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Bipolar neutrosophic soft sets and applications in decision making

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Abstract. Neutrosophic set, proposed by Smarandache considers a truth membership function, an indeterminacy membership function and a falsity membership function. Soft set, proposed by Molodtsov is a mathematical framework which has the ability of independency of parameterizations inadequacy, syndrome of fuzzy set, rough set, probability. Those concepts have been utilized successfully to model uncertainty in several areas of application such as control, reasoning, game theory, pattern recognition, and computer vision. Nonetheless, there are many problems in real-world applications containing indeterminate and inconsistent information that cannot be effectively handled by the neutrosophic set and soft set. In this paper, we propose the notation of bipolar neutrosophic soft sets that combines soft sets and bipolar neutrosophic sets. Some algebraic operations of the bipolar neutrosophic set such as the complement, union, intersection are examined. We then propose an aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft set and develop a decision making algorithm based on bipolar neutrosophic soft sets. Numerical examples are given to show the feasibility and effectiveness of the developed approach.

Keywords: Algebraic operations, bipolar neutrosophic soft sets, decision making, neutrosophic sets, soft sets

1. Introduction

To handle uncertainty, Zadeh [34] proposed fuzzy set which is characterized by a membership degree with range in the unit interval [0, 1]. From several decades, this novel concept is utilized successfully to model uncertainty in several areas of application such as control, reasoning, game theory, pattern recognition, and computer vision. Fuzzy sets, especially, become an important area for the research in medical diagnosis, engineering, social sciences etc. Since in fuzzy set, the degree of association of an element is single value in the unit interval [0, 1], it may not be adequate that the non-association of an element is equal to 1 minus the association degree due to the existence of hesitation degree. Thus Atanassov [4] coined intuitionistic fuzzy set in 1986 to overcome this issue by incorporating the hesitation degree so-called hesitation margin which is define by 1 minus the sum of association degree and non-association degree. Consequently the intuitionistic fuzzy set captured an association degree as well as non-association degree which became the generalization of fuzzy set.

To judge the human decision making ability based on positive and negative effects, Bosc and Pivert [5] said that bipolarity provides the propensity of the human mind to reason and make decisions that depends on positive and negative effects. They argued that both positive information depicts what is possible, satisfactory, permitted, desired, or considered as being acceptable while the negative statements express what is impossible, restricted, rejected, or

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forbidden and negativity of choices correspond to constraints, since they particularize that what kind of values or objects have to be rejected (i.e., those that do not satisfy the constraints or totally opposite), whereas positive preferences correspond to wishes, as they specify which objects are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes. To utilize this idea, Lee [24, 25] defined bipolar fuzzy sets which generalizes the concept fuzzy sets. Kang and Kang [23] applied the bipolar fuzzy set theory to sub-semigroups with operators in semigroups.

Smarandache [32] in 1998, introduced neutrosophic set and neutrosophic logic by considering a truth membership function, an indeterminacy membership function and a falsity membership function. Neutrosophic set has the ability to generalize classical sets, fuzzy sets, intuitionistic fuzzy sets. Smarandache [32] and Wang et al. [33] further developed single valued neutrosophic sets in order to use them in an easy way in scientific and engineering fields. Then, Deli et al. [16] developed bipolar neutrosophic sets and study their application in decicion making. Ali et al. [2] proposed neutrosophic cubic set with application in pattern recognition. Broumi et al. [36, 37] introduced Bipolar Single Valued Neutrosophic Graph theory and its Shortest Path problem. Recently, Ali and Smarandache [1] define complex neutrosophic set to represent the uncertain. Some more literature on neutrosophic set and applications can be found in [7, 8, 17–20, 38–58].

Molodtsov [29] proposed soft set to handle uncertainty in a parameterized way. Soft set is a mathematical framework which has the ability of independency of parameterizations inadequacy, syndrome of fuzzy set, rough set, probability etc.. Soft set applied successfully in several fields to tackle the issues and problems such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Also, Karaaslan and Karatas [22] Aslam et al. [3] studied bipolar soft sets and bipolar fuzzy soft sets, respectively. A huge amount of research work on soft set theory can be seen in [9–12, 14, 21, 26, 30]. Also, some authors studied concept of neutrosophic soft set in [6, 13, 15, 27, 28].

This paper is dedicated to propose bipolar neutrosophic set which is a hybrid structure of soft set and bipolar neutrosophic set. Firstly, we introduce the bipolar neutrosophic soft set and discuss some basic properties with illustrative examples adopting from Kang and Kang [23]. Then, we study some algebraic operations of the bipolar neutrosophic set such as the complement, union, intersection etc. We then propose an aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft set and develop a decision making algorithm based on bipolar neutrosophic soft sets. Numerical examples are given to show the feasibility and effectiveness of the developed approach.

The organization of this paper is as follows. In Section 1, we presented the relevant literature review. Section 2 is dedicated to the fundamental concepts. In Section 3, bipolar neutrosophic set has been presented. We also studied core properties in the same section. Section 4 is about aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft set. In this section the proposed algorithm based on aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft set is presented with a numerical example. Conclusion is given in Section 5.

2. Preliminary

In this section, we give the basic definitions and results of neutrosophic set theory [32], soft set theory [29], neutrosophic soft set theory [13], bipolar fuzzy set [24], bipolar fuzzy soft set [3] and bipolar neutrosophic set [16] that are useful for subsequent discussions.

Definition 1. [32] Let U be a universe. A neutrosophic sets (NS) K in U is characterized by a truth-membership function T_K, an indeterminacy-membership function I_K and a falsity-membership function F_K. $T_K(x)$; $I_K(x)$ and $F_K(x)$ are real standard or non-standard elements of $]0^-, 1^+[$. It can be written as:

$$K = \{< x, (T_K(x), I_K(x), F_K(x)) >: x \in U,$$
$$T_K(x), I_K(x), F_K(x) \in]^-0, 1[^+\}.$$

There is no restriction on the sum of $T_K(x)$, $I_K(x)$ and $F_K(x)$, so $0^- \leq T_K(x) + I_K(x) + F_K(x) \leq 3^+$.

Definition 2. [33] Let E be a universe. A single valued neutrosophic sets (SVNS) A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. It can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E,$$
$$T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

Definition 3. [29] Let U be a universe, E be a set of parameters that describe the elements of U, and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping

$$f_A : E \to P(U) \text{ s.t } f_A(x) = \emptyset \text{ if } x \in E - A \quad (1)$$

where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parameterized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\}$$

Definition 4. [13] Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of parameters that are describing the elements of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping

$$f_N : E \to N(U)$$

where f_N is called an approximate function of the neutrosophic soft set N. For $x \in E$, the set $f_N(x)$ is called x-approximation of the neutrosophic soft set N which may be arbitrary, some of them may be empty and some may have a nonempty intersection. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $N(U)$, and therefore it can be written a set of ordered pairs,

$$N = \{(x, \{< u, T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) >: x \in U\} : x \in E\}$$

where

$$T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) \in [0, 1].$$

Definition 5. [13] Let N_1 and N_2 be two neutrosophic soft sets over neutrosophic soft universes (U, A) and (U, B), respectively.

1. N_1 is said to be neutrosophic soft subset of N_2 if $A \subseteq B$ and $T_{f_{N_1(x)}}(u) \leq T_{f_{N_2(x)}}(u)$, $I_{f_{N_1(x)}}(u) \leq I_{f_{N_2(x)}}(u)$, $F_{f_{N_1(x)}}(u) \geq F_{f_{N_2(x)}}(u)$, $\forall x \in A, u \in U$.
2. N_1 and N_2 are said to be equal if N_1 neutrosophic soft subset of N_2 and N_2 neutrosophic soft subset of N_2.

Definition 6. [13] Let N_1 and N_2 be two neutrosophic soft sets. Then,

1. The complement of a neutrosophic soft set N_1 denoted by N_1^c and is defined by

$$N_1{}^c = \{(x, \{< u, F_{f_{N_1(x)}}(u), 1 - I_{f_{N_1(x)}}(u), T_{f_{N_1(x)}}(u) >: x \in U\} : x \in E\}$$

2. The union of N_1 and N_2 is denoted by $N_3 = N_1 \tilde{\cup} N_2$ and is defined by

$$N_3 = \{(x, \{< u, T_{f_{N_3(x)}}(u), I_{f_{N_3(x)}}(u), F_{f_{N_3(x)}}(u) >: x \in U\} : x \in E\}$$

where

$$T_{f_{N_3(x)}}(u) = max(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)),$$
$$I_{f_{N_3(x)}}(u) = min(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)),$$
$$F_{f_{N_3(x)}}(u) = min(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)).$$

3. The intersection of N_1 and N_2 is denoted by $N_4 = N_1 \tilde{\cap} N_2$ and is defined by

$$N_4 = \{(x, \{< u, T_{f_{N_4(x)}}(u), I_{f_{N_4(x)}}(u), F_{f_{N_4(x)}}(u) >: x \in U\} : x \in E\}$$

where

$$T_{f_{N_4(x)}}(u) = min(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)),$$
$$I_{f_{N_4(x)}}(u) = max(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)),$$
$$F_{f_{N_4(x)}}(u) = max(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)).$$

Definition 7. [24] Let U be a universe. A bipolar fuzzy set Λ in U is defined as;

$$\Lambda = \{(u, T^+(u), T^-(u)) : u \in U\}$$

where $T^+ \to [0, 1]$ and $T^- \to [-1, 0]$. The positive membership degree $T^+(u)$, denotes the truth membership corresponding to a bipolar fuzzy set Λ and the negative membership degree $T^-(u)$ denotes the truth membership of an element $u \in U$ to some implicit counter-property corresponding to a bipolar-fuzzy set Λ.

Definition 8. [3] Let U be a universe and E be a set of parameters that are describing the elements of U. A bipolar fuzzy soft set Θ in U is defined as;

$$\Theta = \{(e, \{(u, T^+(u), T^-(u)) : u \in U\}) : e \in E\}$$

where $T^+ \to [0, 1]$ and $T^- \to [-1, 0]$. The positive membership degree $T^+(u)$, denotes the truth membership corresponding to a bipolar fuzzy soft set Θ and the negative membership degree $T^-(u)$ denotes the truth membership of an element $u \in U$ to some implicit counter-property corresponding to a bipolar fuzzy soft set Θ.

Definition 9. [16] Let U be a universe. A bipolar neutrosophic set $\mathbb{A}$ in U is defined as;

$$\mathbb{A} = \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)) : u \in U\}$$

where $T^+, I^+, F^+ \to [0, 1]$ and $T^-, I^-, F^- \to [-1, 0]$. The positive membership degree $T^+(u)$, $I^+(u)$, $F^+(u)$, denotes the truth membership, indeterminate membership and false membership of an element corresponding to a bipolar neutrosophic set $\mathbb{A}$ and the negative membership degree $T^-(u)$, $I^-(u)$, $F^-(u)$ denotes the truth membership, indeterminate membership and false membership of an element $u \in U$ to some implicit counter-property corresponding to a bipolar neutrosophic set $\mathbb{A}$.

3. Bipolar neutrosophic soft sets

In this section, we propose the concept of neutrosophic soft sets and their operations.

Definition 10. Let U be a universe and E be a set of parameters that are describing the elements of U. A bipolar neutrosophic soft set $\mathbb{B}$ in U is defined as;

$$\mathbb{B} = \{(e, \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)) : u \in U\}) : e \in E\}$$

where $T^+, I^+, F^+ \to [0, 1]$ and $T^-, I^-, F^- \to [-1, 0]$. The positive membership degree $T^+(u)$, $I^+(u)$, $F^+(u)$, denotes the truth membership, indeterminate membership and false membership of an element corresponding to a bipolar neutrosophic soft set $\mathbb{B}$ and the negative membership degree $T^-(u)$, $I^-(u)$, $F^-(u)$ denotes the truth membership, indeterminate membership and false membership of an element $u \in U$ to some implicit counter-property corresponding to a bipolar neutrosophic soft set $\mathbb{B}$.

Example 1. Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2\}$. Then, bipolar neutrosophic soft set $\mathbb{B}_1$ and $\mathbb{B}_2$ over U is given as, respectively;

$$\begin{aligned}\mathbb{B}_1 = \{(e_1, \{&(u_1, 0.5, 0.8, 0.1, -0.5, -0.7, -0.2),\\ &(u_2, 0.6, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &(u_3, 0.6, 0.8, 0.1, -0.5, -0.8, -0.8)\}),\\ &(e_2, \{(u_1, 0.8, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &(u_2, 0.4, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &(u_3, 0.7, 0.8, 0.1, -0.4, -0.7, -0.4)\})\end{aligned}$$

and

$$\begin{aligned}\mathbb{B}_2 = \{(e_1, \{&(u_1, 0.4, 0.8, 0.5, -0.6, -0.7, -0.2),\\ &(u_2, 0.3, 0.6, 0.7, -0.3, -0.7, -0.2),\\ &(u_3, 0.6, 0.2, 0.6, -0.5, -0.5, -0.3)\}),\\ &(e_2, \{(u_1, 0.1, 0.8, 0.7, -0.2, -0.7, -0.2),\\ &(u_2, 0.1, 0.8, 0.7, -0.5, -0.5, -0.5),\\ &(u_3, 0.7, 0.6, 0.1, -0.4, -0.7, -0.3)\})\end{aligned}$$

Definition 11. An empty bipolar neutrosophic soft set $\mathbb{B}^{\emptyset}$ in U is defined as;

$$\mathbb{B}^{\emptyset} = \{(e, \{(u, 0, 0, 1, -1, 0, 0)) : u \in U\}) : e \in E\}$$

Definition 12. An absolute bipolar neutrosophic soft set $\mathbb{B}^{\mathbb{U}}$ in U is defined as;

$$\mathbb{B}^{\mathbb{U}} = \{(e, \{(u, 1, 1, 0, 0, -1, -1)) : u \in U\}) : e \in E\}$$

It is noted that the empty and absolute neutrosophic soft sets form the unit to the proposed system.

Example 2. Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$. Then,

1. Empty bipolar neutrosophic soft set $\mathbb{B}^{\emptyset}$ in U is given as;

$$\begin{aligned}\mathbb{B}^{\emptyset} = \{(e_1, \{&(u_1, 0, 0, 1, -1, 0, 0),\\ &(u_2, 0, 0, 1, -1, 0, 0),\\ &(u_3, 0, 0, 1, -1, 0, 0)\}),\\ &(e_2, \{(u_1, 0, 0, 1, -1, 0, 0),\\ &(u_2, 0, 0, 1, -1, 0, 0),\\ &(u_3, 0, 0, 1, -1, 0, 0)\}),\\ &(e_3, \{(u_1, 0, 0, 1, -1, 0, 0),\\ &(u_2, 0, 0, 1, -1, 0, 0),\\ &(u_3, 0, 0, 1, -1, 0, 0)\})\}\end{aligned}$$

2. Absolute bipolar neutrosophic soft set $\mathbb{B}^{\mathbb{U}}$ in U is given as;

$$\begin{aligned}\mathbb{B}^{\mathbb{U}} = \{(e_1, \{&(u_1, 1, 1, 0, 0, -1, -1),\\ &(u_2, 1, 1, 0, 0, -1, -1),\\ &(u_3, 1, 1, 0, 0, -1, -1)\}),\\ &(e_2, \{(u_1, 1, 1, 0, 0, -1, -1),\\ &(u_2, 1, 1, 0, 0, -1, -1),\end{aligned}$$

$$(u_3, 1, 1, 0, 0, -1, -1)\}),$$
$$(e_3, \{(u_1, 1, 1, 0, 0, -1, -1),$$
$$(u_2, 1, 1, 0, 0, -1, -1),$$
$$(u_3, 1, 1, 0, 0, -1, -1)\})\}$$

Definition 13. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2$ be two bipolar neutrosophic soft sets over U. Then, $\mathbb{B}_1$ is bipolar neutrosophic soft subset of $\mathbb{B}_2$, is denoted by $\mathbb{B}_1 \sqsubseteq \mathbb{B}_2$, if $T_1^+(u) \leq T_2^+(u)$, $I_1^+(u) \geq I_2^+(u)$, $F_1^+(u) \geq F_2^+(u)$, $T_1^-(u) \geq T_2^-(u)$, $I_1^-(u) \leq I_2^-(u)$ and $F_1^-(u) \leq F_2^-(u)$ for all $(e, u) \in E \times U$.

Example 3. Let $U = \{u_1, u_2\}$, $E = \{e_1, e_2\}$. If

$$\begin{aligned}\mathbb{B}_1 = \{&(e_1, \{(u_1, 0.7, 0.8, 0.2, -0.5, -0.9, -0.3),\\ &(u_2, 0.6, 0.8, 0.7, -0.5, -0.7, -0.2)\}),\\ &(e_2, \{(u_1, 0.8, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &(u_2, 0.4, 0.8, 0.7, -0.5, -0.7, -0.2)\})\}\end{aligned}$$

and

$$\begin{aligned}\mathbb{B}_2 = \{&(e_1, \{(u_1, 0.8, 0.1, 0.2, -0.6, -0.8, -0.3),\\ &(u_2, 0.9, 0.2, 0.3, -0.9, -0.7, -0.2)\}),\\ &(e_2, \{(u_1, 0.9, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &(u_2, 0.5, 0.8, 0.7, -0.8, -0.7, -0.1)\})\}\end{aligned}$$

then, we have $\mathbb{B}_1 \sqsubseteq \mathbb{B}_2$.

Definition 14. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2$ be two bipolar neutrosophic soft sets over U. Then, $\mathbb{B}_1$ is bipolar neutrosophic soft equal to $\mathbb{B}_2$, is denoted by $\mathbb{B}_1 = \mathbb{B}_2$, if $T_1^+(u) = T_2^+(u)$, $I_1^+(u) = I_2^+(u)$, $F_1^+(u) = F_2^+(u)$, $T_1^-(u) = T_2^-(u)$, $I_1^-(u) = I_2^-(u)$ and $F_1^-(u) = F_2^-(u)$ for all $(e, u) \in E \times U$.

Definition 15. Let $\mathbb{B}$ be a bipolar neutrosophic soft sets over U. Then, the complement of a bipolar neutrosophic soft set $\mathbb{B}$, is denoted by $\mathbb{B}^c$, is defined as;

$$\begin{aligned}\mathbb{B}^c = \{&(e, \{(u, F^+(u), 1 - I^+(u), T^+(u), F^-(u),\\ &-1 - I^-(u), T^-(u)) : u \in U\}) : e \in E\}\end{aligned}$$

Example 4. Consider the Example 1. Then,

$$\begin{aligned}\mathbb{B}^c = \{&(e_1, \{(u_1, 0.1, 0.2, 0.5, -0.2, -0.3, -0.5),\\ &(u_2, 0.7, 0.2, 0.6, -0.2, -0.3, -0.5),\\ &(u_3, 0.1, 0.2, 0.6, -0.8, -0.2, -0.5)\}),\\ &(e_2, (u_1, 0.7, 0.2, 0.8, -0.2, -0.3, -0.5),\\ &(u_2, 0.7, 0.2, 0.4, -0.2, -0.3, -0.5),\\ &(u_3, 0.1, 0.2, 0.7, -0.4, -0.3, -0.4))\}\end{aligned}$$

Definition 16. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2$ be two bipolar neutrosophic soft sets over U. Then, the union of $\mathbb{B}_1$ and $\mathbb{B}_2$, is denoted by $\mathbb{B}_{\nvDash} \sqcup \mathbb{B}_{\nvDash}$, is defined as;

$$\begin{aligned}&\mathbb{B}_1 \sqcup \mathbb{B}_2\\ &\quad = \{(e, \{(u, max_i\{T_i^+(u)\}, min_i\{I_i^+(u)\},\\ &\qquad min_i\{F_i^+(u)\}, min_i\{T_i^-(u)\}, max_i\{I_i^-(u)\},\\ &\qquad max_i\{F_i^-(u)\}) : u \in U\}) : e \in E,\ and\, i = 1, 2\}\end{aligned}$$

Example 5. Consider the Example 1. Then,

$$\begin{aligned}&\mathbb{B}_1 \sqcup \mathbb{B}_2\\ &\quad = \{(e_1, \{(u_1, 0.5, 0.8, 0.1, -0.6, -0.7, -0.2),\\ &\qquad (u_2, 0.6, 0.6, 0.7, -0.5, -0.7, -0.2),\\ &\qquad (u_3, 0.6, 0.2, 0.1, -0.5, -0.5, -0.3)\}),\\ &\qquad (e_2, (u_1, 0.8, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &\qquad (u_2, 0.4, 0.8, 0.7, -0.5, -0.7, -0.2),\\ &\qquad (u_3, 0.7, 0.6, 0.1, -0.4, -0.7, -0.3))\}\end{aligned}$$

Definition 17. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2, ..., n$ be n bipolar neutrosophic soft sets over U. Then, the union of n bipolar neutrosophic soft set $\mathbb{B}_i$, is denoted by $\sqcup_{i=1}^n \mathbb{B}_i$, is defined as;

$$\begin{aligned}&\sqcup_{i=1}^n \mathbb{B}_i\\ &\quad = \{(e, \{(u, max_i\{T_i^+(u)\}, min_i\{I_i^+(u)\},\\ &\qquad min_i\{F_i^+(u)\}, min_i\{T_i^-(u)\}, max_i\{I_i^-(u)\},\\ &\qquad max_i\{F_i^-(u)\}) : u \in U\}) : e \in E,\\ &\qquad i = 1, 2, ..., n\}\end{aligned}$$

Definition 18. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2$ be two bipolar neutrosophic soft sets over U. Then, the intersection of $\mathbb{B}_1$ and $\mathbb{B}_2$, is denoted by $\mathbb{B}_1 \sqcap \mathbb{B}_2$, is defined as;

$$\begin{aligned}&\mathbb{B}_1 \sqcap \mathbb{B}_2\\ &\quad = \{(e, \{(u, min_i\{T_i^+(u)\}, max_i\{I_i^+(u)\},\\ &\qquad max_i\{F_i^+(u)\}, max_i\{T_i^-(u)\}, min_i\{I_i^-(u)\},\end{aligned}$$

$$min_i\{F_i^-(u)\}) : u \in U\}) : e \in E,\ i = 1, 2\}$$

Example 6. Consider the Example 1. Then,

$$\begin{aligned}
&\mathbb{B}_1 \sqcap \mathbb{B}_2\\
&= \{(e_1, \{(u_1, 0.4, 0.8, 0.5, -0.5, -0.7, -0.2),\\
&(u_2, 0.3, 0.8, 0.7, -0.3, -0.7, -0.2),\\
&(u_3, 0.6, 0.8, 0.6, -0.5, -0.8, -0.8)\}),\\
&(e_2, (u_1, 0.1, 0.8, 0.7, -0.2, -0.7, -0.2),\\
&(u_2, 0.1, 0.8, 0.7, -0.5, -0.7, -0.5),\\
&(u_3, 0.7, 0.8, 0.1, -0.4, -0.7, -0.4))\}
\end{aligned}$$

Definition 19. Let $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ for $i = 1, 2, ..., n$ be n bipolar neutrosophic soft sets over U. Then, the intersection of n bipolar neutrosophic soft set $\mathbb{B}_i$, is denoted by $\sqcap_{i=1}^n \mathbb{B}_i$, is defined as;

$$\begin{aligned}
\sqcap_{i=1}^n \mathbb{B}_i = \{(e, \{(u, &min_i\{T_i^+(u)\}, max_i\{I_i^+(u)\},\\
&max_i\{F_i^+(u)\}, max_i\{T_i^-(u)\},\\
&min_i\{I_i^-(u)\}, min_i\{F_i^-(u)\}) : u \in U\})\\
&: e \in E,\ and\ i = 1, 2, ..., n\}
\end{aligned}$$

Proposition 1. *Let* $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ *for* $i = 1, 2, 3$ *be three bipolar neutrosophic soft sets over U. Then,*

1. $\mathbb{B}_1 \sqcup \mathbb{B}_2 = \mathbb{B}_2 \sqcup \mathbb{B}_1$
2. $\mathbb{B}_1 \sqcap \mathbb{B}_2 = \mathbb{B}_2 \sqcap \mathbb{B}_1$
3. $\mathbb{B}_1 \sqcup (\mathbb{B}_2 \sqcup \mathbb{B}_3) = (\mathbb{B}_1 \sqcup \mathbb{B}_2) \sqcup \mathbb{B}_3$
4. $\mathbb{B}_1 \sqcap (\mathbb{B}_2 \sqcap \mathbb{B}_3) = (\mathbb{B}_1 \sqcap \mathbb{B}_2) \sqcap N_3$

Proof. The proofs can be easily obtained since the max functions and min functions are commutative and associative.

Proposition 2. *Let* $\mathbb{B}_1 = \{(e, \{(u, T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)) : u \in U\}) : e \in E\}$ *be a bipolar neutrosophic soft sets over U. Then,*

1. $\left(\mathbb{B}_1^c\right)^c = \mathbb{B}_1$
2. $(\mathbb{B}^{\mathbb{U}})^c = \mathbb{B}^{\emptyset}$
3. $\mathbb{B}_1 \sqsubseteq \mathbb{B}^{\mathbb{U}}$
4. $\mathbb{B}^{\emptyset} \sqsubseteq \mathbb{B}_1$
5. $\mathbb{B}_1 \sqsubseteq \mathbb{B}_1$

Proposition 3. *Let* $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ *for* $i = 1, 2, 3$ *be three bipolar neutrosophic soft sets over U. Then,*

1. $\mathbb{B}_1 \sqsubseteq \mathbb{B}_2 \wedge \mathbb{B}_2 \sqsubseteq \mathbb{B}_3 \Rightarrow \mathbb{B}_1 \sqsubseteq \mathbb{B}_3$
2. $\mathbb{B}_1 = \mathbb{B}_2 \wedge \mathbb{B}_2 = \mathbb{B}_2 \Leftrightarrow \mathbb{B}_1 = \mathbb{B}_3$
3. $\mathbb{B}_1 \sqsubseteq \mathbb{B}_2 \wedge \mathbb{B}_2 \sqsubseteq \mathbb{B}_1 \Leftrightarrow \mathbb{B}_1 = \mathbb{B}_2$

Proposition 4. *Let* $\mathbb{B}_1 = \{(e, \{(u, T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)) : u \in U\}) : e \in E\}$ *be a bipolar neutrosophic soft sets over U. Then,*

1. $\mathbb{B}_1 \sqcup \mathbb{B}_1 = \mathbb{B}_1$
2. $\mathbb{B}_1 \sqcup \mathbb{B}^{\emptyset} = \mathbb{B}_1$
3. $\mathbb{B}_1 \sqcup \mathbb{B}^{\mathbb{U}} = \mathbb{B}^{\mathbb{U}}$

Proposition 5. *Let* $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ *be a bipolar neutrosophic soft sets over U. Then,*

1. $\mathbb{B}_1 \sqcap \mathbb{B}_1 = \mathbb{B}_1$
2. $\mathbb{B}_1 \sqcap \mathbb{B}^{\emptyset} = \mathbb{B}^{\emptyset}$
3. $\mathbb{B}_1 \sqcap \mathbb{B}^{\mathbb{U}} = \mathbb{B}_1$

Proposition 6. *Let* $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ *for* $i = 1, 2$ *be two bipolar neutrosophic soft sets over U. Then, De Morgan's laws are valid*

1. $(\mathbb{B}_1 \sqcup \mathbb{B}_2)^c = \mathbb{B}_1^c \sqcap \mathbb{B}_2^c$
2. $(\mathbb{B}_1 \sqcap \mathbb{B}_2)^c = \mathbb{B}_1^c \sqcup \mathbb{B}_2^c$

Proof. *i.*

$$\begin{aligned}
&(\mathbb{B}_1 \sqcup \mathbb{B}_1)^c\\
&= \{(e, \{(u, max_i\{T_i^+(u)\}, min_i\{I_i^+(u)\},\\
&\quad min_i\{F_i^+(u)\}, min_i\{T_i^-(u)\},\\
&\quad max_i\{I_i^-(u)\}, max_i\{F_i^-(u)\}) : u \in U\})\\
&\quad : e \in E,\ and\ i = 1, 2\}^c\\
&= \{(e, \{(u, min_i\{F_i^+(u)\}, 1 - min_i\{I_i^+(u)\},\\
&\quad max_i\{T_i^+(u)\}, max_i\{F_i^-(u)\},\\
&\quad -1 - max_i\{I_i^-(u)\}, min_i\{T_i^-(u)\}) : u \in U\})\\
&\quad : e \in E,\ and\ i = 1, 2\}\\
&= \{(e, \{(u, min_i\{F_i^+(u)\}, max_i\{1 - I_i^+(u)\},\\
&\quad max_i\{T_i^+(u)\}, max_i\{F_i^-(u)\},\\
&\quad min_i\{-1 - I_i^-(u)\}, min_i\{T_i^-(u)\}) : u \in U\})\\
&\quad : e \in E,\ and\ i = 1, 2\}\{(e, \{(u, T_1^+(u),\\
&\quad I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u),\\
&\quad F_1^-(u)) : u \in U\}) : e \in E\}^c \sqcap\\
&\quad \{(e, \{(u, T_2^+(u), I_2^+(u), F_2^+(u),\\
&\quad T_2^-(u), I_2^-(u), F_2^-(u)) : u \in U\}) : e \in E\}^c
\end{aligned}$$

$$= \mathbb{B}_1^c \sqcap \mathbb{B}_2^c$$

ii.

$$\begin{aligned}
(\mathbb{B}_1 \sqcap \mathbb{B}_1)^c &= \{(e, \{(u, min_i\{T_i^+(u)\}, max_i\{I_i^+(u)\}, \\
&\quad max_i\{F_i^+(u)\}, max_i\{T_i^-(u)\}, \\
&\quad min_i\{I_i^-(u)\}, min_i\{F_i^-(u)\}) : u \in U\}) \\
&\quad : e \in E,\ and\, i = 1, 2\}^c \\
&= \{(e, \{(u, max_i\{F_i^+(u)\}, \\
&\quad 1 - max_i\{I_i^+(u)\}, min_i\{T_i^+(u)\}, \\
&\quad min_i\{F_i^-(u)\}, -1 - min_i\{I_i^-(u)\}, \\
&\quad max_i\{T_i^-(u)\}) : u \in U\}) \\
&\quad : e \in E,\ and\, i = 1, 2\} \\
&= \{(e, \{(u, max_i\{F_i^+(u)\}, \\
&\quad min_i\{1 - I_i^+(u)\}, min_i\{T_i^+(u)\}, \\
&\quad min_i\{F_i^-(u)\}, max_i\{-1 - I_i^-(u)\}, \\
&\quad max_i\{T_i^-(u)\}) : u \in U\}) \\
&\quad : e \in E,\ and\, i = 1, 2\} \\
&= \{(e, \{(u, T_1^+(u), I_1^+(u), \\
&\quad F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)) \\
&\quad : u \in U\}) : e \in E\}^c \sqcup \\
&\quad \{(e, \{(u, T_2^+(u), I_2^+(u), \\
&\quad F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u)) \\
&\quad : u \in U\}) : e \in E\}^c \\
&= \mathbb{B}_1^c \sqcup \mathbb{B}_2^c
\end{aligned}$$

Proposition 7. *Let* $\mathbb{B}_i = \{(e, \{(u, T_i^+(u), I_i^+(u), F_i^+(u), T_i^-(u), I_i^-(u), F_i^-(u)) : u \in U\}) : e \in E\}$ *for* $i = 1, 2, 3$ *be three bipolar neutrosophic soft sets over U. Then,*

1. $\mathbb{B}_1 \sqcap (\mathbb{B}_2 \sqcup \mathbb{B}_3) = (\mathbb{B}_1 \sqcap \mathbb{B}_2) \sqcup (\mathbb{B}_1 \sqcap \mathbb{B}_3)$
2. $\mathbb{B}_1 \sqcup (\mathbb{B}_2 \sqcap \mathbb{B}_3) = (\mathbb{B}_1 \sqcup \mathbb{B}_2) \sqcap (\mathbb{B}_1 \sqcup \mathbb{B}_3)$

4. Aggregation bipolar neutrosophic soft operator

In this section, we propose an aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft sets. Also, we develope an algorithm based on bipolar neutrosophic soft sets and give numerical examples to show the feasibility and effectiveness of the developed approach.

Definition 20. Let $\mathbb{B} = \{(e, \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)) : u \in U\}) : e \in E\} = \{\{(u, T_e^+(u), I_e^+(u), F_e^+(u), T_e^-(u), I_e^-(u), F_e^-(u)) : u \in U\} : e \in E\}$ be a bipolar neutrosophic soft sets over U. Then, aggregation bipolar neutrosophic soft operator, denoted by $\mathbb{B}_{agg}$, is defined as;

$$\mathbb{B}_{agg} = \{\mu_{\mathbb{B}}(u)/u : u \in U\}$$

$$\mu_{\mathbb{B}}(u) = \frac{1}{2|E \times U|}\sum_{e\in E}(|1 - I_e^+(u)(T_e^+(u) - F_e^+(u)) + I_e^-(u)(T_e^-(u) - F_e^-(u))|)$$

where $|E \times U|$ is the cardinality of $E \times U$.

Now we give a decision algorithm for bipolar neutrosophic soft sets.

Algorithm.

1. Construct the bipolar neutrosophic soft set on U.
2. Compute the aggregation bipolar neutrosophic soft operator.
3. Find an optimum alternative set on U.

Example 7. (It is adopted from [14]) Assume that that a workplace wants to fill a position. There are 5 candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM), that is from the department of human resources.

He want to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the bipolar neutrosophic soft decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates $U = \{u_1, u_2, u_3, u_4, u_5\}$ which may be characterized by a set of parameters $E = \{e_1, e_2, e_3\}$ which is "$e_1 = experience$", "$e_2 = technical\ information$" and "$e_3 = age$". Now, we can apply the method as follows:

1. DM constructs a bipolar neutrosophic soft $\mathbb{B}$ over the alternatives set U as;

$$\begin{aligned}
\mathbb{B} = \{(e_1, \{&(u_1, 0.8, 0.9, 0.4, -0.5, -0.7, -0.6), \\
&(u_2, 0.5, 0.4, 0.8, -0.5, -0.7, -0.5), \\
&(u_3, 0.5, 0.5, 0.8, -0.5, -0.8, -0.9), \\
&(u_4, 0.9, 0.8, 0.3, -0.5, -0.2, -0.7), \\
&(u_5, 0.5, 0.5, 0.4, -0.9, -0.8, -0.8)\}), \\
(e_2, \{&(u_1, 0.8, 0.4, 0.7, -0.4, -0.2, -0.6),
\end{aligned}$$

$$(u_2, 0.5, 0.3, 0.7, -0.9, -0.7, -0.8),$$
$$(u_3, 0.5, 0.9, 0.8, -0.5, -0.7, -0.6)),$$
$$(u_4, 0.5, 0.7, 0.8, -0.9, -0.3, -0.7),$$
$$(u_5, 0.4, 0.1, 0.8, -0.5, -0.8, -0.9)\}),$$
$$(e_3, \{(u_1, 0.7, 0.8, 0.6, -0.5, -0.1, -0.8),$$
$$(u_2, 0.8, 0.9, 0.4, -0.5, -0.4, -0.8),$$
$$(u_3, 0.2, 0.9, 0.5, -0.1, -0.9, -0.4),$$
$$(u_4, 0.5, 0.4, 0.2, -0.5, -0.6, -0.9),$$
$$(u_5, 0.9, 0.8, 0.8, -0.5, -0.7, -0.1)\})\}$$

2. DM finds the aggregation bipolar neutrosophic soft operator $\mathbb{B}_{agg}$ of $\mathbb{B}$ as;

$$\mathbb{B}_{agg} = \{0.0793/u_1, 0.0923/u_2, 0.1010/u_3, 0.0797/u_4, 0.0983/u_5\}$$

3. Finally, DM chooses u_3 for the position from $\mathbb{B}_{agg}$ since it has the maximum degree 0.1010 among the others.

Example 8. (It is adopted from [31]) Let $U = \{o_1, o_2, o_3, o_4, o_5, o_6\}$ be the set of objects having different colors, sizes and surface texture features. The parameter set, $E = \{e_1, e_2, e_3\}$ in which "$e_1 = color\ space$", "$e_2 = size$" and "$e_3 = surface\ texture$". We can apply the algorithm as follows:

1. DM constructs a bipolar neutrosophic soft $\mathbb{B}$ over the alternatives set U as;

$$\mathbb{B} = \{(e_1, \{(o_1, 0.3, 0.4, 0.6, -0.3, -0.5, -0.4),$$
$$(o_2, 0.3, 0.9, 0.3, -0.6, -0.7, -0.4),$$
$$(o_3, 0.4, 0.5, 0.8, -0.5, -0.6, -0.7),$$
$$(o_4, 0.8, 0.2, 0.4, -0.7, -0.3, -0.5),$$
$$(o_5, 0.7, 0.3, 0.6, -0.7, -0.6, -0.6),$$
$$(o_6, 0.9, 0.2, 0.4, -0.7, -0.6, -0.6)\}),$$
$$(e_2, \{(o_1, 0.4, 0.2, 0.8, -0.6, -0.4, -0.8),$$
$$(o_2, 0.8, 0.6, 0.3, -0.7, -0.5, -0.6),$$
$$(o_3, 0.6, 0.4, 0.4, -0.3, -0.7, -0.8),$$
$$(o_4, 0.9, 0.8, 0.2, -0.7, -0.5, -0.6),$$
$$(o_5, 0.2, 0.1, 0.9, -0.3, -0.6, -0.7),$$
$$(o_6, 0.3, 0.2, 0.8, -0.3, -0.5, -0.7)\}),$$
$$(e_3, \{(o_1, 0.3, 0.4, 0.1, -0.7, -0.3, -0.6),$$
$$(o_2, 0.8, 0.9, 0.4, -0.5, -0.4, -0.8),$$
$$(o_3, 0.5, 0.6, 0.3, -0.3, -0.7, -0.6),$$
$$(o_4, 0.7, 0.6, 0.6, -0.3, -0.4, -0.7),$$
$$(o_5, 0.6, 0.8, 0.5, -0.3, -0.5, -0.3),$$
$$(o_6, 0.8, 0.7, 0.7, -0.3, -0.5, -0.3)\})\}$$

2. DM finds the aggregation bipolar neutrosophic soft operator $\mathbb{B}_{agg}$ of $\mathbb{B}$ as;

$$\mathbb{B}_{agg} = \{0.1007/o_1, 0.0803/o_2, 0.0773/o_3, 0.0750/o_4, 0.0927/o_5, 0.930/o_6\}$$

3. Finally, DM chooses o_1 for the position from $\mathbb{B}_{agg}$ since it has the maximum degree 0.1007 among the others.

Example 9. (It is adopted from [27]) We consider the problem to select the most suitable house which Mr. X is going to choose on the basis of his m number of parameters out of n number of houses (we choose $n = 5$ and $m = 5$). Let $U = \{h_1, h_2, h_3, h_4, h_5\}$ be the set of houses having different features $E = \{e_1, e_2, e_3, e_4, e_5\}$ in which in which "$e_1 = beautiful$", "$e_2 = cheap$", "$e_3 = in\ good\ repairing$", "$e_4 = moderate$" and "$e_5 = wooden$". We can apply the algorithm as follow:

1. DM constructs a bipolar neutrosophic soft $\mathbb{B}$ over the alternatives set U as;

$$\mathbb{B} = \{(e_1, \{(h_1, 0.6, 0.3, 0.8, -0.5, -0.7, -0.6),$$
$$(h_2, 0.7, 0.2, 0.6, -0.5, -0.7, -0.5),$$
$$(h_3, 0.8, 0.3, 0.4, -0.5, -0.8, -0.9),$$
$$(h_4, 0.7, 0.5, 0.6, -0.5, -0.2, -0.7),$$
$$(h_5, 0.8, 0.6, 0.7, -0.9, -0.8, -0.8)\}),$$
$$(e_2, \{(h_1, 0.5, 0.2, 0.6, -0.4, -0.2, -0.6),$$
$$(h_2, 0.6, 0.3, 0.7, -0.9, -0.7, -0.8),$$
$$(h_3, 0.8, 0.5, 0.1, -0.6, -0.8, -0.6)),$$
$$(h_4, 0.6, 0.8, 0.7, -0.9, -0.3, -0.7),$$
$$(h_5, 0.5, 0.6, 0.8, -0.5, -0.8, -0.9)\}),$$
$$(e_3, \{(h_1, 0.7, 0.3, 0.4, -0.5, -0.1, -0.8),$$
$$(h_2, 0.7, 0.5, 0.6, -0.5, -0.4, -0.8),$$
$$(h_3, 0.3, 0.5, 0.6, -0.1, -0.9, -0.4),$$
$$(h_4, 0.7, 0.6, 0.8, -0.5, -0.6, -0.9),$$
$$(h_5, 0.8, 0.7, 0.6, -0.5, -0.7, -0.1)\}),$$

$(e_4, \{(h_1, 0.8, 0.5, 0.6, -0.5, -0.1, -0.8),$

$(h_2, 0.6, 0.8, 0.3, -0.5, -0.4, -0.8),$

$(h_3, 0.7, 0.2, 0.1, -0.1, -0.9, -0.4),$

$(h_4, 0.8, 0.3, 0.6, -0.5, -0.6, -0.9),$

$(h_5, 0.7, 0.8, 0.3, -0.5, -0.7, -0.1)\}),$

$(e_5, \{(h_1, 0.6, 0.7, 0.2, -0.5, -0.1, -0.8),$

$(h_2, 0.8, 0.1, 0.8, -0.5, -0.4, -0.8),$

$(h_3, 0.7, 0.2, 0.6, -0.1, -0.9, -0.4),$

$(h_4, 0.8, 0.3, 0.8, -0.5, -0.6, -0.9),$

$(h_5, 0.7, 0.2, 0.6, -0.5, -0.7, -0.1)\})\}$

2. DM finds the aggregation bipolar neutrosophic soft operator $\mathbb{B}_{agg}$ of $\mathbb{B}$ as;

$$\mathbb{B}_{agg} = \{0.1470/h_1, 0.1477/h_2, 0.1137/h_3, 0.1443/h_4, 0.1747/h_5\}$$

3. Finally, DM chooses h_5 for the position from $\mathbb{B}_{agg}$ since it has the maximum degree 0.1747 among the others.

It has been observed in Examples 7–9 that the proposed method requires less steps of computation than the relevant works in [14, 27, 31] whilst provides more information on membership degrees (positive and negative) for decision.

5. Conclusion

In this paper, we introduced the bipolar neutrosophic soft set that combines soft sets and bipolar neutrosophic sets. Some new operations on bipolar neutrosophic soft sets were designed. We developed a decision making method based on bipolar neutrosophic soft sets. Numerical examples taken from the existing works [14, 27, 31] were performed to show the feasibility and electiveness of the developed approach. For further study, we will apply our work to real world problems with realistic data and extend proposed algorithm to other decision making models with vagueness and uncertainty. An extension from Bipolar to Tripolar Neutrosophic Soft Sets and even Multipolar Neutrosophic Soft Sets as inspired in [35] will be our next targets.

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