_\omega + \Delta p$
         else
            $p^b_\omega \leftarrow p^b_\omega + \Delta p$
         end if
      end if
   e) The price-adjustment process is complete when no price changes occur during an iteration.
4) The arrangement $[\tau(\cup_{i\in\mathcal{A}} Y^*_{mi})|p^b]$ is a competitive equilibrium, where p^b is the vector of buyer prices for the trades in Ω_m. This coincides with a stable outcome, $\cup_{i\in\mathcal{A}} Y^*_{mi}$.

Potential trades start at low prices, which are desirable for buyers, but undesirable for sellers. At each iteration of the price-adjustment process, agents select their favourite set of upstream and downstream trades, given the current buyer and seller prices. No agent will select a set of trades it is unable or unwilling to make. Then, trades that are selected by buyers but rejected by sellers have their prices increased. The price-adjustment process ends once all trades have either been

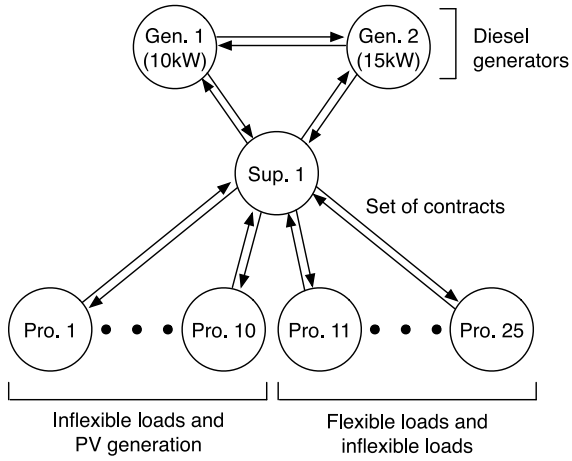


Fig. 1. The contract network for case studies A and B.

TABLE I
CASE STUDY PARAMETERS

c_{g11}	\$0.038/kWh	c_{g21}	\$0.015/(kWh)^2	$\bar{g}_1$	10kW
c_{g12}	\$0.047/kWh	c_{g22}	\$0.008/(kWh)^2	$\bar{g}_2$	15kW
c_{t1}	\$0.01/kWh	$\underline{\alpha}_f$	\$0.05/kWh	$\overline{\alpha}_f$	\$0.15/kWh
$[t_s, t_f]$	$[1, \ldots, 24]$	$D_f(t_s)$	5kWh	$\overline{d}_f$	1kW
Case Study A (risk neutral prosumers)					
k_p^{ra}	0	k_d^{ra}	0	r	1
Case Study B (risk averse prosumers)					
k_p^{ra}	1	k_d^{ra}	10	r	1.2

accepted or rejected by both their buyer and seller. All agents will be able and willing to fulfil their selected trades.

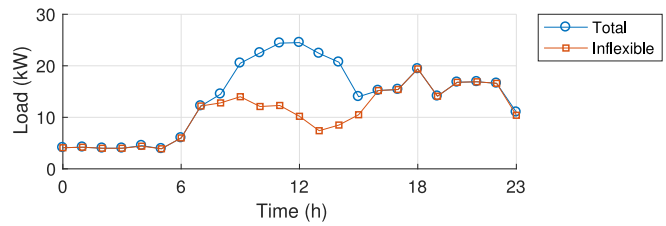
With buyers making offers to sellers, the stable outcome that results is the one that maximises the utility that accrues to agents that only operate as buyers [27]. An alternative price-adjustment process can be similarly constructed, starting at high energy prices, with sellers making progressively lower offers for trades to potential buyers.

V. RESULTS

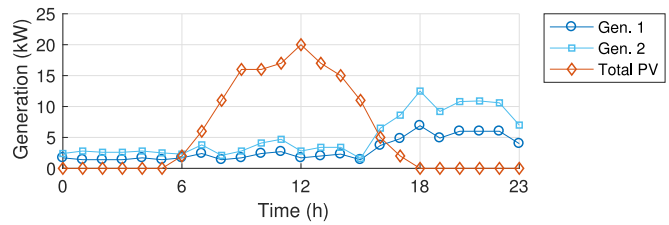
Case study simulations were completed to verify the operation of the proposed energy market design for an islanded microgrid with two diesel generators, an intermediate supplier and 25 prosumers. Fig. 1 shows the contract network between the agents, which structures their trading relationships. The two generators can trade energy directly with one another, and the supplier acts as an intermediary between the generators and the prosumers. This models the situation when a supplier provides the interface and communications which allow the prosumers to engage in P2P energy trading, and thus acts as an intermediary. Note that the proposed methodology does not preclude direct energy trading between generators and prosumers from being introduced.

The case study parameters are shown in Table I. Energy trading occurs at each hour over a day, with 0.1kWh energy trades, and a price increment of \$0.001 (\$0.01 per kWh).

Each prosumer has an inflexible load profile, from data supplied with the IEEE European Low Voltage Test Feeder [40].



(a) Case Study A: Total prosumer consumption and inflexible load.



(b) Case Study A: Power generated by generators 1 and 2, and the total prosumer PV generation.

Fig. 2. Case Study A: Energy trading with risk neutral prosumers.

Prosumers 1–10 have PV generation sources with 2kW nominal capacity. The PV generation profile shared by the prosumers was calculated using irrandiance and temperature data from the NREL Baseline Measurement System for the 1st of June, 2016 [41]. Prosumers 11–25 do not have PV generation sources, but each have a time-coupled flexible load which must be supplied with 5kWh of energy over the day (e.g., an electric hot water tank). These loads have a power limit of 1kW.

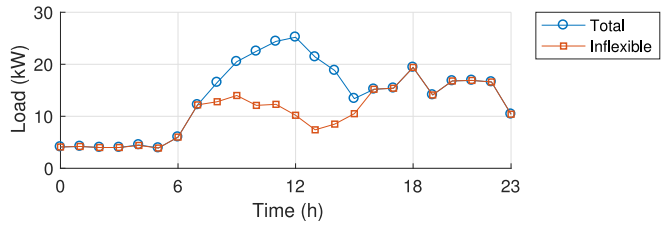
Generators 1 and 2 have diesel generators, with 10kW and 15kW capacity respectively. Quadratic marginal cost functions were obtained using fuel consumption data from Cummins [42]. The supplier has linear transaction costs of \$0.01/kWh.

In the forward market, the agents treat the real-time price of energy as a random variable. The prosumers also treat their inflexible load as a random variable. In practice, the agents would need to 'learn' the distributions for these variables through repeated trading. For this study, the distributions of the prosumer inflexible loads were obtained based on the 100 load profiles the prosumer loads were drawn from. To model the agents having individual inaccurate real-time price estimates, first a real-time market was simulated to obtain a nominal average contract price for each agent p_i^{nom}. Then, in the forward market, each agent treated the real-time price $p_{Ri}(t)$ at each time interval as having mean price $\nu_i(t)p_i^{nom}$ and standard deviation $0.17p_i^{nom}$, where $\nu_i(t) \sim \text{unif}(-0.3, 0.3)$.

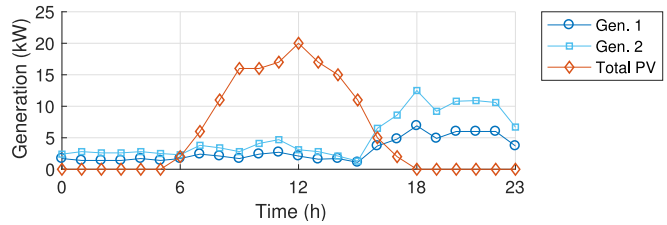
Two case study simulations are presented: Case Study A shows the operation of the forward and real-time markets for risk neutral prosumers, and Case Study B shows the effect of risk averse prosumers.

For Case Study A, Fig. 2(a) shows the prosumers' total consumption, and Fig. 2(b) shows the output power of generators 1 and 2, and the total PV generation for prosumers 1–10.

During the start of the day (12 a.m. to 6 a.m.) the prosumers have relatively low demand, and there is almost no PV generation. The prosumers always satisfy their inflexible load without flexible consumption. Since the overall demand



(a) Case Study B: Total prosumer consumption and inflexible load.



(b) Case Study B: Power generated by generators 1 and 2, and the total prosumer PV generation.

Fig. 3. Case Study B: Energy trading with risk averse prosumers.

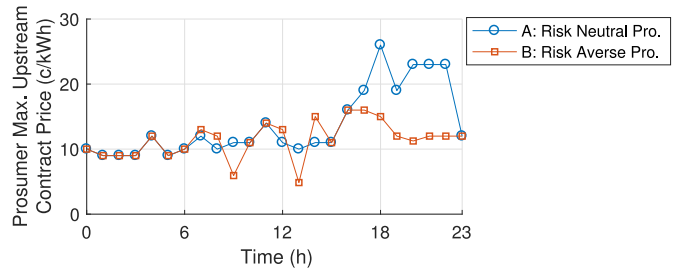


Fig. 4. The maximum price paid for upstream energy contracts by prosumers during case studies A and B.

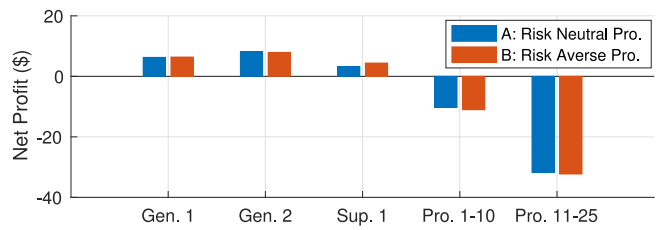


Fig. 5. The net profits (revenue less costs) accumulated by different groups of agents during case studies A and B.

is low, the generators operate with low marginal costs and are willing to sell energy at relatively low prices. As shown in Fig. 2(b), Generator 2 sells more energy than Generator 1, since it has a lower quadratic cost coefficient.

During the middle of the day (6 a.m. to 6 p.m.), PV generation is available, which has zero marginal cost. The prosumers use this energy to satisfy their time-coupled flexible loads.

Towards the end of the day (6 p.m. to 12 a.m.), the prosumers have high demand and there is no PV generation. No flexible consumption occurs during this period since the prosumers have already satisfied their flexible loads.

Fig. 3(a) and 3(b) show the consumption and generation profiles for Case Study B, with risk averse prosumers. The results are similar, with higher flexible consumption earlier in the day.

Fig. 4 compares the maximum price paid for upstream energy contracts by the prosumers during case studies A and B. For Case Study A, with risk neutral prosumers, the maximum price is \$0.26/kWh. For Case Study B, with risk averse prosumers, the maximum price is \$0.16/kWh (38% lower). However, as shown in Fig. 5, the prosumers accumulate less total utility during Case Study B compared with Case Study A, due to their risk aversion. For both case studies, Prosumers 1–10 have lower costs than prosumers 11–25, since they are able to satisfy their loads with PV generation, and they do not have the additional time-coupled flexible loads to satisfy.

Given the agent preferences (7)–(13), the agents' purchasing decisions in the forward market are affected by their estimates of the mean and standard deviation of the real-time energy price, but not the exact shape they expect the distribution to take. However, the shape of the distribution will affect their expected utilities in the forward market.

To compare the effect of prosumers expecting different real-time price distributions on the accuracy of their expected utilities in the forward market, the absolute errors between the expected utility and the actual utility at each trading interval was calculated and summed over the day for each prosumer. This was completed for case studies A and B, assuming uniform and normal distributions (with the same mean and standard deviation). For Case Study A, with risk-neutral prosumers, the average of each prosumer's absolute utility errors summed over the day is \$1.28 when a uniform distribution is expected and \$1.47 when a normal distribution is expected. For Case Study B, with risk averse prosumers, the average of each prosumer's absolute utility errors summed over the day is \$1.15 when a uniform distribution is expected and \$1.32 when a normal distribution is expected. The expected utilities are more accurate in Case Study B, with risk averse prosumers, and for both case studies, the uniform distribution is more accurate than the normal distribution.

VI. Conclusion

A new scalable market design for P2P energy trading has been presented, using bilateral contract networks. The design includes forward and real-time markets and incorporates important characteristics particular to energy trading, including upstream-downstream energy balance and forward market uncertainty. Utility-maximising preferences for real-time contracts and forward contracts have been presented for different types of electricity market participants. It has been shown that these preferences satisfy full substitutability conditions, which ensures a distributed price-adjustment process can be used to find an agreed network of contracts that the agents do not wish to mutually deviate from.

Future research will be important to incorporate additional technical and engineering features into the proposed market design, so that it is suitable for different practical applications. Important features include generator ramp rates and minimum production limits. Also, with the emergence of P2P energy trading markets, new tariff structures will be needed to divide fixed network costs between the participants. Anther promising area for future research would be to investigate alternative risk

measures to model agent risk preferences, for example value at risk and conditional value at risk [43].

Appendix

In this Appendix, it is shown that the agent preferences from Section III satisfy full substitutability. The following definition for an agent's indirect utility function, from [39], is needed.

Definition 4: The indirect utility function of agent i is given by $V_{mi}(p) := \max_{\Psi \subseteq \Omega_{mi}}\{U_{mi}(\kappa([\Psi|p]))\}$, where $\Omega_{mi} = \{\omega \in \Omega_m | i \in \{b(\omega), s(\omega)\}\}$.

The preferences of an agent are fully substitutable, if and only if they induce a submodular indirect utility function V_{mi}. To show V_{mi} is submodular, it is enough to show that for any two trades $\phi, \psi \in \Omega_{mi}$ and prices $p_{\Omega'_{mi}} \in \mathbb{R}^{|\Omega_{mi} \setminus \{\psi,\phi\}|}$, $p^h_\phi \geq p_\phi$, $p^h_\psi \geq p_\psi$ [39],

$$\begin{aligned} &V_{mi}\Big(p_{\Omega'_{mi}}, p_\phi, p^h_\psi\Big) - V_{mi}\Big(p_{\Omega'_{mi}}, p^h_\phi, p^h_\psi\Big) \\ &\quad \geq V_{mi}\Big(p_{\Omega'_{mi}}, p_\phi, p_\psi\Big) - V_{mi}\Big(p_{\Omega'_{mi}}, p^h_\phi, p_\psi\Big). \end{aligned} \tag{14}$$

The following upstream contracts are introduced $y_\phi := (\phi, p_\phi), y_\psi := (\psi, p_\psi)$ as well as downstream contracts $z_\phi := (\phi, p^h_\phi), z_\psi := (\psi, p^h_\psi)$. Also, let Y'_{mi} be a set of upstream contracts and let Z'_{mi} be a set of downstream contracts, where $y_\phi, y_\psi \notin Y'_{mi}$ and $z_\phi, z_\psi \notin Z'_{mi}$.

From (1), the agent utility functions are linear with respect to price, and from (7)–(13) the marginal utility an agent receives for accepting a contract only depends on its price and the number of previously accepted upstream and downstream trades. Therefore, to show that (14) is satisfied, it is sufficient to show the following conditions hold:

$$\begin{aligned} &U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\phi\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}\big) \\ &\quad \geq U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\phi, y_\psi\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\psi\big) \end{aligned} \tag{15}$$

$$\begin{aligned} &U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\phi, z_\psi\big) - U\big(Y'_{mi}, Z'_{mi}, z_\psi\big) \\ &\quad \geq U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\phi\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}\big) \end{aligned} \tag{16}$$

$$\begin{aligned} &U_{mi}\big(Y'_{mi}, Z'_{mi}\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}, z_\phi\big) \\ &\quad \geq U_{mi}\big(Y'_{mi}, Z'_{mi}, y_\psi\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}, z_\phi, y_\psi\big) \end{aligned} \tag{17}$$

$$\begin{aligned} &U_{mi}\big(Y'_{mi}, Z'_{mi}, z_\psi\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}, z_\phi, z_\psi\big) \\ &\quad \geq U_{mi}\big(Y'_{mi}, Z'_{mi}\big) - U_{mi}\big(Y'_{mi}, Z'_{mi}, z_\phi\big) \end{aligned} \tag{18}$$

In other words, the agent receives equal or greater marginal utility for accepting a given upstream or downstream contract when fewer other same-side contracts have been accepted, and when additional cross-side contracts have been accepted.

Proof of Theorem 1: First, consider the real-time market preferences for prosumer $i \in \mathcal{P}$, described by U_{Ri}. Let $c'_y = |Y'_{Ri}| + |Y'_{Fi}| - |Z'_{Ri}| - |Z'_{Fi}| - d_{ri} - \beta$, where $\beta \in \{0, 1, 2\}$. Conditions (15)–(18) require that:

$$\begin{aligned} &\alpha_{fi}\min\Big\{d_{fi}, c'_y + 1\Big\} - \alpha_{fi}\min\Big\{d_{fi}, c'_y\Big\} \\ &\quad \geq \alpha_{fi}\min\Big\{d_{fi}, c'_y + 2\Big\} - \alpha_{fi}\min\Big\{d_{fi}, c'_y + 1\Big\}, \end{aligned} \tag{19}$$

which is true for all $\alpha_{fi} \geq 0$ and $d_{fi}, c'_y \in \mathbb{Z}$.

Now, consider prosumer i's risk averse forward market preferences, described by U^{ra}_{Fi}, and let $k^{ra}_i = k^{ra}_{pi}\sigma_{p_{Ri}} + k^{ra}_{di}\sigma_{d_{Ri}}$. Conditions (16), (17) result in equalities. Conditions (15), (18) require:

$$\begin{aligned} &\mathbb{E}\big[p_{Ri}\big] + k^{ra}_i\big(|Y'_{Fi}| + 1\big)^{\frac{1}{r_i}} - k^{ra}_i\big(|Y'_{Fi}|\big)^{\frac{1}{r_i}} \\ &\quad \geq \mathbb{E}\big[p_{Ri}\big] + k^{ra}_i\big(|Y'_{Fi}| + 2\big)^{\frac{1}{r_i}} - k^{ra}_i\big(|Y'_{Fi} + 1|\big)^{\frac{1}{r_i}}, \end{aligned} \tag{20}$$

$$\begin{aligned} &k^{ra}_i\big(|Z'_{Fi}| + 2\big)^{r_i} - k^{ra}_i\big(|Z'_{Fi}| + 1\big)^{r_i} + \mathbb{E}\big[p_{Ri}\big] \\ &\quad \geq k^{ra}_i\big(|Z'_{Fi}| + 1\big)^{r_i} - k^{ra}_i\big(|Z'_{Fi}|\big)^{r_i} + \mathbb{E}[p_{Ri}], \end{aligned} \tag{21}$$

Equations (20), (21) are true since $r_i \geq 1, k^{ra}_i \geq 0, |Y'_{Fi}| \geq 0, |Z'_{Fi}| \geq 0$. For U_{Fi}, conditions (15)–(18) result in equalities. ■

Proof of Theorem 2: First, consider the real-time market preferences for intermediate supplier $i \in \mathcal{S}$, described by U_{Ri}. Conditions (15)–(18) result in equalities.

Now, consider supplier i's forward market preferences, described by U_{Fi}. Let $c'_z = |Z'_{Fi}| - |Y'_{Fi}| - \beta$, for $\beta \in \{0, 1, 2\}$. Conditions (15)–(18) require that:

$$\begin{aligned} &c_{t1i}\big[c'_z + 2)\big]^+ - c_{t1i}\big[c'_z + 1\big]^+ \\ &\quad \geq c_{t1i}[c'_z + 1]^+ - c_{t1i}[c'_z]^+, \end{aligned} \tag{22}$$

which is true for all $c_{t1i} \geq 0$, $c'_z \in \mathbb{Z}$. ■

Proof of Theorem 3: First, consider the real-time market preferences for traditional generator $i \in \mathcal{G}$, described by U_{Ri}. Let $c'_z = |Z'_{Ri}| + |Z'_{Fi}| - |Y'_{Ri}| - |Y'_{Fi}| - \beta$, where $\beta \in \{0, 1, 2\}$. Conditions (15)–(18) require that:

$$\begin{aligned} &c_{g1i}[c'_z + 2]^+ - c_{g1i}[c'_z + 1]^+ + c_{g2i}\big([c'_z + 2]^+\big)^2 \\ &\quad - c_{g2i}([c'_z + 1)]^+)^2 \geq c_{g1i}[c'_z + 1]^+ - c_{g1i}[c'_z]^+ \\ &\quad + c_{g2i}([c'_z + 1]^+)^2 - c_{g2i}\big([c'_z]^+\big)^2, \end{aligned} \tag{23}$$

which is true for all $c_{g1i}, c_{g2i} \geq 0$, and $c'_z \in \mathbb{Z}$.

For U_{Fi}, conditions (15)–(18) result in equalities. ■

References

[1] J. M. Guerrero *et al.*, "Distributed generation: Toward a new energy paradigm," *IEEE Ind. Electron. Mag.*, vol. 4, no. 1, pp. 52–64, Mar. 2010.

[2] D.-M. Han and J.-H. Lim, "Smart home energy management system using IEEE 802.15.4 and ZigBee," *IEEE Trans. Consum. Electron.*, vol. 56, no. 3, pp. 1403–1410, Aug. 2010.

[3] A. Dimeas *et al.*, "Smart houses in the smart grid: Developing an interactive network," *IEEE Electrification Mag.*, vol. 2, no. 1, pp. 81–93, Mar. 2014.

[4] R. Wilson, "Architecture of power markets," *Econometrica*, vol. 70, no. 4, pp. 1299–1340, 2002.

[5] H.-P. Chao and R. Wilson, *Design of Wholesale Electricity Markets*, Elect. Power Res. Inst., Palo Alto, CA, USA, 2001, p. 115.

[6] D. Pudjianto *et al.*, "Value of integrating distributed energy resources in the U.K. electricity system," in *Proc. IEEE PES Gen. Meeting*, Providence, RI, USA, 2010, pp. 1–6.

[7] P. Grünewald, E. McKenna, and M. Thomson, "Keep it simple: Time-of-use tariffs in high-wind scenarios," *IET Renew. Power Gener.*, vol. 9, no. 2, pp. 176–183, Feb. 2015.

[8] W. Wei, F. Liu, and S. Mei, "Energy pricing and dispatch for smart grid retailers under demand response and market price uncertainty," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1364–1374, May 2015.

[9] A. Mondal, S. Misra, and M. S. Obaidat, "Distributed home energy management system with storage in smart grid using game theory," *IEEE Syst. J.*, vol. 11, no. 3, pp. 1857–1866, Sep. 2015.

[10] D. Papadaskalopoulos and G. Strbac, "Decentralized participation of flexible demand in electricity markets—Part I: Market mechanism," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3658–3666, Nov. 2013.

[11] T. Morstyn, B. Hredzak, R. P. Aguilera, and V. G. Agelidis, "Model predictive control for distributed microgrid battery energy storage systems," *IEEE Trans. Control Syst. Technol.*, to be published, doi: 10.1109/TCST.2017.2699159.

[12] T. Morstyn, B. Hredzak, and V. Agelidis, "Network topology independent multi-agent dynamic optimal power flow for microgrids with distributed energy storage systems," *IEEE Trans. Smart Grid*, to be published.

[13] M. Kraning, E. Chu, J. Lavaei, and S. Boyd, "Dynamic network energy management via proximal message passing," *Found. Trends Optim.*, vol. 1, no. 2, pp. 70–122, 2014.

[14] R. Li, Q. Wu, and S. S. Oren, "Distribution locational marginal pricing for optimal electric vehicle charging management," *IEEE Trans. Power Syst.*, vol. 29, no. 1, pp. 203–211, Jan. 2014.

[15] Z. Yuan and M. R. Hesamzadeh, "Implementing zonal pricing in distribution network: The concept of pricing equivalence," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Boston, MA, USA, Jul. 2016, pp. 1–5.

[16] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nat. Energy*, vol. 1, no. 4, pp. 1–6, Mar. 2016.

[17] L. Einav, C. Farronato, and J. Levin, "Peer-to-peer markets," *Annu. Rev. Econ.*, vol. 8, no. 1, pp. 615–635, Oct. 2016.

[18] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Başar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1439–1450, May 2014.

[19] M. N. Faqiry and S. Das, "Double-sided energy auction in microgrid: Equilibrium under price anticipation," *IEEE Access*, vol. 4, pp. 3794–3805, 2016.

[20] N. Liu *et al.*, "Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3569–3583, Sep. 2017.

[21] J. Ma, J. Deng, L. Song, and Z. Han, "Incentive mechanism for demand side management in smart grid using auction," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1379–1388, May 2014.

[22] W. Lee, L. Xiang, R. Schober, and V. W. S. Wong, "Direct electricity trading in smart grid: A coalitional game analysis," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 7, pp. 1398–1411, Jul. 2014.

[23] B. Zhang, C. Jiang, J.-L. Yu, and Z. Han, "A contract game for direct energy trading in smart grid," *IEEE Trans. Smart Grid*, to be published, doi: 10.1109/TSG.2016.2622743.

[24] E. Cazalet, D. M. Paul, J. Price, E. Woychik, and J. Caldwell, "Transactive energy models," NIST Transactive Energy Challenge: Business and Regulatory Models Working Group, Gaithersburg, MD, USA, 2016.

[25] A. S. Kelso and V. P. Crawford, "Job matching, coalition formation, and gross substitutes," *Econometrica*, vol. 50, no. 6, pp. 1483–1504, Nov. 1982.

[26] M. Ostrovsky, "Stability in supply chain networks," *Amer. Econ. Rev.*, vol. 98, no. 3, pp. 897–923, May 2008.

[27] J. W. Hatfield, S. D. Kominers, A. Nichifor, M. Ostrovsky, and A. Westkamp, "Stability and competitive equilibrium in trading networks," *J. Polit. Econ.*, vol. 121, no. 5, pp. 966–1005, Oct. 2013.

[28] T. Fleiner, Z. Janko, A. Tamura, and A. Teytelboym, "Trading networks with bilateral contracts," in *Proc. 3rd Conf. Auctions Market Mech. Appl.*, 2015, pp. 1–39.

[29] S. I. Vagropoulos and A. G. Bakirtzis, "Optimal bidding strategy for electric vehicle aggregators in electricity markets," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4031–4041, Nov. 2013.

[30] T. Morstyn, B. Hredzak, and V. G. Agelidis, "Control strategies for microgrids with distributed energy storage systems: An overview," *IEEE Trans. Smart Grid*, to be published, doi: 10.1109/TSG.2016.2637958.

[31] A. J. Conejo, J. M. Arroyo, N. Alguacil, and A. L. Guijarro, "Transmission loss allocation: A comparison of different practical algorithms," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 571–576, Aug. 2002.

[32] D. Papadaskalopoulos, G. Strbac, P. Mancarella, M. Aunedi, and V. Stanojevic, "Decentralized participation of flexible demand in electricity markets—Part II: Application with electric vehicles and heat pump systems," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3667–3674, Nov. 2013.

[33] M. Carrion and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, Aug. 2006.

[34] W.-T. Huang, K.-C. Yao, and C.-C. Wu, "Using the direct search method for optimal dispatch of distributed generation in a medium-voltage microgrid," *Energies*, vol. 7, no. 12, pp. 8355–8373, 2014.

[35] G. Hug, S. Kar, and C. Wu, "Consensus+innovations approach for distributed multiagent coordination in a microgrid," *IEEE Trans. Smart Grid*, vol. 6, no. 4, pp. 1893–1903, Jul. 2015.

[36] D. Gregoratti and J. Matamoros, "Distributed energy trading: The multiple-microgrid case," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2551–2559, Apr. 2015.

[37] W. Tang and R. Jain, "Dynamic economic dispatch game: The value of storage," *IEEE Trans. Smart Grid*, vol. 7, no. 5, pp. 2350–2358, Sep. 2016.

[38] M. H. K. Tushar and C. Assi, "Optimal energy management and marginal-cost electricity pricing in microgrid network," *IEEE Trans. Ind. Informat.*, vol. 13, no. 6, pp. 3286–3298, Dec. 2017.

[39] J. W. Hatfield, S. D. Kominers, A. Nichifor, M. Ostrovsky, and A. Westkamp, "Full substitutability in trading networks," in *Proc. 16th ACM Conf. Econ. Comput. (EC)*, Portland, OR, USA, 2015, pp. 39–40.

[40] European Low Voltage Test Feeder. *IEEE PES Distribution Systems Analysis Subcommittee Test Feeders*. Accessed: Jun. 1, 2017. [Online]. Available: ewh.ieee.org/soc/pes/dsacom/testfeeders

[41] A. Andreas and T. Stoffel, "NREL solar radiation research laboratory (SRRL): Baseline measurement system (BMS); Golden, Colorado (Data)," NREL, Golden, CO, USA, Rep. DA-5500-56488, 1981.

[42] *Cummins Diesel Light Commercial Generators*, Cummins, Columbus, IN, USA, 2014. [Online]. Available: power.cummins.com/light-commercial-generators/generators/diesel-generators

[43] R. T. Rockafellar and S. Uryasev, "Conditional Value-at Risk for general loss distributions," *J. Bank. Financ.*, vol. 26, pp. 1443–1471, 2002.

Thomas Morstyn (S'14–M'16) received the B.E. (Hons.) degree in electrical engineering from the University of Melbourne, Australia, in 2011 and the Ph.D. degree in electrical engineering from the University of New South Wales, Australia, in 2016.

He was an Electrical Engineer with the Rio Tinto Technology and Innovation Group from 2012 to 2014. He is currently an Oxford Martin Fellow with the Department of Engineering Science, University of Oxford. His research interests include multiagent control and market design for coordinating distributed energy resources in power systems.

Alexander Teytelboym received the B.Sc. degree in economics from the London School of Economics and Political Science in 2008, and the M.Phil. and D.Phil. degrees in economics from the University of Oxford in 2010 and 2013, respectively.

He is an Economist interested in market design as well as social and economic networks. Before joining the Department of Economics, he was the Otto Poon Research Fellow with the Institute for New Economic Thinking, Oxford Martin School and prior to that a Post-Doctoral Fellow with the Laboratory for Information and Decision Systems, M.I.T.

Malcolm D. McCulloch (SM'89) received the B.Sc. (Eng.) and Ph.D. degrees in electrical engineering from the University of the Witwatersrand, Johannesburg, South Africa, in 1986 and 1990, respectively.

In 1993, he joined the University of Oxford, Oxford, U.K., to head up the Energy and Power Group, where he is currently an Associate Professor with the Department of Engineering Science. He is active in the areas of electrical machines, transport, and smart grids. His work addresses transforming existing power networks, designing new power networks for the developing world, developing new technology for electric vehicles, and developing approaches to integrated mobility.