

Bipolar Preference Modelling and Aggregation in Decision Support.

Meltem Öztürk, Alexis Tsoukiàs

LAMSADE - CNRS, Université Paris Dauphine

{ozturk,tsoukias}@lamsade.dauphine.fr

Abstract

The paper discusses the use of positive and negative reasons when preferences about alternative options have to be considered. Besides explaining the intuitive and formal situations where such a bipolar reasoning is used, the paper shows how it is possible to generalise the concordance/discordance principle in preference aggregation and apply it to the problem of aggregating preferences expressed under intervals.

1 Introduction

Preference modelling, aggregation and exploitation constitute three main steps in elaborating an evaluation model within a decision aiding process [Bouyssou et al., 2006]. In the preference modelling step we are interested in finding a suitable way to translate “preference statements” (of the type “I prefer x to y ”), expressed by a decision maker, into formal statements enabling to establish an evaluation model for decision support purposes. We then may need to aggregate such preference models in the case they represent several criteria or opinions. The result is then used in the exploitation step where we try to establish a final recommendation for a choice or a ranking problem.

Preferences (about a set of objects) can be explicitly stated by the decision maker or implicitly obtained through other information (prices, evaluations, measures etc.). In both cases we may face the situation where the information is expressed in a bipolar form which distinguishes positive information from negative one: positive or negative assessments, positive or negative impacts are typical examples. Moreover, the way by which preferences are manipulated may be based on a bipolar reasoning, from voting procedures of large bodies¹ to well known decision support methods using the example/counter-example principle, the ideal/anti-ideal solution principle or the concordance/discordance principle, etc. There can be different types of bipolarity

¹see the Nice treaty about decision procedures in the new European Union bodies.

([Dubois and Prade, 2006]) and a conjoint treatment of positive and negative information may not fit the decision situation and information at hand. We therefore, need specific procedures in such cases.

In this paper we consider such bipolar information as positive and negative reasons supporting or denying a possible preference statement. We are thus interested in the use of independent positive and negative reasons where negative information is not the complement of the positive one. Such reasons should account for the representation of bipolarity in the preference modelling, aggregation and exploitation steps, while constructing an evaluation model.

The paper is organised as follows. In section 2, we briefly review the use of “bi-polar” scales in value theory and deontic logic, some first attempts to capture the possibility to use positive and negative reasons when reasoning about values and preferences. Then in section 3, we present several real world decision situations where the principle of considering positive and negative reasons independently is current practice. In Section 4, we introduce our notation and set our problem. In Section 5, we show how this can be handled generalising the concepts of concordance and discordance. Section 6, is dedicated to an example of aggregating preferences expressed on intervals where the use of positive and negative reasons is essential. We conclude showing further research directions of this work.

2 Bipolarity in value theory: Historical discussion

In many fields, such as economics, social sciences, psychology, political science and decision aiding, we need to represent values. *Value theory*, introduced in the sixties by ([Rescher, 1969]), is one of the first attempts to develop a general theory proposing an axiology and trying to establish some relations between "evaluation" and "value". Questions in which value theory is interested are such as: what is a value?, is it a *property of an object* (like its size)?, or is it a *relationship* that arises out of circumstances linking the value object with the valuing subject in some special way? The last question shows that there exists a connection between "value" and "preference" since value may be determined as a result of a preference comparison.

The realisation of a value can be smaller or larger in one instance of its application as compared with another. Rescher emphasises that "*Evaluation*" in the strictest sense is *e-value-ation: a comparative assessment or measurement of something with respect to its embodiment of a certain*

value and must be understood as the result of application of a valuation to certain items in a specific case. He differentiates two types of value scales: bipolar and monopolar.

- **Bipolar scale:** A bipolar value scale covers the entire range between negative, neutral and positive. The value and the corresponding disvalue are presented on the same scale as opposite evaluations (example: ugly - indifferent - beautiful, disloyal - lukewarm - loyal).
- **Monopolar scale:** A monopolar value scale covers only half of range covered by bipolar ones: neutral-positive. It does not permit to express a disvalue because of the lack of the negative part, only the absence of something positive (like the wealth) can be expressed in this covered spectrum (example: unimaginative - imaginative, unintelligent - intelligent). In this case neutral point does not have a "neutral sense" since it represents a complete lack of something positive.

Rescher points out that such a difference between scales may be necessary in order to differentiate two notions: *worth* and *value*. The notion of *value* which is broader than the one of *worth* requires the expressivity of disvalue while the one of *worth* does not admit the negative pole since something having negative worth is not meaningful.

Another domain, related to value theory and preference, considering comparative evaluation of items and covering some bipolarity, is "the logic of preference". Different researchers have been interested in this subject; we undertake here the approach of Von Wright ([von Wright, 1963], [von Wright, 1972]) who was the first to introduce (to our knowledge) basic notions of such a logic. Von Wright defined five principles of a logic of preference which are related to the asymmetry and transitivity of preference relation, the connection between preference and the change of the state of affairs, the definition of preference between a disjunction of two objects and another one and the holistic nature of preference. The central and most interesting principle of his approach concerns the connection between *change* and *preference*. He defines the preference between two objects from four situations covering all the possibilities concerning their states: $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$ and $\neg p \wedge \neg q$. Object p is preferred to object q if and only if the world with $p \wedge \neg q$ is preferred to the one with $\neg p \wedge q$ as *end-state of contemplated possible changes in his present situation*². The indifference between two different objects is interpreted as in the following: *under*

²That means that a change from the second one to the first one is preferable

some circumstances the state $(p \wedge \neg q)$ is *not* preferred to the state $(\neg p \wedge q)$ and *under some circumstances* the state $(\neg p \wedge q)$ is *not* preferred to the state $(p \wedge \neg q)$. It should be observed that the two occurrences of "some" need not refer to the *same* circumstances. Indifference as defined above is not unconditional. As a consequence, the indifference of two states between themselves does not entail that the two states have the same value relative to any other state: $(pIq) \wedge (pPr) \longrightarrow (qPr)$ is not a tautology (here P represents a strict preference and I an indifference).

Such an approach has some bipolar properties:

- Concerning indifference relation, two poles can be interpreted as: one pole related to the circumstances for an affirmation and the other one against the same affirmation.
- Concerning preference relation, we can see two poles: one pole related to the states of p and the other related to the states of q .

To define the preference in such a way, it can be useful also to interpret some notions like goodness, badness or indifference:

A state p is *good* if it is unconditionally preferred to its negation $\neg p$.

A state p is *bad* if its negation $\neg p$ is unconditionally preferred to it.

As it can be remarked, bipolarity has been interpreted since the sixties by researchers who are interested in the representation of values and who proposed some tools (like bipolar scales) for this purpose.

Bipolarity can be also found in real life decision problems. In the following section, we present different decision problem situations (with a group of decision makers, one decision maker, with multiple criteria or one criterion, with or without veto, etc.) where bipolarity can be found under different aspects.

3 Bipolarity in intuitive decision making and in practice: Examples

Consider the very common situation where the faculty has to deliberate on the admission of candidates to a course (let's say a management course). Then consider two candidates: the first, x , having quite good grades, systematically better than the second, y , but with a very bad grade in management science; then candidate y , who is systematically worse than x , but has an excellent grade in management science. Several faculty members will claim that, although candidate y is

not better than candidate x , it is also difficult to consider x better than y due to their inverse quality concerning the key class of the course, management science. The same faculty members will also claim that the two candidates cannot be considered indifferent because they are completely different. These members are intuitively adopting the same decision rule: *candidate x is better than candidate y iff (s)he has a majority of grades in (her)his favor and is not worse in a number of key classes*. For an extensive discussion on the question of grades in decision support see [Bouyssou et al., 2000].

If we consider a class grade of a candidate as (her)his value on a criterion, the reader will observe that in the above decision rule there exist criteria having a “negative power”. Such a “negative power” is not compensated by the “positive power” of the majority of criteria. It acts independently and only in a negative sense.

Consider an individual, facing a comparison problem between two objects which are evaluated by intervals. Suppose that the price of the first one is between 100 and 130 and the one of the second object is between 110 and 120. Which should (s)he prefer? Let’s suppose that we are interested in the preference of the first one over the second one. In this case, the difference between 100 and 110 has a “positive power” since in the left end points of intervals the first one is cheaper, but the difference between 130 and 120 occurs as “negative power” since this time the first one is more expensive. Positive and negative powers are independent and there can be cases where one can not compensate the other.

Consider now a Parliament. The government has the support of the majority of seats, although not a very strong one. Suppose now that a bill on a very sensitive issue (such as education, religion, national defence, minority rights etc.) is introduced for discussion by the government. Several political, social and ethical issues are involved. Suppose finally that the opposition strongly mobilises, considering that this bill is a major attack against “something”. Massive demonstrations are organised, an aggressive media campaign is pursued etc.. It is quite reasonable that the government will try to find a compromise on some aspects of the bill in order to improve its “acceptability”. Note, however, that such a compromise concerns aspects argued by the minority and not the majority.

Which decision rule is the government using to choose an appropriate law proposal in such a situation? *A law proposal x is considered “better” than y iff it meets the majority will and*

does not mobilises the minority aversion. It should be observed that the minority is considered here as an independent decision power source. Such a “decision rule” is a regular practice in all mature democracies. Although the minority does not have the power to impose its political will, it has the possibility of expressing a “veto”, at least occasionally. Such a “negative power” may not necessarily be codified somewhere, but is accepted. Actually, it is also a guarantee of the democratic game. When the present majority becomes a minority it will be able to use the same “negative power”.

Finally, consider the Security Council of the United Nations. Here, a number of nations are officially endowed with a veto power such that resolutions taken with a majority of votes (even the highest ones) can be withdrawn if such a veto is used. We observe that in this case the decision rule “*x is better than y if it is the case for the majority and no veto is used against x*” is officially adopted. Again we observe that the countries having a veto power do not have a “positive power” (impose a decision), but only a “negative” one.

Within all the cited examples, there exist two different not compensated types of power, positive and negative ones, each one representing a different pole.

As it can be noted the use of independent positive and negative reasons within decision rules is common practice for electoral bodies, commissions, boards, etc., besides being an intuitive rule for comparing alternatives described under multiple attributes. It is therefore necessary to consider a specific model to handle them.

4 Notation and Problem

In the following A will represent a finite (countable) set of objects (candidates, alternatives, actions etc.) on which preferences are expressed and from which a choice or a ranking is expected to be established.

We are going to note with \succeq (possibly subscribed \succeq_i) preference relations on the set A to be read as “ x is at least as good as y ” ($x \succeq y$ or $\succeq(x, y)$). We only impose reflexivity on such a relation. If necessary we may add other specific properties. \succ will represent as usually the asymmetric part of \succeq . We will also use capital letters $P, Q, I \dots$ in order to represent specific preference relations (characterised by their properties). As usual P^{-1} will represent the inverse relation of P ($P^{-1}(x, y) \equiv P(y, x)$).

We are going to use \succeq^+ in order to represent preference sentences of the type “there are positive reasons for considering x at least as good as y ”, while \succeq^- will represent sentences of the type “there are negative reasons for which it should not be the case that x is at least as good as y ”. Both \succeq^+ and \succeq^- are binary relations.

Given a set H of such preference relations (a set of criteria) and for each couple $(x, y) \in A$ we note as H_{xy}^+ the subset of H for which $x \succeq^+ y$ holds (the coalition of criteria for which there are positive reasons for which x is at least as good as y : positive coalition). In the same way we are going to note as H_{xy}^- the subset of H for which $x \succeq^- y$ holds (the coalition of criteria for which there are negative reasons for which it should not be the case that x is at least as good as y : negative coalition).

Our problem can be summarized in two steps.

1. Establish for each couple $(x, y) \in A$ an overall preference relation (\succeq), possibly separating it in \succeq^+ and \succeq^- . This should correspond to a general rule to be applied recursively in the case there is a hierarchy of criteria to take into account. We call this the *preference aggregation step*.
2. Given such an overall preference relation, establish a final recommendation under form of a choice or a ranking on the set A , whenever this is required. We call this the *preference exploitation step*.

The reader can see an extensive discussion about the above two steps in classic Multiple Criteria Decision Analysis in [Bouyssou et al., 2006].

5 Generalizing Concordance and Discordance

5.1 Preference Aggregation

We introduce the general rule

$$x \succeq^+ y \iff \mathcal{P}^+(H_{xy}^+) \geq \gamma \quad (1)$$

$$x \succeq^- y \iff \mathcal{P}^-(H_{xy}^-) \geq \delta \quad (2)$$

where \mathcal{P}^+ (\mathcal{P}^-) represents a measure of the importance of the “positive” (respectively negative) coalition and γ and δ represent two thresholds.

We are not going to discuss in this paper how \mathcal{P}^+ (\mathcal{P}^-) is established, but without loss of generality we can assume that is a real valued function to the interval $[0, 1]$. Of course the thresholds γ and δ are defined within the same interval.

The first rule should be read as: *when comparing x to y under all criteria, there are sufficient positive reasons to claim that x is at least as good as y iff the coalition of criteria where it is the case that x is at least as good as y is sufficiently strong.*

The second rule should be read as: *when comparing x to y under all criteria, there are sufficient negative reasons to claim that it is not the case that x is at least as good as y iff the coalition of criteria where it is not the case that x is at least as good as y is sufficiently strong.*

In principle \mathcal{P}^+ and \mathcal{P}^- are independently evaluated and therefore the strength of the positive and negative coalitions are not computed in the same way nor can be considered one the complement of the other. If we interpret the above rule within a social choice setting, we can consider \mathcal{P}^+ as the strength of the majority coalition, the γ threshold being the majority required to approve a bill, while \mathcal{P}^- should be considered as the minority strength, the δ threshold representing the situation where a veto could be expressed. Consider again the United Nations Security Council example. Positive power for each member is $1/15$. The strength of the positive coalition is computed additively, the γ threshold being $3/5$. The negative power of each member is 0 or 1 (depending on their status: permanent, not permanent). The strength of the negative coalition is computed using the max operator, the δ threshold being 1.

The idea of using \mathcal{P}^+ and \mathcal{P}^- has been already introduced in Multiple Criteria Decision Making methods. In the so called “outranking methods”, the global preference relation S (to be read as “at least as good as”) is generally established as

$$S(x, y) \iff C(x, y) \wedge \neg D(x, y) \quad (3)$$

where $C(x, y)$ is the concordance test (is there a weighted majority of criteria in favor of x wrt to y ?) and $D(x, y)$ is the discordance test (is there a veto against x wrt to y ?).

Example 1. *A typical application of the above rule can be seen in one of the oldest ”outranking*

methods” (see [Roy, 1968]) where:

$$C(x, y) \iff \frac{\sum_{j \in J_{xy}} w_j}{\sum_j w_j} \geq \gamma, \quad (4)$$

$$D(x, y) \iff \exists j : g_j(y) - g_j(x) > v_j \quad (5)$$

where:

- g_j is a real valued function representing the evaluation of alternatives with respect to the criterion c_j (to be maximised);
- w_j is a non negative coefficient which represents the importance of the criterion c_j ;
- J_{xy} represents the set of criteria for which x is at least as good as y ; more precisely, $J_{xy} = \{j : g_j(y) - g_j(x) \leq q_j\}$ where q_j is the indifference threshold associated to criterion c_j ; therefore, $J_{xy} = H_{xy}^+$;
- γ is a majority threshold;
- v_j is a veto threshold on criterion c_j ;
- consequently H_{xy}^- will be the set of criteria where a veto is expressed against $S(x, y)$.

In this case a sufficiently strong positive coalition is any subset of criteria for which the sum of the importance coefficients is at least γ . If such a coalition exists, it means that we have positive reasons to consider that x is at least as good as y . On the other hand, we have negative reasons to consider that x is at least as good as y when y is largely better than x on at least one criterion.

However, this way to interpret the concordance/discordance principle presents a number of weak points. Using the definition in equation (3), both the concordance and the non-discordance tests have to be verified in order to establish the outranking relation. Indeed, the negative reasons (discordance) are sufficient to invalid the positive ones (concordance). However, there is a big semantic difference between a situation where a majority of criteria supports that “ x is at least as good as y ”, but there is a veto, a situation where there is neither majority nor veto and a situation where there is a minority of criteria in favour of the outranking relation. In other words, when comparing two alternatives x and y , the use of the concordance/discordance principle introduces

four different epistemic situations but only two possible cases can occur (either the outranking relation holds or not).

Moreover, the principle does not work recursively. There is no way to consider the existence of positive and negative reasons for each single criterion which should be aggregated separately. This prevents the use of this method in a hierarchical structure of criteria and agents.

Remark 1. *The two functions \mathcal{P}^+ and \mathcal{P}^- are supposed to be measures of the strength of the positive and negative coalition of criteria respectively. It is reasonable to consider such functions as “fuzzy measures” or “valued binary relations” instead of using their “cuts” represented by the thresholds γ and δ . This is the approach adopted by several authors including [Figueira and Greco, 2004], [Grabisch and Labreuche, 2005], [Fernandez and Olmedo, 2005] or [Öztürk and Tsoukiàs, 2006]. The result will be a “bi-polar” (positive/negative) measure of the strength of preference for each pair of objects in A .*

5.2 Preference Exploitation

Aggregating preferences will generally result in a binary relation which is neither necessarily complete nor transitive (see [Bouyssou, 1996], [Vincke, 1982]). The global relations \succeq^+ and \succeq^- obtained after aggregating preferences are not necessarily orders. Thus, is difficult, if not impossible, to identify a best choice or a ranking of the set A , just using these relations. In order to obtain such a result (which we may call a final recommendation) it is necessary to further elaborate the information obtained from the aggregation step.

The literature offers a large variety of procedures for this purpose when conventional preference structures are considered (see for instance [Vincke, 1992]). The interested reader can see more details in [Bouyssou et al., 2006], chapter 7. However, very few, if any, procedures exist when positive and negative procedures are considered separately (see [Greco et al., 1998]). In this paper we present two procedures:

- the positive/negative net flow procedure;
- the positive/negative dominance ranking procedure.

Let us recall that the input of such procedures are the two binary relations \succeq^+ and \succeq^- on the set A and the output is a ranking of the set A .

1. *The positive/negative net flow.* For each element $x \in A$ we compute a score

$$\begin{aligned} \sigma(x) = & |\{y \in A : x \succeq^+ y\}| + |\{y \in A : y \succeq^- x\}| \\ & - |\{y \in A : y \succeq^+ x\}| - |\{y \in A : x \succeq^- y\}| \end{aligned} \quad (6)$$

We then rank the set A by decreasing values of σ . In other terms, for each element x we count the elements for which there are positive reasons such that x should be at least as good as them plus the elements for which there are negative reasons for which they should not be at least as good as x and we subtract the number of elements for which there are positive reasons for which they should be at least as good as x and the elements for which there are negative reasons for which x should not be at least as good as them. This procedure generalizes the net flow procedure used in MCDM (see [Bouyssou, 1992]).

2. *The positive/negative dominance ranking.* The procedure establishes two distinct rankings, one for the positive and one for the negative reasons and works as follows:

- consider the graph associated to the relation \succeq^+ ;
- identify the subset A_1^+ of A such that there are no entering arcs to any of its elements (the elements of A for which there are no other elements having positive reasons for which they should be at least as good as them);
- establish A_1^+ as an equivalence class (the best), eliminate it from A and apply the same procedure to $A \setminus A_1^+$; this will identify the second best equivalence class A_2^+ ;
- proceed until the set A is totally ranked from A_1^+ (the best) to A_n^+ (the least best);
- consider the graph associated to the relation \succeq^- ;
- identify the subset A_1^- of A such that there are no entering arcs to any of its elements (the elements of A for which there are no other elements having negative reasons for which they should not be at least as good as them);
- establish A_1^- as an equivalence class (the worst), eliminate it from A and apply the same procedure to $A \setminus A_1^-$; this will identify the second worst equivalence class A_2^- ;
- proceed until the set A is totally ranked from A_n^- (the least worst) to A_1^- (the worst);
- the two rankings do not necessarily coincide. A partial ranking of A can be obtained from the intersection of these two rankings.

Several other procedures can be conceived. We limit ourselves in this paper to these two

examples just in order to show how it is possible to obtain a final ranking after preferences have been aggregated using positive and negative reasons independently. Concluding this section we can make the following remarks.

Remark 2. *In the case \mathcal{P}^+ and \mathcal{P}^- are considered as fuzzy measures the preference exploitation step will require different procedures. For examples see [Öztürk and Tsoukiàs, 2006].*

Remark 3. *The idea of final recommendation adopted in this paper focusses on obtaining a choice or a ranking for the set A . However, in real decision support situations a final recommendation can be richer than that. For instance it can identify conflicts and incomparabilities to analyze before any further decision. A clear representation of the positive and negative reasons behind such critical issues is extremely beneficial in such situations.*

In the following we are going to show how the above ideas apply to a specific preference aggregation problem: the case where preferences are expressed through comparison of intervals.

6 Aggregating preferences on Intervals

Consider a set of four objects $A = \{a, b, c, d\}$ and a set of four criteria $H = \{h_1, h_2, h_3, h_4\}$. In order to simplify the presentation we consider that to all four criteria correspond attributes where the objects in A can take numerical values on a scale from 1 to 8. However, in this particular case we make the hypothesis that due to uncertainties on the real values of the objects these will take a value under form of an interval. Figure 1 shows how the objects in A are evaluated on the four attributes.

Where is the problem? Conventionally, comparison of intervals is based on the hypothesis that “ x is preferred to y ”, $P(x, y)$, iff the lowest value of x is larger than the greatest value of y (the intervals thus being disjoint). In all other cases “ x is indifferent to y ” ($I(x, y)$) (for more on interval orders see [Fishburn, 1985], [Pirlot and Vincke, 1997], [Trotter, 1992]). If we interpret as usual the binary relation \succeq as $P \cup I$ then the graphs representing this binary relation for the four criteria can be seen in table 1 (here represented under form of a 0-1 matrix).

Aggregating these preferences with a simple majority rule: “ x is at least as good as y iff it is the case for at least three out of four criteria”, will return all four objects indifferent. This is

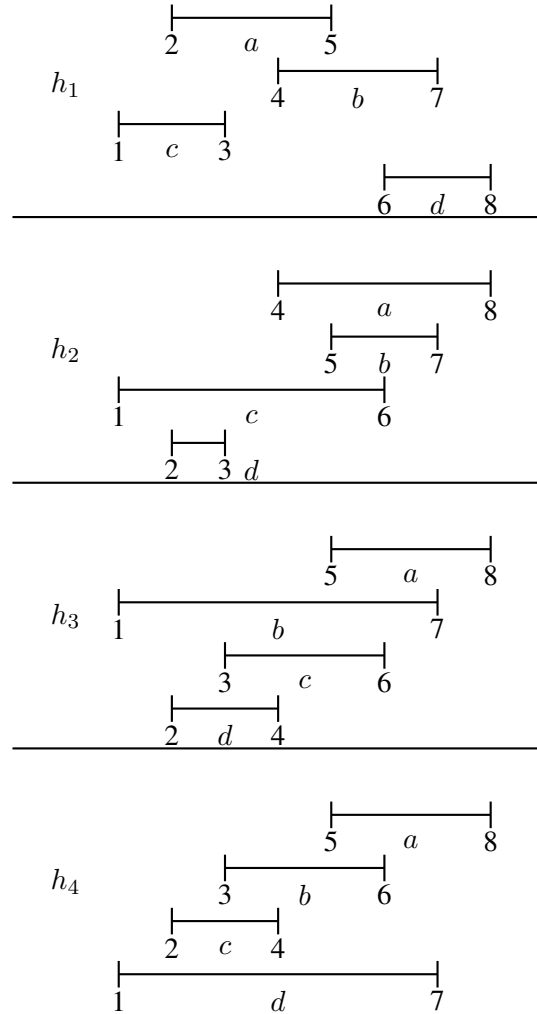


Figure 1: The values of A on the four attributes

not surprising, since conventional interval orders only use positive information and are unable to differentiate between sure indifference and hesitation between indifference and preference.

In order to overcome this problem we are going to introduce positive and negative reasons both in comparing objects at each criterion level as well as at the aggregated one. For this purpose we are going to use a preference structure called *PQI*-interval order ([Tsoukiàs and Vincke, 2003]). In this structure we consider three possibilities when comparing intervals:

- strict preference (P): when an interval is completely to the right of the other (exactly as in

h_1	a	b	c	d	h_2	a	b	c	d
a	1	1	1	0	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	0	1	0	c	1	1	1	1
d	1	1	1	1	d	0	0	1	1
h_3	a	b	c	d	h_4	a	b	c	d
a	1	1	1	1	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	1	1	1	c	0	1	1	1
d	0	1	1	1	d	1	1	1	1

Table 1: The four Interval Orders

conventional interval orders);

- indifference (I): when one interval is completely included in the other;

- hesitation between preference and indifference or weak preference (Q): when an interval is to the right to the other, but they have a non empty intersection.

Applying this structure to the information previously presented we obtain the preference relations in table 2.

We are now going to interpret such preference relations in terms of positive and negative reasons. For this purpose we are going to use the \succeq^+ and \succeq^- relations and define:

$$P(x, y) \iff \succeq^+(x, y) \wedge \not\succeq^-(x, y) \wedge \not\succeq^+(y, x) \wedge \succeq^-(y, x) \quad (7)$$

$$I(x, y) \iff \succeq^+(x, y) \wedge \not\succeq^-(x, y) \wedge \succeq^+(y, x) \wedge \not\succeq^-(y, x) \quad (8)$$

$$Q(x, y) \iff \succeq^+(x, y) \wedge \not\succeq^-(x, y) \wedge \succeq^+(y, x) \wedge \succeq^-(y, x) \quad (9)$$

In other words, while P and I represent “sure” situations of interval comparison, the relation Q represents an hesitation between them: indeed when comparing x to y we have positive reasons claiming that x is at least as good as y and no negative reasons claiming the opposite; when comparing y to x we have both positive and negative reasons (due to the fact that the larger value of y is larger than the smaller value of x , but smaller than the larger value of x).

h_1	a	b	c	d	h_2	a	b	c	d
a	I	Q^{-1}	Q	P^{-1}	a	I	I	Q	P
b	Q	I	P	Q^{-1}	b	I	I	Q	P
c	Q^{-1}	P^{-1}	I	P^{-1}	c	Q^{-1}	Q^{-1}	I	I
d	P	Q	P	I	d	P^{-1}	P^{-1}	I	I
h_3	a	b	c	d	h_4	a	b	c	d
a	I	Q	Q	P	a	I	Q	P	Q
b	Q^{-1}	I	I	I	b	Q^{-1}	I	Q	I
c	Q^{-1}	I	I	Q	c	P^{-1}	Q^{-1}	I	I
d	P^{-1}	I	Q^{-1}	I	d	Q^{-1}	I	I	I

Table 2: The four PQI interval orders

For further details on such models the reader can see [Öztürk , 2005], [Tsoukiàs et al., 2002] and [Tsoukiàs and Vincke, 1998]. Applying this reasoning to the information concerning the set A we get the results in table 3 (for the positive reasons) and in table 4 (for the negative reasons).

In order to aggregate these positive and negative reasons let us apply now the principle introduced in equations 1-2. In this precise case we use the following specific rule:

$$x \succeq^+ y \iff \frac{|\{h_j : x \succeq_j^+ y\}|}{|H|} \geq \frac{3}{4} \quad (10)$$

$$x \succeq^- y \iff \frac{|\{h_j : x \succeq_j^- y\}|}{|H|} \geq \frac{1}{2} \quad (11)$$

Actually we use a very simple aggregation rule. Both \mathcal{P}^+ and \mathcal{P}^- are additive and for both the positive and negative distribution of power we consider the criteria equivalent. The results of this aggregation can be seen in table 5.

What do we get from these results?

First of all we are able to reconstruct a PQI preference structure at the aggregated level. We can establish the type of preference relation holding for any pair of objects in the set A . More precisely, applying equations 7-9 we get the results shown in table 6.

We can now check whether such a PQI preference structure is also a PQI interval order

γ_1^+	a	b	c	d	γ_2^+	a	b	c	d
a	1	1	1	0	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	0	1	0	c	1	1	1	1
d	1	1	1	1	d	0	0	1	1

γ_3^+	a	b	c	d	γ_4^+	a	b	c	d
a	1	1	1	1	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	1	1	1	c	0	1	1	1
d	0	1	1	1	d	1	1	1	1

Table 3: Positive Reasons

γ_1^-	a	b	c	d	γ_2^-	a	b	c	d
a	0	1	0	1	a	0	0	0	0
b	0	0	0	1	b	0	0	0	0
c	1	1	0	1	c	1	1	0	0
d	0	0	0	0	d	1	1	0	0

γ_3^-	a	b	c	d	γ_4^-	a	b	c	d
a	0	0	0	0	a	0	0	0	0
b	1	0	0	0	b	1	0	0	0
c	1	0	0	0	c	1	1	0	0
d	1	0	1	0	d	1	0	0	0

Table 4: Negative Reasons

γ^+	a	b	c	d	γ^-	a	b	c	d
a	1	1	1	1	a	0	0	0	0
b	1	1	1	1	b	1	0	0	0
c	1	1	1	1	c	1	1	0	0
d	0	1	1	1	d	1	0	0	0

Table 5: Positive and Negative Reasons after Aggregation

PQI	a	b	c	d
a	I	Q	Q	P
b	Q^{-1}	I	Q	I
c	Q^{-1}	Q^{-1}	I	I
d	P^{-1}	I	I	I

Table 6: The PQI preference structure after aggregation

and if it is the case we can try to reconstruct a numerical representation for each element of A under form of interval. Using results known in the literature ([Ngo The and Tsoukias, 2005], [Ngo The et al., 2000]) we can prove that in this precise case this is indeed a PQI interval order, a numerical representation of which can be seen in figure 2.

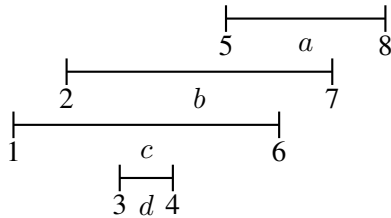


Figure 2: The global PQI interval order

In the case we definitely need a more operational result such as a ranking we can use any of the procedures introduced in section 4.2. More precisely adopting the positive/negative net flow procedure (see equation 7) we get $a > b > d > c$ ($>$ representing the ranking relation). If we use the positive/negative dominance ranking we obtain a different result: $a > b > d, c$. This is not

surprising, if we consider the nature of the aggregation procedure and the information available. Concluding this section we may make the following remarks.

Remark 4. *To our knowledge this is the only way through which is possible to aggregate preferences expressed on intervals without ending with only indifference, losing precious information (see [Pirlot and Vincke, 1992]). The identification of positive and negative reasons in intervals comparison allows to exploit information which in conventional preference modelling is usually neglected. The specific suggestions done in this paper should be considered as examples, since several other possibilities can be considered depending on the problem on hand.*

Remark 5. *The reader can check that modifying the parameters and rules in the aggregation and exploitation steps one can obtain significantly different results. This is not surprising since these are not preferential information obtained from the decision maker and are more or less arbitrary. Care should be taken to tune them robustly.*

Remark 6. *Fortunately the algorithmic part of the above procedures is “easy”. Indeed as shown in [Ngo The and Tsoukias, 2005] and [Ngo The et al., 2000], checking if a PQI preference structure is a PQI interval order and finding a numerical representation are all problems in \mathbf{P} .*

7 Conclusions

In this paper we focus on the advantages of using independent positive and negative reasons in preference aggregation. More precisely:

- aggregating independent positive and negative reasons allows to clearly distinguish situations of sure preference from situations of hesitation as well as between incomparabilities due to conflicts (presence of both positive and negative reasons) and incomparabilities due to ignorance (absence of both positive and negative reasons);
- modelling independently positive and negative reasons allows to use the same principle for any level of preference modelling (single criterion, single agent, multiple criteria, multiple agents and their combinations), thus generalising the concordance/discordance principle;

- the use of positive and negative reasons when objects evaluated on intervals are compared allows to solve the problem of aggregating such preferences, a situation encountered not only in decision aiding, but in several other fields (see [Öztürk et al., 2005]).

Several research problems remain open in the paper. Among these we note the following:

- axiomatise preference aggregation and exploitation procedures based on the independent use of positive and negative reasons (of the type presented in this paper);
- study appropriate formalisms (multiple valued logics, argumentation theory etc.) enabling elegant and compact representations besides further extending the potentialities of this approach;
- further investigate the problem of aggregating *PQI* preference structures: under what conditions the aggregation of such preference structures will result in a *PQI* interval order or any other order representable by intervals?

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