Binary Power Control for Sum Rate Maximization over Multiple Interfering Links

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***Abstract*— We consider allocating the transmit powers for a wireless multi-link (N-link) system, in order to maximize the total system throughput under interference and noise impairments, and short term power constraints. Employing dynamic spectral reuse, we allow for centralized control. In the two-link case, the optimal power allocation then has a remarkably simple nature termed *binary* power control: Depending on the noise and channel gains, assign full power to one link and minimum to the other, or full power on both.**

Binary power control (BPC) has the advantage of leading towards simpler or even distributed power control algorithms. For $N > 2$ we propose a strategy based on checking the corners of the domain resulting from the power constraints to perform BPC. We identify scenarios in which binary power allocation can be proven optimal also for arbitrary N. Furthermore, in the general setting for $N > 2$, simulations demonstrate that a throughput performance with negligible loss, compared to the best non-binary scheme found by geometric programming, can be obtained by BPC. Finally, to reduce the complexity of optimal binary power allocation for large networks, we provide simple algorithms achieving 99% of the capacity promised by exhaustive binary search.

***Index Terms*— Power control, Cellular systems, Information rates, and Adaptive modulation**

I. Introduction

The need for ever higher spectrum efficiency motivates the search for system-wide optimization of the wireless resources. A key example of multi-link resource allocation is that of power control, which serves as means for both battery savings at the mobile, and for interference management. Traditional power control solutions are designed for voice-centric networks, hence aiming at guaranteeing a target signal-to-noise-and-interference ratio (SNIR) level to the users [1]–[3]. In modern wireless data networks, adaptive coding and modulation with power control [4], [5] is or will be implemented, and throughput maximization becomes a more relevant metric.

The simultaneous optimization of transmission rates and power with the aim of maximizing the multi-link sum capacity is a difficult problem, which perhaps explains why the problem has received relatively little attention in the past, although now it is clearly gaining interest [6], [7]. Considering the problem of optimally allocating the transmit power for N concurrent communication links, a common approach is to use a high SNIR approximation to establish convexity in the sum-throughput objective function [6], [7]. However, this approximation by construction prohibits completely turning off the power of any link at any time. This extra constraint may in fact cause the resulting power vector to steer away from the optimum solution in certain cases. Indeed one of the major points emphasized in the present work is that in the context of multi-link capacity maximization, the ability of shutting down one or more links (or transmitting at minimum allowed power > 0) in certain slots can be instrumental in approaching maximum network throughput.

By restricting the scenario to interference limited systems, i.e., neglecting noise sources, in [7] the high SNIR assumption is modified so that links contributing less than a fixed amount to the total throughput are dropped. For the remaining links the high SNIR approximation is still used. Although improvements over the schemes are presented in [6], "the proposed method is still inferior to maximization of the actual aggregate throughput" according to [7]. In [8], specializing to the case of uplink single-cell CDMA, i.e., N transmission links with a common receiver, and enforcing quality of service constraints, results on simplifying the power control search space are derived. However, in this paper we shall consider a more general system model, for which the uplink single-cell CDMA setting can be seen as a special case. Thus the results and conclusions from [8] are not in general applicable to our model.

Under a sum power constraint, the authors of [9] neglect noise sources and use waterfilling to maximize the network capacity, while in [10], under the assumption of symmetric interference, a two-user power allocation that depends on the level of interference is derived. Due to the sum power constraint, the neglection of noise, and the symmetry-of-interference assumption these results are not applicable to our cellular system power allocation analysis. In our opinion, it is more reasonable to instead assume individual power constraints at every link, and that the received interference in

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general will be different for different users. Further, in [11], game-theoretic approaches are used to analyze a symmetric one-dimensional two-cell network, assuming the received power to be a function of the transmitter-receiver distance only. However, here we model any geometric setup, as well as allowing for arbitrary signal degradation, e.g., caused by path loss, multipath fading, or shadowing.

When modelling the transmission rate as a *linear* function of the received power, [12] shows that a link when active should transmit at maximum power for optimality. This result has the merit of showing potential benefits of an on/off power control, but in general, the assumed linear relationship between rate and power is however unfortunately far from the truth since the rate is known to have a $\log(\cdot)$ behavior. The proof does not extend to arbitrarily increasing rate-power relations, and the results will not in general yield throughput-optimal power allocation. Nevertheless, here we show that when using a *low SNIR approximation*, the linear relation in [12] is indeed obtained, and thus the conclusions from that paper holds in this case, and can be extended to include a minimum power constraint at each base station.

In this paper we tackle the problem of sum rate maximizing power allocation in multi-link networks with orthogonal MAC protocols without resorting to the previously described restricting assumptions of high SNIR, interference-limited systems, or interference symmetry. The application we have in mind is a wireless data access network with best-effort type of quality of service, and the total aggregate throughput (sum rate) across the network is the figure of merit. The system is assumed to be enabled with a perfect link adaptation protocol, so the user rate is adapted instantaneously as a function of the user's signal to noise and interference ratio, thus always achieving Shannon capacity in any resource slot. Extending [13], our contributions are as follows: In the two-link case, the optimal power allocation is analytically shown to be remarkably simple; transmit at full power at link 1, minimum at link 2, vice versa, or at full power at both links. Next, we consider the $N > 2$ case, and show that when either a geometric-arithmetic mean or low-SNIR approximation is applicable, binary power control[1] is still optimal (as is always true for any SNIR in the $N = 2$ case). In the general case for $N > 2$, we utilize the mathematical framework of geometric programming [14] (GP) in order to establish a sum capacity benchmark, to compare our proposed binary power allocation with, through exhaustive simulations. Empirically, we demonstrate that the loss associated with restriction to binary power levels is negligible. On the other hand, discretizing the optimization space is highly beneficial: the feedback rate needed to communicate between network nodes is reduced, transmitter design is simplified, and finally, limiting the potential solutions to search over better facilitates distributed resource allocation [15].

For networks with a large number of links, we consider clustering groups of links as a way of lowering the power control complexity, as well as reducing the required channel knowledge. Through clustering significant complexity reduction is possible, at the cost of only a small reduction in network capacity. Finally, we propose a simple greedy approach to binary power control, and demonstrate that its throughput performance in a wide range of communication environments is virtually indistinguishable from that of exhaustive binary power search.

[1] On/off power control and binary power control are equivalent if the minimum transmit power is zero.

The remainder of the present paper is organized as follows. We introduce the wireless system model under investigation in Section II. In Section III we derive optimal power control schemes that maximizes the sum throughput. Algorithms for reducing the complexity of binary power control by clustering and greedy approaches are presented in Section IV. In Section V numerical results are presented, and finally conclusions are given in Section VI.

II. System Model

We consider a wireless network featuring a number of transmitters and receivers, of which there are N active pairs selected for transmission by a scheduling (MAC) protocol. In order to focus solely on power control, we do not explicitly consider scheduling or MAC protocols here. However, note that the results presented in this paper are valid for any scheduling algorithm, as the effect of one such algorithm over another is simply to induce different channel statistics for the selected links [16]. We also emphasize that our analysis is valid for any geometry, even for non-cellular systems such as ad-hoc networks, as long as the sum of link capacities is a relevant performance metric. To facilitate exposition, we shall however adopt a cellular terminology from here on; see Fig. 1.

In the network considered, the spectral resource slots are shared by all cells, leading in general to an interference and noise impaired system. The communication links are considered to be downlink, but the results can also be generalized to an uplink scenario. The data destined for user u_n is transmitted with power P_n. Each base station is in general assumed to operate under both minimum and peak short term (per-slot) power constraints,

$$P_{\min,n} \le P_n \le P_{\max,n}, \quad n = 1, 2, \ldots, N. \tag{1}$$

Letting $P_{\min,n} > 0$ might be necessary in some scenarios to ensure that a user u_n receive a minimum transmission, such as control information or pilot symbols. Further, the different cells can be given priorities by assigning individual values of $P_{\max,n}$.

Now, denote by $G_{n,i}(m)$ the channel power gain to the selected mobile user $u_n(m)$ in cell n from the cell i base station, in resource slot m. We will suppress the slot index from now on, concentrating on one arbitrary slot. The channel gains are assumed to be constant over each such resource slot, i.e., we have a block fading scenario. Note that the gains $G_{n,n}$ correspond to the desired communication links, whereas the $G_{n,i}$ for $n \neq i$ correspond to the unwanted interference links. Assuming the transmitted symbols to be independent random variables with zero mean and a variance of P_n, the signal to noise-plus-interference ratio (SNIR) for each user is given by

$$\mathrm{SNIR}_{u_n} = \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}}, \tag{2}$$

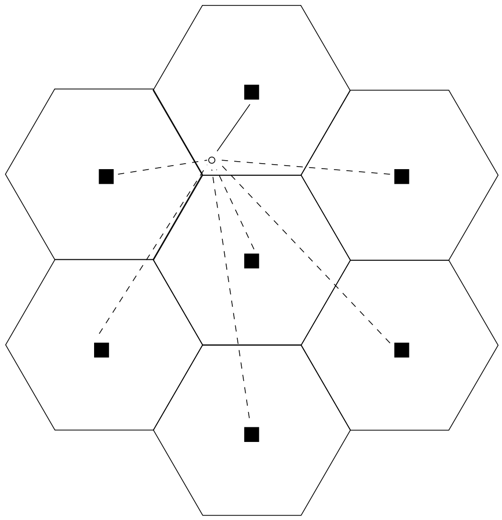

Fig. 1. N-cell wireless system model, where $N = 7$. Base stations are shown as solid squares. For the user (shown as a circle) in the top cell, the desired communication link is shown as a solid line, whereas the interference links are shown as dashed lines.

where $\sigma^2_{Z_n}$ is the variance of the independent zero-mean AWGN in cell n. Under the assumption of Gaussian distributed signal transmission in all cells, the interference terms will be Gaussian, also after being weighted by the (constant) interference gains in the current block and subsequently summed. Then channel experienced by each user within a given time slot is AWGN, and thus the capacity for each user is given by the AWGN Shannon capacity, i.e., the achievable rate (in information bits/s/Hz) for user u_n is given by

$$R_{u_n} = \log_2(1 + \text{ SNIR}_{u_n}). \tag{3}$$

From (2) and (3) the total achievable throughput (sum rate) $R = \sum_{n=1}^{N} R_{u_n}$ is then found as

$$R = \sum_{n=1}^{N} \log_2 \left(1 + \frac{P_n G_{n,n}}{\sigma^2_{Z_n} + \sum_{j \neq n} P_j G_{n,j}} \right). \tag{4}$$

Finally, we note that our system model with (possibly different) noise levels $\{\sigma^2_{Z_n}\}_{n=1}^{N}$ also accommodates the modeling of additional interfering sources disturbing the users differently, contrary to previous works. As will be discussed later, one important application of this is when, for complexity reduction, joint multi-cell power allocation is undertaken over a subnet (cluster) of neighboring cells only. In this case $\sigma^2_{Z_i}$ represents the combined effect of noise and interference received from out-of-cluster cells by the i^{th} user.

III. Transmit Power Analysis

This section presents the general optimal power allocation scheme $\mathbf{P}^* = (P_1^*, \ldots, P_N^*)$, which has as inputs the channel gains $\{G_{n,i} > 0\}$, and the AWGN variances $\{\sigma^2_{Z_n} > 0\}$. We search for the optimal power allocation by approaching the following optimization problem,

$$\mathbf{P}^* = \arg \max_{\mathbf{P} \in \Omega^N} R, \tag{5}$$

where $\Omega^N = \{\mathbf{P} | \; P_{\text{min},n} \leq P_n \leq P_{\text{max},n}, \; n = 1, \ldots, N\}$ is the feasible set and R is given in (4). Since Ω^N is a closed and bounded set and $R : \Omega^N \to \mathbb{R}$ is continuous, (5) has a solution [17, Theorem 0.3]. Before we proceed, we note the following lemma.

Lemma 1: The optimal transmit power vector will have at least one component equal to $P_{\text{max},n}$.

Proof: From (4) we have that, for $\beta > 1$ and $\mathbf{P} \in \Omega^N$:

$$R(\beta \mathbf{P}) = \log_2 \Big(\prod_{n=1}^{N} \big(1 + \frac{P_n G_{n,n}}{\frac{\sigma^2_{Z_n}}{\beta} + \sum_{j \neq n} P_j G_{n,j}} \big) \Big) > R(\mathbf{P}). \tag{6}$$

Thus, we can increase the sum throughput R, by increasing all components of $\mathbf{P}$ by a factor β, until one component hits the boundary $P_{\text{max},n}$. Hence, the solution of (5) will have at least one component equal to $P_{\text{max},n}$. The interpretation of (6) is that increasing all the transmit powers by a factor β, is equivalent to reducing the noise in each cell by the same factor. ■

Note that for one or more $P_{\text{min},n} = 0$, Ω^N admits solutions where some base stations shut down the power completely. Since one or more of the N base stations may then be turned off in a resource slot, from a cellular engineering point of view this scheme can be interpreted as a form of dynamic channel reuse. Allowing the network to completely turn off base stations will be sum-throughput optimal, but this optimality comes at the expense of fairness between the users in the various cells. Fairness can be restored by increasing $P_{\text{min},n}$, achieving full fairness at $P_{\text{min},n} = P_{\text{max},n}$, $\forall n$, analogously to the time horizon parameter in proportional fair scheduling [18]. Additionally, fairness can be targeted through introducing appropriate scheduling criteria [16].

A. Trivial Solutions

By inspection of (4) we can identify some trivial (not necessarily unique) solutions of (5). Firstly, if the system is noise limited, i.e., the interference can be neglected, then $\mathbf{P}^* = (P_{\text{max},1}, \ldots, P_{\text{max},N})$. Secondly, for the case of an interference limited system (noise set to zero), we see that $R \to \infty$ if any one of the base stations is turned on with any power $P_{\text{min},n} \leq P_n \leq P_{\text{max},n}$. However, in our analysis we will assume that noise is present, as in all practical systems.

B. The 2-Cell Case

We shall deal with the 2-cell case separately, as it allows us to derive analytically the optimal power allocation. By Lemma 1, the optimal power allocation is found among the following alternatives:

- Extremal points on the boundaries of Ω^2: I.e., for $P_2 = P_{\text{max},2}$, P_1's corresponding to $\frac{\partial R(P_1, P_{\text{max},2})}{\partial P_1} = 0$, or for $P_1 = P_{\text{max}}$, P_2's such that $\frac{\partial R(P_{\text{max},1}, P_2)}{\partial P_2} = 0$.
- Corner points of Ω^2: $(P_{\text{max},1}, P_{\text{min},2})$, or $(P_{\text{min},1}, P_{\text{max},2})$, or $(P_{\text{max},1}, P_{\text{max},2})$.

Since the logarithm is a monotonically increasing function, we can look for extreme points on the boundary by considering the function $J(P_1, P_2) \triangleq (1 + \text{SNIR}_{u_1})(1 + \text{SNIR}_{u_2})$, i.e.,

$$J(P_1, P_2) = \big(1 + \frac{P_1 G_{1,1}}{\sigma^2_{Z_1} + P_2 G_{1,2}} \big) \big(1 + \frac{P_2 G_{2,2}}{\sigma^2_{Z_2} + P_1 G_{2,1}} \big). \tag{7}$$

Now, by differentiating $J(P_1, P_{\max,2})$ with respect to P_1 we find

$$\frac{\partial J}{\partial P_1} = \frac{CP_1^2 + 2DP_1 + E}{F}, \tag{8}$$

where

$$C = G_{1,1}G_{2,1}^2 > 0, \quad D = G_{1,1}G_{2,1}\sigma_{Z_2}^2 > 0, \tag{9a}$$
$$E = -P_{\max,2}G_{2,1}G_{2,2}(\sigma_{Z_1}^2 + P_{\max,2}G_{1,2}) + G_{1,1}\sigma_{Z_2}^2(\sigma_{Z_2}^2 + P_{\max,2}G_{2,2}), \tag{9b}$$
$$F = (\sigma_{Z_1}^2 + P_{\max,2}G_{1,2})(\sigma_{Z_2}^2 + P_1G_{2,1})^2 > 0. \tag{9c}$$

Since F is always strictly positive, a P_1 such that $\frac{\partial J}{\partial P_1} = 0$ can be found as the solution to $CP_1^2 + 2DP_1 + E = 0$. Now, since also $C, D > 0$, this quadratic equation either has no zero for $P_1 \in [P_{\min,1}, P_{\max,1}]$, or has one zero there, and changes sign from $-$ to $+$. In either case it is clear that the maximum is attained at a boundary point $P_{\min,1}$ or $P_{\max,1}$. Due to symmetry, the above analysis also hold for P_2. Thus, we can conclude that (P_1^*, P_2^*) is found in the set of corner points of the feasible domain, $\Delta\Omega^2 = \{(P_{\max,1}, P_{\min,2}), (P_{\min,1}, P_{\max,2}), (P_{\max,1}, P_{\max,2}).\}$. Hence, we have the following theorem.

Theorem 1: For the two-cell case, the sum throughput maximizing power allocation is binary[2]. Mathematically,

$$\arg \max_{(P_1,P_2)\in\Delta\Omega^2} R(P_1, P_2) = \arg \max_{(P_1,P_2)\in\Omega^2} R(P_1, P_2). \tag{10}$$

Proof: See above. ■

Inspecting (10) we see that, for zero minimum powers, of the two users in question, the user with the highest signal to noise ratio (SNR), defined as $\frac{P_{\max,n}G_{n,n}}{\sigma_{Z_n}^2}$, will always receive transmission at full power $P_{\max,n}$. For $(P_1, P_2) = (P_{\max,1}, P_{\max,2})$ this is trivially true. Furthermore, from (4), the decision of $(P_1, P_2) = (P_{\max,1}, 0)$ or $(0, P_{\max,2})$ is decided by each user's SNR alone, since there will be no interference for these power allocations.

C. Binary Power Control in the N-Cell Case

For $N > 2$, analytical treatment of the optimization problem (5) proves to be challenging, because of the lack of convexity and the fact that the above analysis from the two-cell case does not generalize to N cells. However, motivated by the optimality of binary power allocation for the two-cell case, reduced feedback requirements, and as well as by its potential as the key simplification in design of simple or even distributed algorithms, we will investigate the properties of binary power control also in the N-cell case.

Binary power control for N cells is done by evaluating $R(\mathbf{P})$ at the corners of Ω^N, and picking the maximum value. Mathematically formulated,

$$\mathbf{P}_{\text{bin}} = \arg \max_{\mathbf{P}\in\Delta\Omega^N} R(\mathbf{P}), \tag{11}$$

where $\Delta\Omega^N$ is the set of $2^N - 1$ corner points of Ω, excluding the all-$P_{\min,n}$ point.

[2]For the case of $P_{\min,n} = 0$, $\forall n$, this result was independently reported both by the authors in [13], and in [19], [20].

Unfortunately, a seemingly pessimistic theoretical result is obtained there: It can be shown that binary power allocation is no longer optimal for $N > 2$. However, as we shall see it appears to still be very well approximating the capacity obtained by the optimal solution resulting from continuous power control, as indicated by the example below.

Example 1: We simulated a $N = 3$ cell network with the following parameters. Common peak and minimum power constraints of $P_{\max} = 10^{-3}$, and $P_{\min} = 0$, respectively, assuming identical noise figures for the different receivers, the AWGN power is found as kT_0B, where k is Boltzmann's constant, $T_0 = 290$ Kelvin is the ambient temperature, and $B = 1$ MHz is the equivalent noise bandwidth, i.e., $\sigma_{Z_1}^2 = \sigma_{Z_2}^2 = \sigma_{Z_3}^2 = 4.0039\times10^{-15}$. As an example of the randomly generated channel gain matrix, based on path loss, shadowing and multipath effects we have

$$G = 10^{-9} \times \begin{pmatrix} 0.0432 & 0.0106 & 0.0012 \\ 0.0004 & 0.2770 & 0.0043 \\ 0.0045 & 0.0137 & 0.1050 \end{pmatrix}.$$

Then, by the best binary power allocation $(P_1, P_2, P_3) = (1, 1, 1)P_{\max}$, a sum throughput of 9.4555 bits/s/Hz is obtained, while by assigning the optimal powers $(P_1, P_2, P_3) = (1, 0.8595, 1)P_{\max}$ we get a throughput of 9.4590 bits/s/Hz. As we will see later, this example is quite typical in the sense that binary power control, though suboptimal, very often yields a throughput close to that obtained by optimally allocating the powers. While achieving only marginally higher sum throughput under the given power constraints, optimal continuous control can however offer some savings in terms of sum transmit power.

We shall now consider binary power control for N cells in three cases, 1) approximation by the arithmetic-geometric means inequality, 2) the low-SNIR regime, and 3) the general case.

1) Arithmetic mean-geometric mean approximation: From the *arithmetic - geometric means inequality* we have, for positive numbers $x_1, \cdots, x_N$ [21],

$$G_N = \left(\prod_{n=1}^{N} x_n\right)^{\frac{1}{N}} \leq \frac{1}{N}\sum_{n=1}^{N} x_n = A_N, \tag{12}$$

where G_N and A_N are the geometric mean (GM) and arithmetic mean (AM) of $x_1, \cdots, x_N$, respectively. Equality in (12) can be obtained if and only if $x_1 = \cdots = x_N$. Writing (4) as a log of products, and letting $x_n = (1 + \text{SNIR}_n)$, we can apply the above inequality to obtain

$$\begin{aligned} R(\mathbf{P}) &= \log_2\left(\prod_{n=1}^{N} 1 + \frac{P_nG_{n,n}}{\sigma_{Z_n}^2 + \sum_{j\neq n} P_jG_{n,j}}\right) \\ &\leq N\log_2\left(1 + \frac{1}{N}\sum_{n=1}^{N} \frac{P_nG_{n,n}}{\sigma_{Z_n}^2 + \sum_{j\neq n} P_jG_{n,j}}\right). \end{aligned} \tag{13}$$

Now, in scenarios where the right hand side of the above inequality can be used as an *approximation* of $R(\mathbf{P})$, i.e.,

$R(\mathbf{P}) \approx N\log_2\left(1+\frac{1}{N}\sum_{n=1}^{N}\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}}\right)$, the optimization problem (5) simplifies to

$$\mathbf{P}^* = \arg\max_{\mathbf{P}\in\Omega^N} N\log_2\left(1+\frac{1}{N}\sum_{n=1}^{N}\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}}\right), \tag{14}$$

and we can analytically find a closed form solution. As is always true in the two-cell case, the optimal power control in the N-cell case is binary when the AM-GM approximation is accurate.

Theorem 2: The solution $\mathbf{P}^*$ from (14) is binary, i.e., $\mathbf{P}^* \in \Delta\Omega^N$.

Proof: Due to the monotonicity of the log-function, we establish the result by showing that the argument of the logarithm in the cost function of (14) is convex in each variable P_k.

$$\begin{aligned}&\frac{\partial^2}{\partial P_k^2}\left(1+\frac{1}{N}\sum_{n=1}^{N}\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}}\right)\\ &=\frac{1}{N}\sum_{n\neq k}\frac{2P_nG_{n,n}G^2_{n,k}}{(\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j})^3}\geq 0.\end{aligned} \tag{15}$$

Now, for any $\mathbf{P}$ where at least one of its components is not an endpoint of its interval, there is another point $\mathbf{P}'$ with $R(\mathbf{P}') \geq R(\mathbf{P})$ such that one more component is at an endpoint of its interval. ■

An obvious question is for which scenarios the sum throughput is well approximated using the AM-GM inequality. The quality of the approximation can in general be quantified by inspecting the difference between the right hand and left hand side of (13), which can be written as

$$\begin{aligned}&N\log_2\left(1+\frac{1}{N}\sum_{n=1}^{N}\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}}\right)\\ &-\log_2\left(\prod_{n=1}^{N}1+\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}}\right)\\ &=N\log_2\left(\frac{A_N}{G_N}\right).\end{aligned} \tag{16}$$

From (12), $\log_2\left(\frac{A_N}{G_N}\right)\geq 0$, and using Specht's ratio $S(h)$ [22] we find

$$\frac{A_N}{G_N}\leq S(h)\triangleq\frac{(h-1)h^{\frac{1}{h-1}}}{e\ln h}, \tag{17}$$

where $h \triangleq \max_{1\leq j,k\leq N}\frac{x_k}{x_j}$. Using these bounds together we have the following result:

$$0\leq\log_2\left(\frac{A_N}{G_N}\right)\leq\log_2\left(\frac{(h-1)h^{\frac{1}{h-1}}}{e\ln h}\right). \tag{18}$$

Inspecting (18), we see that the quality of the approximation largely depends on the spread of the x_n values; indeed the more concentrated the $(1+\mathrm{SNIR}_n)$ factors are, the better the approximation is, reaching equality between the arithmetic and geometric mean with all SNIRs equal. As an example application, consider the case of low SNIR. Then, by default the SNIR is low in all cells, providing concentrated values of $\{x_n\}_{n=1}^N$, and the optimal power control in this scenario is binary.

2) Low-SNIR regime: The optimality of binary power control in the low-SNIR case can also be derived using another argument as we now investigate. In the low-SNIR regime we can apply an approximation of the achievable rate of each user, thus simplifying the problem. Specifically, when the SNIR is low, the following approximation obtained by Taylor expansion holds [23]: $\log_2(1+\mathrm{SNIR})\approx\frac{\mathrm{SNIR}}{\ln 2}$. Thus, we have

$$\begin{aligned}R(\mathbf{P})&=\sum_{n=1}^{N}\log_2(1+\mathrm{SNIR}_{u_n})\\ &\approx\frac{1}{\ln 2}\sum_{n=1}^{N}\frac{P_nG_{n,n}}{\sigma^2_{Z_n}+\sum_{j\neq n}P_jG_{n,j}},\end{aligned} \tag{19}$$

and again find that binary power control is optimal, which is easily seen from the proof of Theorem 2[3]. In fact, the objective function obtained by both the low-SNIR approximation and the arithmetic-geometric means approximation is maximized by the same binary power values.

In the low-SNIR case the binary power allocation is also optimal for a weighted sum rate criterion, $R_{\mathrm{w}} = \sum_{n=1}^{N} w_nR_n, w_n\geq 0$, which we state as a corollary.

Corollary 1: In the low-SNIR regime, for a weighted sum rate criterion, the sum throughput maximizing power control is binary.

Proof: The result follows by the rules of differentiation. ■

3) General case: In general, when none of the above approximations hold, unfortunately we have not found mathematical relations establishing the performance of binary power control, and hence we resort to exhaustive numerical simulations, trying to cover typical settings for cellular networks. To evaluate the performance of our proposed binary power control against a non-binary benchmark we capitalize on recent developments in geometric programming [14], [24], as discussed in the next subsection.

Independent of whether any of the above approximations hold, we still have to solve the discrete maximization (11), which has a worst-case complexity of $O(2^N)$ for exhaustive search. For small to moderate values of N, the globally optimal solution to (11) can easily be found by simply checking the corner points. For large N, since the cost function is non-linear and the optimization search space is spanned by binary variables, (11) may be approached using $0-1$ nonlinear programming [25], clustering, or greedy approaches. We discuss clustering and greedy approaches in Section IV.

Currently, there is also ongoing research on finding simple, distributed solutions to the present power control problem (also including scheduling) [15]. Finally, we note that the previously mentioned and more general case where the objective function

[3]For the case of $P_{\min,n}=0$, $\forall n$, and identical peak power constraints, this was independently reported also by the authors in [19], [20]. Further, in [12], also with $P_{\min,n}=0$, $\forall n$, the optimization problem (5) with R as a linear function of the received power, similar to (19), is considered and an alternative proof for on/off power control is given.

to be maximized is a sum of weighted rates is also an interesting problem, but this is a subject for future research[4].

D. Geometric Programming Power Control for N Cells

As mentioned above, to evaluate the performance of binary power control, we will use power control by geometric programming as the yardstick[5]. First, we therefore provide a brief background on geometric programming [14]. A *monomial* is a function $f : \mathbf{R}^n_{++} \to \mathbf{R}$: $g(\mathbf{P}) = cP_1^{a^{(1)}} P_2^{a^{(2)}} \cdots P_n^{a^{(n)}}$, where $\mathbf{R}^n_{++}$ is the strictly positive quadrant of $\mathbf{R}^n$, $c > 0$ is a constant, and $a^{(i)} \in \mathbf{R}$, $i = 1, \ldots, n$. A sum of monomials is called a *posynomial*:

$$f(\mathbf{P}) = \sum_{k=1}^{K} c_k P_1^{a_k^{(1)}} P_2^{a_k^{(2)}} \cdots P_n^{a_k^{(n)}}. \tag{20}$$

Then, a geometric program (GP) in standard form is written as:

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{P}), \\ \text{subject to} \quad & f_i(\mathbf{P}) \le 1,\, i = 1, \ldots, I \\ & g_m(\mathbf{P}) = 1,\, m = 1, \ldots, M, \end{aligned} \tag{21}$$

where f_i, $i = 0, \ldots, I$ are posynomials and g_m, $m = 1 \ldots M$ are monomials.

Using the results in [24], the optimization problem in (5) can be written as follows[6]:

$$\begin{aligned} \text{minimize} \quad & \prod_{n=1}^{N} \frac{1}{1 + \text{SNIR}_{u_n}}, \\ \text{subject to} \quad & \frac{P_n}{P_{\max,n}} \le 1,\, n = 1, \ldots, N, \\ & \frac{P_{\min,n}}{P_n} \le 1,\, n = 1, \ldots, N. \end{aligned} \tag{22}$$

Inspecting (22), we see that the constraints are monomials (and hence posynomials), but the objective function is a *ratio* of posynomials, as shown by

$$\prod_{n=1}^{N} \frac{1}{1 + \text{SNIR}_{u_n}} = \prod_{n=1}^{N} \frac{\sigma^2_{Z_n} + \sum_{j \ne n} G_{n,j} P_j}{\sigma^2_{Z_n} + \sum_{j=1}^{N} G_{n,j} P_j}, \tag{23}$$

and the fact that posynomials are closed under multiplication. Hence, (22) is not a GP in standard form, but a *signomial* programming (SP) problem [14]. Following the iterative procedure from [24], (22) is solved by constructing a series of GPs, each of which can easily be solved. The GP in iteration l of the series is constructed by approximating the denominator posynomial (23) by a monomial, using the value of $\mathbf{P}$ from the previous iteration, while the series is initialized by any feasible $\mathbf{P}$. Specifically, denote the denominator posynomial of (23) as $g(\mathbf{P})$. Since a posynomial is a sum of monomials, write $g(\mathbf{P}) = \sum_i u_i(\mathbf{P})$ where $u_i(\mathbf{P})$ is a monomial. Then, in iteration l, $g(\mathbf{P})$ is approximated by a monomial $\tilde{g}_l(\mathbf{P})$ as follows [24]:

$$g(\mathbf{P}) \ge \tilde{g}_l(\mathbf{P}) = \prod_i \Big(\frac{u_i(\mathbf{P})}{\alpha_i^l} \Big)^{\alpha_i^l}, \tag{24}$$

where $\alpha_i^l = u_i(\mathbf{P}_{l-1})/g(\mathbf{P}_{l-1})$, and $\mathbf{P}_l$ is the value of $\mathbf{P}$ in iteration l. By using (24), (23) is now a ratio of a posynomial and a monomial. This ratio is again a posynomial, and hence (22) is approximated and transformed to standard form, and can be solved using GP techniques. The iteration is terminated at the l'th loop if $||\mathbf{P}_l - \mathbf{P}_{l-1}|| < \epsilon$, where ϵ is the error tolerance. This procedure is provably convergent and empirically almost always computes the optimal power allocation [24], and thus represents an upper bound against which we can measure the performance of binary power control.

IV. Low-complexity Power Control Algorithms

Despite the promise of binary power control in terms of near throughput optimality and key implementation simplifications, solving the exhaustive binary power allocation problem (11) for large networks presents the system designer with an exponentially complex task. In this section we study two approaches towards reducing the search complexity.

The underlying idea behind lowering the complexity in both approaches is to split the original problem into smaller subproblems, each of which can easily be solved. However, since the problem does not exhibit an *optimal substructure property*, i.e., an optimal solution to the problem does not contain within it optimal solutions to subproblems [27], in general, we will not be able to derive simple algorithms for finding the globally optimal binary solution. As such, our algorithms seek to a achieve a good performance versus complexity compromise, rather than obtaining a global performance optimum.

A. Grouping Clusters of Cells

We now investigate a setting where the total number of N cells in the network are clustered into groups of $K << N$ cells, and each cell either transmits with full or minimum power. For a given cluster Q, the interference from the remaining $N - K$ cells will simply contribute as noise, i.e. the sum throughput of the cells in Q is given as

$$R_{\text{cluster, } Q} = \sum_{q \in Q} \log_2 \big(1 + \frac{P_q G_{q,q}}{\sigma^2_{Z_q} + \sigma^2_{\mathrm{I}_q} + \sum_{\substack{j \in Q \\ j \ne q}} P_j G_{q,j}} \big), \tag{25}$$

where $\sigma^2_{\mathrm{I}_q} = \sum_{j \notin Q} P_j G_{q,j}$ is the interference from cells in the network which are *not* part of the cluster. Assuming that this interference term can be estimated or averaged from the knowledge of the power activity in other clusters, the idea is to do power control only *locally* within each cluster. Hence, the following problem is solved for each cluster Q,

$$\mathbf{P}_Q = \arg \max_{\mathbf{P} \in \Delta\Omega^K} R_{\text{cluster, } Q}. \tag{26}$$

To solve the cluster based maximization problem, we have to investigate $\frac{N}{K}$ subproblems each with maximally 2^K evaluations, hence yielding a complexity in N and K of $O(N\, 2^K)$,

[4]The weighted sum solution presented in [24] is only valid in the high-SNIR regime.

[5]The content in this section is largely based on [24], [26], where geometric power control for wireless networks is given a formal mathematical treatment.

[6]For $P_{\min,n} = 0$, in theory the strictly positive quadrant assumption can be violated. However, numerically this is not a problem in practice as the geometric programs are solved using interior-point methods, searching *inside* the feasible domain [26].

while keeping K fixed, the complexity is $O(N)$. Also, since we only need to know the sum of the out-of-cluster interference, not its terms, compared to the exhaustive binary search problem (11), the required channel knowledge is reduced.

B. A Greedy Approach to Power Control

Now, consider approaching the binary power control problem by a greedy method, i.e., we are looking for a simple and efficient algorithm which in each step makes the choice that appears best at the moment. This greedy algorithm belongs to the class of "local search" methods, popular due to its wide range of applications in non-linear integer programming problems [25], an example being allocating traffic channels in OFDMA systems [28].

A key point in a greedy algorithm is the *selection function* which chooses the best candidate to be added to the solution. Here, we start with all cells assigned minimum power $P_{\min,n}$, and look for candidate cells which should have maximum power $P_{\max,n}$. Inspecting (4), and denoting the set of cells assigned maximum power as S, we note that at each stage, in deciding whether a cell $n \notin S$ should be operated at maximum power, an obvious selection function is the capacity that would be obtained by letting cell n transmit at maximum power $P_{\max,n}$, while keeping the transmit powers obtained in the previous stages fixed.

Summarizing, we arrive at Algorithm 1 looping once over N cells, where R_l and $(\mathbf{P}_l)_j$ respectively denote the sum throughput and the j'th component of the power allocation vector $\mathbf{P}_l$, at step l. After traversing the N cells, the power allocation vector is found by inspecting at which step the sum rate achieved its maximum. Complexitywise, the proposed greedy algorithm makes N choices, and needs to solve the optimization problem in line 4, with complexity $O(N)$. Thus, it runs in $O(N^2)$ time, achieving a significant reduction compared to the exhaustive search.

Algorithm 1 Greedy power control

1: $\mathbf{P}_0 = \mathbf{P}_{\min}$, $R_0 = R(\mathbf{P}_0)$, $S = \emptyset$
2: **for** $l = 1$ to N **do**
3: $\quad \mathbf{P}_l = \mathbf{P}_{l-1}$
4: $\quad n^* = \arg\max_{n \notin S} R\big((\mathbf{P}_l)_n = P_{\max,n}, (\mathbf{P}_l)_{j \neq n} = (\mathbf{P}_l)_j\big)$
5: $\quad (\mathbf{P}_l)_{n^*} = P_{\max,n}$
6: $\quad S = S + \{n^*\}$
7: $\quad R_l = R(\mathbf{P}_l)$
8: **end for**
9: $\mathbf{P}^* = \mathbf{P}_{\arg\max_{1 \le l \le N} R_l}$

V. Numerical Results

In this section we present numerical results on the achievable sum network capacities for an N-cell wireless system utilizing the various schemes of power control we have analyzed.

TABLE I
CELLULAR SYSTEM PARAMETERS

Parameter	Suburban Macro	Urban Macro	Urban Micro LOS
Cell layout	Hexagonal	Hexagonal	Hexagonal
Carrier frequency	1900 MHz	1900 MHz	1900 MHz
$P_{\max,n}, \forall n$	10 W	10W	1 W
$P_{\min,n}, \forall n$	0 W	0 W	0 W
BS to BS distance	3000 m	3000 m	1000 m
Exclusion disc radius	35 m	35 m	20 m
Operating temperature	290 Kelvin	290 Kelvin	290 Kelvin
Shadowing st. dev.	8 dB	8 dB	4 dB
Equiv. noise BW	1 MHz	1 MHz	1 MHz

A. Simulation Model

Based on the system model described in Section II, we will now evaluate a cellular system through Monte Carlo simulations, assuming that the user distribution is uniform in each cell. To most accurately model a typical cellular system we follow the spatial channel models for use in system level simulations developed by the 3GPP-3GPP2 working group [29]. Specifically the following environments are considered: the suburban macrocell, the urban macrocell, and the urban microcell line of sight (LOS). In the macrocell environments the base station antennas are above rooftop height, while for the urban microcell setting it is at rooftop height. Depending on the model, BS-to-user distances should exceed $20-35$ meters, thus we exclude users from being located in a circular disk of radius $20-35$ m around each base station. Further simulation details can be found in Table I.

B. Description of Transmission Schemes

We consider two link adaptation schemes; adaptive coded modulation using capacity-achieving codes with, and without, power control. Without power control, the power at all base stations is held constant at $P_{\max,n}$, $\forall n$. Based on the current received SNIR level the modulation and coding rates are then selected. Allowing for power control, adaptive coded modulation is used to transmit at power levels that are optimized respectively according to GP power control (5), binary power control (11), the clustering approach (26), and greedy power control.

C. Network Capacity Statistics

To obtain the system throughput statistics for an average user in each cell, we ran 10000 independent trials, in each trial drawing user locations and path gain matrices from their corresponding distributions. Table II depicts the average per-cell capacity, defined as $\frac{R}{N}$, for the three simulation settings, in bits/s/Hz versus the number of cells N. It is clear that introducing power control improves the throughput performance for $N \geq 2$, in particular for the urban microcell environment. However, note the only marginal improvement in going from binary power control to optimal GP power control based on geometric programming. As seen from the table, the average per cell capacity decreases as the number of cells increase. This is to be expected since all cells share the same spectral resources. As an example of how instrumental it is to be able

TABLE II
NETWORK CAPACITY STATISTICS

	Average pr. cell capacity $\frac{R}{N}$ (bits/s/Hz/cell) shown in (GP, Binary, Full) triplets		
N	Suburban Macro	Urban Macro	Urban Micro
1	$(6.02, 6.02, 6.02)$	$(5.13, 5.13, 5.13)$	$(11.96, 11.96, 11.96)$
2	$(4.93, 4.93, 4.74)$	$(4.40, 4.40, 4.27)$	$(6.64, 6.64, 4.54)$
3	$(4.41, 4.40, 4.02)$	$(4.03, 4.03, 3.75)$	$(6.03, 6.03, 3.39)$
4	$(4.03, 4.01, 3.53)$	$(3.70, 3.69, 3.33)$	$(4.66, 4.65, 2.91)$
5	$(3.98, 3.95, 3.45)$	$(3.68, 3.67, 3.28)$	$(3.88, 3.85, 2.75)$
6	$(3.81, 3.78, 3.25)$	$(3.54, 3.53, 3.11)$	$(3.41, 3.36, 2.58)$
7	$(3.67, 3.64, 3.08)$	$(3.42, 3.41, 2.97)$	$(3.06, 3.00, 2.40)$

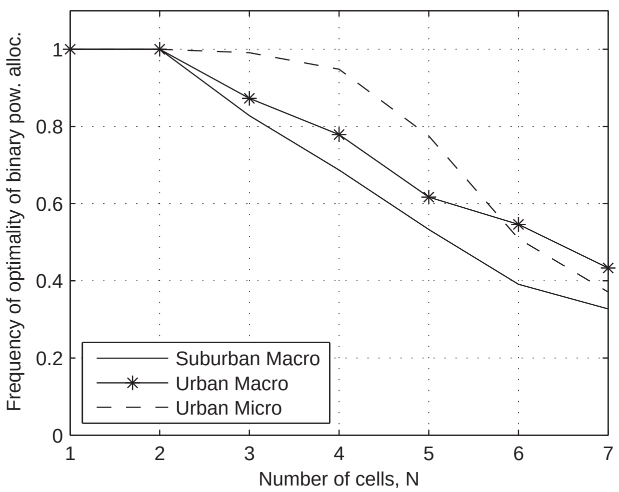

Fig. 2. Frequency of optimality of binary power allocation, relative to optimal (GP) power allocation, plotted versus the number of cells N for all three simulation environments.

to operate some cells at minimum power, we see that the *system capacity* in the urban microcell environment is less for two cells than for one cell when using full power. However, using binary and GP power control, we observe an *increase* in system capacity when going from one to two cells, due to better management of interference.

In Fig. 2 we have plotted the frequency of optimality of binary power control, i.e., the percentage of simulations where binary power control is still optimal. It is seen that for one and two cells, binary power control is indeed always optimal, while for more than two cells it is optimal only in a certain fraction of the cases. When the number of cells increases, binary power allocation is more seldom optimal. However, as shown in Table II, the gap between the optimal (GP) power control and the suboptimal binary power control is still marginal. This demonstrated near-optimality of binary power control has several potential implications in the design and analysis of wireless networks. Firstly, the complexity of the transmitter design is reduced, since only a two-level power control is required. Secondly, binary power control provides a key simplification of the problem by enabling *distributed* control of the power allocation [15].

D. Average transmit power

To improve further our understanding of the power control problem, in Fig. 3 we have plotted the average transmit power for the suburban macrocell and the urban microcell environments, as a function of the number of cells. In the macrocell setup, we see that the average transmit power of binary and GP power allocation are approximately the same, and that on average for a 7-cell network, 4 cells should be on. On the other hand, for the LOS microcell setup, significantly fewer cells should be turned on. This is due to the fact that the cells are much smaller, combined with a lower path loss due to a line of sight environment, hence the interference caused by turning on a cell dominate more. Also, we note that for increasing N, the GP power control uses less transmit power than binary control. Hence, even though there are no significant throughput gains in using continuous power control in these settings, it is possible to reduce the average transmit power, while achieving the same network capacity.

E. Reducing Complexity by Clustering Cells and Greedy Power Control

Now, we consider the results obtained by grouping the total number of cells in a large system into smaller clusters of neighboring cells[7]. In Table III the *normalized capacity* from (26), i.e., the capacity obtained by using binary power allocation in clusters of size K relative to a binary exhaustive search over the total network of 19 cells, is plotted versus the number of cells in the clusters. It is seen that even with small clusters of 3 cells more than 90% of the capacity can be achieved. Increasing the cluster size K yields improvements in sum throughput. Although not shown here, it is clear that as $K \to N$, the clustering scheme will be identical to that of the binary exhaustive search.

Finally, we evaluate the results of the greedy power control scheme. As seen from Table III, the greedy scheme performs extremely well, achieving more than 99% of the capacity from binary exhaustive search in all simulation environments. Using the previously derived running time expressions, we see that the complexity is reduced by a factor of $\frac{N^2}{2^N} = \frac{19^2}{2^{19}}$, or equivalently 99.99% complexity reduction, while sacrificing less than 1% of the network capacity. Comparing the clustering approach to the greedy approach, it is clear the greedy algorithm yields better results, which is to be expected, and can be explained as follows. While the greedy approach is a fully centralized scheme, the clustering based power control operates locally in clusters using an interference average from the other clusters, and has also for a fixed cluster size lower complexity. Also, from Table IV we see the average number of cells turned on by the clustering, greedy, and binary exhaustive schemes. We see as the number of cells in a cluster K increases, the fraction of cells turned on decreases. For the greedy and binary exhaustive search less cells are on, which helps explain their strong performance. Nonetheless, we see

[7]Clustering by grouping neighboring cells is not claimed to be optimal; indeed optimal clustering is a research problem in itself, beyond the scope of this paper.

TABLE III
NETWORK CAPACITY STATISTICS FOR COMPLEXITY REDUCED SCHEMES, $N = 19$

	Normalized capacity relative to binary exhaustive search								
	Clusters of K cells							Greedy	Binary exhaustive
	1	2	3	4	5	6	7		
Suburban Macro	0.82	0.88	0.91	0.93	0.95	0.96	0.95	0.997	1
Urban Macro	0.84	0.89	0.92	0.94	0.95	0.96	0.96	0.998	1
Urban Micro	0.84	0.87	0.9	0.92	0.93	0.95	0.94	0.994	1

TABLE IV
AVERAGE NUMBER OF CELLS TURNED ON FOR COMPLEXITY REDUCED SCHEMES, $N = 19$

	Average number of cells turned on								
	Clusters of K cells							Greedy	Binary exhaustive
	1	2	3	4	5	6	7		
Suburban Macro	19	16.88	15.19	14.55	13.99	13.47	13.5	11.3	11.3
Urban Macro	19	16.98	15.38	14.79	14.21	13.75	13.75	11.7	11.7
Urban Micro	19	17.93	16.45	15.72	15.1	14.23	14.26	10.7	10.5

that the promised benefits of discretizing the power levels can be achieved at a low complexity.

VI. CONCLUSIONS

We have analyzed transmit power allocation for an N-cell wireless system under a sum-capacity maximization criterion and minimum and peak power constraints at each base station. Assuming perfect channel gain information to be available, we have investigated the system capacity without power control, with binary power control, and with GP-based power control. We show that the optimal power control is binary for two cells, as well as when the network throughput can be approximated either by a geometric-arithmetic means inequality or by a low-SNIR assumption. In the general case when $N > 2$, it was demonstrated by extensive computer simulations that a restriction to binary power levels yields only a negligible capacity loss.

To reduce the complexity of exhaustively searching for the optimal binary power allocation for large networks, simple algorithms based on clustering and greedy approaches were derived. Using these algorithms a significant complexity reduction is possible at only a small penalty in network capacity. For practical systems, these results are of importance since the transmitter design is simplified, overhead feedback signalling is reduced, and the search for distributed algorithms becomes more manageable. Finally, we note that power control should complemented with scheduling to further improve the performance of the system. In certain scenarios this has previously been explored in [15], [30].

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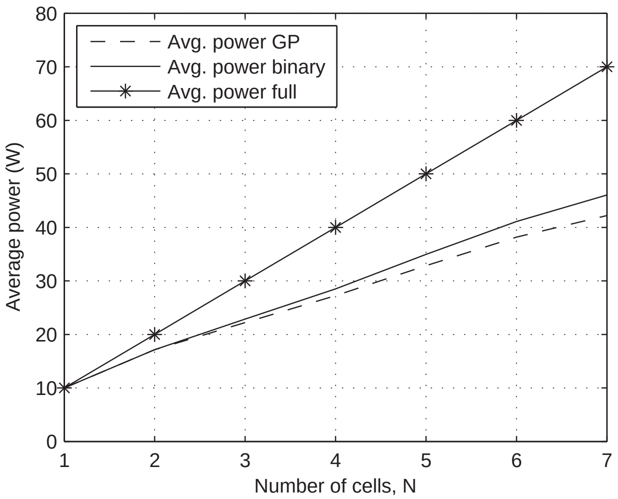
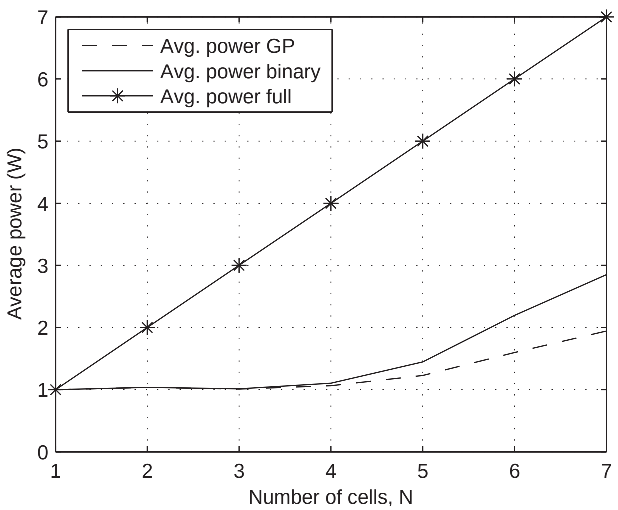


(a) Suburban macrocell environment. Maximum transmit power per cell $P_{\max,n} = 10\,\mathrm{W}, \forall n$



(b) Urban microcell environment. Maximum transmit power per cell $P_{\max,n} = 1\,\mathrm{W}, \forall n$.

Fig. 3. Average sum transmit power for macro and microcell setups versus the number of cells N.

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