

Birefringent left-handed metamaterials and perfect lenses for vectorial fields

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Abstract. We describe the properties of specific non-reflecting birefringent left-handed metamaterials and demonstrate *a birefringent perfect lens* for vectorial fields. We predict that, in a sharp contrast to the concept of a conventional perfect lens realized at $\epsilon = \mu = -1$ (where ϵ is the dielectric permittivity and μ is the magnetic permeability), the birefringent left-handed slab possesses the property of negative refraction either for TE- or TM-polarized waves or for both of them simultaneously. This allows selective focusing and a spatial separation of the images created at different polarizations. We discuss several applications of the birefringent left-handed lenses such as the beam splitting and near-field diagnostics at the sub-wavelength scale.

Contents

1. Introduction	2
2. Birefringent non-reflecting metamaterial	3
3. Perfect lenses for vectorial fields	4
4. Conclusions	8
Acknowledgments	9
References	9

1. Introduction

Current interest in the physics of sub-wavelength imaging is explained by many potential applications of this effect, including lithography and data storage. The concept of a perfect lens [1] suggests one of the potential ways for improving the resolution of imaging systems based on the unique properties of the so-called left-handed metamaterials or materials with negative refractive index [2, 3]. The possibility of a perfect lens whose resolution is not limited by the classical diffraction limit has been a subject of intense debates by the scientific community in the past 5 years, and at first it was met with considerable opposition [4]. However, many difficulties raised by the critics have been answered and have clarified the concept itself and its limitations [5] through the results of numerical simulations [6] and recent experiments on negative refraction, e.g. with a negative-index material assembled from discrete elements arranged on a planar circuit board [7].

The so-called *perfect lens* is created by a slab of a left-handed metamaterial with $\epsilon = \mu = -1$, where ϵ is the dielectric permittivity and μ is the magnetic permeability. Veselago [2] predicted that such a material would have a negative refractive index of $n = -\sqrt{\epsilon\mu} = -1$, and a flat slab of this material would act as a lens refocusing *all rays* from a point source on one side of the slab into a point on the other side of the slab (see figure 1). Later, Pendry [1] suggested that such a lens can reconstruct the near field of a source, and as a result it can create an ideal image [1].

Thus, a slab of the left-handed metamaterial can be used for a sub-wavelength imaging because it amplifies all evanescent (near field) modes inside the slab, and therefore it allows to preserve the information about the source structure with a resolution better than the radiation wavelength. However, to satisfy the conditions for such a perfect lens to operate, the distance between the source and the slab surface, a , and the distance between the second surface of the slab and the image point, b , should be connected with the slab thickness d by the relation [1] (see figure 1),

$$a + b = d. \quad (1)$$

The relation (1) means that it is impossible to create an image at the distances larger than the slab thickness, and this is one of the serious limitations for the applicability of left-handed perfect lenses.

In this paper, we demonstrate that birefringent non-reflecting left-handed metamaterials can be used for *birefringent perfect lenses*, which can focus vectorial fields. In particular, we show that, in contrast to the condition $\epsilon = \mu = -1$ for the conventional perfect lens, the birefringent left-handed lens can focus either TE- or TM-polarized waves or both, with a *varying distance* between the TE- and TM-images, and this property allows us to expand dramatically the applicability limits of the perfect lenses. We suggest specific structures for the dielectric and the magnetic tensors that allows to create this type of birefringent media, which is transparent for an arbitrary angle of incidence. In addition, we show that such a birefringent lens is free from the limitations imposed by the condition (1), and we also discuss some other applications of the birefringent left-handed metamaterials for the beam polarization splitting and sub-wavelength imaging.

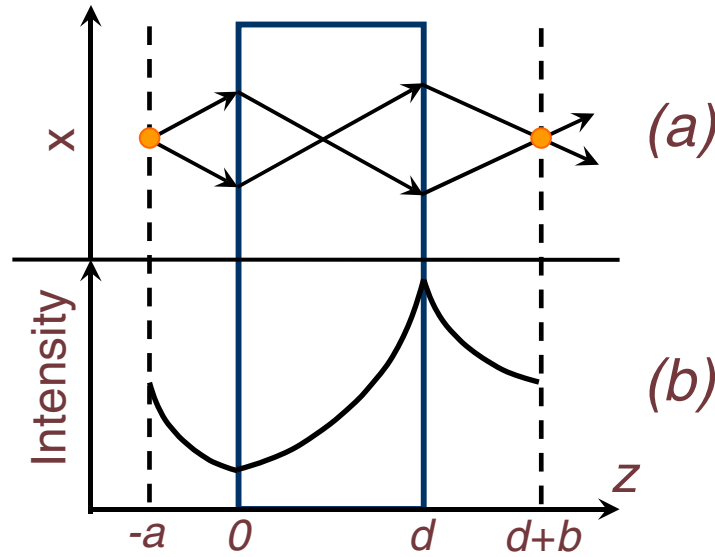


Figure 1. Schematic presentation of (a) ray focusing and (b) reconstruction of evanescent waves by a left-handed lens.

2. Birefringent non-reflecting metamaterial

Anisotropic metamaterials have been studied previously in several papers [8], where different regimes of wave propagation are discussed. Here, we consider a specific linear birefringent medium described by the following tensors of dielectric permittivity $\hat{\epsilon}$ and magnetic permeability $\hat{\mu}$, which in the main axes of the crystal have the form

$$\hat{\epsilon} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A^{-1} \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & B^{-1} \end{pmatrix}, \quad (2)$$

where A and B are generally arbitrary complex functions of the frequency. We substitute the expressions (2) into Maxwell's equations and obtain the equations for the transverse spatial harmonics of the monochromatic ($\sim \exp(i\omega t - ik_x x)$) electromagnetic waves, for the case of (i) *TM polarization*, when $\mathbf{E} = (E_x, 0, E_z)$ and $\mathbf{H} = (0, H_y, 0)$

$$\frac{d^2 H_y}{dz^2} + A^2(k_0^2 - k_x^2)H_y = 0, \quad (3)$$

$$E_x = -\frac{1}{ik_0 A} \frac{dH_y}{dz}, \quad E_z = -\frac{k_x}{k_0} A H_y \quad (4)$$

and for the case of (ii) *TE polarization*, when $\mathbf{E} = (0, E_y, 0)$ and $\mathbf{H} = (H_x, 0, H_z)$

$$\frac{d^2 E_y}{dz^2} + B^2(k_0^2 - k_x^2)E_y = 0, \quad (5)$$

$$H_x = -\frac{1}{ik_0 B} \frac{dE_y}{dz}, \quad H_z = \frac{k_x}{k_0} B E_y, \quad (6)$$

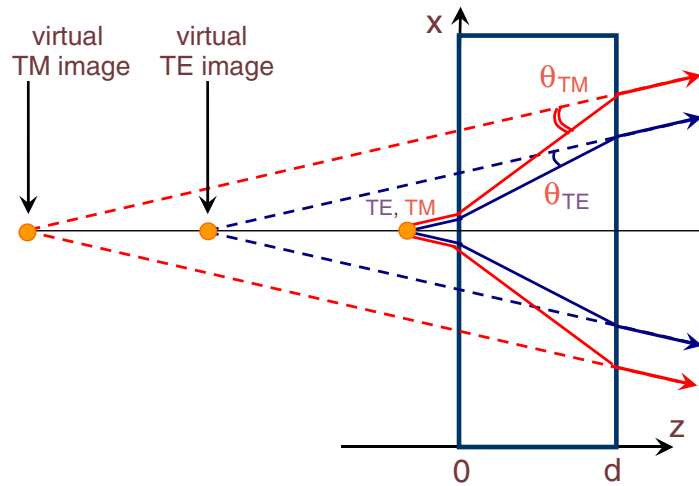


Figure 2. Ray diagram showing the creation of two separate TE- and TM-polarized virtual images of a source; θ_{TE} and θ_{TM} are the angles between the directions of the group velocity and wavevector of the TE and TM waves inside the slab.

where $k_0 = \omega/c$ is the wavenumber in vacuum, k_x is the wave vector component along the x -axes and c is the speed of light. It is easy to verify that the wave impedance of this birefringent medium matches exactly the impedance of vacuum, for both polarizations with any transverse wavenumbers k_x , and for *arbitrary* (including complex) values of A and B . Therefore, the medium described by the tensors (2) is ideally impedance-matched with vacuum, and it is reflectionless⁴. Such a birefringent medium was suggested to create a perfectly matched layer in the finite-difference time-domain simulations [10]. In a general case, when the vacuum is substituted by a medium with some ϵ_s and μ_s , the impedance matching conditions would require some modification of equation (2), namely $\hat{\epsilon} \rightarrow \epsilon_s \hat{\epsilon}$ and $\hat{\mu} \rightarrow \mu_s \hat{\mu}$. Below we consider $\epsilon_s = \mu_s = 1$ without loss of generality.

3. Perfect lenses for vectorial fields

We consider a slab of the metamaterial with the dielectric and magnetic properties characterized by the tensors (2). The slab is surrounded by vacuum, and it has the thickness d ($0 \leq z \leq d$). We assume that a point source is located at the distance $z = -a$ from the nearest surface of the slab, as shown in figure 2, the source generates both TE- and TM-polarized waves, and it is described by the corresponding distribution of the electric field $E_y(x, z = -a)$, for the TE polarization, or the magnetic field $H_y(x, z = -a)$, for the TM polarization, in the plane $z = -a$. We denote the spatial spectra of these fields as $\alpha_e(k_x)$ and $\alpha_m(k_x)$, respectively. Using equations (3)–(6) for describing the electromagnetic field in the slab, and satisfying the boundary conditions for the tangential components of the fields, we obtain general expressions for the spatial harmonics of

⁴ Reflectionless in this kind of medium but with $A = B > 0$ has been mentioned earlier in [9].

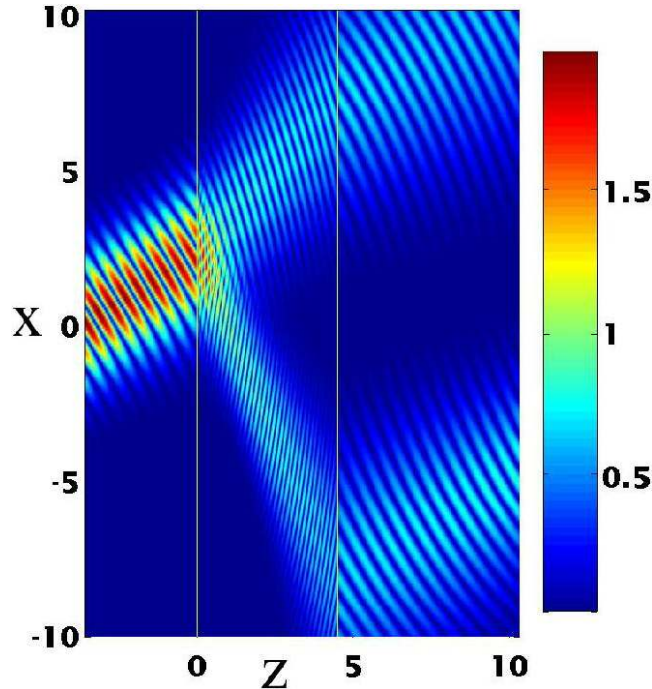


Figure 3. Beam scattering by a slab of the birefringent metamaterial ($A = -4$, $B = +2$, $d = 5$). The TM-polarized component experiences negative refraction, while the TE-polarized component refracts normally. Coordinates are normalized to the free-space wavelength.

the fields behind the slab, i.e. for $z > d$,

$$H_y(z, k_x) = \alpha_m(k_x) \exp \left[-i\sqrt{k_0^2 - k_x^2}(a + Ad + z') \right], \quad (7)$$

for the TM-polarized waves, and

$$E_y(z, k_x) = \alpha_e(k_x) \exp \left[-i\sqrt{k_0^2 - k_x^2}(a + Bd + z') \right] \quad (8)$$

for the TE-polarized waves, where $z' = z - d$. Thus, for real A and B , equations (7) and (8) reproduce the field structure of the source in the region $z' > 0$ shifted from the source position by the distance $(A - 1)d$ (TM waves) or $(B - 1)d$ (TE waves). A typical ray diagram for this case is shown in figure 2 for $A > 1$, $B > 1$ and $A > B$, where we show the position of the source for both the polarizations, as well as spatially separated virtual images created by the lens. In general, for $A \neq B$, the virtual images of the TE and TM sources are shifted relative to each other. For $0 < A$ and $B < 1$, the virtual images can be located either between the slab and the source or inside the metamaterial slab.

More interesting cases of the medium (2) correspond to *negative* values of A and/or B . When $A < 0$ and $B > 0$, negative refraction occurs for the TM-polarized waves only, whereas the TE-polarized waves refract normally, see figure 3. For $A > 0$ and $B < 0$, the opposite effect occurs, i.e. negative refraction is possible for the TE-polarized waves only. This property can be

used for the polarization-sensitive beam separation. Figure 3 shows an example of this separation for the slab with parameters $A = -4$ and $B = +2$. A two-dimensional beam propagates at the angle of incidence 30° and is refracted. Initially, the beam is composed of two polarizations with the same partial intensities. When the beam is refracted at the surface, the TM-polarized wave undergoes negative refraction, and it becomes separated from the normally refracted TE beam.

Another specific feature of the birefringent lenses is a possibility to form *two separate perfect images* for the TE- and TM-polarized waves. This property follows from the results (7) and (8). In particular, for $A < 0$ and $B > 0$, the transverse spatial spectrum of the TM-polarized field in the plane $z'_m = |A|d - a$ coincides with the spectrum of the source

$$H_y(z'_m, k_x) = \alpha_m(k_x), \quad (9)$$

while the TE-polarized component of the beam is positively refracted. In the case $A > 0$ and $B < 0$, the image is created by the TE-polarized waves at $z'_e = |B|d - a$,

$$E_y(z'_e, k_x) = \alpha_e(k_x), \quad (10)$$

whereas the TM-polarized waves experience positive refraction, and they do not create an image. Thus, in the case of the birefringent lens, additional parameters appear, which mitigate the strict limitations for the isotropic lens imposed by equation (1). As a result, the source and the image can be located further away from the slab. More importantly, when both A and B are negative and $A \neq B$, both TE and TM images appear, and they are separated by the distance

$$h = |z'_e - z'_m| = (|B| - |A|)d, \quad (11)$$

which, in the absence of dissipative losses, can be arbitrary large. This allows novel possibilities for the sub-wavelength resolution, diagnostics and microscopy.

In the case $|A| = |B|$, the TE and TM images coincide and in a particular case $A = -1$ and $B = -1$, we recover the results for the *isotropic perfect lens* discussed by Pendry [1] and Veselago [2]. In general, the basic physics for operating these birefringent perfect lenses is similar to the isotropic case, and it is defined by two major factors: (i) negative refraction and (ii) amplification of evanescent waves. Figure 1(b) shows schematically the structure of the evanescent waves in the slab for the case of Pendry's lens, which is similar for both isotropic and birefringent left-handed media. Figures 4(a) and (b) show schematically the ray diagram in two special cases, when a single source generates both TE- and TM-polarized waves (see figure 4(a)) creating two separate images, and when the TE and TM sources are separated and they create a combined image (see figure 4(b)).

A possibility of the sub-wavelength resolution of a pair of sub-wavelength sources by using the birefringent left-handed lens has also been verified numerically, and some examples are presented in figure 5 for the case of a lossy medium when $\hat{\epsilon}_l = \hat{\epsilon} - i\delta_{ik} \times 10^{-8}$ and $\hat{\mu}_l = \hat{\mu} - i\delta_{ik} \times 10^{-8}$, where $\delta_{ik} = 1$ for $i = k$ and it is 0 otherwise. The mixed-polarized source consists of two beams of the width $\lambda/5$, separated by the distance $2\lambda/5$, where λ is the free-space wavelength, metamaterial parameters are $A = -2.5$ and $B = -1.5$ and the slab thickness is $\lambda/2$. A difference in the resolution for the TE- and TM-polarized waves for $A \neq B$ is explained by different effective optical thickness of the slab for two polarizations. An increase of the losses decreases the resolution abilities of the lens dramatically. Figures 6(a) and (b) show the spatial

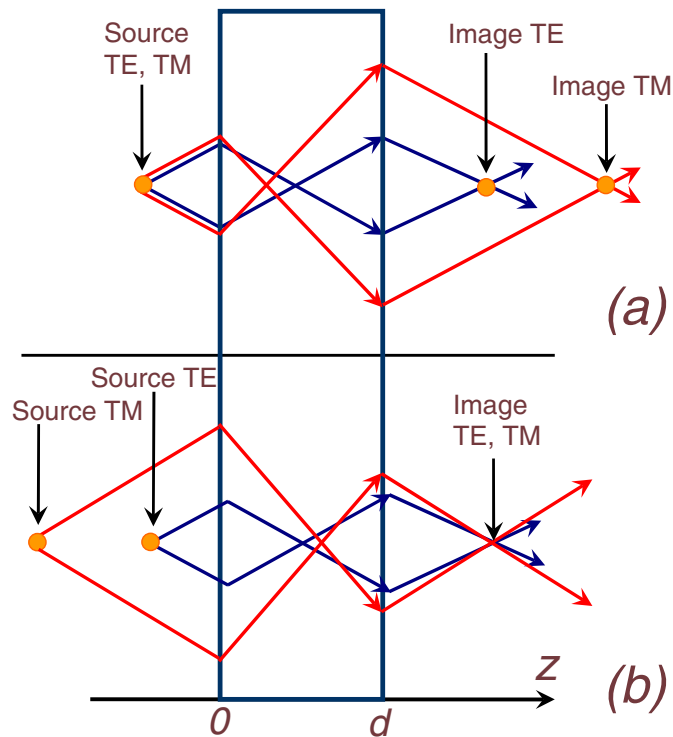


Figure 4. Ray diagrams of the birefringent left-handed lens imaging. (a) A single source imaging by a metamaterial slab characterized by negative $A \neq B$. TE and TM images are separated by the distance h defined by equation (11). (b) Separated sources with two different polarizations can create the images in the same plane, provided the sources are separated by the distance (11).

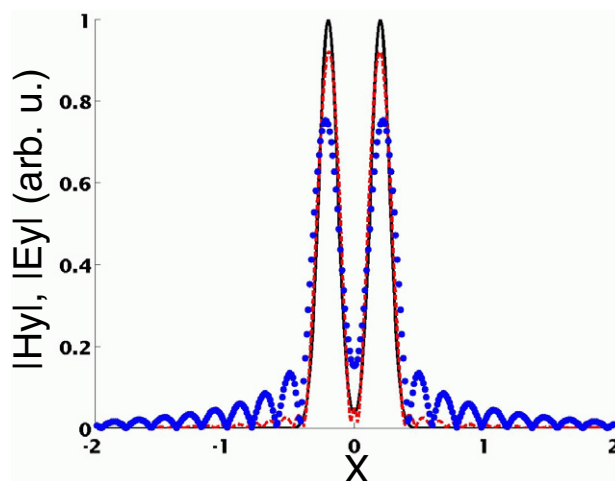


Figure 5. Transverse structure of the TE and TM fields of the source (—), electric field of the TE image (- - -) and magnetic field of the TM image (•••). The coordinate is normalized to the free-space wavelength.

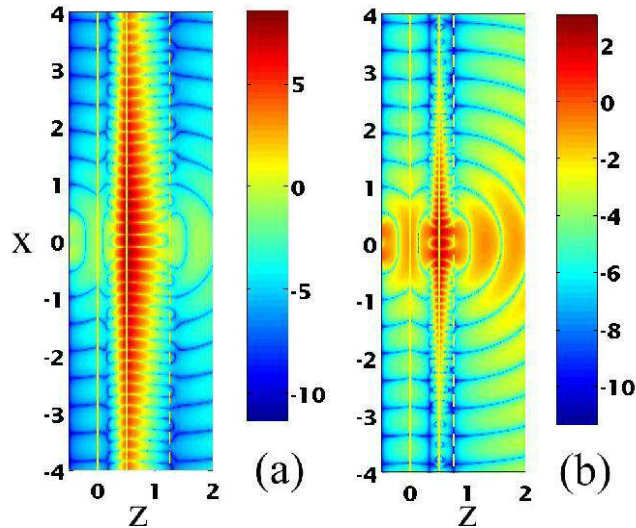


Figure 6. Spatial distribution of the absolute values of (a) magnetic and (b) electric fields (logarithmic scale) generated by two sub-wavelength sources of the width $\lambda/5$ separated by the distance $2\lambda/5$. Parameters are $A = -2.5$ and $B = -1.5$, and the slab thickness is $\lambda/2$. Solid lines mark the metamaterial slab, dashed lines show the image planes. Coordinates are normalized to the free-space wavelength.

distribution of the magnetic field, $\mathcal{R}e(H_y(x, z))$, for the TM polarization and spatial distribution of electric field, $\mathcal{R}e(E_y(x, z))$, for the TE polarization, respectively.

Different examples presented above clearly demonstrate that the birefringent left-handed metamaterials and birefringent perfect lenses are novel objects with many unusual properties and, more importantly, they may demonstrate much broader spectrum of potential applications, in comparison with the isotropic metamaterials and perfect lenses [1, 2]. Although the birefringent perfect lenses are not yet realized experimentally, we believe that the ideas and results presented here are quite realistic and will initiate strong efforts in creating the composite metamaterials with *substantially birefringent* properties, including those that satisfy the specific conditions for the tensor components of equation (2). This would require a new thinking in applying the traditional approaches of the papers [11, 12] where the fabrication of isotropic metamaterials has been reported. Such an anisotropy can be achieved by using more complicated elementary cells made of wires and split-ring resonators, instead of the traditional symmetric cubic lattice [12], in order to engineer both the electric and magnetic response in three different directions. We also note that in order to realize a birefringent lens, which is able to create an image of a three-dimensional source (compared to the two-dimensional case considered above), one should take the metamaterial with $A = B$, and it can simplify the design of the composite. Such a lens creates an image of both polarizations in the same plane.

4. Conclusions

We have introduced a novel type of left-handed media with reflectionless and birefringent properties for refraction of vectorial fields. In particular, we have shown that the flat-slab focusing

and negative refraction of electromagnetic waves may occur under different conditions for the TE- and TM-polarized waves or simultaneously with two spatially separated TE and TM images. Our results suggest novel directions in the study of the intriguing properties of left-handed metamaterials and photonic crystals, and they provide novel ideas for a design of composite materials with negative refraction.

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