Journal of Classical Analysis
Volume 1, Number 2 (2012), 135–146
doi:10.7153/jca-01-14

BIORTHOGONAL p–WAVELET PACKETS RELATED TO THE WALSH POLYNOMIALS

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Abstract. This paper deals with the construction of biorthogonal p-wavelet packets on $\mathbb{R}^+$ related to the Walsh polynomials and their properties are investigated by means of Walsh-Fourier transform. Three biorthogonal formulas regarding these p-wavelet packets are derived. Moreover, it is shown how to obtain several new Riesz bases of the space $L^2(\mathbb{R}^+)$ by constructing a series of subspaces of these p-wavelet packets.

Mathematics subject classification (2010): 42C40, 42C15, 42C10.
Keywords and phrases: p-Multiresolution analysis, p-wavelet packets, Riesz basis, Walsh polynomials, Walsh-Fourier transform.

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