IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received November 2, 2019, accepted December 6, 2019, date of publication December 13, 2019, date of current version December 31, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2959719

Bipartite Consensus for Second Order Multi-Agent Systems With Exogenous Disturbance via Pinning Control

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This work was supported in part by the National Natural Science Foundation of China under Grant 61971181, Grant 61602163, Grant 61973110, and Grant 61471163, in part by the the Science Fund for Distinguished Young Scholars of Hubei Province under Grant 2017CFA034, and in part by the Youth Technology Innovation Team of Hubei Province under Grant T201710.

ABSTRACT This paper focuses on the leader-following bipartite consensus problem for second order multi-agent systems (MASs) subject to the disturbance generated from exosystem. Different from some related works on this topic, the upper bound of disturbance is not required and the disturbance observer is proposed to estimate the exogenous disturbance. To guarantee the bipartite consensus of nonlinear MASs, both pinning control and disturbance observer strategy are employed. With the help of linear matrix inequality and Lyapunov stability theory, it is demonstrated that leader-following bipartite consensus for nonlinear MASs can be realized if a fraction of the agents are controlled under some sufficient conditions. The effectiveness of the developed approach is verified via simulations.

INDEX TERMS Bipartite consensus, MASs, exogenous disturbance, pinning control.

I. INTRODUCTION

In the past years, intensive attentions have been paid to the field of control systems owing to its broad applications in real-world systems, especially for networked systems [1]–[5] and MASs [6]–[10]. Compared with networked systems, the MASs not only provide theoretical basis for explaining complex nature phenomena, but also provide excellent model for analyzing interconnecting behaviors among agents. There are diverse coordinated control problems of MASs, such as synchronization [11], containment [12], flocking [13] and consensus [14]–[16]. Consensus aimed at designing appropriate protocol to reach a state agreement by interacting with their neighbors has attracted growing interests.

Particularly, leader-following consensus is a special case of consensus, where there exists a leader. Xu *et al.* [17] addressed the leader-following consensus based on event-triggered for MASs under different topologies. Li *et al.* [18] investigated the consensus problems for MASs with linear and nonlinear dynamics. Additionally, the case with a leader-following communication graph is also studied. Although one can design algorithms to drive the agents to follow the leader, it has difficulty in adding controllers to all agents for a large complex network. To amend the drawbacks of commonly leader-following method, the pinning control is developed to apply local feedback injects to part agents, which not only reduce the number of controllers but also achieve expected tracking consensus. Zhou *et al.* [19] solved pinning-based consensus for second order MASs in finite time. Wang *et al.* [20] discussed the cluster consensus for robotic systems whose dynamics are modeled by a Lagrangian equation.

Most of the literature under the premise that agents negotiate friendly. Nevertheless, the agents may also negotiate in a hostile way, and the edge corresponding to the interactions represented as signed graphs can be also negative weights. In the context of signed networks [21]–[24], a bipartite consensus problem was introduced, which distinguished consensus from enabling the agents to reach a value with identical quantities but opposite sign. Valcher et al. [25] considered consensus and bipartite consensus for high-order MASs. Hu *et al.* [26] dealt with consensus problem for

The associate editor coordinating the review of this manuscript and approving it for publication was Mou Chen.

MASs with antagonistic interactions and noises. However, external disturbance widely exists in real processes and is a main source of instability and poor performance. Therefore, the disturbance rejection is a particularly vital issue in the controller design of MASs. References [27] and [28] investigated leader-follower bipartite consensus of linear MASs subjected to bounded disturbance over signed graph. Wang *et al.* [29] proposed a disturbance observe-based control (DOBC) for high order MASs with Lipschitz dynamic and input delay. Zhang et al. [30] presented further results on consensus for second-order MASs in the presence of exogenous disturbance. As far as we know, there are very few results on the bipartite consensus problem of nonlinear MASs when the external disturbances are generated from an exogenous system.

In view of the foregoing concerns, based on our prior work on bipartite consensus problem [31], [32], this paper deals with leader-following bipartite consensus problem of second-order nonlinear MASs subject to exogenous disturbance. The contributions of this paper are summarized into threefold: i) Compared with the previous results in [25]–[28], the disturbance is taken into account and the upper bound of external disturbance is not required in control law design. Moreover, a disturbance observer-based control approach is applied to estimate the external disturbance generated by a linear exogenous system, which seems effective and practicable. ii) A pinning-based control scheme is constructed by selecting agents and the lower bound of the pinning gains is obtained. The obtained results provide a novel solution to the leader-following bipartite consensus of nonlinear multi-agent systems, which is more meaningful in theory and application. iii) Both cooperative and antagonistic interactions between agents are considered in the nonlinear MASs, achieving bipartite consensus on signed digraph is challenging as the Laplacian matrix is neither M-matrix nor a symmetric matrix to exploit their mathematical properties.

The rest of this paper is organized as follows. In Section II, necessary preliminaries are introduced primarily. In Section III, the main results about second order bipartite tracking consensus for MASs with and without disturbances are shown. Section IV provides the illustrative example to demonstrate the effectiveness of the proposed methodology. Finally, the conclusion is in Section V.

Notations: Throughout this study, let 1_N be the N-dimensional column vector with all ones. A $\otimes$ B represents the Kronecker product of matrices A and B, $sgn(\cdot)$ is the standard sign function.

II. PRELIMINARIES

A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ with $a_{ij} \neq 0$ if $(v_j, v_i) \in \mathcal{E}$. Self-loops are not allowed, i.e., $a_{ii} = 0, \forall i = 1, \ldots, N$. The neighbor set of node i is denoted by $N_i = \{j : a_{ij} \neq 0\}$. We say $\mathcal{G}$ has a path means that there exists a sequence of nodes $i_1, \ldots, i_m$ such that $(i_l, i_{l+1}) \in \mathcal{E}$, $\forall l = 1, \ldots, m-1$. Moreover, $\mathcal{G}$ is structurally balanced if it provides two subsets $\mathcal{V}_1$ and $\mathcal{V}_2$, where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geq 0$ *if* $\forall v_i, v_j \in \mathcal{V}_q\ (q \in \{1, 2\})$, $a_{ij} \leq 0$ *if* $\forall v_i \in \mathcal{V}_q, v_j \in \mathcal{V}_r$, $q \neq r\ (q, r \in \{1, 2\})$. The Laplacian matrix L is given in the form of

$$l_{ij} = \begin{cases} \sum_{k=1}^{N} |a_{ik}|, & j = i. \\ -a_{ij}, & j \neq i. \end{cases}$$

Denote by

$$\mathcal{D} = \{D = diag\{d_1, d_2, \ldots, d_N\}, d_i \in \{-1, +1\}\},$$

then we have the next lemma.

Lemma 1 ([33]): For structurally balanced $\mathcal{G}$, $\exists D \in \mathcal{D}$, such that $D\mathcal{A}D$ has all elements nonnegative. Besides, D exists two partitions, i.e. $\mathcal{V}_1 = \{i|d_i > 0\}$ and $\mathcal{V}_2 = \{i|d_i < 0\}$.

Let's begin with the second order kinematics of MASs

Followers:

$$\begin{aligned} \dot{r}_i &= v_i, \\ \dot{v}_i &= f(r_i, v_i) + u_i + G_0 w_i. \end{aligned}$$

Leader:

$$\begin{aligned} \dot{r}_0 &= v_0, \\ \dot{v}_0 &= f(r_0, v_0), \end{aligned} \tag{1}$$

where $r_i \in R^m$, $v_i \in R^m$ and $u_i \in R^m$ are known as position, velocity and input of followers, respectively. Correspondingly, $r_0 \in R^m$ and $v_0 \in R^m$ are position and velocity of virtual leader. w_i is the distrbance, $G_0 \in R^{m\times m}$ is coefficient matrix. Moreover, $f : R^m \times R^m \to R^m$ is a continuous and differentiable vector-valued function representing the intrinsic nonlinear dynamics.

Assumption 1: In network (1), the nonlinear function $f(r_i, v_i)$ is an odd function on r_i and v_i, that is, $f(-r_i, -v_i) = -f(r_i, v_i)$.

Assumption 2 ([34]): Suppose that there exist non-negative constants h_1, h_2, h_3, and h_4 such that for any $r_1, r_2, v_1, v_2 \in R^m$, there hold

$$\begin{aligned} &(r_1 - r_2)^T [f(r_1, v_1) - f(r_2, v_2)] \\ &\quad \leq h_1(r_1 - r_2)^T (r_1 - r_2) + h_2(v_1 - v_2)^T (v_1 - v_2), \end{aligned}$$

$$\begin{aligned} &(v_1 - v_2)^T [f(r_1, v_1) - f(r_2, v_2)] \\ &\quad \leq h_3(r_1 - r_2)^T (r_1 - r_2) + h_4(v_1 - v_2)^T (v_1 - v_2). \end{aligned}$$

Remark 1: It seems that Assumption 2 is strict. Actually, Assumption 2 is a semi-Lipschitz condition, which is satisfied by Lorenz system [35], Chua oscillator [36] and so on.

Assumption 3: Assume the graph describing communication topology is connected and structurally balanced.

Definition 1 ([37]): The leader-following bipartite consensus is said to be guaranteed if the following equalities are satisfied

$$\begin{cases} \lim_{t\to\infty} \|r_i - r_0\| = 0, & \forall i \in \mathcal{V}_1 \\ \lim_{t\to\infty} \|r_i + r_0\| = 0, & \forall i \in \mathcal{V}_2 \end{cases} \quad \begin{cases} \lim_{t\to\infty} \|v_i - v_0\| = 0, & \forall i \in \mathcal{V}_1 \\ \lim_{t\to\infty} \|v_i + v_0\| = 0, & \forall i \in \mathcal{V}_2 \end{cases}$$

Lemma 2 ([38]): For a symmetric matrix $M \in R^{N\times N}$ and diagonal matrix $S = \{s_1, \ldots, s_l, 0, \ldots, 0\}$ with $s_i > 0, i = 1, \ldots, l\ (1 \le l \le N)$, let

$$M - S = \begin{bmatrix} G - \tilde{S} & Q \\ Q^T & M_l \end{bmatrix},$$

where M_l is the minor matrix of M by removing its first p row-column pairs, G and Q are matrices with appropriate dimensions, $\tilde{S} = diag\{s_1, s_2, \ldots, s_l\}$. If $s_i > \lambda_{\max}\left(G - QM_l^{-1}Q\right), i = 1, 2, \ldots, l$, then $M - S < 0$ is equivalent to $M_l < 0$.

Lemma 3 ([38]): Given linear matrix inequality

$$\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} < 0,$$

where $J_{11} = J_{11}^T, J_{12} = J_{21}^T, J_{22} = J_{22}^T$, is equivalent to either of following conditions:

$$1) J_{11} < 0, J_{22} - J_{21}J_{11}^{-1}J_{12} < 0.$$
$$2) J_{22} < 0, J_{11} - J_{12}J_{22}^{-1}J_{21} < 0.$$

Lemma 4: For matrices A, B, C and D with appropriate dimensions, one has

$$1)\ (A + B) \otimes C = A \otimes C + B \otimes C,$$
$$2)\ (A \otimes B)(C \otimes D) = (AC) \otimes (BD).$$

III. MAIN RESULTS

In this section, we are at the position to tackle the leader-following bipartite consensus problem for second order MASs by pinning control under disturbances case and no disturbances case, respectively.

A. LEADER-FOLLOWING BIPARTITE CONSENSUS WITH DISTURBANCES

Assume that the disturbance w_i is generated by the linear exogenous system

$$\begin{aligned} \dot{\xi}_i &= A\xi_i, \\ w_i &= C\xi_i, \end{aligned} \tag{2}$$

where $\xi_i \in R^{m_2}$ is the internal state of the exogenous system, $A \in R^{m_2\times m_2}$ and $C \in R^{m\times m_2}$ are the matrices of the disturbance system, and (A, C) is observable.

The pinning protocol for system (1) is proposed as

$$\begin{aligned} u_i = -\alpha \sum_{j=1}^{N} |a_{ij}| \left[\left(r_i - sgn\left(a_{ij}\right) r_j\right) + \left(v_i - sgn\left(a_{ij}\right) v_j\right)\right] \\ - \alpha s_i \left[(r_i - d_i r_0) + (v_i - d_i v_0)\right] - G_0 w_i^*, \end{aligned} \tag{3}$$

where $\alpha > 0$, and $s_i > 0, i = 1, 2, \ldots, l$; $s_i = 0, i = l + 1, \ldots, N$.

Let $y_i = (r_i, v_i)^T$, the MASs (1) can be rewritten as

$$\dot{y}_i = L_i y + T_i u_i + G_i w_i,$$

where $T_i = [0, I_m]^T \in R^{2m\times m}$, $G_i = [0, G_0]^T \in R^{2m\times m}$ and L_i is a matrix with its rows are selected from $(i-1)m + 1$ to im and from $mn + m(i-1) + 1$ to $mn + mi$ of the following matrix

$$\begin{pmatrix} 0 & I_n \otimes I_m \\ -\alpha (L + S) \otimes I_m & -\alpha (L + S) \otimes I_m \end{pmatrix}.$$

Inspired by [39], in order to estimate the unknown disturbance w_i, a basic disturbance observer is suggested as

$$\begin{aligned} \dot{z}_i &= (A + KG_iC)(z_i - Ky_i) + K(L_i y + T_i u_i), \\ \hat{\xi}_i &= z_i - Ky_i, \\ w_i^* &= C\hat{\xi}_i, \end{aligned} \tag{4}$$

where $\hat{\xi}_i$ and w_i^* are the estimates of ξ_i and w_i, respectively. $K \in R^{m_2\times 2m}$ is the observer gain matrix to be designed.

Denote

$$e_i = \xi_i - \hat{\xi}_i.$$

Based on (2), (3) and (4), one has

$$\dot{e}_i = (A + KG_iC)\, e_i. \tag{5}$$

According to the above analysis, we obtain the following lemma for the exponentially stability of the error system (5)

Lemma 5 ([39]): Consider system (1) with the disturbance produced by exogenous system (2). The estimate error system (5) is globally exponentially stable, if and only if there exists gain matrix K satisfying

$$A + KG_iC < 0.$$

Theorem 1: Presume that Assumptions 1-3 hold. The leader-following bipartite consensus for system (1) is achieved by utilizing protocol (3), if there exists matrix $P > 0$, such that

$$\Sigma = \begin{bmatrix} W & \frac{\Pi}{2} \\ \frac{\Pi^T}{2} & I_n \otimes H \end{bmatrix} < 0, \tag{6}$$

where

$$W = \begin{bmatrix} (\rho I_N - \alpha (L + S)) \otimes I_m & 0 \\ 0 & (\rho I_N - \alpha (L + S)) \otimes I_m \end{bmatrix}$$

$$\begin{aligned} \rho &= \max\{h_1 + h_3, h_2 + h_4 + 1\}, \\ \Pi &= [0, \Psi]^T \otimes G_0 C, \\ \Psi &= \begin{pmatrix} \alpha\left(L + L^T\right) + 2\alpha S & I_N \\ I_N & I_N \end{pmatrix}, \\ H &= \bar{A}^T P + P\bar{A}, \\ \bar{A} &= A + KG_iC. \end{aligned}$$

Proof: Since $sgn\left(a_{ij}\right)d_i = d_j$, one can obtain

$$\begin{aligned} u_i = & -\alpha \sum_{j=1}^{N} \left|a_{ij}\right| \left[\left(r_i - d_i r_0\right) - sgn\left(a_{ij}\right)\left(r_j - d_j r_0\right) \right. \\ & + \left. \left(\left(v_i - d_i r_0\right) - sgn\left(a_{ij}\right)\left(v_j - d_j r_0\right) \right) \right] \\ & - \alpha s_i \left[\left(r_i - d_i r_0\right) + \left(v_i - d_i v_0\right) \right] - G_0 w_i^*. \end{aligned} \quad (7)$$

Let $\tilde{r}_i = r_i - d_i r_0$, $\tilde{v}_i = v_i - d_i v_0$. With (1) and (7), one yields

$$\begin{aligned} \dot{\tilde{r}}_i &= \tilde{v}_i, \\ \dot{\tilde{v}}_i &= -\alpha \sum_{j=1}^{N} \left|a_{ij}\right| \left[\left(\tilde{r}_i - sgn\left(a_{ij}\right)\tilde{r}_j\right) + \left(\tilde{v}_i - sgn\left(a_{ij}\right)\tilde{v}_j\right) \right] \\ & \quad - \alpha s_i \left(\tilde{r}_i + \tilde{v}_i\right) + f\left(r_i, v_i\right) - d_i f\left(r_0, v_0\right) + G_0 C e_i. \end{aligned} \quad (8)$$

Equation (8) can be expressible in matrix form as

$$\begin{aligned} \dot{\tilde{r}} &= \tilde{v}, \\ \dot{\tilde{v}} &= -\left(\alpha\left(L+S\right) \otimes I_m\right)\left(\tilde{r} + \tilde{v}\right) + F\left(r, v\right) \\ & \quad - D 1_N \otimes f\left(r_0, v_0\right) + \left(I_N \otimes G_0 C\right) e, \end{aligned}$$

where $\tilde{r}$ and $\tilde{v}$ are, respectively, the column stack vectors of $\tilde{r}_i$ and $\tilde{v}_i$.

Let $\tilde{y} = \left(\tilde{r}^T, \tilde{v}^T\right)^T$, then

$$\dot{\tilde{y}} = F\left(r, v, r_0, v_0\right) + B\tilde{y} + \Gamma e, \quad (9)$$

where

$$\begin{aligned} F\left(r, v, r_0, v_0\right) &= \begin{pmatrix} 0 \\ \left[F\left(r, v\right) - D 1_N \otimes f\left(r_0, v_0\right)\right] \end{pmatrix}, \\ B &= \begin{pmatrix} 0_N & I_N \\ -\alpha\left(L+S\right) \otimes I_m & -\alpha\left(L+S\right) \otimes I_m \end{pmatrix}, \\ \Gamma &= \left[0, I_n\right]^T \otimes G_0 C. \end{aligned}$$

Construct Lyapunov functional candidate as

$$V = \frac{1}{2}\tilde{y}^T\left(\Psi \otimes I_m\right)\tilde{y} + \sum_{i=1}^{N} e_i^T P e_i, \quad (10)$$

where

$$\Psi = \begin{pmatrix} \alpha\left(L + L^T\right) + 2\alpha S & I_N \\ I_N & I_N \end{pmatrix}.$$

By Lemma 3 and condition (6), $\Sigma < 0$ is equivalent to $W < 0$, which results in

$$\rho I - \alpha\left(L + S\right) < 0.$$

Further, one can yield

$$\alpha\left(L + L^T\right) + 2\alpha S > 0,$$

which indicates that $\Psi > 0$ and $V \geq 0$.

Differentiating $V\left(t\right)$ as

$$\dot{V} = \dot{V}_1 + \dot{V}_2,$$

where

$$\dot{V}_1 = \tilde{y}^T\left(\Psi \otimes I_m\right)\left(F\left(r, v, r_0, v_0\right) + B\tilde{y}\right), \quad (11)$$

$$\dot{V}_2 = \tilde{y}^T\left(\left[0, \Psi\right]^T \otimes G_0 C\right)e + \sum_{i=1}^{N} e_i^T\left(\bar{A}^T P + P\bar{A}\right)e_i. \quad (12)$$

With Assumption 1, expanding equation (12) as

$$\begin{aligned} \dot{V}_1 &= \tilde{r}\left[\alpha\left(L + L^T + 2S\right) \otimes I_m\right]\tilde{v} \\ & \quad - \tilde{r}^T\left[\left(\alpha\left(L+S\right) \otimes I_m\right)\left(\tilde{r} + \tilde{v}\right)\right] \\ & \quad + \tilde{r}^T\left[F\left(r, v\right) - D 1_N \otimes f\left(r_0, v_0\right)\right] + \tilde{v}^T\tilde{v} \\ & \quad + \tilde{v}^T\left[F\left(r, v\right) - D 1_N \otimes f\left(r_0, v_0\right)\right] \\ & \quad - \tilde{v}^T\left[\left(\alpha\left(L+S\right) \otimes I_m\right)\left(\tilde{r} + \tilde{v}\right)\right] \\ &= -\tilde{r}^T\left[\alpha\left(L+S\right) \otimes I_m\right]\tilde{r} \\ & \quad + \tilde{v}^T\left[\left(I_N - \alpha\left(L+S\right)\right) \otimes I_m\right]\tilde{v}^T \\ & \quad + \sum_{i=1}^{N} \tilde{r}_i^T\left(f\left(r_i, v_i\right) - f\left(\bar{r}_0, \bar{v}_0\right)\right) \\ & \quad + \sum_{i=1}^{N} \tilde{v}_i^T\left(f\left(r_i, v_i\right) - f\left(\bar{r}_0, \bar{v}_0\right)\right), \end{aligned}$$

where $\bar{r}_0 = d_i r_0$, $\bar{v}_0 = d_i v_0$.

From Assumption 2, one can derive

$$\begin{aligned} \dot{V}_1 &\leq \tilde{r}^T\left[\left(\left(h_1 + h_3\right)I_N - \alpha\left(L+S\right)\right) \otimes I_m\right]\tilde{r} \\ & \quad + \tilde{v}^T\left[\left(\left(h_2 + h_4 + 1\right)I_N - \alpha\left(L+S\right)\right) \otimes I_m\right]\tilde{v}^T \\ & \leq \tilde{r}^T\left[\left(\rho I_N - \alpha\left(L+S\right)\right) \otimes I_m\right]\tilde{r} \\ & \quad + \tilde{v}^T\left[\left(\rho I_N - \alpha\left(L+S\right)\right) \otimes I_m\right]\tilde{v}^T \\ &= \tilde{y}^T W \tilde{y}. \end{aligned} \quad (13)$$

Combined with (11), (12) and (13), one finds that

$$\begin{aligned} \dot{V} &\leq \tilde{y}^T W \tilde{y} + \tilde{y}^T \Pi e + \sum_{i=1}^{N} e_i^T\left(\bar{A}^T P + P\bar{A}\right)e_i \\ &= \left(\tilde{y}^T, e^T\right)\Sigma\begin{pmatrix} \tilde{y} \\ e \end{pmatrix}. \end{aligned}$$

It is easy to conclude $\dot{V} \leq 0$ under condition (6), and $\dot{V} = 0$ if and only if $\tilde{y} = 0$ and $e = 0$, which follow that $\tilde{r} = 0$ and $\tilde{v} = 0$. From LaSalle's invariance principle, one has $\|r_i - d_i r_0\| = 0$ and $\|v_i - d_i v_0\| = 0$, $i = 1, \ldots N$, as $t \to \infty$. This implies that the leader-following bipartite consensus with disturbances is obtained. This completes the proof.

Remark 2: For pinning control, it is common to select the pinning nodes and design the controller gains for multi-agent systems under unsigned digraph. Compared with [19], [20], provided a pinning scheme for unsigned digraph, we develop bipartite consensus protocol for multi-agent systems under signed digraph based on pinning control. And under transformation, the pinning nodes and the controller gains can be easily designed according to the scheme presented in [38].

Remark 3: Unlike the multi-agent system in [30]–[32], [40], [41], the system (1) depends on the nonlinear term $f\left(r_i, v_i\right)$. Therefore, the bipartite consensus problem of the nonlinear multi-agent systems with disturbance is nontrivial because it is hard to solve the nonlinear term. Motivated by [29], it is possible to use the dynamic gain technique to overcome the nonlinearities. Hence, by integrating dynamic gain technique and disturbance observer-based control (DOBC) method, we can deal with bipartite consensus problem for nonlinear multi-agent systems subject to external disturbance.

B. LEADER-FOLLOWING BIPARTITE CONSENSUS WITHOUT DISTURBANCES

In this subsection, let $w_i = 0$, which implies that there are no disturbances. The system (1) reduces to

Followers:

$$\begin{aligned} \dot{r}_i &= v_i, \\ \dot{v}_i &= f(r_i, v_i) + u_i. \end{aligned}$$

Leader:

$$\begin{aligned} \dot{r}_0 &= v_0, \\ \dot{v}_0 &= f(r_0, v_0). \end{aligned} \tag{14}$$

Correspondingly, the pinning protocol of system (14) is formulated as

$$\begin{aligned} u_i = -\alpha \sum_{j=1}^{N} |a_{ij}| \left[\left(r_i - sgn\left(a_{ij}\right) r_j\right) + \left(v_i - sgn\left(a_{ij}\right) v_j\right) \right] \\ - \alpha s_i \left[(r_i - d_i r_0) + (v_i - d_i v_0) \right]. \end{aligned} \tag{15}$$

Let $\rho I_N - \alpha \frac{L+L^T}{2} = M$, and

$$\rho I_N - \alpha \frac{L+L^T}{2} - \alpha S = \begin{bmatrix} G - \alpha \tilde{S} & Q \\ Q^T & M_l \end{bmatrix}. \tag{16}$$

Theorem 2: Presume that Assumptions 1-2 hold. The leader-following bipartite consensus for system (14) is achieved by utilizing protocol (15), if

$$(i)\ \lambda_{\max}\left(\left(-\frac{L+L^T}{2}\right)_l\right) < -\frac{\rho}{\alpha}, \tag{17}$$

$$(ii)\ s_i > \lambda_{\max}\left(G - Q M_l^{-1} Q^T\right), i = 1, \ldots, l. \tag{18}$$

Proof: Based on the proof of Theorem 1, one has

$$\begin{aligned} \dot{\tilde{r}}_i &= \tilde{v}_i, \\ \dot{\tilde{v}}_i &= -\alpha \sum_{j=1}^{N} |a_{ij}| \left[\left(\tilde{r}_i - sgn\left(a_{ij}\right) \tilde{r}_j\right) + \left(\tilde{v}_i - sgn\left(a_{ij}\right) \tilde{v}_j\right) \right] \\ &\quad - \alpha s_i \left(\tilde{r}_i + \tilde{v}_i\right) + f(r_i, v_i) - d_i f(r_0, v_0). \end{aligned} \tag{19}$$

Equation (19) can be written in compact form as

$$\begin{aligned} \dot{\tilde{r}} &= \tilde{v}, \\ \dot{\tilde{v}} &= -\left(\alpha (L + S) \otimes I_n\right)(\tilde{r} + \tilde{v}) + F(r, v) \\ &\quad - D 1_N \otimes f(r_0, v_0). \end{aligned}$$

Let $\tilde{y} = \left(\tilde{r}^T, \tilde{v}^T\right)^T$, construct Lyapunov functional candidate as

$$V = \frac{1}{2} \tilde{y}^T (\Psi \otimes I_m) \tilde{y}, \tag{20}$$

where

$$\Psi = \begin{pmatrix} \alpha\left(L + L^T\right) + 2\alpha S & I_N \\ I_N & I_N \end{pmatrix}.$$

It follows from condition (17) that

$$\alpha \lambda_{\max}\left(\left(-\frac{L+L^T}{2}\right)_l\right) + \rho < 0.$$

Due to $\rho \geq 1$, one obtains

$$\lambda_{\max}\left(\left(\rho I_N - \alpha \frac{L+L^T}{2}\right)_l\right) \leq \alpha \lambda_{\max}\left(\left(-\frac{L+L^T}{2}\right)_l\right) + \rho < 0.$$

Namely, one can derive

$$M_l = \left(\rho I_N - \alpha \frac{L+L^T}{2}\right)_l < 0. \tag{21}$$

According to Lemma 2 and (21), there hold

$$\rho I_N - \alpha \frac{L+L^T}{2} - \alpha S < 0.$$

Namely,

$$\alpha\left(L + L^T\right) + 2\alpha S - I_N > 0.$$

By Lemma 3, one concludes that $\Psi > 0$, which indicates that $V \geq 0$.

Rewrite (20) as

$$V = \frac{\alpha}{2} \tilde{r}\left(\left(L + L^T + 2S\right) \otimes I_m\right) \tilde{r} + \tilde{r}^T \tilde{v} + \frac{1}{2} \tilde{v}^T \tilde{v}.$$

Differentiating V gives

$$\begin{aligned} \dot{V}(t) &= \tilde{r}\left[\alpha\left(L + L^T + 2S\right) \otimes I_m\right] \tilde{v} + \tilde{r}^T \dot{\tilde{v}} + \tilde{v}^T \tilde{v} + \tilde{v}^T \dot{\tilde{v}} \\ &= -\tilde{r}^T \left[\alpha (L + S) \otimes I_m\right] \tilde{r} + \tilde{v}^T \left[\left(I_N - \alpha (L + S)\right) \right. \\ &\quad \left. \otimes I_n \right] \tilde{v}^T + \sum_{i=1}^{N} \tilde{r}_i^T \left(f(r_i, v_i) - f(\bar{r}_0, \bar{v}_0)\right) \\ &\quad + \sum_{i=1}^{N} \tilde{v}_i^T \left(f(r_i, v_i) - f(\bar{r}_0, \bar{v}_0)\right). \end{aligned}$$

From Assumption 2, one has

$$\begin{aligned} \dot{V}(t) &\leq \tilde{r}^T \left[\left((h_1 + h_3) I_N - \alpha (L + S)\right) \otimes I_n\right] \tilde{r} \\ &\quad + \tilde{v}^T \left[\left((h_2 + h_4 + 1) I_N - \alpha (L + S)\right) \otimes I_n\right] \tilde{v}^T \\ &\leq \tilde{r}^T \left[\left(\rho I_N - \alpha (L + S)\right) \otimes I_n\right] \tilde{r} \\ &\quad + \tilde{v}^T \left[\left(\rho I_N - \alpha (L + S)\right) \otimes I_n\right] \tilde{v}^T. \end{aligned} \tag{22}$$

By condition (17) and (18), we have shown that $\rho I_N - \alpha \frac{L+L^T}{2} - \alpha S < 0$. According to (22), we conclude $\dot{V} \leq 0$ and $\dot{V} = 0$ if and only if $\tilde{r} = 0$ and $\tilde{v} = 0$. In view of LaSalle's invariance principle, there hold $\|r_i - d_i r_0\| = 0$ and $\|v_i - d_i v_0\| = 0, i = 1, \ldots N$ as $t \to \infty$ Hence, the leader-following bipartite consensus for system (14) under protocol (15) is indeed realized. This completes the proof.

Remark 4: Pinning criterion is obtained in Theorem 2. It is worth mentioning that information on control gains is

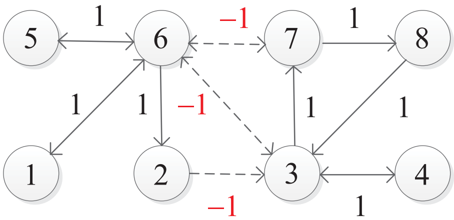

FIGURE 1. Diagraph with eight agents.

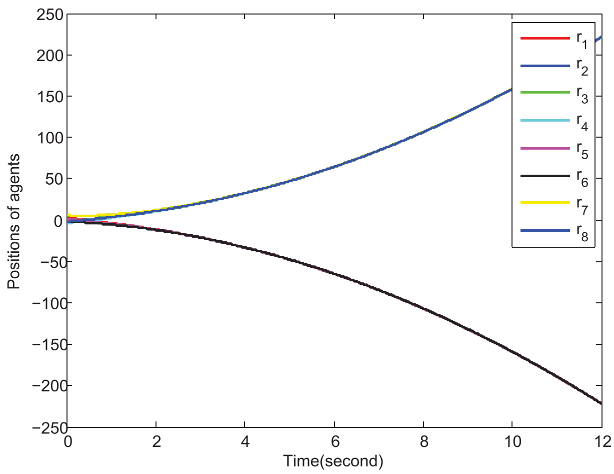

FIGURE 2. Positions of agents with disturbance.

separate and the low bound of parameter α can be obtained easily, from which we can design the pinning feedback gains conveniently.

IV. NUMERICAL SIMULATIONS

This section provides some simulations to investigate the leader-following bipartite consensus of second order MASs with exogenous disturbances under pinning control. Suppose that the signed network consisting of eight agents shown in Fig.1, the solid and dashed lines, respectively, represent the collaborative and antagonistic relationships between agents. One can easily verify that the topology corresponding to the diagraph is structurally balanced by letting $\mathcal{V}_1 = \{1, 2, 5, 6\}$, $\mathcal{V}_2 = \{3, 4, 7, 8\}$. Moreover, L can be calculated as

$$L=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 4 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

and $D = diag\,(1, 1, -1, -1, 1, 1, -1, -1)$.

Selecting the agents 3, 7 and 8 to be pinned. The nonlinear function f is

$$f\,(r_i, v_i) = \begin{pmatrix} k_1\,(v_{i2} - v_{i1} - \theta\,(v_{i1})) \\ v_{i1} - v_{i2} + v_{i3} \\ -k_2 v_{i2} - k_3 v_{i3} \end{pmatrix},$$

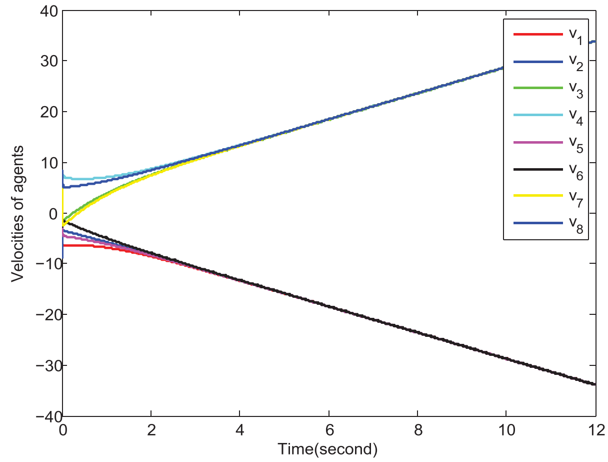

FIGURE 3. Velocities of agents with disturbance.

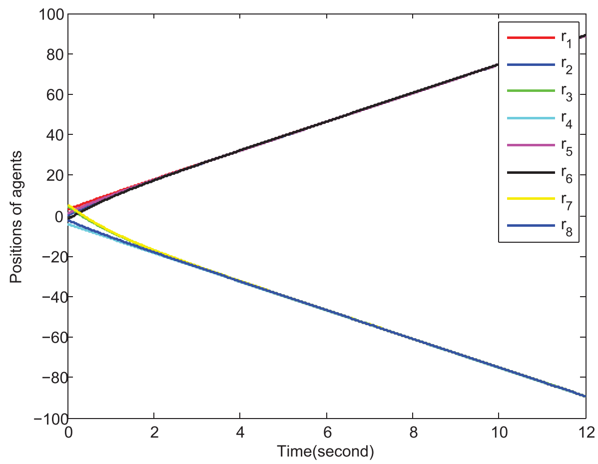

FIGURE 4. Positions of agents with disturbance by pinning control.

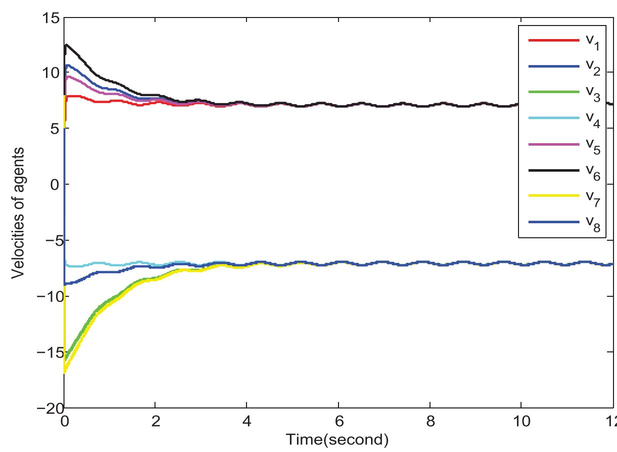

FIGURE 5. Velocities of agents with disturbance by pinning control.

where $k_1 = 10$, $k_2 = 19.53$, $k_3 = 0.1636$, $\theta\,(v_{i1}) = -0.7831 v_{i1} - 0.0.3247\,(|v_{i1} + 1| - |v_{i1} - 1|)$. By means of Assumption 1, one has $h_1 + h_3 = 11.2845$, $h_2 + h_4 = 32.1$, ρ=33.1. Let α=130, condition (17) is satisfied with

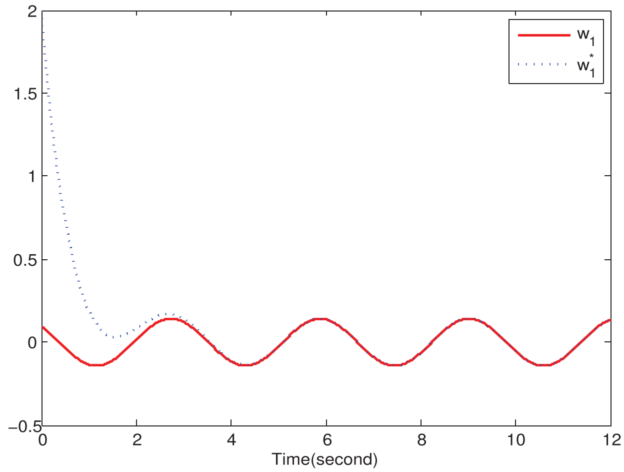

FIGURE 6. The disturbance w_1 and its estimation w_1^*.

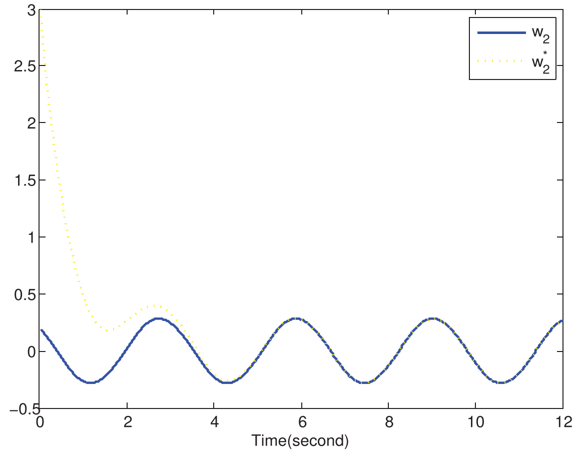

FIGURE 7. The disturbance w_2 and its estimation w_2^*.

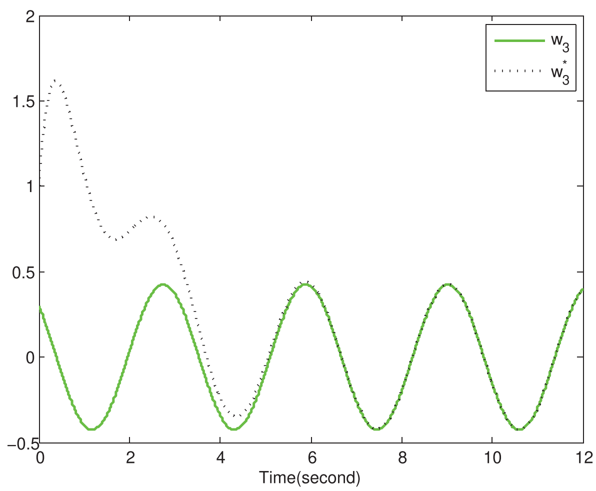

FIGURE 8. The disturbance w_3 and its estimation w_3^*.

$\lambda_{\max}\left(\left(-\frac{L+L^T}{2}\right)_l\right) = -0.2576 < -0.2546$. Combined with matrix decomposition (16) and condition (18), we derive the lower bound of pinning feedback gain as $s_i > 5.06$.

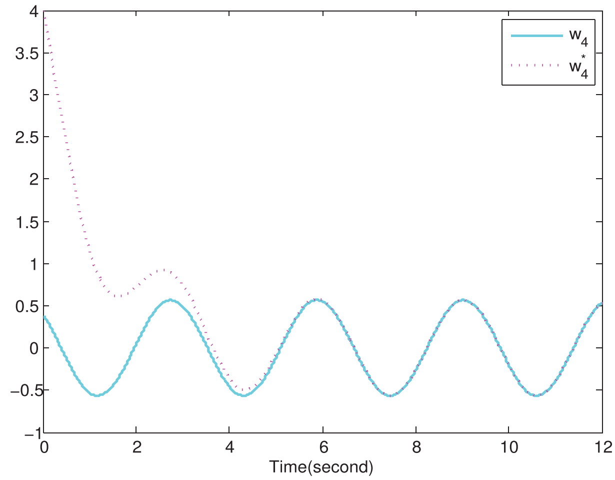

FIGURE 9. The disturbance w_4 and its estimation w_4^*.

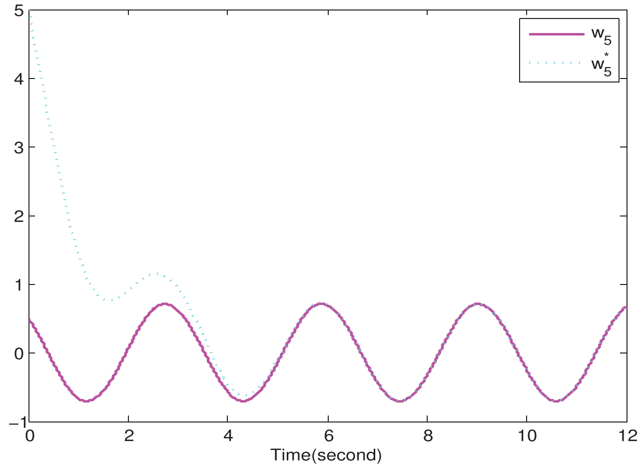

FIGURE 10. The disturbance w_5 and its estimation w_5^*.

In view of Theorem 2, the leader-following bipartite consensus is achieved if we choose $s_i = 5.06$, $i = 1, 2, 3$. Here, set $s_i=6$, $i = 1, 2, 3$, $s_i = 0$, $i = 4, 5, 6, 7, 8$. Moreover, assume the matrix $G_0 = 1$, the initial states and velocities of agents are given randomly in $[1, 8]$ and $[-1, 6]$, respectively. Consider the parameters of the exogenous disturbance system are

$$A=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

and $C = [1, 0]$ with initialized value $\xi_i = [0.6\sin 2, 0.6\cos 2]^T$. In Fig. 2, the leader-following bipartite consensus of MASs subject to disturbance is reached. By pinning control, we can obtain the states and velocities of MASs with expected bipartite consensus track in Fig. 3. By solving inequality (6), we derive $P = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$, $K = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$. From Figs. 4-11, the disturbances generated from exosystem (2) and its estimation are plotted, which show that the disturbance observer exhibits excellent tracking performance.

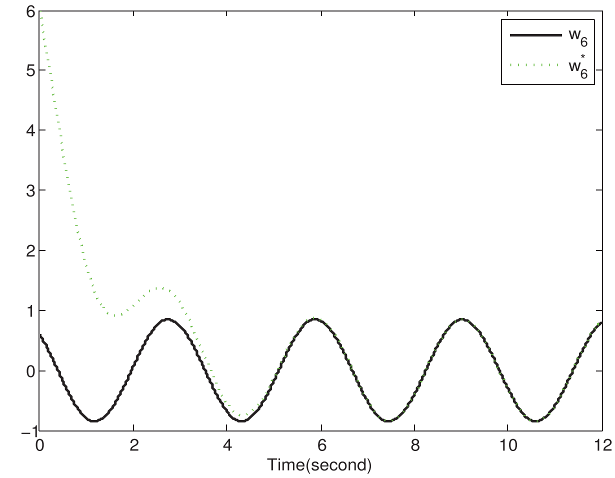

FIGURE 11. The disturbance w_6 and its estimation w_6^*.

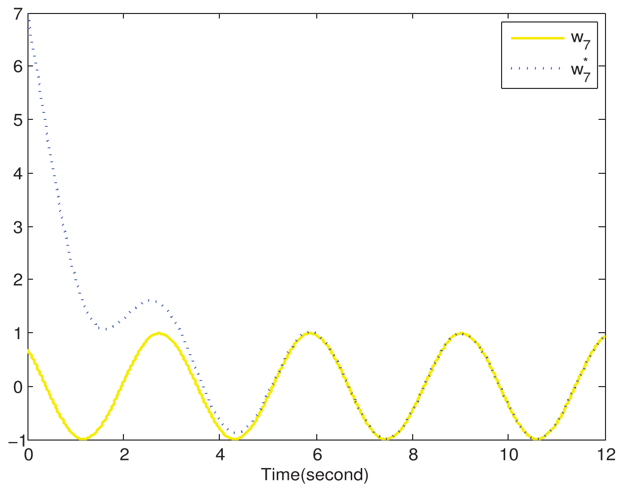

FIGURE 12. The disturbance w_7 and its estimation w_7^*.

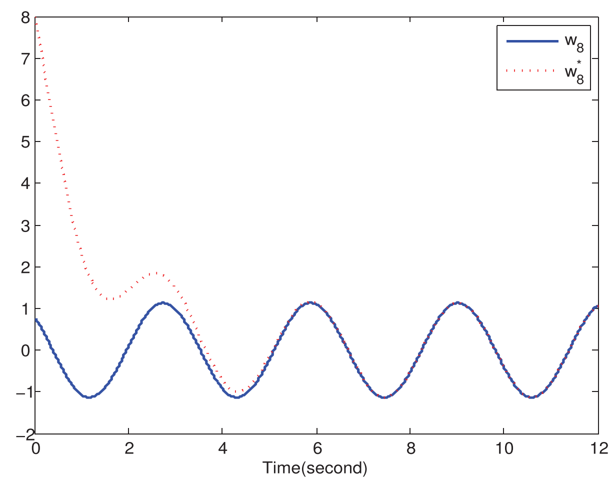

FIGURE 13. The disturbance w_8 and its estimation w_8^*.

V. CONCLUSION

The problem of leader-following bipartite consensus for second order nonlinear MASs in the presence of exogenous disturbances is studied. The method includes pinning control and disturbance observer strategy, where the disturbance observer is designed to estimate the disturbance yield by an exogenous system. Finally, numerical results are presented to verify the effectiveness of proposed approach.

For future work, it would be interesting to see if the current analysis can be generalized to group consensus in more complicated networks, such as switching networks, time-varying networks and so on.

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