S. Fleishman, I. Drori, D. Cohen-Or Tel Aviv University

Presented by Derek Bradley

## Outline

- Motivation
- Previous Work
- Bilateral Mesh Denoising
- Image Processing Background
- Bilateral Image Filtering
- Transforming from Images to Meshes
- Mesh Denoising
- Results
- Discussion
Previous Work
$\square$ Laplacian Smoothing
$v_{i}=v_{i}+\lambda \cdot \Delta v_{i}$
$\left.\Delta v_{i}=\frac{1}{|(i, j)|} \sum_{(i, j)}\left(v_{j}-v_{i}\right) \right\rvert\,$
$V_{i}$


## Previous Work

- Weighted Laplacian

$$
\begin{gathered}
v_{i}=v_{i}+\lambda \cdot \Delta v_{i} \\
\Delta v_{i}=\frac{1}{\sum_{(i, j)} w_{i j}} \sum_{(i, j)} w_{i j}\left(v_{j}-v_{i}\right)
\end{gathered}
$$



## Previous Work

- Laplacian + Expansion



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## Previous Work

- Implicit Fairing (IF) [Desbrun 1999]
- Implicit integration of the diffusion equation

$$
X_{\text {Explicit }}^{X^{n+1}=(I+\lambda d t L) X^{n}} \quad(I+\lambda d t L) X^{n+1}=X^{n}
$$

- Anisotropic Feature-Preserving Denoising (AFP) [Desbrun 2000]
- Features detected using local curvature
- Denoise using weighted mean curvature smoothing
- Penalize vertices with large ratio between principle curvatures


## Image Processing Background

- An image is an array of integers (0-255)


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## Bilateral Image Filtering

- Goal: Smooth the image intensities, but preserve strong edges (features)
- New intensity = weighted average of neighbours
- Two weights:
- Geometric: Closer pixels weighted higher (closeness smoothing filter)
- Photometric: Strong changes in intensity penalized (similarity weight function)

Image Processing Background

| N | N | N | N | N | N | N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | N | N | N | N | N | N |
| N | N | N | N | N | N | N |
| N | N | N | V | N | N | N |
| N | N | N | N | N | N | N |
| N | N | N | N | N | N | N |
| N | N | N | N | N | N | N |

- v is the current pixel
- $N(v)$ is the set of neighbouring pixels of $v$
- $I(v)$ is the intensity of $v$


## Bilateral Mesh Denoising

- Application of an image smoothing technique
- Vertices are moved along their normal direction

$$
v_{i}=v_{i}+d \cdot n_{i}
$$

- Scalar value $\boldsymbol{d}$ to be computed for each vertex
- Feature preserving
- Can be iterative or single-pass
- But first ... some image processing basics


## Bilateral Image Filtering

Closeness Smoothing Filter

- 1d Gaussian Function

[wikipedia.org]



## Bilateral Image Filtering

Similarity Weight Function

- Another Gaussian function
$W_{s}(x)=e^{-x^{2} / 2 \sigma_{s}^{2}}$
- $\mathbf{x}=$ absolute difference in intensity values
- Result: pixels with large changes in intensity are weighted lower

Bilateral Image Filtering

- Combining the weights and normalizing:
$\hat{I}(v)=\frac{\sum_{p \in N(v)} W_{c}(\|p-v\|) \cdot W_{s}(\| I(v)-I(p) \mid) \cdot I(p)}{\sum_{p \in N(v)} W_{c}(\|p-v\|) \cdot W_{s}(I(v)-I(p) \mid)}$
- In practice, $\mathbf{N}(\mathrm{v})$ is defined by the set of points:

$$
\left\{q_{i}\right\} \text {, where }\left\|\nu-q_{i}\right\|<\left|2 \sigma_{c}\right|
$$

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Transforming from Images to
Meshes

- Vertices instead of pixels
- Neighbourhood N(v), defined the same
- Closeness smoothing filter:
- 3D Euclidean distance instead of 2D
- Similarity weight function:
- Heights of neighbouring vertices $=$ pixel intensities


## Mesh Denoising

DenoisePoint(Vertex v, Normal n)

$$
\begin{aligned}
& \left\{\mathrm{q}_{\mathrm{i}}\right\}=\text { neighbourhood }(\mathrm{v}) \\
& \mathrm{K}=\left|\left\{\mathrm{q}_{\mathrm{i}}\right\}\right| \\
& \text { sum }=0 \text {, normalizer }=0 \\
& \text { for } \mathrm{i}:=1 \text { to } \mathrm{K} \\
& \qquad \mathrm{t}=\left|\left|\mathrm{v}-\mathrm{q}_{\mathrm{i}}\right|\right| \\
& \left.\quad \mathrm{h}=<\mathrm{n}, \mathrm{v}-\mathrm{q}_{\mathrm{i}}\right\rangle \\
& \mathrm{W}_{\mathrm{c}}=\exp \left(-\mathrm{t}^{2} /\left(2 \sigma_{\mathrm{c}}^{2}\right)\right) \\
& \mathrm{W}_{\mathrm{s}}=\exp \left(-\mathrm{h}^{2} /\left(2 \sigma_{\mathrm{s}}^{2}\right)\right) \\
& \quad \operatorname{sum}+=\left(\mathrm{W}_{\mathrm{c}} * \mathrm{~W}_{\mathrm{s}}\right) * \mathrm{~h} \\
& \text { normalizer }+=\mathrm{W}_{\mathrm{c}} * \mathrm{~W}_{\mathrm{s}}
\end{aligned}
$$

return Vertex $\mathrm{v}=\mathrm{v}+\mathrm{n} *($ sum/normalizer $)$


## Discussion

- Issues when using an image-based technique on a mesh:
- Only applies to manifold meshes
- Irregularity of meshes
- Shrinkage
- Vertex drift
- Handling boundaries
- Mirror neighbours at boundary vertices
- Virtual vertices at infinity (used in this algorithm)


## Discussion

- Setting the parameters $\left(\sigma_{\mathrm{c}}, \sigma_{\mathrm{s}}\right.$, \# iterations)
- User-assisted method
- $\sigma_{c}$ and $\sigma_{s}$ :
- User selects smooth point and radius on the mesh
$\square$ Large $\sigma_{\mathrm{c}}=$ few iterations, small $\sigma_{\mathrm{c}}=$ more iterations
- Small $\sigma_{\mathrm{c}}$ makes sense
- large values can cross features
- smaller neighbourhood leads to faster iterations
$\square<6$ iterations for all results in the paper


## Discussion

- Independently, Jones et al. present the same algorithm with minor differences:
- Surface predictor
- Single pass



## Discussion

- Disadvantages
- Assumes well-behaved meshes
- Can result in self-intersection

$$
\text { Derek Bradley } 2006
$$

## References

- S. Fleishman, I. Drori, D. Cohen-Or. Bilateral mesh denoising. SIGGRAPH 2003.
- C. Tomasi, R. Manduchi. Bilateral filtering for gray and color images. ICCV 1998.
- T. Jones, F. Durand, M. Desbrun. Non-iterative featurepreserving mesh smoothing. SIGGRAPH 2003.
- M. Desbrun, M. Meyer, P. Schroder, A.H. Barr. Implicit fairing of irregular meshes using diffusion and curvature flow. SIGGRAPH 1999.
- M. Desbrun, M. Meyer, P. Schroder, A.H. Barr. Anisotropic feature-preserving denoising of height fields and bivariate data. Graphics Interface 2000.
- www.wikipedia.org

