

## Research Article

# Bioheat Transfer Equation with Protective Layer

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The human thermal comfort is the state of mind, which is affected not only by the physical and body's internal physiological phenomena but also by the clothing properties such as thermal resistance of clothing, clothing insulation, clothing area factor, air insulation, and relative humidity. In this work, we extend the one-dimensional Pennes' bioheat transfer equation by adding the protective clothing layer. The transient temperature profile with the clothing layer at the different time steps has been carried out using a fully implicit Finite Difference (FD) Scheme with interface condition between body and clothes. Numerically computed results are bound to agree that the clothing insulation and air insulation provide better comfort and keep the body at the thermal equilibrium position. The graphical representation of the results also verifies the effectiveness and utility of the proposed model.

## 1. Introduction

The study of thermal comfort in the complex vascular geometry of the human body with a protective clothing system is not only the subjective domain but also the physiological factors inside the body. According to the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE), thermal comfort is the state of the mind when one can feel and express the satisfaction with the thermal environment [1]. Thermoregulation is the process of controlling the internal body temperature through the hypothalamus heat production and heat loss center. The body also uses other processes of thermodynamical systems that constantly produce energy by metabolic activity together with dilating or constricting blood vessels, shivering, and sweating [2, 3].

The presence of clothes, on the other hand, plays a vital role for maintaining the thermal comfort at the equilibrium condition of heat production and heat loss by the body. Clothing, the interface between the skin surface and the environment, works as an insulator and also transports the heat from the body to the outer environment around us. The key factors affecting thermal comfort are categorized as

- (1) Personal factors
- (2) Environmental factors

*1.1. The Personal Factors.* Metabolism, determined by age, sex, health, etc., is one of the human personal factors which makes a difference in thermal comfort. Besides, another major factor which affects thermal comfort is clothing. A significant difference in thermal comfort can be caused by small changes in clothing layers. In the winter season, wearing a sweater and socks makes better comfort, whereas in summer, wearing light clothes makes better comfort in the workplace.

*1.2. Environmental Factors.* Air temperature, moving air, radiant temperature, and relative humidity are the environmental factors that help to maintain thermal comfort by keeping the room comfortable. The radiation heat flux, on the other hand, penetrates a certain depth passing entirely through the fabric, depending on the fabric structure and radiative wavelength. In contrast, the convection portion of the heat source could reach the fabric surface only [4].

Usually, heat transfer in the body through the garment occurs from heat conduction and heat radiation; then, the temperature rises and transfers into the air gap. Suitable clothing along with these environmental factors, therefore, maintains better comfort at home and workplace (business, office, study room, etc.).

## 2. Role of Clothing

**2.1. Thermal Resistance and Clothing Insulation.** The ratio of temperature difference between two faces of material to the rate of heat flow per unit area is defined as thermal resistance. In the study of thermal insulation of clothing, thermal resistance is a very important parameter defined as the function of thickness and thermal conductivity of clothes. Thermal Resistance  $R_{th} \text{ m}^2 \cdot (^\circ\text{C}/\text{W})$  is given by [5]

$$R_{th} = \frac{\Delta T}{q} = \frac{L}{k}, \quad (1)$$

where  $L$  is the thickness of cloth (m),  $T$  is the temperature difference ( $^\circ\text{C}$ ), and  $q$  is the heat flow rate ( $\text{W}/\text{m}^2$ ). Though the heat transfer in the clothed body consists of conduction and radiation, the primary determinant of the insulation is the thickness of clothes on which the insulation is very much dependent. The limbs of human body with insulation parameters can be seen in Figure 1.

Thermal insulation, together with the air gap, does not only provides comfort at the skin surface and body from a cold environment but also protects from burn injuries. While sitting by the fire, the air gap provides thermal insulation that limits heat transfer to skin and protects the skin from excessive heat.

**2.1.1. Clo Unit.** Clo unit is the measure of thermal resistance and includes the insulation provided by any layer of trapped air between skin and the insulation value of clothing itself. 1 Clo is defined as the insulation of the clothing system that requires maintaining a sitting-resting average male comfortable in a normally ventilated room. Clo is the thermal insulation of overall clothing worn by a person. It has 0.1 m/s air velocity at air temperature  $21^\circ\text{C}$  and relative humidity less than 50%. Among the total heat produced by the metabolic reaction, 24% heat is lost through evaporation and respiration. As  $1 \text{ met} = 50 \text{ kcal}/\text{m}^2\text{h}$ , the evaporative and respiratory heat loss =  $1 \text{ met} \times 24\% = 50 \times 0.24 = 12 \text{ kcal}/\text{m}^2\text{h}$ . Remaining  $38 \text{ kcal}/\text{m}^2\text{h}$  is transmitted through the clothing system by conduction, convection, and radiation. The comfortable skin temperature is  $33^\circ\text{C}$ , so the total insulation of clothing and air layer ( $I_T = I_{cl} + I_a$ ) is given by [8]

$$I = \frac{33 - 21}{38} = 0.32,$$

$$I_a = 0.14 \text{ m}^2 \cdot (^\circ\text{C}/\text{h/kcal}),$$

$$(I_{cl} = 0.32 - 0.14) \text{ m}^2 \cdot (^\circ\text{C}/\text{h/kcal}) = 0.18 \text{ m}^2 \cdot (^\circ\text{C}/\text{h/kcal}), \quad (2)$$

where  $I_a$  is the insulation of air and  $I_{cl}$  is the insulation cloth.

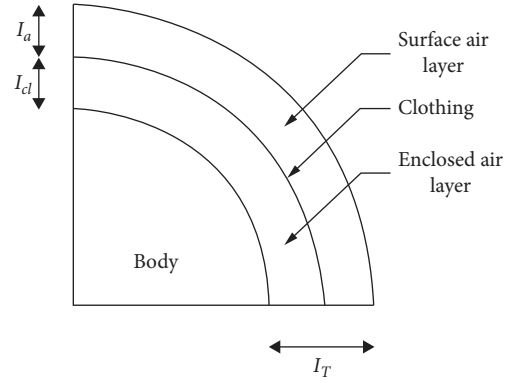


FIGURE 1: A cylindrical model with clothing and air insulation [6, 7].

Since  $1 \text{ kcal}/\text{h} = 1.163 \text{ watt (W)}$ , so 1 Clo unit is defined as

$$0.18 \text{ m}^2 \cdot (^\circ\text{C}) = \frac{0.18}{1.163} \equiv 0.155 \text{ m}^2 \cdot (^\circ\text{C}/\text{W}). \quad (3)$$

**2.2. Convective and Radiative Heat Transfer.** Clothing works as the mediator of heat exchange through convection as well as radiation. The standard measurement condition in ISO 9920, four different manikins in three different laboratories, were used and determined the male and female clothing thermal insulation values of 52 nonwestern clothing configuration [9]. If  $h_c$  ( $\text{m}^2 \cdot (^\circ\text{C}/\text{W})$ ) and  $h_r$  ( $\text{m}^2 \cdot (^\circ\text{C}/\text{W})$ ) are the heat exchange due to convection radiation, respectively, then the convective and radiative heat exchange is defined in [7, 9] by

$$h_c + h_r = \frac{\Delta T}{I_{cl}}. \quad (4)$$

**2.3. Clothing Area Factor.** The dimensionless parameter, clothing area factor  $f_{cl}$ , is the ratio between the surface area of clothed human body  $A_b$  ( $\text{m}^2$ ) and the surface area of nude human body  $A_n$  ( $\text{m}^2$ ) which is given by

$$f_{cl} = \frac{A_{cl}}{A_n}. \quad (5)$$

The prediction equation for clothing area factor  $f_{cl}$ , based on western clothing and listed in ISO Standard 9920-2009 (ISO 2009), can also be found in various publications. The equation for  $f_{cl}$  is given [1, 9, 10] as

$$\begin{aligned} \text{In Clo, } f_{cl} &= 1 + 0.31 I_{cl}, \\ \text{In } (\text{m}^2 \cdot (^\circ\text{C}/\text{W})), f_{cl} &= 1 + 1.97 I_{cl}. \end{aligned} \quad (6)$$

**2.4. Clothing Efficiency Factor.** The clothing efficiency factor (dimensionless) depends upon the air insulation  $I_a$  ( $\text{m}^2 \cdot (^\circ\text{C}/\text{W})$ ) and the thermal insulation of overall clothing, not only the particular garment but also the entire garment, including tops, bottoms, innerwear, and everything

(including even socks and gloves). The total clothing insulation ( $I_T = I_{cl} = \sum_i I_{cli}$ ) (where  $I_{cli}$  is the insulation of each fabric item) for a person in summer and winter is given in Table 1. So, the clothing efficiency factor  $F_{cl}$  (dimensionless) is provided by [4, 7, 8]

$$F_{cl} = \frac{I_a}{I_{cl} + (I_a/f_{cl})}. \quad (7)$$

Pennes [11], a famous researcher, established a bioheat transfer model in 1948 based on experimental observation incorporating the blood perfusion term for heat flow within tissue. Various methods related to the biological model using Pennes' equation is tackled by many researchers one after another. Zhao et al. [12] developed a two-level finite difference scheme for one-dimensional Pennes' bioheat equation and established the stability and convergence condition taking only one initial condition. Gurung and Saxena [13], Luitel et al. [2, 14, 15], Agrawal et al. [3], Acharya et al. [16], and Parsons [17] used Pennes' model for steady-state and unsteady-state temperature distribution within the human dermal part using finite element and finite difference methods.

So far as the study of clothing is concerned, a number of research studies [1, 4, 9, 10, 18] have turned significant stones in regards of thermal comfort in the human body under the protective clothing by taking the dry and evaporative heat loss with the significant clothing area factor within the different human manikins only by the experimental method taking the equation only in the clothing part. Havenith et al. [9] published a database of static clothing thermal insulation and vapor permeability values' non-western ensembles for ASHARE standard 55, ISO-7730 (International Organization for Standardization), and ISO-9920.

Gurung and Saxena [13] studied the transient temperature distribution in the human dermal part with a protective layer at low atmospheric temperature, considering the bioheat equation to observe only the effect of thickness

and mass of the protective layer. So, the mathematical study of the bioheat equation along with the impact of clothing insulation, air insulation, and clothing area factor is still lacking and is one of our research interests.

On the contrary, in the cold environment, the cold receptor becomes active and vasoconstriction of blood vessel takes place. Shivering of muscle also appears in this case. If the body core temperature goes down around 30°C, then one goes in the unconscious condition, and, if the body temperature goes down up to 27°C, his/her heart beating stops. Similarly, in the hot climatic condition, the hot receptor becomes active and vasodilation takes place and sweating occurs. The increment in body temperature up to 42°C causes hyperthermia [14–16, 19]. To overcome all these unusual conditions, the human body needs proper well-designed protective clothing because only the internal physiological conditions cannot cope up with such extreme cold and hot climatic condition without wearing sufficient clothing in winter and putting on only light clothing in summer. So, this study, heat transfer in the human body with protective clothing, is indispensable for proper management of clothing to keep the body highly comfortable condition.

The heat transfer in the human body with a protective clothing system is a heterogeneous phenomenon. The analytical solution of such material is not an easy task. So, the study of the numerical approach for getting an approximation solution is essential for this purpose. The recent paper aims to extend one-dimensional Pennes' bioheat equation with unconditionally stable state and convergence which incorporates various personal and environmental factors on the one hand and protective clothing on the other hand. The graphical representation of convergence for the FD scheme will be shown for the use of the model.

### 3. Mathematical Formulation of the Model

The bioheat transfer equation with the protective clothing system is given by

$$\rho c \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + w_b c_b (T_a - T) + q_m + P(T_{sk} - T_{cl}), \quad (8)$$

where  $\rho$  is tissue density ( $\text{kg/m}^3$ ),  $c$  is tissue specific heat ( $\text{J/kg}^\circ\text{C}$ ),  $k$  is thermal conductivity ( $\text{W/m}^\circ\text{C}$ ),  $W_b$  is blood perfusion rate ( $\text{kg/m}^2 \cdot \text{s}$ ),  $c_b$  is blood specific heat ( $\text{J/kg}^\circ\text{C}$ ),  $T_a$  is arterial blood temperature ( $^\circ\text{C}$ ),  $q_m$  is metabolic heat generation ( $\text{W/m}^3$ ), and the symbol  $P$  in the last term is given by

$$P = \frac{k_{cl}}{A_{cl}} \left( \frac{\text{W}}{\text{m}^3 \cdot ^\circ\text{C}} \right), \quad (9)$$

where  $A_{cl} = A_b f_{cl}$ ,  $k_{cl}$  is thermal conductivity of clothes ( $\text{W/m}^\circ\text{C}$ ),  $T_{sk}$  is the skin temperature ( $^\circ\text{C}$ ), and ( $T_{cl}$ ) is the cloth temperature ( $^\circ\text{C}$ ).

The bioheat equation (8) is the extension of the equation suggested by Pennes' in 1948. The left-hand side is the total heat storage; the first, second, and third terms of right-hand side are diffusion, perfusion, and metabolic heat generation, respectively. The extra term  $P(T_{sk} - T_{cl})$ , with clothing parameters, is the heat transfer from the skin to clothing layer.

One-dimensional unsteady-state temperature profiles in the cylindrical shape of the human body with the clothing system and the bioheat equation (8) in radial direction are performed and given below:

TABLE 1: Total insulation of clothes in Clo unit 8, 10.

Cloth item (summer)	$(I_{cli})$ Clo value	Cloth item (winter)	$(I_{cli})$ Clo value
Half shirt	0.19	Full shirt	0.28
Underwear	0.04	Underwear	0.04
Pants	0.11	Pants/trousers	0.24
Socks	0.02	Socks	0.03
Shoes	0.02	Shoes	0.04
—	—	Suit jacket	0.48
Total	0.38	Total	1.11

$$\rho c \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + w_b c_b (T_a - T) + q_m + P (T_{sk} - T_{cl}), \quad (10)$$

where  $r$  is radial distance from the center of core towards skin surface (m).

The study of heat transfer in such a nonhomogeneous phenomenon, the human body, and the protective clothing system is really cumbersome. So, we decouple equation (10) first and then combine by using the interface condition between the body and clothing part.

**3.1. Heat Transfer Equation for Body.** As  $P = 0$  in the body part so the bioheat equation for the body part is given by

$$\rho c \frac{\partial T_t}{\partial t} = k_t \left( \frac{\partial^2 T_t}{\partial r^2} + \frac{1}{r} \frac{\partial T_t}{\partial r} \right) + w_b c_b (T_a - T_t) + q_m. \quad (11)$$

**3.2. Heat Transfer Equation for Clothing.** The heat equation for clothing with  $W_b = 0$  and  $q_m = 0$  is therefore given by

$$\rho c \frac{\partial T_{cl}}{\partial t} = k_{cl} \left( \frac{\partial^2 T_{cl}}{\partial r^2} + \frac{1}{r} \frac{\partial T_{cl}}{\partial r} \right) + P (T_{sk} - T_{cl}). \quad (12)$$

**3.3. Boundary Conditions.** The inner boundary condition of the living tissue is considered uniform and taken as

$$\text{at } r = 0, \quad \frac{\partial T_t}{\partial r} = 0. \quad (13)$$

There is continuous heat flux between clothing surface and atmospheric environment as the outer surface of clothes is exposed to external environment [7, 17]. In this case, heat loss from the body via clothes is caused by convection and radiation. The Robin boundary condition due to convection condition is guided by Newton's law of cooling, and the term due to radiation is guided by the Stefan Boltzmann law and is given as

$$\text{at } r = R, \quad -k_{cl} \frac{\partial T_{cl}}{\partial r} = F_{cl} [h_c (T_{cl} - T_\infty) + \epsilon \sigma (T_{cl}^4 - T_\infty^4)], \quad (14)$$

where  $F_{cl}$  is the effective clothing area factor given in equation (14),  $h_c$  is the heat transfer coefficient due to convection,  $\epsilon$ , the emissivity that lies between 0 to 1,  $\sigma = 5.67 \times 10^{-8}$  is Stefan Boltzmann constant, and  $T_\infty$  is the atmospheric temperature.

The bioheat problem becomes nonlinear when the nonlinear radiation term in the boundary condition appears. In this case, it becomes difficult to formulate. To avoid such complexity, we apply the simplified form of boundary condition as

$$\begin{aligned} -k_{cl} \frac{\partial T_{cl}}{\partial r} &= F_{cl} [h_c (T_{cl} - T_\infty) + \epsilon \sigma (T_{cl} - T_\infty)(T_{cl} + T_\infty)(T_{cl}^2 - T_\infty^2)], \\ &= F_{cl} (T_{cl} - T_\infty) [h_c + \epsilon \sigma (T_{cl} + T_\infty)(T_{cl}^2 - T_\infty^2)], \\ &= F_{cl} (h_c + h_r)(T_{cl} - T_\infty), \\ &= h_A (T_{cl} - T_\infty). \end{aligned} \quad (15)$$

**3.4. Initial Condition.** For the time dependent boundary value problem, the initial condition is given by

$$T(r, 0) = T_0(r), \text{ where, } T = T(T_t, T_{cl}). \quad (16)$$

## 4. Solution of the Model

For the solution of model (10), we perform the following steps:

- (1) Construction of the Finite Difference (FD) scheme for models (11) and (12)
- (2) Getting solution of (11)
- (3) Using the solution of (11) to get the solution of (12) and applying the interface and boundary conditions
- (4) Representing the combined results in graph by computer algebraic software

**4.1. Construction of FD Scheme.** One-dimensional form of cylindrical tissue for the body part is divided into  $N$  discrete points uniquely specified by spatial indices,  $r_i = i r$ , in the radial direction. The discretization of circular cross-section of peripheral human limb, where the temperature flow in axial direction is uniform, is shown in Figure 2.

In the time discretization,  $t$  is denoted by the discrete time step size, and the total time to evaluate the temperature is  $t^n = n\Delta t$ .

**4.2. FD Scheme for the Nodes in Body Part.** We use FD scheme by writing equation (11) using implicit finite difference (central difference) scheme for right-hand terms and forward difference for left-hand term.

**4.2.1. FD Scheme at Boundary  $r=0$  (Body Core).** The cylindrical thickness  $r$  is measured from body core, as shown in Figure 2. At the body core, both  $r$  and the heat flux  $(\partial T_t / \partial r)$  are zero; then,  $(1/r)(\partial T / \partial r)$  approaches to indeterminate form  $(0/0)$  as  $r \rightarrow 0$ .

The use of Hospital rule then gives

$$\frac{1}{r} \frac{\partial T_t}{\partial r} \Big|_{r=0} = \frac{(\partial/\partial r)(\partial T_t/\partial r)}{(\partial/\partial r)(r)} \Big|_{r=0} = \frac{\partial^2 T_t}{\partial r^2} \Big|_{r=0}. \quad (17)$$

Now, equation (11) becomes

$$\rho c \frac{\partial T_t}{\partial t} = 2D \left( \frac{\partial^2 T_t}{\partial r^2} \right) + M(T_a - T_t) + S, \quad (18)$$

$$E_0 T_0^{n+1} - 4\alpha T_1^{n+1} - F = T_0^n, \quad (19)$$

where  $D = (k_t/\rho c)$ ,  $\alpha = (D\Delta t/\Delta r^2)$ ,  $M = (w_b c_b/\rho c)$ ,  $S = (q_m/\rho c)$ , and  $F = \Delta t(MT_a + S)$ .

As shown in Figure 2, we solve the problem in two phase body part and clothing part. For the body part, we take the interior nodes from  $i = 1, 2, \dots, N - 1$ , and the FD scheme of equation (11) is given by

$$D_i T_{i-1}^{n+1} + E_i T_i^{n+1} + B_i T_i^{n+1} - F = T_i^n, \quad (20)$$

where  $D_i = (-\alpha + (\alpha/2i))$ ,  $E_i = (1 + 2\alpha + M\Delta t)$ , and  $B_i = (-\alpha - (\alpha/2i))$ .

The construction in (20) implies that FD schemes have a truncation error in the order  $O(\Delta r^2 + \Delta t)$  for each interior point  $(t^n, r_i)$ ,  $n \geq 1, 0 < i < N$ .

At  $i = N$ , the skin surface is the interface between body and clothes. So, the FD scheme at  $i = N$  is

$$D_N T_{N-1}^{n+1} + E_N T_N^{n+1} + B_N T_{N+1}^{n+1} - F = T_N^n. \quad (21)$$

**4.2.2. Interface Condition at  $i = N$  (at Skin Surface).** The interface temperature between the skin surface and clothes at  $i = N$ . The right-most Dirichlet boundary at this point is

$$T_t|_{i=N} = T_{cl}|_{i=N} = T_{int}. \quad (22)$$

FD scheme, at  $i = N$ , is

$$D_N T_{N-1}^{n+1} + E_N T_N^{n+1} + B_N T_{int}^{n+1} - F = T_N^n. \quad (23)$$

This construction implies that FD schemes have an error in the order  $O(\Delta r^2 + \Delta t)$  for the gride point  $(t^n, r_N)$ , at  $i = N$ .

The system of equations (19), (20), and (23) can be written in the matrix form as

$$A_1 T_t^{n+1} = T_t^n + B_1, \quad (24)$$

where  $T_t^n = [T_0^n, T_1^n, T_3^n, \dots, T_N^n]^T$ .

$A_1$  is the corresponding tridiagonal matrix of order  $(N + 1) \times (N + 1)$  and  $T_t^{n+1}$  and  $B_1$  are of column vectors of  $(N + 1) \times 1$ .  $A_1$  matrix is diagonally dominant since the absolute value of each leading diagonal element satisfies the relation:

$$|a_{jj}| \geq \sum_{i=0, i \neq j}^N |a_{ij}|. \quad (25)$$

**4.3. FD Scheme for Clothing Part.** As we have mentioned above, the skin surface is the interface between the body and clothing part at  $i = N$ ; the FD scheme for (12) yields

$$D_N T_{N-1}^{n+1} + E_N T_N^{n+1} + B_N T_{N+1}^{n+1} = T_N^n, \quad (26)$$

where  $E_N = E_{N+1} = E_{N+2} = \dots = E_R = (1 + 2\alpha + P_1)$ .

**4.3.1. Interface Condition at  $i = N$  (between Skin and Cloth).** Before taking the interface temperature between skin surface and clothes, it is necessary to evaluate the interface thermal conductivity  $K$ . The nonhomogeneous material such as body and clothes which consists different physiological properties, has nonuniform thermal conductivity. Thus, the proper formulation for nonuniform  $K$  is highly desirable. The interface conductivity  $K$  is to assume a linear variation of  $K$  between two points  $N$  and  $N + 1$  is given by [20]

$$K = f_N k_t + (1 - f_N) k_{cl}, \quad (27)$$

where  $f_N$  is the interpolation factor defined by  $f_N = (\Delta r_c / \Delta r)$  and  $r_c$  is the mesh size in protective layer.

For the interface grid point  $N$ , we consider the control volume surrounding  $N$  is filled with the uniform conductivity  $k_t$  of body tissue, one around  $N + 1$  with a conductivity  $k_{cl}$  of clothes. The good representation for a heat flux over the composite domain between  $N$  and  $N + 1$  leads to

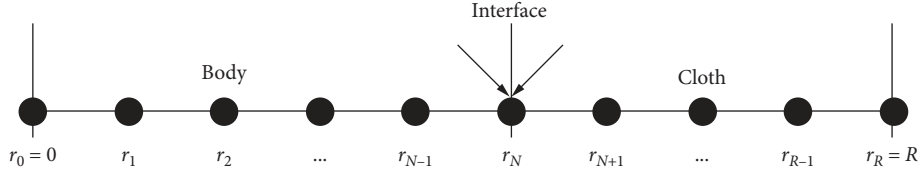


FIGURE 2: Discretization in radial direction with interface.

$$q_N = \frac{T_{N+1}^{n+1} - T_N^{n+1}}{((\Delta r/k_t) + (\Delta r_c/k_{cl}))}. \quad (28)$$

4.3.2. *Interface Conditions at  $i = N$  (Left Boundary for Cloth Part).* The heat flux occurs at the skin surface, and the left most boundary of clothes at  $i = N$  is given by

$$-k_t \frac{\partial T}{\partial r} = q_N. \quad (29)$$

FD formulation for (29):

$$\frac{T_{N+1}^{n+1} - T_{N-1}^{n+1}}{2 \Delta r} = -K \frac{T_{N+1}^{n+1} - T_N^{n+1}}{((\Delta r/k_t) + (\Delta r_c/k_{cl}))}, \quad (30)$$

$$T_{N-1}^{n+1} = T_{N+1}^{n+1} - 2R_c q_N.$$

Now, equation (26) with left boundary condition can be written as

$$D_N T_{N-1}^{n+1} + (2\alpha_1 - P_1) T_{N+1}^{n+1} - F_N = T_N^n, \quad (31)$$

where  $F_N = 2D_N R_c q_N$ .

FD scheme for each interior grid point  $(t^n, r_i)$ ,  $n \geq 1, N < i < R$  in the clothing part with a truncation error of the order  $O(\Delta r^2 + \Delta t)$  is given by

$$D_i T_{i-1}^{n+1} + E_i T_i^{n+1} + B_i T_{i+1}^{n+1} - F = T_i^n, \quad (32)$$

where  $D_i = (-\alpha_1 + (\alpha_1/2i))$ ,  $E_i = (1 + 2\alpha_1 + M\Delta t)$ ,  $B_i = (-\alpha_1 - (\alpha_1/2i))$ , and  $\alpha_1 = (D_{cl}\Delta t/(\Delta r_c)^2)$ .

4.3.3. *Boundary Conditions at  $i = R$  (at the Surface of the Cloth).* The heat flux occurs at the outer surface of the right-most boundary of clothes at  $i = R$  which is given by

$$-k_{cl} \frac{\partial T_{cl}}{\partial r_c} = h_A (T_{cl} - T_\infty), \quad (33)$$

$$T_{R+1}^{n+1} = T_{R-1}^{n+1} - 2h_A R_c (T_{cl} - T_\infty).$$

FD scheme at  $i = R$  is now given by

$$(-2\alpha_1 - P_1) T_{R-1}^{n+1} + (E_R - 2B_R h_A R_c) T_R^{n+1} - F_R = T_R^n. \quad (34)$$

The system of equations (31), (32), and (34) can be written in the matrix form as

$$A_2 T_{cl}^{n+1} = T_{cl}^n + B_2, \quad (35)$$

where  $T_{cl}^n = [T_N^n, T_{N+1}^n, T_{N+2}^n, \dots, T_{R-1}^n, T_R^n]^T$ .

$A_2$  is the corresponding tridiagonal matrix of order  $(R - N) \times (R - N)$ , and  $T_{cl}^{n+1}$  and  $B_2$  are of column vectors of order  $(R - N) \times 1$ .

$A_2$  is diagonally dominant matrix since the absolute value of each leading diagonal element of  $A_2$  satisfies the relation:

$$|a_{jj}| \geq \sum_{i=N, i \neq j}^R |a_{ij}|. \quad (36)$$

Since each matrices  $A_1$  and  $A_2$  are diagonally dominant, so  $A_1^{-1}$  and  $A_2^{-1}$  exist and systems (24) and (35) are separately solvable.

The constructed FD scheme (20) and (31) in our model has truncation error  $\tau(\Delta r, \Delta t) = O(\Delta r^2 + \Delta t)$ . So, as  $\Delta r, \Delta t \rightarrow 0$  as  $\tau(\Delta r, \Delta t) \rightarrow 0$  separately for body part ( $0 \leq i \leq N$ ) and clothes part ( $N \leq i \leq R$ ), hence, the model is consistent.

The notion of the second matrix norms of invertible matrices  $\|A_1\|_2, \|A_2\|_2$  and their inverses  $\|A_1^{-1}\|_2, \|A_2^{-1}\|_2$  together with Gregorian Theorem [12] imply the relation

$$\|E^{n+1}\|_2 \leq \|E^0\|_2 \quad (37)$$

where  $E^{n+1} = \begin{cases} T_t^{(n+1)} - T_t^{*(n+1)}, \\ T_{cl}^{(n+1)} - T_{cl}^{*(n+1)}. \end{cases}$

$T_t^{*(n+1)}$  and  $T_{cl}^{*(n+1)}$  are small perturb in  $T_t^{(n+1)}$  and  $T_{cl}^{(n+1)}$ , respectively.

Then, by Lax-Richtmyer theorem, one can claim that the model is unconditionally stable with respect to initial data. Finally, the stability and consistent imply the convergence. Hence, the model is unconditionally convergent.

## 5. Numerical Results and Discussion

The numerical solution of heat transfer in human body with a protective clothing layer is obtained from the bioheat equation (10) by decoupling it into equations (11) and (12) and applying finite difference scheme separately. The temperature obtained from the system of (11) (skin surface) is used in equation (12) to calculate the temperature for clothes layer on the body. For the numerical experiment, the following parametric values in Tables 2 and 3 are chosen.

5.1. *Graphical Representation.* The effects of different parameters mentioned in Tables 1–3 have been investigated for heat transfer in a cylindrical-shaped clothed human body. Different mesh sizes are taken to demonstrate the validity and applicability of the developed numerical FD schemes (20) and (32). The tests of the combined solution of systems

TABLE 2: Thermophysical parameters related to the body part 15, 21.

Parameters	Symbols	Values	Units
Thermal conductivities	$k_t$	0.48	$W/m^{\circ}C$
Blood specific heat	$c_b$	3850	$J/kg^{\circ}C$
Blood density	$\rho_b$	1000	$kg/m^3$
Blood perfusion rate	$W_b$	3	$kg/s \cdot m^3$
Metabolism	$q_m$	1085	$W/m^3$
Arterial temperature	$T_a$	37	$^{\circ}C$
Thickness of tissue (domain)	$N$	0.03	$m$
Temperature at right boundary	$T_b$	24	$^{\circ}C$

TABLE 3: Physical parameters related to clothing properties.

Parameters	Symbols	Values	Units	References for the values
Thermal conductivities	$k_{cl}$	0.305	$W/m^{\circ}C$	Abbas et al. [22]
Thickness of clothes	$L_{cl}$	0.0050	$m$	Gurung and Saxena [13]
Total thickness (tissue and cloth)	$R$	0.035	$m$	—
Density of clothes	$\rho_{cl}$	1550	$kg/m^3$	Holmer et al. [4]
Specific heat of clothes	$c_{cl}$	1340	$J/kg^{\circ}C$	Holmer et al. [4]
Clothing insulation	$I_{cl}$	0.17	$m^2 \cdot (^{\circ}C/W)$	Holmer et al. [4], Havenith et al. [9]
Air insulation	$I_a$	0.0992	$m^2 \cdot (^{\circ}C/W)$	Havenith et al. [9]
Area of nude body	$A_b$	1.7	$(m^2)$	<a href="http://www.medicinenet.com">http://www.medicinenet.com</a>

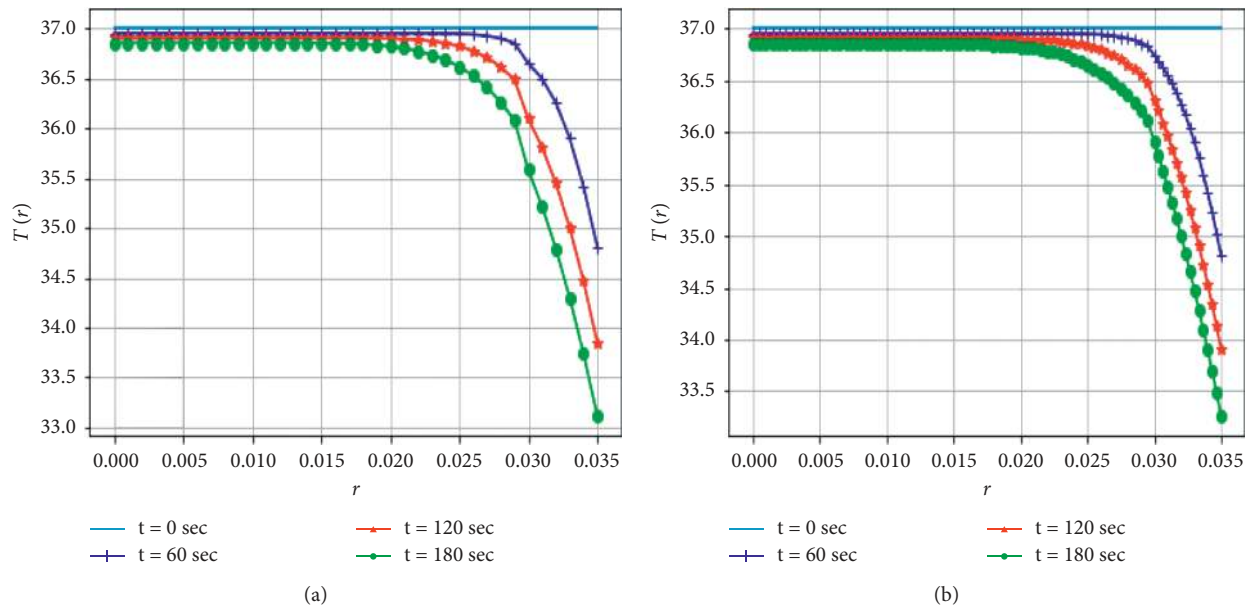


FIGURE 3: Continued.

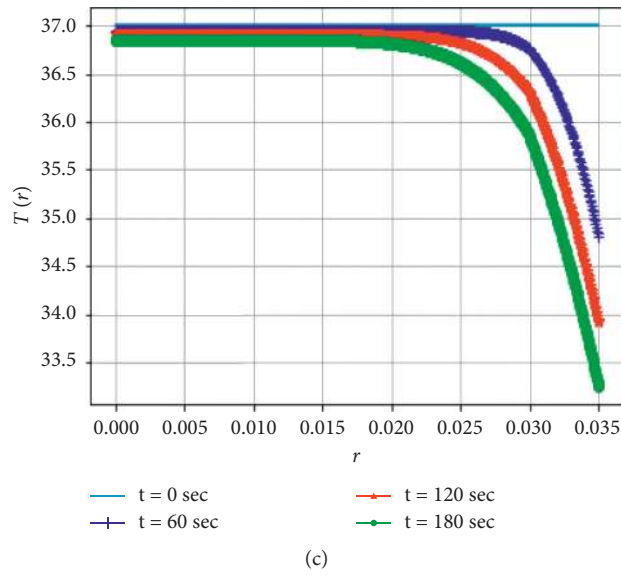


FIGURE 3: Temperature profile with various mesh sizes at time step  $\Delta t = 0.01$  sec. (a) Mesh size  $\Delta r = 0.001$  m,  $\Delta t = 0.01$  s. (b) Mesh size  $\Delta r = 0.0005$  m,  $\Delta t = 0.01$  s. (c) Mesh size  $\Delta r = 0.00005$  m,  $\Delta t = 0.01$  s.

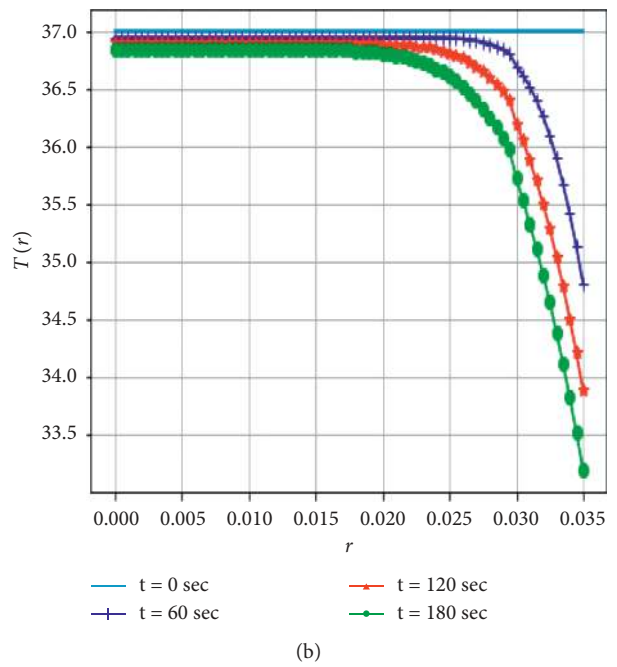
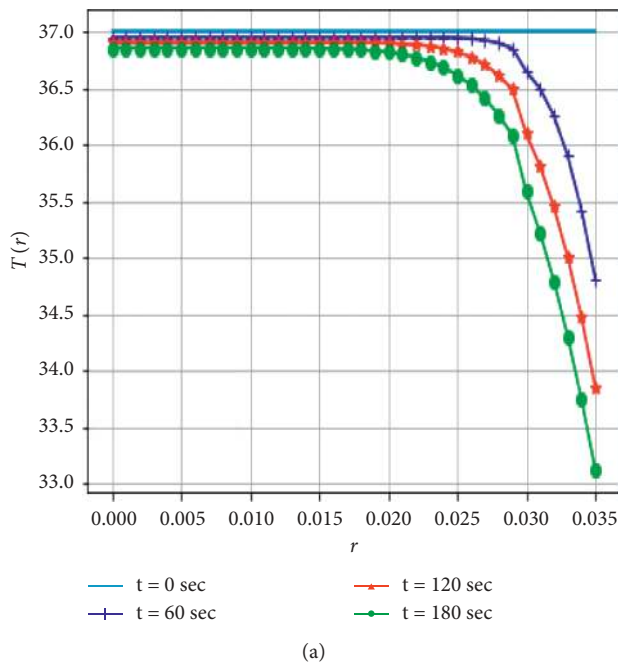


FIGURE 4: Continued.



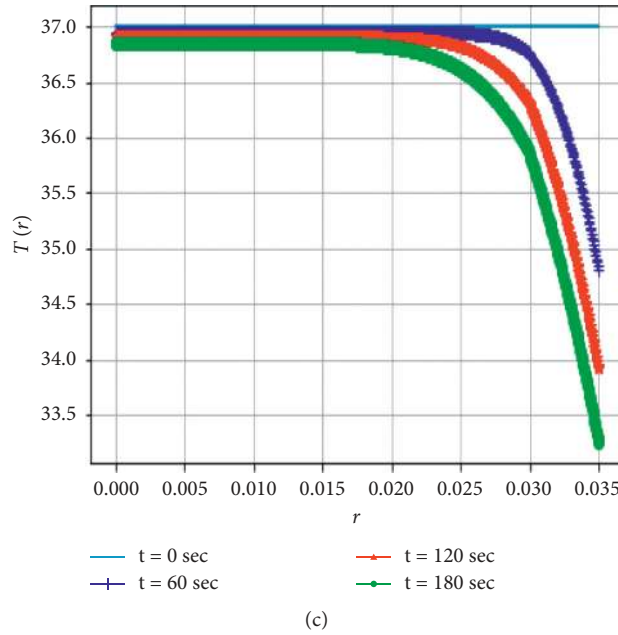


FIGURE 4: Temperature profile with various mesh sizes at time step  $\Delta t = 0.4$  sec. (a) Mesh size  $\Delta r = 0.001$  m,  $\Delta t = 0.4$  s. (b) Mesh size  $\Delta r = 0.0005$  m,  $\Delta t = 0.4$  s. (c) Mesh size  $\Delta r = 0.00005$  m,  $\Delta t = 0.4$  s.

TABLE 4: Temperature profile at the interface (skin surface) when  $\Delta t = 0.01$  s.

$r$ (m)	Temperature in 60 (s)	Temperature in 120 (s)	Temperature in 180 (s)
0.001	36.76374598	36.30866076	35.85715129
0.0001	36.76037207	36.30920285	35.86597663
0.00005	36.7602856	36.309688591	35.86747444

TABLE 5: Temperature profile at the interface (skin surface) when  $\Delta t = 0.4$  sec.

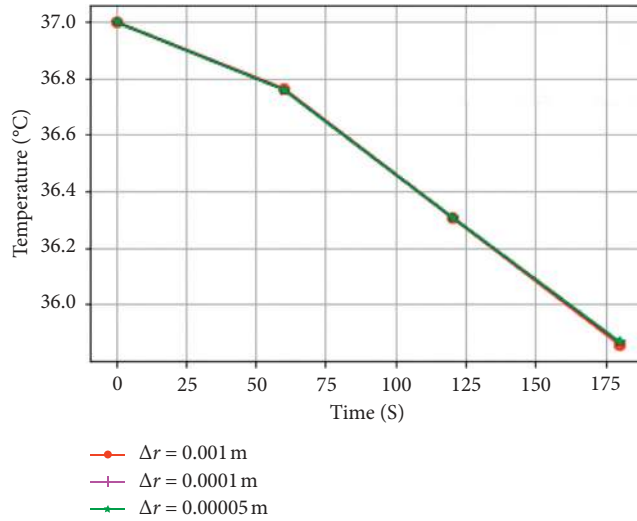
$r$ (m)	Temperature in 60 (s)	Temperature in 120 (s)	Temperature in 180 (s)
0.001	36.75829838	36.30752624	35.85570826
0.0001	36.76037207	36.30920285	35.86597663
0.00005	36.7582809	36.30809916	35.866122888

(24) and (35) have been performed and tabulated in (a–c) of Figure 3 in time step  $\Delta t = 0.01$  sec and (a–c) of Figure 4 in time step  $\Delta t = 0.4$  sec with different mesh sizes 0.001 m, 0.0005 m, and 0.00005 m respectively.

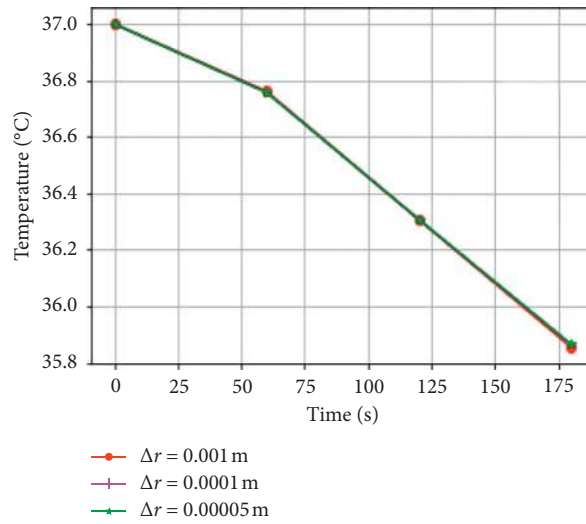
In Figures 3(a) and 4(a), the curves are slightly deviated when  $r_N = N\Delta r_N = 0.030$  m at the skin surface due to the interface condition between two materials having nonhomogeneous behavior. The curves in Figures 3(b) and 4(b) are less deviated than Figures 3(a) and 4(a), while the graphs in Figure 3(c) and Figure 4(c) are smoother than in previous four Figures. On the one hand, the comparison in graphs concerns that the increment in numbers of grid points makes the graphs smoother, more accurate, and reliable; on the other hand, the graphs are independent of mesh sizes. All the graphs in Figures 3 and 4 indicate that the temperature remains steady up to certain distance (0.02 m) from the body core, then decreases towards the skin surface and further

then towards clothes. The temperature profile obtained in 60 seconds, in 120 seconds, and in 180 seconds are, respectively, the same no matter the mesh sizes are how small and large. The interface temperature (skin surface temperature) obtained from results having different mesh sizes at time steps 0.01 second and 0.4 second are, respectively, shown in Tables 4 and 5 and graphically shown in Figures 5(a) and 5(b). Similarly, the temperature in Tables 6 and 7 and Figures 6(a) and 6(b) represent these results exactly same way as in the previous case at the skin surface. As the graphs presented in Figure 7 and obtained temperature profile in Table 6 for body part and in Table 7 for clothes coincide, respectively, it can be concluded that the numerical solution of the model is stable and convergence with respect to the grid.

The temperatures in Table 8 and graphs in Figure 7 at time steps  $\Delta t = 0.05$  s,  $\Delta t = 0.1$  s,  $\Delta t = 0.4$  s, and  $\Delta t = 0.5$  s with  $\Delta r = 0.0005$  m, respectively, coincide. So, all graphs are



(a)



(b)

FIGURE 5: (a) Temperature profile at the interface (skin surface) at  $\Delta t = 0.01$  s. (b) Temperature profile at the interface (skin surface) at  $\Delta t = 0.4$  s.

TABLE 6: Temperature profile when  $\Delta t = 0.01$  s.

$r(m)$	Temperature in 60 (s)	Temperature in 120 (s)	Temperature in 180 (s)
0.001	34.8052701	33.88093058	33.19336681
0.0001	34.80522479	33.9068697	33.25005773
0.00005	34.80551148	33.91018652	33.25706422

TABLE 7: Temperature profile when  $\Delta t = 0.4$  sec.

$r$ (m)	Temperature in 60 (s)	Temperature in 120 (s)	Temperature in 180 (s)
0.001	34.79948362	33.87632483	33.18921782
0.0001	34.80522479	33.9068697	33.25005773
0.00005	34.79994767	33.90584859	33.25312605

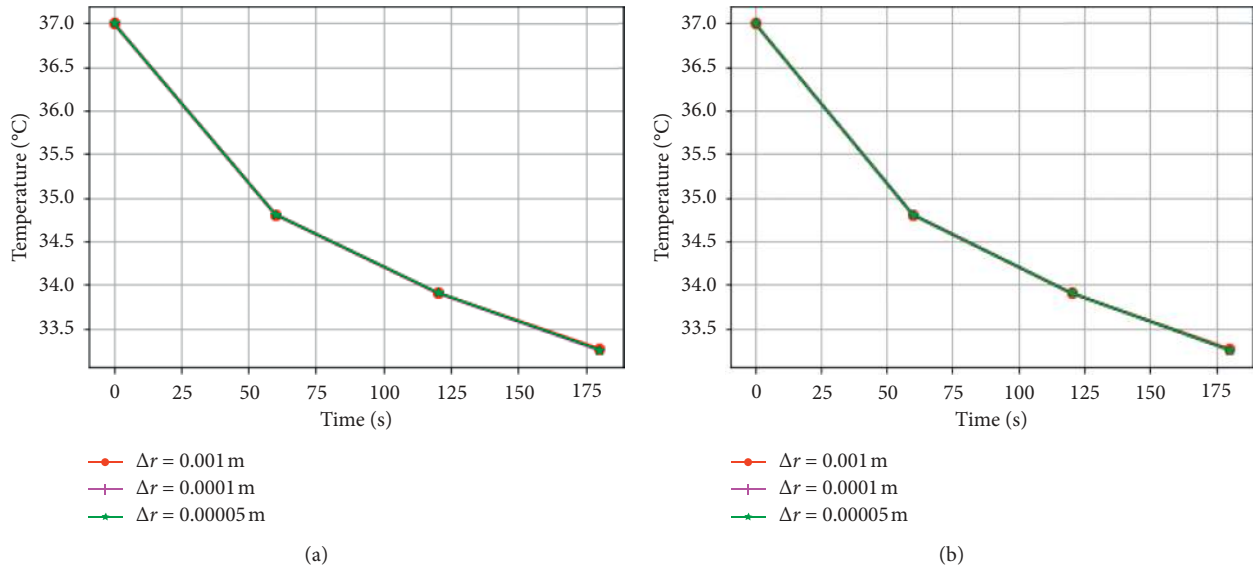


FIGURE 6: (a) Temperature profile at the interface (skin surface) at  $\Delta t = 0.01$  s. (b) Temperature profile at the interface (skin surface) at  $\Delta t = 0.4$  s.

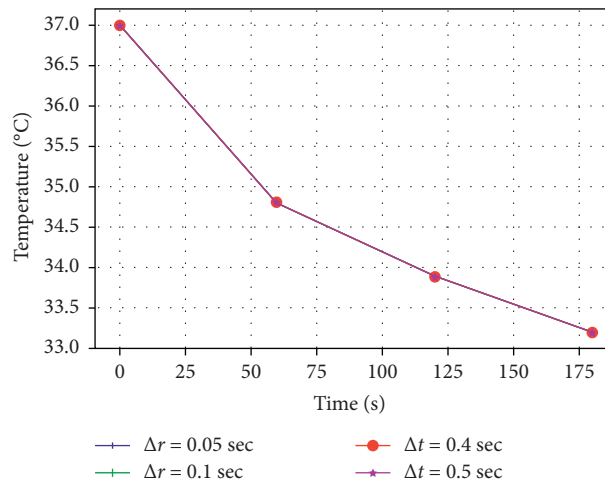


FIGURE 7: Temperature profile when  $\Delta r = 0.0005$  m at different time steps.

TABLE 8: Temperature profile when  $\Delta r = 0.0005$  m at different time steps.

$t$ (s)	Temperature in 60 (s)	Temperature in 120 (s)	Temperature in 180 (s)
0.05	34.80467674	33.8804583	33.19294127
0.1	34.803935	33.87986792	33.19240934
0.4	34.79948362	33.87632483	33.18921782
0.5	34.79799948	33.8751435	33.18815399

independent of the time step sizes as well. These results help to verify the stability and convergence of the FD scheme for the model.

### 6. Conclusion

A one-dimensional time dependent bioheat transfer model with a protective clothing system has been established and

solved using the fully implicit, unconditionally stable finite difference method. Because of the heterogeneous domain having two distinct physiological and physical behaviors of body and clothes, the differential equation models for two distinct parts (body and clothes) are solved separately using implicit scheme and then combined by using interface condition. The model is the extension of Pennes' bioheat equation with nonlinear Robin's boundary condition. The

developed model with the clothing phenomena is stable, consistent, and convergent on the basis of grid points as well as time step sizes. The numerical verification of the convergence and stability of the model has also been represented graphically. The result shows that suitable management of clothing, in cold and hot climatic condition, keeps the body in a highly satisfactory and comfortable level. The numerical computational results in this paper seem to agree with the similar values of clothing parameters which are experimentally verified in [1, 9]. The proposed model may be useful for the clothing and environmental designers as well as biomedical researchers.

Future work should include extension of the model to higher dimensions and use of fractional derivatives as studied in [23–25].

### Data Availability

The data used for supporting the findings of this study are included within the article.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

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