

# Bilateral Filter Approach and Fast Discrete Curvelet Transform for Poisson Noise Removal from Images

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**Abstract**— We analyse two methods of removing poisson noise from images using a bilateral filter and by Fast discrete Curvelet Transform (FDCT). The Variance stabilizing transform (VST) is the main feature of the noise removal as it converts the Poisson distribution to the Gaussian domain, which makes the noise removal process relatively simple. Once the Gaussian distribution is obtained, the bilateral filter (BF) can be used for removing noise. We can also use the FDCT instead of bilateral filter, as it is capable of sparse representation of image intrinsic features. We implement both the methods separately, compare them and demonstrate simulations for monitoring their effectiveness in poisson noise removal. The results show that FDCT is more efficient for preserving image features, while bilateral filter is much faster and simple to implement.

**Index Terms**— Bilateral Filter, Fast discrete curvelet transform, Filter banks, Gaussian distribution, Optimisation, Poisson domain, Variance stabilizing transform.

## 1 INTRODUCTION

THE issue of poisson noise removal has always been a concern as far as image processing is considered. Such images with poisson noise have to be changed to Gaussian distribution for effective noise removal.

Usually, images are affected by poisson noise when photonic detection is used for image acquisition [1]. The arrivals of photons to the sensor are independent of each other and results in uncertainty about the arrival rate. This gives rise to poisson distribution. The properties of such image like signal-to-noise ratio (SNR) are signal dependent and keep varying along the image plane. Anscombe transform is an example of variance stabilizing transform (VST) which transforms the Poisson domain into the Gaussian domain [2]. Such a transformation enables us to apply denoising methods based on Gaussian domain which are widely used. VST can be inverted after performing the denoising methods. Such VST has poor performance when the image intensity is low. To overcome this setback, we rely on Multi-scale VSTs (MS-VST) [3]. MS-VSTs combine the VSTs with lowpass filters so that they can be used with curvelets. The curvelet feature of sparse representation of image features makes FDCT a good denoising method.

The bilateral filter is a weighted averaging filter used for removing Gaussian noise. Bilateral filter is applied with the MS-VST for denoising. It is an effective method for noise removal as

universal noise like impulse noise, salt and pepper noise and Gaussian noise, but is unable to deal with poisson noise. So by incorporating the switching bilateral filter with MS-VST we can extend the noise removal limits to universal noise. In this paper, as we are interested with poisson noise only the bilateral filter is considered which removes Gaussian noise. The MS-VST helps the change of domain from poisson to Gaussian to facilitate the use of Bilateral filter.

The structure of the paper is as follows. Section 2 mentions about MS-VST. Section 3 discusses about bilateral filter. Section 4 refers to FDCT. Section 5 presents the simulations and results processed using matlab. Section 6 contains conclusion of paper.

## 2 MULTISCALE VARIANCE STABILISING TRANSFORM (MS-VST)

The Multi-scale variance stabilizing transform is used for stabilising the variance of a poisson image, resulting in a gaussian domain. This new transform overcomes the drawback of Anscombe transform of deteriorating when low intensity images are processed. We can eliminate this drawback by incorporating lowpass filters to enhance the SNR before applying VST.

We use the lowpass filters initially to average out the SNR. The filtered Poisson image is taken as  $X$  and defines  $\tau_k = \sum_i (h[i])^k$ , for  $k=1,2,3,\dots$ . In this paper  $k$  upto 3 is taken.  $h$  is the impulse response of the filter. The VST of the image after passing through lowpass filter is given by [3]

$$T(X) \approx b \cdot \text{sign}(X+c) \sqrt{|X+c|} \quad (1)$$

where  $\text{sign}$  is the signum function. Constants  $a$  and  $b$  are given by

$$c \approx 7\tau_2 / 8\tau_1 - \tau_3 / 2\tau_2$$

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it is relatively fast and simple.

The switching bilateral filter [4] can be used for removal of

$$b \approx 2\sqrt{|\tau_1|} / \tau_2 \quad (2)$$

This MS-VST output can be combined with either a bilateral filter or FDCT. If FDCT method is used, it requires a final stage of optimization for removing the error of FDCT coefficients. The algorithm for poisson noise removal using FDCT can be defined in three steps which are transformation, detection and estimation. Transformation involves the change from poisson domain to Gaussian domain. Detection using test like hard thresholding to detect significant coefficients. Estimation which includes optimization techniques. For the bilateral method optimization methods are not required. The SNR and time required for program execution of two methods are taken into consideration.

### 3 BILATERAL FILTER

The Bilateral filter was proposed by Tomasi and Manduchi which has the peculiarity of preserving the edges along with Gaussian noise removal. The filter follows non-linear behavior. In this paper, we take a 3\*3 window. This window defines the characteristics of the bilateral filter. A weighting function is used which calculates the weighted average of the window. Each pixel is replaced by this particular value. In the weighting function, preference is given to neighborhood pixels that are near and similar to the central pixel i.e the Euclidean distance and luminance difference is given priority.

If  $x(i, j)$  is the current pixel and  $x(i+s, j+t)$  is the surrounding pixel in the processing window of size  $(2N+1, 2N+1)$ , then output of bilateral filter  $y(i, j)$  is given by

$$y(i, j) = \frac{\sum_{s=-NtoN} \sum_{t=-NtoN} G(s, t) R(s, t) x(i+s, j+t)}{\sum_{s=-NtoN} \sum_{t=-NtoN} G(s, t) R(s, t)} \quad (3)$$

$$G(s, t) = \frac{\exp\left(-\left((i-s)^2\right) + \left((j-t)^2\right)\right)}{2\sigma_w^2} \quad (4)$$

$$R(s, t) = \frac{\exp\left(-\left(x(i, j) - x(i+s, j+t)\right)^2\right)}{2\sigma_q^2} \quad (5)$$

$G(s, t)$  is the Gaussian filter which calculates Euclidean distance and  $R(s, t)$  is the range filter for calculating luminance difference as mentioned above. The properties or nature of the bilateral filter can be varied using the range filter. Gaussian noise is uniform in its distribution and affects the image pixels in a uniform manner. The weighting function smooths out this noise.  $\sigma_w$  and  $\sigma_q$  are standard deviations.

### 3.1 Combining MS-VST with Bilateral filter

The image with poisson noise is initially passed through a lowpass filter to increase the SNR before applying it to the MS-VST stage. Filtering has to be done in spatial domain. The maximally flat equiripple design in matlab can be used for the lowpass filter implementation. Subsequently, the filtered image is subjected to MS-VST. The impulse response of the filter is used to execute (1) and (2). Finally, the VST transformed image is processed using a bilateral filter. The steps are depicted below in fig1

Image with poisson noise → Lowpass filter →  
MS-VST → Bilateral filter → Noise reduced image.

Fig. 1. Bilateral filter approach for poisson noise removal

### 4 FAST DISCRETE CURVELET TRANSFORM (FDCT)

Fast discrete curvelet transforms [5] are used for optimal sparse representation of objects with discontinuities along  $C^2$  edges. Curvelets are highly anisotropic waveforms at fine scales which follows the parabolic law ( $\text{length} \approx \text{width}^2$ ). FDCT are second generation curvelets which unlike first generation curvelets do not rely on ridgelets. There are two implementations of FDCT. They are 1) using Unequispaced fast Fourier transform (USFFT) and 2) using frequency wrapping. Both the methods differ by choice of spatial grid that is used for translation of curvelets having different scale and angle. The common steps involved in both methods are shown in fig2.

Image → FFT → windowing for different scales and angles → IFFT → FDCT coefficients

Fig. 2. Common steps involved in the two methods

The image is applied with fast Fourier transform. As we are dealing with images 2D FFT is used. There are FDCT windows which are used as separate functions in a curvelet toolbox that applies curvelet frequency windows at various scales and angles. The scales are to be decided by the user initially and set accordingly. The FDCT coefficients are obtained by computing the inverse FFT of each window output separately. The output contains coefficients of different scales. Separation of this output according to their scales give us a complete coefficient output in a cell form.

This process steps constitutes only the forward FDCT. These coefficients can be modified based on our working

criteria. For obtaining the initial image, inverse FDCT must be implemented. We obtain the inverse FDCT by inverting each step given in fig2.

In the FDCT approach for poisson noise removal, optimisation steps are required. The FDCT coefficients contain errors due to the noise. So after optimization, the errors are minimised. FDCT is faster and less redundant than the first generation curvelets. They have clear advantage over wavelets that they can efficiently preserve intrinsic features of an image using minimum coefficients.

#### 4.1 Combining MS-VST with FDCT

Combination of MS-VST with FDCT [6] begins with the design of lowpass filter banks which depends on the number of scales used. The lowpass filters must be dyadic in nature and filtering is performed in spatial domain. The maximally flat equiripple design is used for lowpass filters. The frequency range of the lowpass filters are dyadic i.e. they are expressed in powers of 2. The scales determine the property of the filter. If the scale,  $J=4$ , then the frequency will be  $\pi/(2^J)$ . Likely, the frequency of each lowpass filter will vary based on the scale. This dyadicity enables the FDCT coefficients to be extracted efficiently. We must initially choose the number of scales. According to the scales, there must be equal number of lowpass outputs. The frequency range of the filters used must be strictly dyadic. The output of each such lowpass filters is passed through the respective MS-VST stage. The impulse response of the lowpass filters is used in this stage. Finally, the FDCT extracts the coefficients. The 2D-FFT of the MS-VST output is taken and then the FDCT window is applied. There will be FDCT window for each curvelet of varying scales. Windowing is followed by 2D-IFFT. These coefficients are combined together for the final coefficient output. Optimisation is performed for these coefficients. The different stages are detailed in fig3.

Image with noise → lowpass filters with dyadic frequencies → MS-VST for each lowpass filter output → 2D-FFT → windowing for curvelets with varying scales and angles → 2D-IFFT → FDCT coefficients

Fig. 3. FDCT combined with MS-VST

After optimization, the inverse FDCT must be applied to receive the original image.

#### 4.2 Optimisation

There are many optimization techniques available. In this paper, we are using the least mean square error (LMSE) method. The LMSE filter has two inputs, the input image and the error between the actual image and corrupted image. A

sets of weights is also applied which is initially assumed. The LMSE performs number of iterations according to the initial conditions we specify. The weights are updated at each iteration. The rate of updation is also specified by the user. After execution, we obtain an error matrix and the output matrix whose error is minimized. The forward FDCT of the output matrix is performed to obtain reference coefficients. These coefficients are compared with the FDCT coefficients obtained before. This is the thresholding process. Based on the reference, the FDCT coefficients are updated. Also other methods like steepest descent algorithm can be used.

### 5 SIMULATIONS AND RESULTS

The Barbara image of size 512\*512 was used for processing. The simulations were conducted in Matlab. The SNR and time taken for the two methods were noted. These parameters were used for evaluation. The lowpass filters were designed using the inbuilt commands in matlab. The FIR lowpass equiripple design can be used. The FDCT implementations were executed using the Curvelab Toolbox designed by D.L. Donoho. The results are shown in figures below



Fig. 4.(a) Barbara image of size 512\*512



.(b) Barbara image with impulse noise

The actual Barbara image of size 512\*512 is shown in fig.4 (a). Fig.4(b) is the image corrupted with poisson noise. Poisson

noise can be added using matlab commands or such images can be downloaded.



(c) Output using FDCT method



(d) Output using bilateral filter method

Eventhough the output using bilateral filter appeared to be clearer, the SNR of the FDCT method was found to be higher. The SNR of FDCT method is 24db and that of bilateral filter is 14db. So the picture quality of FDCT output is higher. But the complexity of bilateral filter is much simple compared to FDCT method. Execution time of FDCT and bilateral filter is 56 seconds and 46 seconds respectively (execution was performed using Intel atom processor, it gets reduced when higher processors are used, but still the difference does not change.)

## 6 CONCLUSION

Two methods of poisson noise removal from images were introduced in this paper, the bilateral filter method and the FDCT method. The intrinsic features of the image are better preserved using FCDT method which has a higher SNR. But the execution time of bilateral filter is lesser than FDCT method. These two methods can be switched in applications according to the prevailing situations. In every commercial

application there will be a high performance situation where time constrain is not an issue. There will also be low performance conditions where only speed of execution is required i.e. fast processing is expected. Combined implementation of these two methods results in higher performance than the individual performance. There are many possibilities of future research in this area. Methods for removing Poisson noise without change of domain are seriously considered.

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